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Dynamics of Love-Type Waves in Orthotropic Layer Under the Influence of Heterogeneity and Corrugation

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ABSTRACT

The present problem deals with the propagation of Love-type surface waves in a bedded structure comprises of an inhomogeneous orthotropic layer and an elastic half-space. The upper boundary and the interface between two media are considered to be corrugated. An analytical method (separation of variables) is adapted to solve the second order PDEs, which governs the equations of motion. Equations for particle motion in the layer and half-space have been formulated and solved separately. Finally, the frequency relation has been established under suitable boundary conditions at the interface of the orthotropic layer and the elastic half-space. Obtained relation is found to be in good agreement with the classical case of Love wave propagation. Remarkable effects of heterogeneity and corrugation parameters on the phase velocity of the considered wave have been represented by the means of graphs. Moreover, the group velocity curves are also plotted to exhibit the profound effect of heterogeneity considered in the layer. Results may be useful in theoretical study of wave propagation through composite layered structure with irregular boundaries.

Keywords : Love-type waves; Orthotropic layer; Corrugation; Heterogeneity; Elastic half-space.

1 INTRODUCTION

SHEAR waves propagation in a certain class of anisotropic medium with or without heterogeneity has long been a topic of interest. In the recent past, the elastodynamics response of anisotropic media has attracted many researchers. Orthotropic materials exhibit mechanical and thermal properties that differ along three mutually perpendicular directions. Many fiber-reinforced composites, wood, sheet metal comprises of orthotropic materials. These materials find their use in many engineering applications. As per Hooke's law orthotropic materials involve nine elastic constants. Several attempts have been made to highlight the response of material anisotropy on the velocity profile of surface seismic waves. The wave propagation at the boundary of orthotropic thermoelastic materials with voids and isotropic elastic half-space has been studied by Kumar and Kumar [1]. Love waves propagation in an orthotropic granular layer under initial stress was studied by Ahmed [2]. Kundu et al. [3]

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investigated the propagation of SH-type waves in sandwiched structure. As it is a well-known fact that earth has an irregular structure, so it is always not possible to deal with all the engineering problems taking the earth as a planar structure. It motivates us to study the impact of non-planar boundaries (corrugation) on different Earth models. Tomar and Kaur [4-5] adopted the Rayleigh's method of approximation to obtain expressions for the scattering of SH-waves at a corrugated interface between two anisotropic heterogeneous elastic solid half spaces. They have also investigated the reflection and transmission of a plane SH-wave incident at a corrugated interface between a dry sandy half-space and an anisotropic elastic half-space. Chattopadhyay et al. [6] studied the propagation of horizontally polarized shear waves in a magneto-elastic monoclinic stratum having rectangular irregularity in lower interface, sandwiched between two semi-infinite isotropic elastic media. Propagation of Love-type waves in a heterogeneous medium was studied by Kundu et al. [7]. In this paper influences of heterogeneity and initial stress on the propagation behaviour of the considered wave have been marked. Moreover, a good amount of research works have been carried out to model the propagation of Stonley waves, Rayleigh waves and Torsional waves in different geometry. Ahmed [8] investigated the propagation of Stoneley waves in a non-homogeneous orthotropic elastic medium under the influence of gravity. An explicit secular equation has been derived for Rayleigh waves propagation in an orthotropic half-space lying over an orthotropic elastic layer by Vinh et al. [9]. Using the Fourier transform method Abd-Alla [10] studied the propagation of Love waves in a non-homogeneous orthotropic elastic medium under the influence of initial stress and magnetic field. Singh et al. [11] studied the propagation of Love-type waves in a corrugated heterogeneous orthotropic layer lying over a fiber-reinforced half-space under a hydrostatic state of stress. Singh and Singh [12] investigated the effect of corrugation on the incident q-SV wave at the interface of two dissimilar elastic half-spaces. An explicit secular equation for Stoneley waves in a non-homogeneous orthotropic elastic medium under the influence of gravity was established by Vinh and Seriani [13]. Ayatollahi et al. [14] studied the dynamic behavior of an orthotropic substrate weakened by moving cracks and reinforced by a non-homogenous coating. Recently, Alam et al. [16-17] have investigated the nature of Love-type waves in different models. Singhal and Sahu [18] studied the Rayleigh wave propagation on corrugated orthotropic layer resting over a semi-infinite orthotropic medium. Behavior of surface seismic waves in piezo-composite structures has been marked distinctly by Singhal et al. [19-20]. Torsional wave propagation is dealt by Alam et al. [21-22]. They have considered the layer and half-space with heterogeneities.

