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Research Paper

Numerical and Experimental Investigation on the Effect of Geometric Discontinuity on Frequency Response of Composite Lattice Conical Structures

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ABSTRACT

Composite structures are increasingly being used in various engineering structures such as automotive, aerospace, and civil structures due to their superior properties, namely, high strength-to-weight ratio, impact resistance, and durability. For the purpose of accessibility to other components and possibility of installation, aerospace structures encounters geometrical discontinuities which can lead to a complex structural analysis due to non-isotropic behavior. This paper aims to study the frequency response behavior of a composite lattice conical structure considering the effect of geometric discontinuity stiffened by a circular ring. The lattice structures are made of glass fiber reinforced polymers (GFRP) fabricated using filament winding method and cured in an autoclave. Numerical analysis and experimental modal testing was performed to obtain frequency response of the structures considering geometrical discontinuities. The results showed that the natural frequency values of structures with cutout in free-free boundary conditions are lower than those without cutout. Furthermore, comparing the mode shapes of structures indicated that these shapes were similar to each other and only some slight differences in discontinuity area were observed in some modes. Finally, the highest difference of numerical analysis results in structures with or without cutout was 2.61% while the highest difference of experimental analysis results in the structures was 3.73%. The greatest difference in the numerical and experimental analysis results is pertinent to the second mode in the structure without cutout is 15.64% and in the structure with cutout is 12.48%.

Keywords : Conical structure; Composite lattice structure; Geometric discontinuity; Modal analysis; Finite element.

1 INTRODUCTION

COMPOSITE structures require various static and dynamic analyses in practical conditions to be used in different industries. Achieving low weight and also high special strength of composite structures can be

