An Analytic Study on the Dispersion of Love Wave Propagation in Double Layers Lying Over Inhomogeneous Half-Space

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ABSTRACT

In this work, attempts are made to study the dispersion of Love waves in dry sandy layer sandwiched between fiber reinforced layer and inhomogeneous half space. Inhomogeneity in half space associated with density and rigidity and considered in exponential form. Displacement components for fiber reinforced layer, dry sandy layer and inhomogeneous half-space have been obtained by using method of separable variables. Boundary conditions are defined at the free surface of the fiber reinforced layer and at the interfaces between layers and half space. The dispersion equation has derived in closed form. Numerical calculations for dispersion equation are performed. The study results show the effect of parameters on the velocity of Love waves and presented graphically. Graphs are plotted between wave number and phase velocity to show the effect of reinforced parameter, sandiness parameter and inhomogeneity on the phase velocity of Love waves. From the graphs, it can be concluded that phase velocity decreases with respect to wave number.

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Keywords: Love waves; Reinforced parameter; Sandiness parameter; Inhomogeneity; Wave number; Phase velocity.

1 INTRODUCTION

THE present study deals with the propagation of Love wave in dry sandy layer fastened between fiber-reinforced layer and inhomogeneous half-space, such study conveys the rich consequences, and the results are useful to theoretical and technical aspects in support to the propagation of surface wave in layered media. Analysis of elastic mediums subjected to propagation of surface waves are of significance importance to few branches, e.g. engineering geology, earthquake, mining, sedimentology, geophysics and civil engineering. It is described based on theoretical understanding of the literature that provides extensive information on seismology. Broadly speaking, the seismic waves propagation in layered media is described by some authors in terms of theoretical framework and experimental knowledge of dynamics of strained bodies that embodied in mathematical theory [1] [2] [3]. Compositions of fiber-reinforced are framed of axial particulates fixed in matrix material. The motive of fibre-reinforced compound is to obtain a material with more precise strength and modulus. The components of fibre-

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reinforced have the property of anisotropic unit, until these components are in elastic condition. The theory of finite deformation for fibre-reinforced materials was developed by Pipkin and Rogers [4]. Constitutive equation for fibre-reinforced linearly anisotropy elastic medium in preferred direction was described and a theory developed on the stress in elastic plates reinforced by fibres lying in concentric [5]. It is remained an area of interest for researchers and wave profiles in fiber-reinforced medium have been discussed extensively, several papers were published. The Impact of anisotropy on the scattering of love wave in a self-reinforced layer and compared the SH-wave velocity ratio between heterogeneous and homogeneous half-space demonstrated by Pradhan et al. [6]. Kumar and Gupta [7] observed the consequences of anisotropy and thermal effect on the wave profile in fibre-reinforced layer. Singh and Singh [8] studied the impact of reinforced and anisotropy on the reflection of plane waves in free surface of the half-space. Paswan et.al. [9] discussed the effect of reinforcement and heterogeneity on the velocity of torsional surface wave. Materials present inside the Earth may not always be elastic and isotropic. It is supposed that the layers of the soil within the Earth are additionally sandy than elastic. Also, a dry sandy mantle consists of sand particles and these particles are without moisture or water vapours. The crust of the earth may not be precisely elastic but could be approximated as sandy particles. Some notable works have been carried out on sandy layer, that provide us strong theoretical understanding. Kar et al. [10] discussed the effect of irregularity on Love wave propagation in dry sandy layer. In addition, Tomar and Kaur [11] investigated the reflection and transmission of SH-wave incident at the corrugated interface between two half-spaces. Propagation of Love waves in initially stressed dry sandy layer lying over gravitating porous half-space discussed by Pal and Ghorai [12], as well as the similar study is carried out by Dey et al. [13] for torsional surface wave and explained the favouring effect of gravity in the dry sandy layer on phase velocity of surface wave. Based on assumptions on the Earth's dissimilar layered structure and spherical in shape, many theoretical and experimental research works have been conducted regarding the inhomogeneity in the crust and mantle [14]. Inhomogeneity has been central attraction to researchers, as these works help to understand the interior of the earth and some notable studies are mentioned. It has been studied for several variations with respect to depth and density on the Love wave generation in a homogeneous elastic layer resting an inhomogeneous half-space and plausible variations are discussed [15]. Datta [16] studied the Love wave propagation in non-homogeneous stratum sandwiched between two isotropic half spaces. Abd-Alla and Ahmed [17] discussed Love wave propagation in changeable initially stressed heterogeneous layer and observed that velocity of Love waves depends on inhomogeneity of two mediums. Selim [18] also, observed that the velocity of propagation of torsional surface waves depends on the heterogeneity present in the medium. Saroj et.al. [19] outlined the impacts of gradient variation of material constant and initial stress on Love wave generation in piezoelectric layer constrained between layer and substrate. Seismic waves in unlike layers also have been studied extensively and still considered as an area of interest for research. Abo-Dahab examined the effect of electromagnetic field and rotation on the surface waves in the presence of voids and viscosity in fiber-reinforced media and obtained vital consequences that existence of surface is not possible under the effect of strongly applied electromagnetic field and rotation [20]. Sahu et al. [21] also, studied the shear wave propagation in an inhomogeneous fiber-reinforced layer resting over a semi-infinite medium under gravity. Pal et al. [22] investigated the effect of sandiness on the phase velocity of surface wave in dry sandy layer clamped between liquid layer and porous substrate. Some studies are carried out in different inhomogeneous layers to show the various aspects of parameter notably initial stress with varying properties, inhomogeneity, anisotropy and thermo elasticity on the phase velocity of surface wave [23-25].