In the present paper, the propagation of Love-type waves in an orthotropic layer resting over elastic half-space has been studied. Influences of various parameters like corrugation and heterogeneities (considered in the layer as well as in the half-space) on the phase velocity of considered wave are shown graphically. Moreover, Group velocity curves have also been plotted to show the variation of group velocity with respect to the wave number.

2 FORMULATION OF THE PROBLEM

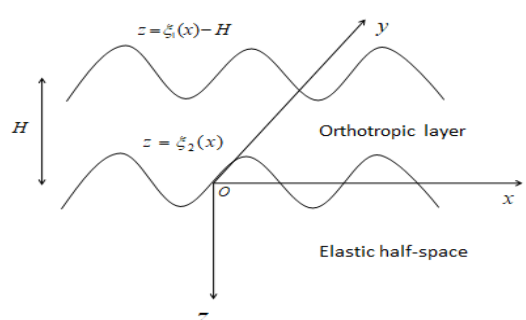

Fig.1
Geometry of the problem.

A layered model is taken into consideration which consists of an orthotropic layer resting over an elastic half-space. The Cartesian coordinate system (x, y, z) is chosen in such a way that the direction of propagation of the Love-type wave is taken along the x axis with z axis vertically downwards in the half-space and the origin is considered at the common interface of the layer and half space. The width of the layer is taken as H and the half space lies in the region $x > 0$. The upper surface of the layer is given as $z = \xi_1(x) - H$ and $z = \xi_2(x)$ defines the common interface of the orthotropic layer and the elastic half-space. $\xi_1(x)$ and $\xi_2(x)$ are continuous periodic functions of *x* and are independent of *y*. The heterogeneity in the orthotropic layer is due to the exponential variation of space variable which is pointing positively downwards.

The Fourier expansions of the periodic function $\xi_1(x)$ and $\xi_2(x)$ may be given by

$$\xi_m(x) = \sum_{n=1}^{\infty}(\xi_r^m e^{irlx} + \xi_{-r}^m e^{-irlx}), i = 1,2 \tag{1}$$

where ξ_r^m and ξ_{-r}^m are the Fourier series expansion coefficients, the series expansion order is represented by *r* and $i = \sqrt{-1}$. Moreover, the wavelength of corrugation is considered to be $2\pi / l$.

We assume $\xi_{\pm 1}^{(1)} = \frac{a}{2}$, $\xi_{\pm 1}^{(2)} = \frac{b}{2}$, $\xi_{\pm r}^{(m)} = \frac{P_r^{(m)} \mp iQ_r^{(m)}}{2}$, $\begin{matrix} m = 1,2 \\ r = 2,3 \end{matrix}$. Where $a, b, P_r^{(m)}, Q_r^{(m)}$ are constants.

In view of these inequalities, the series in Eq. (1) may be written as:

$$\xi_1 = a\cos(lx) + \sum_{r=2}^{\infty}[P_r^{(1)}\cos(rlx) + Q_r^{(1)}\sin(rlx)]$$

$$\xi_2 = b\cos(lx) + \sum_{r=2}^{\infty}[P_r^{(1)}\cos(rlx) + Q_r^{(1)}\sin(rlx)]$$

where $P_r^{(m)}$ and $Q_r^{(m)}$ are cosine and sine Fourier coefficients respectively.