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considered as the main properties that make them suitable to be used in aerial and aerospace industries. Grid-stiffened composite structure can be taken as a rigid arrangement of composite ribs stiffened by fiber that have been connected to each other and formed a two and three dimensional consistent set. In case of damage in one region of the structure, it would not affect the other regions. This is partly due to the nature of composite structure, which consists of fiber and resin, and partly because of the blank space in the structure. Modal analysis is of great importance to locate natural frequencies, damping indexes, and also their mode shapes. Hematnezhad, et al. [1] conducted analytical, numerical, and experimental investigations to evaluate the vibrational properties of stiffened GFRP cylindrical shells. The analytical formulation was based on Sander's thin shell theory. Using Ritz method, the equations of special value are obtained and are then solved with GFRP to assess the natural frequencies of stiffened composite shells. To confirm analytical results, experimental modal analysis was also implemented on the stiffened cylinder. Li and Qiao [2] offered a nonlinear vibration analysis of geodesically-stiffened laminated composite cylindrical shells in an elastic medium. The shell was inserted in an elastic environment which was modeled as the Pasternak elastic basis. It was hypothesized that materials of each shell layer is linearly elastic, anisotropic, and stiffened with fiber. They developed the movement equations using Donnell shell theory with nonlinear type from Von Karman. Results revealed that the different types of lay-up, the shell geometric parameters including the properties of stiffener, boundary conditions, elastic foundation, and excitation magnitude significantly affect the fundamental frequency, nonlinear to linear frequency ratios, and parametric responses of laminated cylindrical shells. Lopatin, et al. [3] reported an analytical expression for fundamental frequency of the composite lattice cylindrical shell with clamped edges. The lattice shell was composed of a large number of helical and hoop ribs which was modelled as a continuous orthotropic thin cylinder with effective stiffness parameters. A solution of the equations of motion of the shell was based on the Fourier decomposition and the Galerkin method leading to an analytical formula for the calculation of a fundamental frequency. Result was verified using the finite-element analysis. The efficiency of the analytical formula was evaluated using numerical examples. Khadem and Nezamoleslami [4] investigated the the free vibrations of composite anisogrid lattice conical shells formed by geodesically helical and circumferential ribs. The lattice part of conical shell was modeled as beam. Thus, in addition to the axial loads, ribs also stand shear and bending moment loads. The theory of first time shell shear transformation was used to respond to the deflection and lateral shear transformations. The final results revealed that increase in the angle of cone vertex leads to an increase in the natural frequencies of conical shell. Zhao, et al. [5] examined a free vibration analysis of multi-layer composite oval cylinder with general boundary conditions. The Ritz method is employed to obtain the frequency parameters associated with the mode shapes. Numerical results revealed that the volume fractions of CNTs, distribution types of CNTs, boundary restraint parameters, and geometrical parameter have an a considerable influence effect on the vibration behavior of the FG-CNTRC truncated conical panels. Wang, et al. [6] presented an analytical model to examine the nonlinear vibrations of a cylindrical shell in environments with different temperatures. To model the stiffened FG cylindrical shells, Von Karman nonlinear theory, Bolotin method, and first time shear transformation theory were used. In addition, to gain nonlinear differential equations, Galerkin and mode analysis methods were employed. As temperature difference (thermal load) increased, the dynamic instable zone of stiffened FG cylindrical shells also increased quickly. It was found that the stiffeners can increase the dynamic stability. Ahmadifar, et al. [7] carried out an experimental and numerical buckling analysis on carbon fiber composite lattice conical structure before and after lateral impact. In this research, a kind of shelled composite lattice conical structure was analyzed experimentally and numerically to determine the buckling stability of the structure before and after damage. Given the axial symmetry of lattice conical structures, the shape of modes in modal analysis is similar to each other (two by two) and their natural frequencies are also equal. Hemmatnezhad, et al. [8] dealt with the integrated analysis of the vibrational behavior of GFRP-stiffened composite cylindrical shells. They used coating method to add the stiffness of stiffeners to the general stiffness of shell and specifying the corresponding stiffness parameter. Zarei, et al. [9] used experimental, numerical, and analytical methods to investigated the free vibrational behavior of grid-stiffened truncated composite conical shells. Given the special geometry of conical shell, the whole structure changed into a conical shell with variable stiffness and thickness. Stiffeners were selected to be of beam type in a way that they can stand not only axial loads, but also bending and shear loads. Trivial differences in results indicate the structure complexity and probable errors might also occur over the sample production process. Comparison of results showed that analytical and numerical models can predict vibration natural frequencies very well. For thicknesses of fewer shells and more modes, the natural frequency of stiffened conical shell was not stiffened with higher than structure network. Zarei, et al. [10] studied the free vibrations of grid-stiffened composite conical-cylindrical shells. The stiffness contribution of the shells and helical stiffeners have been superimposed by means of a smeared approach in order to calculate the total stiffness coefficients of the whole structure. In order to verify the present model, several comparisons have been made which indicate a good agreement between the analytical, numerical and experimental results. Furthermore, the effects of several design parameters such as the cone semi-vertex and stiffeners' orientation angles on the modal parameters have been thoroughly investigated. Zaretabar, et al. [11] modeled and analyzed the conical composite grid structure with and without geometric discontinuity with the approach of determining the buckling stability by both numerical and experimental methods under axial load. Results showed that the presence of openings in the structure has caused the strength of the structure to decrease. Zamani, et al. [12] study aims to the investigation of the effect of grid geometry on the modal response and buckling strength of a composite conical lattice structure under static axial loading by Finite Element Method (FEM). For this purpose, four structures with similar geometry have been designed through four grid structures. Their results showed Changing the grid does not affect the shape of the modes. The isogrid structure bears a higher buckling loading than the anisogrid. Reducing the rib angle is an effective parameter, which increases the buckling loading on the structure.

The present study looked into the numerical and experimental analysis of the effect of geometric discontinuity on frequency response and mode shapes of composite grid-stiffened conical structures. The intended structure had geometric discontinuity in the lateral part which was stiffened with circular ring. The structure was made of glass composite, type *E*, and epoxy fabricated by damp winding method and was cured in autoclave at 70 degrees for 8 hours. In continuation, cutout occurred in the structure which was stiffened by the winding ring from the structure material. To achieve the frequency response of the structures with and without discontinuity, numerical simulation and experimental modal test were implemented. Furthermore, to reach natural frequencies and shape of modes, the structures were analyzed in free-free boundary conditions and the results obtained from finite element method were compared with those of experimental modal analysis.

2 SPECIMEN FABRICATION PROCESS

One method of fabricating grid-stiffened structures is filament winding into silicone molds. Using these grooved silicone molds has some advantages over other methods. Easy wrapping of fiber in the groove positions, which leads to high quality of ribs, and also low cost of mold fabrication are among these advantages. Additionally, separation of structure after curing is typically easy if conditions and necessities of geometric designing of molds are carefully observed. In this case, silicone mold is employed to fabricate further samples. To fabricate silicone mold, a grid rigid plate is made to frame the silicone mold. This plate was selected from plexi-glass in this study. It was then used to fabricate the silicone mold. The constituents used to fabricate silicone mold include a viscose liquid and a stiffening liquid that are combined in a specific rate. The mandrel fabricated in this study is a conical metal piece installed on the automatic winding device and the silicone mold is placed on it.