The aim of the present paper is to derive a dispersion relation for the surface type Love wave in sandwiched layer of dry sandy layer. Using analytic method, non-dimensional phase velocity of Love wave is plotted against non-dimensional wave number to show the effect of parameters associated in layers and half-space.

2 FORMULATION OF THE PROBLEM

In this paper, three layered structure is considered and assumed model consist of dry sandy layer (M_2) clamped between fiber-reinforced layer (M_1) and inhomogeneous half space (M_3). Finite thickness of the Layers M_1, M_2 are h_1 and h_2 respectively. In the *xz*-plane of Cartesian co-ordinate system, origin ($z = 0$) is set at the interface between sandy layer and inhomogeneous half-space. *x-axis* is taken in the direction of wave propagating and *z*-axis is considered positively vertically downward. Occupying regions by layers and half-space can be described as follows, fiber-reinforced layer (M_1) (occupies the region $-h_1 \le z < -h_2$, $-\infty < x < \infty$), dry sandy layer (M_2)

(occupies the region $-h_2 \le z < 0$, $-\infty < x < \infty$) and inhomogeneous half-space (M_3)(occupies the region $0 \le z$, $-\infty < x < \infty$) as shown in the geometry (Fig.1).
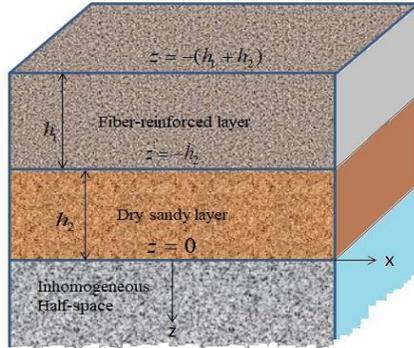
Fig.1
Geometry of the problem.

3   DYNAMICAL BEHAVIOUR OF FIBER REINFORCED LAYER AND ITS SIMULATION (M_1)

The constitutive equation for a fiber reinforced elastic body along the preferred direction is given by Spencer [26]

$$\sigma_{ij}^{(1)} = [\lambda e_{kk} \delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km} \delta_{ij} + a_i a_j e_{kk}) + 2(\mu_L - \mu_T)(a_i a_k e_{kj}) + \beta(a_k a_m e_{km} a_i a_j)] \tag{1}$$

where $\sigma_{ij}^{(1)}$ = stress components, $e_{ij} \left\{ = \frac{1}{2}(u_{i,j} + u_{j,i}) \right\}$ = infinitesimal strain components, u_j = the components of

displacement vector.

The coefficient $\lambda, \mu_T, \mu_L, \alpha$ and β are elastic constant with the dimension of stress. Coefficient α and β are the specific stress components to take into account different layers of concrete part of the composite material, $\vec{a} = \vec{a}(a_1, a_2, a_3)$ such that $a_1^2 + a_2^2 + a_3^2 = 1$.

In this paper, we have assumed the direction of fiber along the *x*-axis and *z*-axis, i.e. $\vec{a} = \vec{a}(a_1, 0, a_3)$.