Let $\xi_{\pm r}^{(m)} = \begin{cases} 0 & ,if \quad r \neq 1 \\ \frac{a}{2} & ,if \quad r = 1, m = 1 \\ \frac{b}{2} & ,if \quad r = 1, m = 2 \end{cases}$

In light of the above representation of $\xi_{\pm r}^{(m)}$, the corrugated boundary surfaces in the considered problem may be expressed in cosine terms only i.e.

$$\xi_1 = a\cos lx \quad \text{and} \quad \xi_2 = b\cos lx \tag{2}$$

2.1 Governing equation for orthotropic layer

For the orthotropic layer the stress components are related to the strain components as:

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 & 0 & 0 \\ M_{12} & M_{22} & M_{23} & 0 & 0 & 0 \\ M_{13} & M_{23} & M_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2N_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2N & 0 \\ 0 & 0 & 0 & 0 & 0 & 2N_2 \end{bmatrix} \begin{bmatrix} S_{11} \\ S_{22} \\ S_{33} \\ S_{23} \\ S_{13} \\ S_{12} \end{bmatrix} \tag{3}$$

where σ_{ij} represents the stress components, S_{ij} are the strain components and the elastic constants are represented by $M_{ij}\ (i, j = 1,2,3)$, N_1, N and N_2 . Now let us assume the displacement components along *x, y* and *z* directions as u_1, v_1 and w_1 respectively. We have considered the propagation of the Love-type wave in the $x-$direction which causes displacement along $y-$ direction only, so we get

$$u_1 = 0, w_1 = 0, v_1 = v_1(x, z, t) \tag{4}$$

By Eq. (4) the strain displacement relation for upper layer becomes

$$S_{11} = 0, S_{22} = 0, S_{33} = 0, S_{23} = \frac{1}{2}\frac{\partial v_1}{\partial z}, S_{13} = 0, S_{12} = \frac{1}{2}\frac{\partial v_1}{\partial x} \tag{5}$$

In light of Eq. (5), Eq. (3) gives

$$\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{13} = 0,\ \sigma_{23} = N_1\frac{\partial v_1}{\partial z},\ \sigma_{21} = N_2\frac{\partial v_1}{\partial x} \tag{6}$$

Using Eq. (6) we get the equation of motion for the orthotropic layer, given by

$$\frac{\partial \sigma_{21}}{\partial x} + \frac{\partial \sigma_{23}}{\partial z} = \rho\frac{\partial^2 v_1}{\partial t^2} \tag{7}$$

where ρ is the density of the upper inhomogeneous corrugated orthotropic layer.

We assume the elastic constants of the corrugated inhomogeneous orthotropic layer to vary exponentially with depth i.e.

$$N_1 = N_1^* e^{\lambda(z+H)},\ N_2 = N_2^* e^{\lambda(z+H)},\ \rho = \rho^* e^{\lambda(z+H)} \tag{8}$$

where N_1^*, N_2^* and ρ^* are the values of N_1, N_2 and ρ at the upper surface i.e. at $z = -H$ and λ is the heterogeneity parameter. Using Eqs. (6), (7), and (8) we have

$$\frac{\partial^2 v_1}{\partial z^2} + \lambda\frac{\partial v_1}{\partial z} + \frac{N_2^*}{N_1^*}\frac{\partial^2 v_1}{\partial x^2} = \frac{1}{\beta^2}\frac{\partial^2 v_1}{\partial t^2} \tag{9}$$

where $\beta^2 = \dfrac{N_1^*}{\rho^*}$.

2.2 Governing equation for half-space

For the elastic half-space the equation of motion in absence of body forces are taken as:

$$\tau_{ij,j} = \rho_2 \ddot{u}_i \tag{10}$$

where τ_{ij} are the stress tensor components, u_i are the displacement vector components. The components along *x, y* and *z* directions are given as u_2, v_2 and w_2 respectively and ρ_2 represents the density of the medium.