The used winding device was a fully automatic device with three degrees of freedom. It was equipped with a computer whose work was to make the winding process automatic with the maximum precision. This study used the wet winding method to wrap the fiber around mandrel. To be more specific, the fiber was coated with resin in a pool (resin bath) before reaching the silicon mold. Then, it is wrapped around the mandrel as shown in Fig. 1.

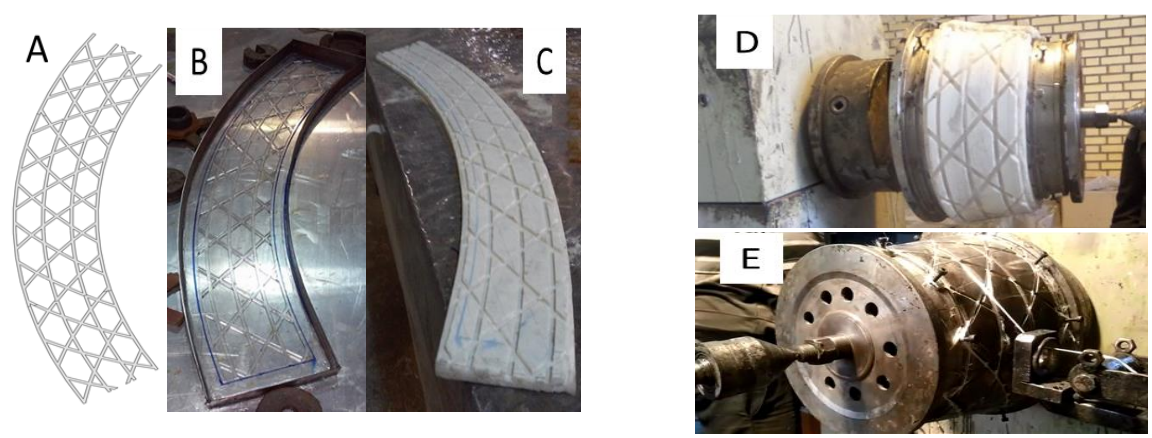


Fig.1
A) Schematic diagram of a conical lattice structure. B) Plexi-glass mold. C) Silicone mold. D) Silicone mold fixed on conical mandrel. E) Fully automatic winding machine.

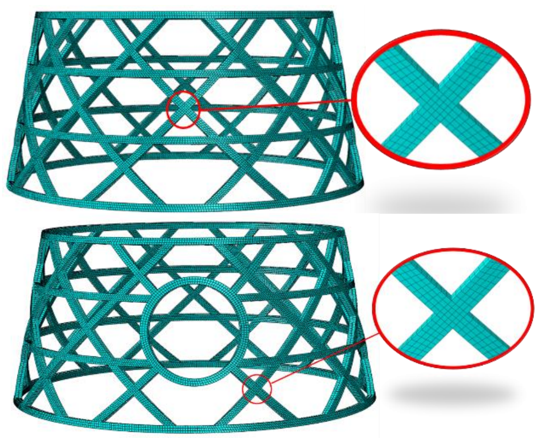
3 FINITE ELEMENT MODELING AND SIMULATION

The first step to analyze the models is that the specimens should be sampled in geometric modeling software completely and accurately. To model the conical grid-stiffened structure of this study, shell element modeling method has been used. Stiffeners are from composites and to make them, glass fiber (type E) and epoxy were used. Table 1 shows the properties used in the finite element model.

Table 1
Mechanical properties of epoxy/glass composite.

Properties	Values
density ρ (kg/m^3)	1.7
Longitudinal Young's Modulus $E1$ (Gpa)	34
Lateral Young's Modulus (perpendicular to fiber) $E2$ (Gpa)	8
Composite Poisson's Ratio (ϑ_{12})	0.24
$G12$, $G13$ Shear module (Gpa)	3
$G23$ Shear Module (Gpa)	3.4

As Fig. 2 presents, quadrilateral shell elements have been used for numerical modeling of the structure. For these structures, S4R element types was considered. To achieve the best numerical results, the mesh convergence analysis was carried out with different number of elements. At the end, as the number of elements increased, the obtained responses showed suitable convergence. According to Fig. 2, 14010 elements are enough to reach suitable response.