For the propagation of Love wave along the *x*-axis and having displacement of particles along *y*-direction only, we have $u_1 = 0 = w_1$ and $v_1 = v_1(x, z, t)$. Using the above result, the non-zero stresses given by Eq. (2), are

$$\left. \begin{aligned} \sigma_{xy}^{(1)} &= \mu_T [P \frac{\partial v_1}{\partial x} + Q \frac{\partial v_1}{\partial z}], \\ \sigma_{yz}^{(1)} &= \mu_T [R \frac{\partial v_1}{\partial x} + Q \frac{\partial v_1}{\partial z}]. \end{aligned} \right\} \tag{2}$$

where,

$$P = (1 + \mu')a_1^2, Q = (\mu' - 1)a_1 a_3, R = (1 + \mu')a_3^2 \quad \text{and} \quad \mu' = \frac{\mu_L}{\mu_T} \tag{3}$$

Equation of motion for fibre-reinforced medium can be written in the absence of body forces as,

$$\frac{\partial \sigma_{xy}^{(1)}}{\partial_x} + \frac{\partial \sigma_{yy}^{(1)}}{\partial_y} + \frac{\partial \sigma_{yz}^{(1)}}{\partial_z} = \rho \frac{\partial^2 v_1}{\partial t^2} \tag{4}$$

where ρ_1 is the density of the fiber reinforced layer, using non-zero stresses given by Eq. (2) in Eq. (4), we have

$$P \frac{\partial^2 v_1}{\partial x^2} + 2Q \frac{\partial^2 v_1}{\partial x \partial z} + R \frac{\partial^2 v_1}{\partial z^2} = \frac{\rho_2}{\mu_T} \frac{\partial^2 v_1}{\partial t^2} \tag{5}$$

We consider the harmonic solution of Eq. (5), as:

$$v_1(x,z,t) = \phi_1(z) e^{\iota k (x-ct)} \tag{6}$$

where k is wave number and c is the phase velocity. On substituting the solution Eq.(6) into Eq(5), we get

$$R \frac{d^2 \phi_1}{dz^2} + 2Q \iota k \frac{d\phi_1}{dz} + k^2 (\frac{c}{c_1} - P)\phi_1 = 0 \tag{7}$$

where $c_1 = \sqrt{\frac{\mu_T}{\rho_2}}$ is the shear wave velocity of fiber-reinforced layer. The solution of Eq. (7) can be expressed as:

$$\psi(z) = A e^{-\iota k m_1 z} + B e^{-\iota k m_2 z} \tag{8}$$

where, $m_1 = \frac{1}{R} \left[Q + \sqrt{Q^2 + R\left(\frac{c^2}{c_1^2} - P\right)} \right]$, $m_2 = \frac{1}{R} \left[Q - \sqrt{Q^2 + R\left(\frac{c^2}{c_1^2} - P\right)} \right]$.

From Eqs. (6) and (8), we get displacement component for fiber reinforced layer,

$$v_1(x,z,t) = (A e^{-\iota k m_1 z} + B e^{-\iota k m_2 z}) e^{\iota k (x-ct)} \tag{9}$$

4 DYNAMICAL BEHAVIOUR OF DRY SANDY LAYER AND IT'S SIMULATION (M_2)

The equations of motion of a dry sandy layer in the absence of body forces in compound form, as given by Biot [27], are

$$\left. \begin{array}{l} \frac{\partial \sigma_{xx}^{(2)}}{\partial x} + \frac{\partial \sigma_{xy}^{(2)}}{\partial y} + \frac{\partial \sigma_{xz}^{(2)}}{\partial z} = \rho_1 \frac{\partial^2 v_2}{\partial t^2} \\ \frac{\partial \sigma_{yx}^{(2)}}{\partial x} + \frac{\partial \sigma_{yy}^{(2)}}{\partial y} + \frac{\partial \sigma_{yz}^{(2)}}{\partial z} = \rho_1 \frac{\partial^2 v_2}{\partial t^2} \\ \frac{\partial \sigma_{zx}^{(2)}}{\partial x} + \frac{\partial \sigma_{zy}^{(2)}}{\partial y} + \frac{\partial \sigma_{zz}^{(2)}}{\partial z} = \rho_1 \frac{\partial^2 v_2}{\partial t^2} \end{array} \right\} \tag{10}$$

where $\sigma_{ij}^{(2)}$ are stress components and ρ_2 are density of layer. For the Love wave propagation along the x-axis, causing displacement in y-direction only as: $u_2 = 0 = w_2$ and $v_2 = v_2(x,z,t)$