The stress-strain relations are described by

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \tag{11}$$

where λ and μ are the lame's elastic coefficients and are functions of *x, y, z* with

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \text{ and } e_{kk} = \dim\vec{u} \tag{12}$$

For the propagation of Love-type wave we have

$$u_2 = w_2 = 0, v_2 = v_2(x,z,t) \tag{13}$$

Using Eq. (13) in Eqs. (11) and (12) we get

$$\tau_{21} = \mu\frac{\partial v_2}{\partial x}\ ,\ \tau_{23} = \mu\frac{\partial v_2}{\partial z} \tag{14}$$

$$\tau_{21,1} = \mu\frac{\partial^2 v_2}{\partial x^2}\ ,\ \tau_{23,3} = \mu\frac{\partial^2 v_2}{\partial z^2}. \tag{15}$$

By using Eq. (14) and Eq. (15), we get

$$\mu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)v_2 + \left(\frac{\partial \mu}{\partial z}\right)\left(\frac{\partial v_2}{\partial z}\right) = \rho_2\frac{\partial^2 v_2}{\partial t^2} \tag{16}$$

3 ANALYTICAL SOLUTIONS

3.1 Solution for orthotropic layer

We consider the solution of Eq. (9) of the form $v_1\left(x,z,t\right) = V_1(z)e^{ik(x-ct)}$. With this assumption Eq. (9) becomes

$$\frac{d^2V_1}{dz^2} + \lambda\frac{dV_1}{dz} + k^2\left(\frac{c^2}{\beta^2} - \frac{N_2^*}{N_1^*}\right)V_1 = 0 \tag{17}$$

$$V_1(z) = e^{\frac{-\lambda}{2}z}\left(A_1\cos pz + A_2\sin pz\right) \tag{18}$$

Therefore the expression for displacement in the upper layer is obtained as:

$$v_1(z) = e^{\frac{-\lambda}{2}z}\left(A_1\cos pz + A_2\sin pz\right)e^{ik(x-ct)} \tag{19}$$

where A_1 and A_2 are arbitrary constants and

$$p = \frac{1}{2}\sqrt{-\lambda^2 + 4k^2\left(\frac{c^2}{\beta^2} - \frac{N_2^*}{N_1^*}\right)} \tag{20}$$

3.2 Solution for half-space

We assume the solution of Eq. (16) of the form

$$v_2\left(x,z,t\right) = V_2(z)e^{ik(x-ct)} \tag{21}$$

and we take the inhomogeneity for the non-homogeneous elastic half-space in the form

$$\mu = \mu_0 e^{\alpha z} \tag{22}$$

$$\rho_2 = \rho_0 e^{\alpha z} \tag{23}$$

where μ_0 and ρ_0 are the constant values of shear modulus μ and mass density ρ_2 at the interface $z = 0$ and α is a constant (heterogeneity parameter). Using the Eqs. (21), (22) and (23), Eq. (16) becomes

$$\left(\frac{d^2 V_2}{dz^2} + \alpha \frac{dV_2}{dz} - \psi_1^2 V_2 \right) = 0 \tag{24}$$

where $\psi_1^2 = \left(k^2 - \frac{k^2 c^2}{\mu_0 / \rho_0} \right) = k^2 - \frac{k^2 c^2}{\beta_2^2}$. Therefore the solution of Eq. (24) is obtained as:

$$V_2(z) = A_3 e^{-\left(\frac{\alpha}{2} + \sqrt{A + \text{Im} k^2}\right) z} + A_4 e^{\left(\frac{\alpha}{2} + \sqrt{A + \text{Im} k^2}\right) z}. \tag{25}$$

Now considering the fact that displacement component vanishes (i.e., $v_2(z) \to 0$) as $z \to \infty$, we obtain $V_2(z) = A_3 e^{-\eta z}$, where $\eta = \left(\frac{\alpha}{2} + \sqrt{A + \text{Im} k^2} \right)$ and A_3 is an arbitrary constant.