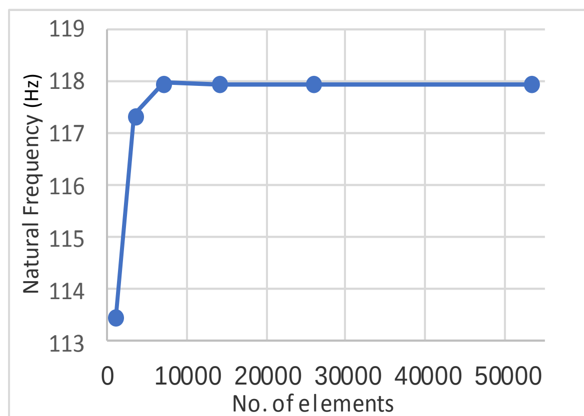


Fig.2
Numerical modeling of structure with and without cutout and mesh convergence diagram.

4 EXPERIMENTAL MODAL ANALYSIS

One of the uses of modal analysis is achieving the modal model of structure which is, in turn, utilized to correspond with finite element model for updating. The sensitivity of modal parameters against variations in any physical parameters of the system can be predicted via the modal del of a dynamic system. The general purpose of modal test is to extract the dynamic and vibrational properties of the structure such as natural frequency, shape of modes, and damping ratio. Modal responses can help to pinpoint the harmful and destructive frequencies for intended applications. In the present study, structural frequency responses from zero to 1000 *Hz* were examined. Modal analysis is carried out via SIMO method which is a uni-input and multi-output method. It means that there is a stimulation point by which the responses of other points can be received and their frequency can be measured.

To run the experimental modal analysis, the structure is tested in free-free boundary conditions. To meet these boundary conditions, the structure is kept hanging in the air via two elastic threads and then the point of the shaker is connected to and integrated with the structure. To do the modal test, finding the natural frequency of any object needs its own physical elementing. To element the structure in this study, a set of points at equal and regular intervals on its side surface were determined. To be more exact, ten points along the structure periphery and four ones along its longitudinal were considered which resulted to creation of an integrated and equal network as shown in Fig. 3. It should be pointed out that the accuracy of the mode shapes are is a function of the number of these networks. Likewise, a physical modeling was performed in the software based on the number of nodes and the properties of the node into which the shaker was also defined.

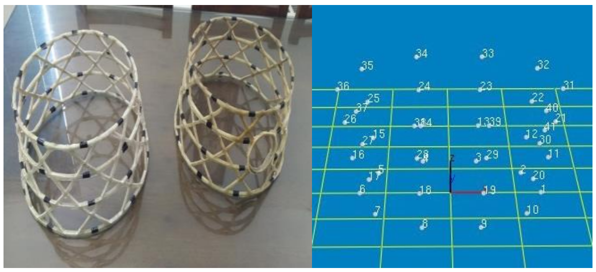


Fig.3
Specified points in structures with and without cutout to run modal analysis.

The experimental modal analysis has been done after setting the modal software. Laser was focused on the node perpendicular and the test operation commenced. Having analyzed and processed all the nodes via the device software, the researchers extracted modal analysis outputs including natural frequencies and mode shapes. This process was undertaken separately for each of the experimental samples of conical grid-stiffened structure.

5 FINITE ELEMENT RESULTS

Table 2 represents the values of numerical analysis natural frequency for composite grid-stiffened conical structure with and without cutout in modes one to six. The first six frequencies that are close to zero has been ignored. From the seventh frequency onwards, the first natural frequency and mode shape have occurred in the structure. In close frequencies, the shape of modes has not changed significantly. Accordingly, repetitive mode shapes have been ignored.

Table 2
Numerical analysis results of natural frequency of structures with and without cutout.

Mode	Structure without cutout (*Hz*)	Structure with cutout (*Hz*)	Difference Percent (%)
First	117.96	114.87	2.61
Second	185.22	185.55	0.9
Third	329.91	323.52	1.93
Fourth	488.74	485.53	0.65
Fifth	624.63	611.37	2.12
Sixth	858.68	845.79	1.5

Fig. 4 shows the results related to the first mode shape obtained from numerical analysis for structure with and without cutout (regardless of repetitive mode shapes). The table results clearly show that the presence of cutout has no effect on the shape of modes.

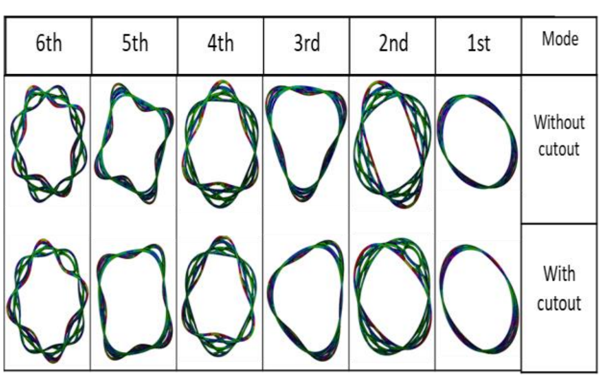


Fig.4
Comparison of numerical analysis mode shapes in structure with and without cutout.

6 EXPERIMANTAL MODAL ANALYSIS RESULTS

In this test, the structure started to vibrate using a shaker connected to it. Then, the frequency response of every node was separately measured and recorded in a diagram by the program. Accumulate nodes responses led to extraction of the whole structure frequency response diagram. Fig. 5 shows the FRF diagram and frequency response cumulative diagram of the structure without cutout. Also, Fig. 6 displays the FRF diagram and frequency response cumulative diagram of the structure with cutout obtained from experimental modal analysis. As it can be seen, the first frequency has the highest amplitude. Also, in subsequent mode shapes, amplitude of frequencies is less than that of the previous mode.

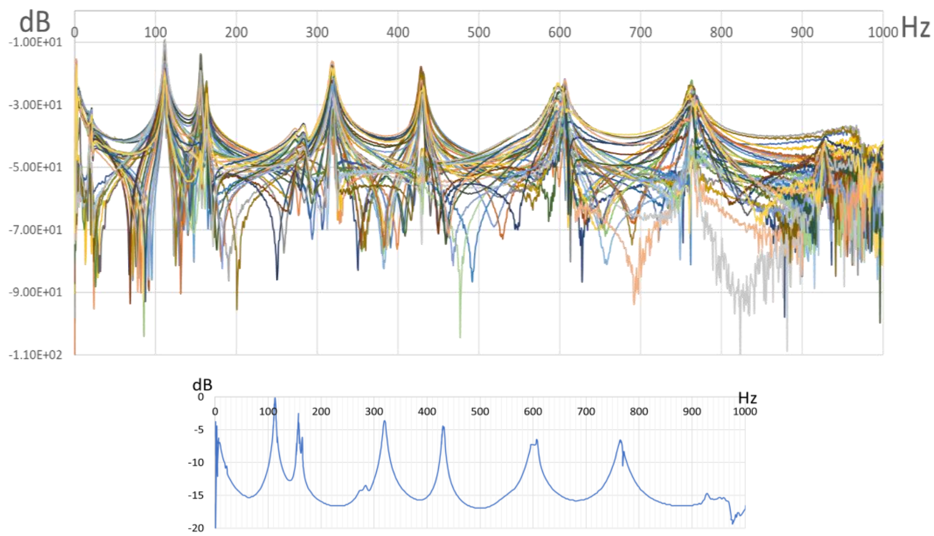


Fig.5
Cumulation of FRF diagram in structure without cutout.