Therefore the expression for displacement in the lower half-space is obtained as:

$$v_2(z) = A_3 e^{-\eta z} e^{ik(x-ct)} \tag{26}$$

4 BOUNDARY CONDITIONS AND FREQUENCY RELATION

The upper surface of the layer is stress free at $z = \xi_1(x) - H$

$$\sigma_{23} - \xi_1' \sigma_{12} = 0 \tag{27}$$

Displacements are continuous at $z = \xi_2(x)$

$$v_1 = v_2 \tag{28}$$

Stresses are continuous at $z = \xi_2(x)$

$$\sigma_{23} - \xi_1' \sigma_{12} = \tau_{23} - \xi_2' \tau_{12} \tag{29}$$

Using the conditions given in Eqs. (27),(28) and (29) we get the following equations

$$A_1 = -A_2 \frac{L_1 \sin\left(p(\xi_1 - H)\right) - L_2 \cos\left(p(\xi_1 - H)\right)}{L_1 \cos\left(p(\xi_1 - H)\right) + L_2 \sin\left(p(\xi_1 - H)\right)} \tag{30}$$

$$A_3 = \left(A_1 \cos p\xi_2 + A_2 \sin p\xi_2\right) e^{-\left(\frac{\lambda}{2} - \eta\right)\xi_2} \tag{31}$$

$$A_1\left(L_2 \sin pg_2 + L_3 \cos pg_2\right) + A_2\left(L_3 \sin pg_2 - L_2 \cos pg_2\right) - \mu L_4 A_3 = 0 \tag{32}$$

On eliminating the arbitrary constants A_1, A_2 and A_3 from Eqs. (30), (31) and (32) we get

$$\tan\left(p\left(\xi_2 - \xi_1 + H\right)\right) = \frac{\mu L_2 L_4 e^{-\left(\frac{\lambda}{2}-\eta\right)\xi_2} + L_1 L_2 - L_2 L_3}{L_1^{\,2} + L_1 L_3 - \mu L_1 L_4 e^{-\left(\frac{\lambda}{2}-\eta\right)\xi_2}} \tag{33}$$

where

$$p = \frac{1}{2}\sqrt{-\lambda^2 + 4k^2\left(\frac{c^2}{\beta^2} - \frac{N_2^{\,*}}{N_1^{\,*}}\right)}$$

$$L_1 = N_1^{\,*}\frac{\lambda}{2} + ik\,\xi_1' N_2^{\,*},\; L_2 = N_1^{\,*} p,\; L_3 = N_1^{\,*}\frac{\lambda}{2} + ik\,\xi_2' N_2^{\,*},\; L_4 = \eta e^{-\eta\xi_2}$$

$$\eta = \left(\frac{\alpha}{2} + \sqrt{A + \operatorname{Im} k^2}\right)$$

$$A = \left\{\left(\frac{\alpha}{2}\right)^2 + \operatorname{Re} k^2 - \frac{\omega^2}{\beta_2^{\,2}}\right\}$$

The Eq. (33) is referred as dispersion equation, which relates the phase velocity of propagation $c = \omega / k$ to the wave number.

5 VALIDATION OF THE PROBLEM

When the heterogeneity of the layer is neglected and the layer and the half-space is taken as isotropic, and also the corrugation is removed from the lower boundary surfaces i.e. $\lambda = 0, N_1^{\,*} = N_2^{\,*} = \mu_1, \alpha = 0, \mu = \mu_2, \xi_2 = 0, \xi_1 = 0$. We have

$$\tan\left(kH\sqrt{\frac{c^2}{\bar{\beta}^2} - 1}\right) = \frac{\mu_2\sqrt{1 - \frac{c^2}{\beta_2^{\,2}}}}{\mu_1\sqrt{\frac{c^2}{\bar{\beta}^2} - 1}} \tag{34}$$

where $\bar{\beta} = \frac{\mu_1}{\rho^*}$. The Eq. (34) is the classical Love wave equation. This expression also validates the condition $\bar{\beta} < c < \beta_2$.