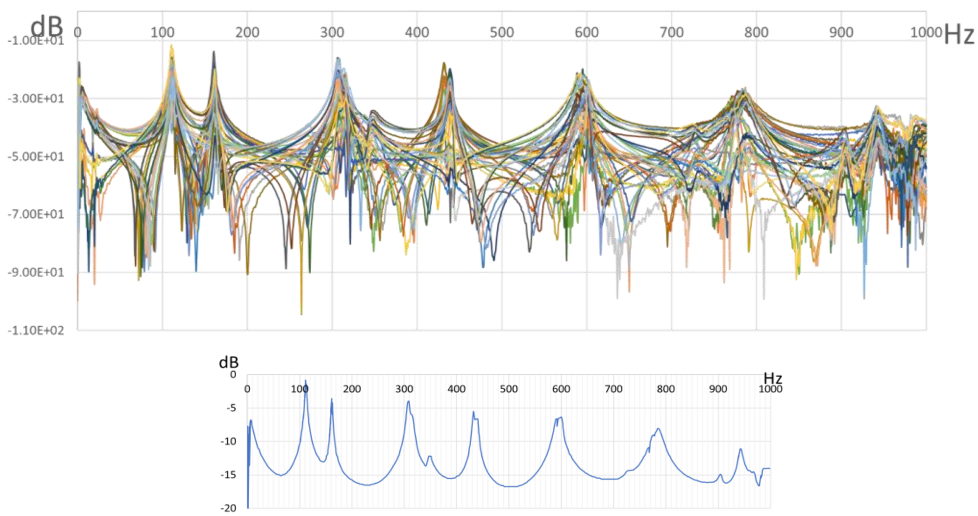


Fig.6
Cumulation of FRF diagram in structure with cutout.

Fig. 7 shows the Comparison of the experimental analysis mode shapes in the structure with and without cutout. As it can be seen, cutout in the first mode shapes has not affected the general shape of these modes. However, slight changes are apparent in the cutout area of mode shapes. The effect of cutout has increased in higher modes and changes in the general shape of modes are also seen.

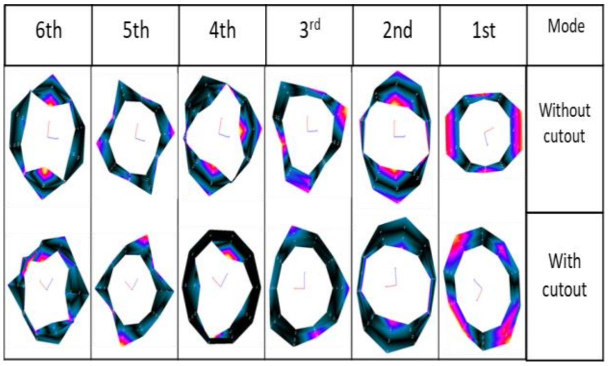


Fig.7
Comparison of the experimental analysis mode shapes in the structure with and without cutout.

7 COMPARISON OF NUMERICAL AND EXPERIMENTAL MODAL ANALYSIS RESULTS

This section deals with comparing and examining the results of modal analysis through experimental and numerical approaches in grid-stiffened conical structures with and without cutout. Table 3 reveals the results of natural frequency of the first six modes of the experimental modal analysis for the structures with and without cutout. Presence of cutout caused negligible change in the natural frequency. This natural frequency difference in the experimental modal analysis from the first to the sixth mode is 1.11%, 1.27%, 3.73%, 0.43%, 1.75% and 2.55% respectively. Results indicated that while the highest difference of numerical analysis in structure with and without cutout is 2.61%, the highest difference in the experimental analysis of the structure with and without cutout is 3.73%. This lower value of natural frequency in the structure with cut out indicates the lower stiffness of this structure compared with the structure without cutout.

Table 3
Experimental analysis results of natural frequency in structure with and without cutout.

Mode	Structure without cutout (*Hz*)	Structure with cutout (*Hz*)	Difference Percent (%)
First	111.88	110.63	1.11
Second	156.25	160.36	1.27
Third	318.13	306.25	3.73
Fourth	430	431.88	0.43
Fifth	605.63	595	1.75
Sixth	763.75	783.75	2.55

Table 4 shows the natural frequency values related to the numerical and experimental analyses along with the difference percentage among the analyses in the structure without cutout. Likewise, Table 5 presents the natural frequency values related to the numerical and experimental analyses along with the difference percentage among the analyses in the structure with cutout. In the structure without cutout, the most difference in the numerical and experimental analyses results is pertinent to the second mode (15.64%). Similarly, in the structure with cutout, the highest difference belongs to the second mode (12.48%). The higher values of natural frequency results in numerical analysis compared with those in experimental analysis is due to the higher stiffness of the structure and also the ideality of finite element analysis compared with the experimental analysis.

Table 4
Comparison of the numerical and experimental results of natural frequency in structure without cutout.

Mode	Experimental (*Hz*)	Numerical (*Hz*)	Difference Percent (%)
First	111.88	117.96	5.15
Second	156.25	185.22	15.64
Third	318.13	329.91	3.75
Fourth	430	488.74	12.01
Fifth	605.63	624.63	3.04
Sixth	763.75	858.68	11.05

Table 5
Comparison of the numerical and experimental results of natural frequency in structure with cutout.