6 NUMERICAL ILLUSTRATIONS

We have taken the following values for our numerical calculation. For orthotropic medium (Kundu et al. [3])

$$N^* = 5.82\times 10^{10} N/m^2,\; N_1^{\,*} = 3.99\times 10^{10} N/m^2,\; \rho = 4500\,kg/m^3$$

For elastic half-space (Singh and Singh [12])

$\mu = 6.5 \times 10^{11} \ N/m^2, \ \rho = 4600 \ kg/m^3$

7 GRAPHICAL DISCUSSIONS

To exhibit the influence of affecting parameters on phase and group velocity of Love-type wave in considered model, graphs are plotted and shown through Fig. 2 to Fig. 8. The general trend in variation of phase velocity is found to be decreasing with wave number (Figs. 2 to 7). In more contrast, Fig.2 shows the variation of phase velocity of Love-type wave with respect to dimensionless wave number for increasing value of height of the orthotropic layer. We observe that the phase velocity is favored by the height of the layer. Fig.3 exhibits the variation of phase velocity of the Love-type wave with respect to dimensionless wave number for increasing value of heterogeneity parameter (λ) of the layer. We observe that the phase velocity increases with increase in values of λ . Figs. 4 and 5 illustrate the variation of phase velocity of Love-type wave with respect to dimensionless wave number for increasing corrugation parameter of the surface and for increasing corrugation parameter of the interface. A comparative study of the graphs in Figs. 4 and 5 declare that the two of corrugations have different effects on the velocity of considered surface wave, i.e. the phase velocity increases with increase in corrugation of the surface of the layer (aH) whereas the phase velocity decreases with the increase in corrugation of the interface (bH) . Fig. 6 shows the variation of phase velocity of Love-type wave with respect to dimensionless wave number for increasing value of heterogeneity parameter (α) of the half-space. It is observed that the phase velocity increases with the increase of the heterogeneity in the half-space (α). Fig. 7 elucidates the variation of phase velocity of Love-type wave with respect to dimensionless wave number for different values of corrugation parameter. Fig. 8 depicts the variation of the group velocity of Love-type wave with respect to dimensionless wave number for different values of heterogeneity parameter of the surface of the orthotropic layer. The figure portrays that the group velocity decreases with the increase in heterogeneity parameter (λ) of the surface of the layer.

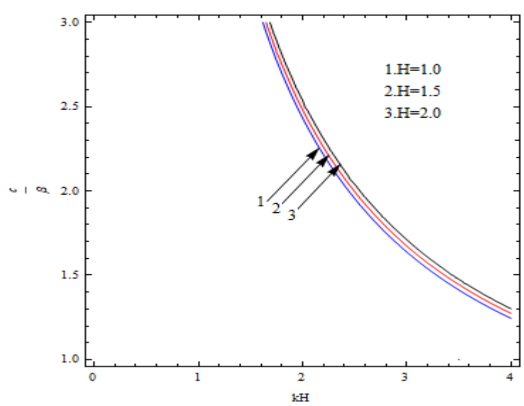

Fig.2
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of height (H) of the orthotropic medium.

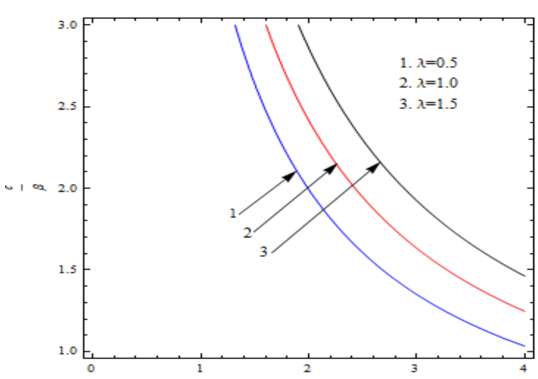

Fig.3
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of heterogeneity parameter (λ) of the layer.

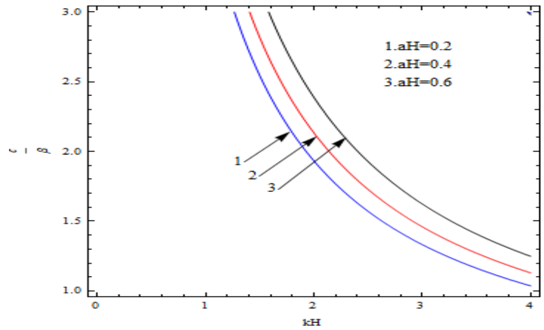

Fig.4
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of corrugation of the surface of the layer (aH).

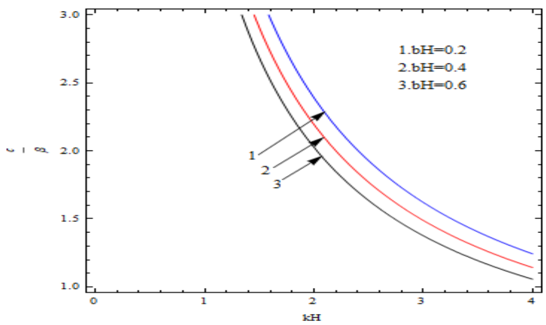

Fig.5
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of corrugation at the interface (bH).

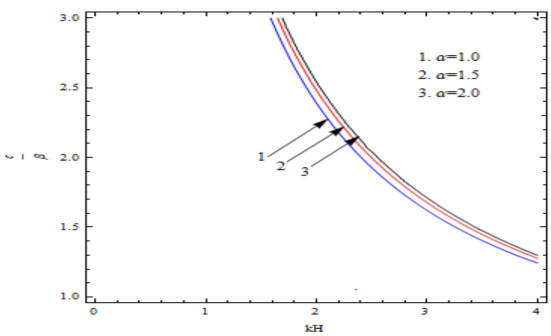

Fig.6
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of heterogeneity parameter (α) of the half-space.

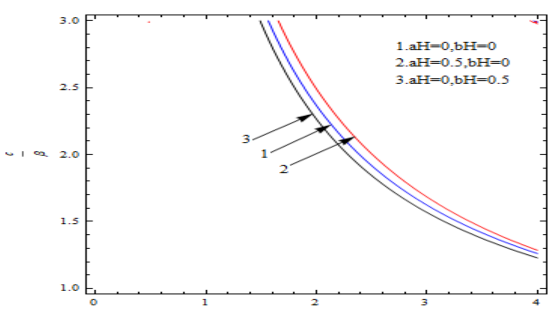

Fig.7
Variation of dimensionless phase velocity (c/β) against dimensionless wave number (kH) for different values of corrugation parameter.

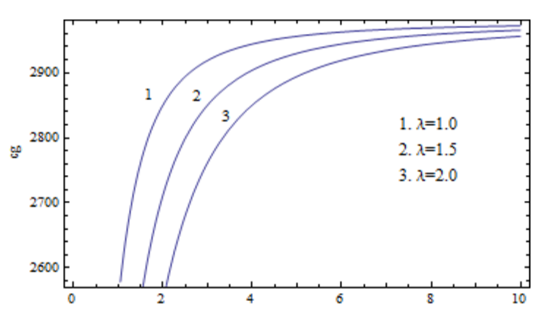

Fig.8
Variation of dimensionless group velocity (cg) against dimensionless wave number (kH) for different values of heterogeneity of the layer (λ).

8 CONCLUSIONS

The propagation of Love-type waves has been investigated by considering a structure where an orthotropic medium is lying over an elastic half-space. Dispersion relation has been derived in closed form and found in good agreement with the classical case of Love wave. Considerable effects of heterogeneity and corrugation parameters on the phase velocity of the considered wave have been observed. Also, the effect of heterogeneity of the layer on the group velocity has been shown by means of graphs. Finally, the following outcome can be drawn from the present study:

1. Dimensionless phase velocity (c/β) of the Love-type waves decreases with respect to the dimensionless wave number (kH) in all the cases as shown in Fig. 2 to Fig. 8.
2. As we increase the height of the layer (H), the phase velocity of the considered wave is found to be increasing.
3. The corrugation parameter of the upper surface of orthotropic layer (aH) favours the phase velocity whereas corrugation parameter at the interface (bH) decreases the phase velocity of the considered waves.
4. Both the heterogeneity associated with the layer (λ) as well as heterogeneity considered in the half-space (α) favours the phase velocity of the Love-type waves.
5. It can be depicted from Fig. 8 that the increase in heterogeneity parameter of the layer (λ) decreases the group velocity of the Love-type waves.

From the above discussion, we can conclude that the heterogeneities in the layer and half-space have remarkably affected the velocity of Love-type waves. Moreover, corrugation also affects the velocity profile of considered waves significantly.

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