Mode	Experimental (*Hz*)	Numerical (*Hz*)	Difference Percent (%)
First	110.63	114.87	3.69
Second	160.36	185.55	12.48
Third	306.25	323.52	5.23
Fourth	431.88	485.53	11.04
Fifth	595	611.37	2.67
Sixth	783.75	845.79	7.33

Fig. 8 shows the mode shapes obtained from experimental analysis and also those obtained from finite element method. As it can be seen, the mode shapes from these two methods conform with each other. This confirms the correctness and coordination of numerical and experimental methods. Stiffened cutout has caused slight change in the mode shape of the structure in the cutout area.

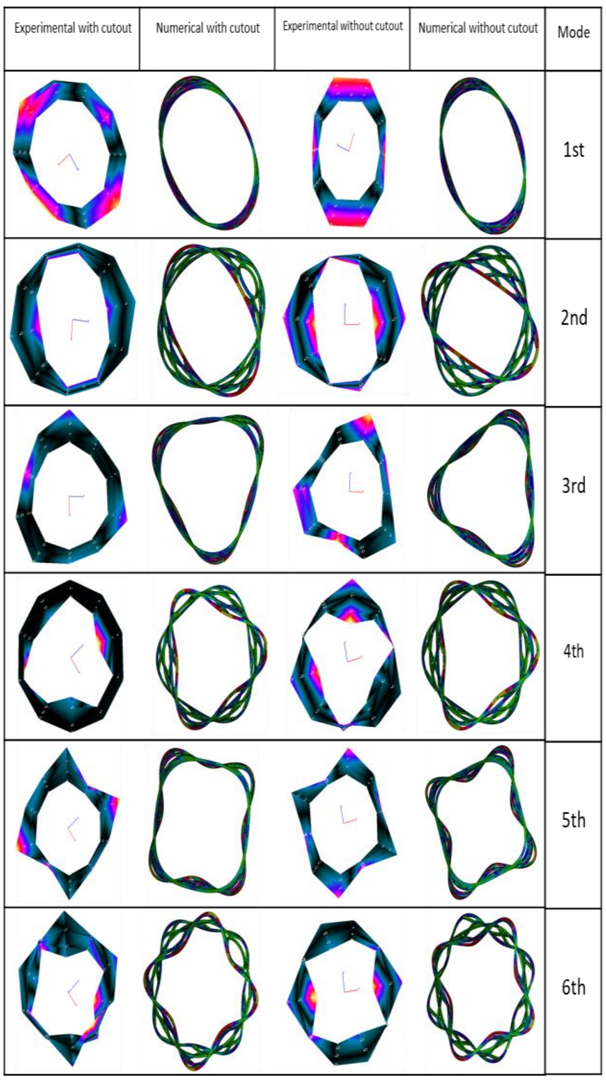


Fig.8
Comparing the mode shapes of structures with and without cutout.

8 EFFECT OF BOUNDARY CONDITIONS ON THE VIBRATIONAL RESPONSE

To develop and complete the present research study and also in view of the practical restrictions of doing experimental modal analysis for different types of support conditions and also given the positive results of finite element analysis validation with experimental tests, this section deals with the modal analysis of other numerical method support conditions. Fig. 9 presents the shape of different modes in fixed-free and fixed-fixed support conditions. In the fixed-fixed support, movement in any direction has been closed for the bigger and smaller diameters. As with the fixed-free support, the bigger diameter of the structure is fixed. As it can be seen from the figure, the shape of modes is different in completely different support conditions. In fixed-free support, the shape of modes in the structure with cutout are slightly different from those in the structure without cutout. By contrast, in the fixed-fixed support, the shape of modes are significantly different and this difference is largely due to the cut out in the structure. Fig. 10 shows the first, second, and mode shapes in fixed-fixed support and under compressive axial load for the structure with and without cutout. The sections with the highest change are the vulnerable areas and the structure is likely to fail at these points in the buckling test. As the figure clearly shows, the presence of cutout has brought about considerable change in the buckling mode shapes.

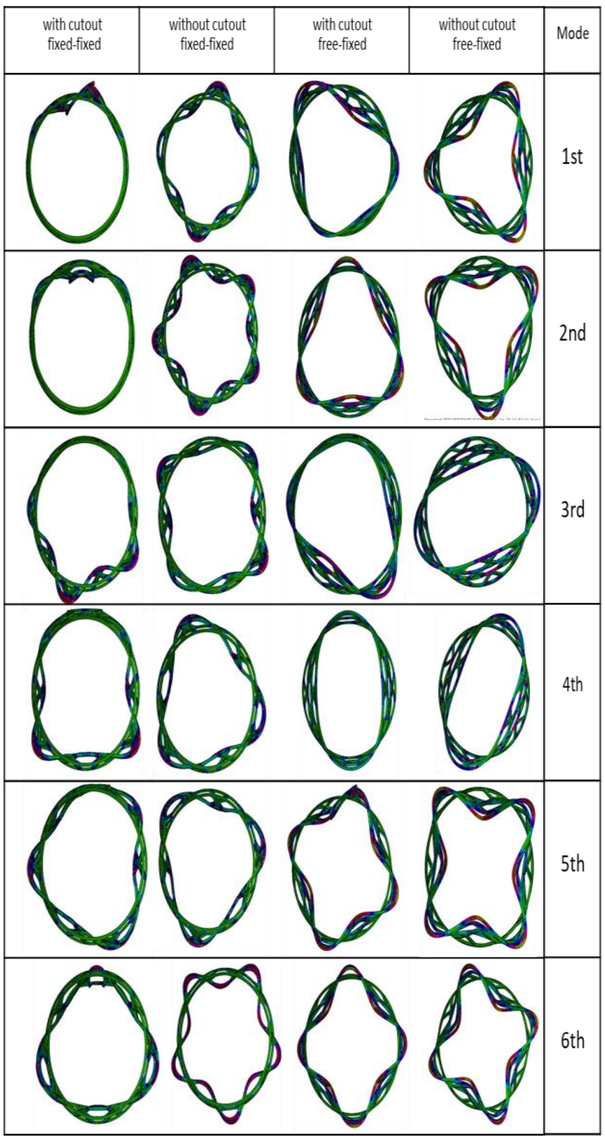


Fig.9
Six shapes of the first mode of numerical analysis for composite grid-stiffened conical structure with and without cutout in fixed-free and fixed-fixed conditions.

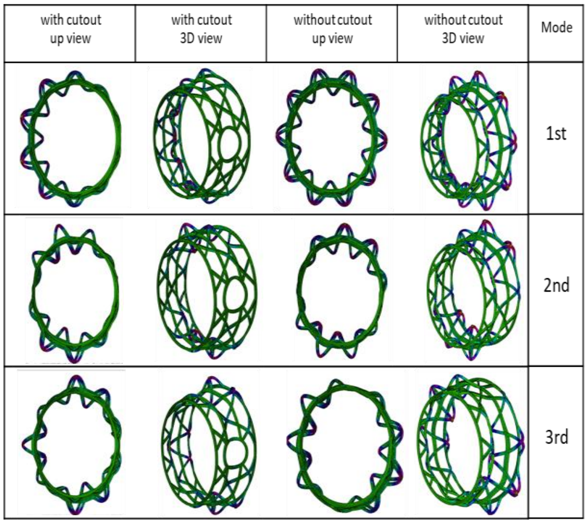


Fig.10
The first three mode shapes of numerical analysis buckling for composite grid-stiffened conical structure with and without cutout.

9 CONCLUSION

Given the similar geometry of grid-stiffened conical structures with or without cutout, the shape of modes in numerical analysis were similar to each other and their corresponding natural frequencies were also close to each other. As with experimental modal analysis of grid-stiffened conical structures with or without cutout in free-free support conditions, the shape of modes had trivial differences from each other in cutout area; however, the general view of their shapes was highly close to each other. The highest difference of numerical analysis results in structures with or without cutout was 2.61% while the highest difference of experimental analysis results in the structures was 3.73%. In fixed-free support, the shape of modes had little difference in the cutout area of the structure with cutout compared with the structure without cutout. Additionally, comparing the shape of fixed-fixed support modes of structures with and without cutout uncovered that the shapes were significantly different. This difference is due to the existence of cutout in the structure. Also, comparing the buckling mode of structures with and without cutout showed that cutout in the structure might give rise to a change in the structure buckling behavior. Given the few errors in comparison and validation of numerical and experimental methods, the obtained numerical method can be utilized to reach natural frequency and shape of modes in composite grid-stiffened conical structures in that this method requires less time and cost. To put it another way, the results of this study indicated that the presented numerical model is an acceptable and reliable model for similar corresponding structures.

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