According to the theory of elasticity

$$\frac{E}{\mu_1} = 2(1+\rho_1) \tag{11}$$

where E, μ_1 and ρ_1 are Young's modulus, rigidity and Poisson's ratio respectively. Weiskopf [28] suggested that relation given by Eq. (11) is not true for a dry sandy layer. So, relation between Young's modulus, rigidity and

Poisson's ratio in sandy layer is $\dfrac{E}{\mu_1} > 2(1 + \rho_1)$ and we consider $\dfrac{E}{\mu_1} = 2\eta(1 + \rho_1)$, when $\eta > 1$ may be called the sandiness parameter. Consider the difference between a sandy layer and classical elastic medium is only rigidity, the rigidity of sandy layer is $\dfrac{\mu_1}{\eta}$.

where μ_1 is rigidity of classical elastic medium.

Stress-Strain relations are

$$
\left.
\begin{aligned}
\sigma_{xx}^{(2)} &= (\lambda + \frac{2\mu_1}{\eta})\frac{\partial u_2}{\partial x} + \lambda \frac{\partial v_2}{\partial y} + \lambda \frac{\partial w_2}{\partial z} \\
\sigma_{yy}^{(2)} &= \lambda \frac{\partial u_2}{\partial x} + (\lambda + \frac{2\mu_1}{\eta})\frac{\partial v_2}{\partial y} + \lambda \frac{\partial w_2}{\partial z} \\
\sigma_{zz}^{(2)} &= \lambda \frac{\partial u_2}{\partial x} + \lambda \frac{\partial v_2}{\partial y} + (\lambda + \frac{2\mu_1}{\eta})\frac{\partial w_2}{\partial z} \\
\sigma_{yz}^{(2)} &= \frac{\mu_1}{\eta}(\frac{\partial w_2}{\partial y} + \frac{\partial v_2}{\partial z}) \\
\sigma_{zx}^{(2)} &= \frac{\mu_1}{\eta}(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}) \\
\sigma_{xy}^{(2)} &= \frac{\mu_1}{\eta}(\frac{\partial u_2}{\partial y} + \frac{\partial v_2}{\partial x})
\end{aligned}
\right\}
\tag{12}
$$

where λ and $\dfrac{\mu_1}{\eta}$ are the Lame's parameter for Sandy layer. Substitute the value of Eq. (12) in Eq. (10). We have the equation of motion for the propagation of Love waves in dry sandy layer with density ρ_2 is,

$$
\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} = \frac{1}{c_2^2}\frac{\partial^2 v_2}{\partial t^2}
\tag{13}
$$

where $c_2 = \sqrt{\dfrac{\mu_1}{\eta \rho_2}}$ and μ_1 is constant. We consider the harmonic solution of Eq. (13)

$$
v_2(x,z,t) = \phi_2(z)e^{ik(x-ct)}
\tag{14}
$$

where k is wave number and c is phase velocity, on substituting the value of Eq. (13) in Eq. (14), we get

$$
\frac{d^2\phi_2}{dz^2} + k^2(\frac{c^2}{c_2^2} - 1)\phi_2(z) = 0
\tag{15}
$$

Solution for the Eq. (15) can be written as,

$$
\phi(z) = C e^{ik m_3 z} + D e^{-ik m_3 z}
\tag{16}
$$

where $m_3 = \sqrt{\dfrac{c^2}{c_2^2} - 1}$. Hence from Eq. (14) and (16), we can have the displacement component for dry sandy layer,

$$
v_2(x,z,t) = (C e^{ik m_3 z} + D e^{-ik m_3 z})e^{ik(x-ct)}
\tag{17}
$$

5   DYNAMICAL BEHAVIOUR OF INHOMOGENEOUS HALF-SPACE AND IT'S SIMULATION (M_3)

Let u_3, v_3, w_3 be the displacement along x, y and z-axes respectively. The propagation of Love waves taken along x-direction, so the displacement components along y-direction are $u_3 = 0 = w_3$ and $v_3 = v_3(x, z, t)$.

Inhomogeneity in the half-space associated to rigidity and density is considered in exponential form, varying with the respect to depth i.e., $\mu = \mu_2 e^{sz}$ and $\rho = \rho_3 e^{sz}$, where 's' is inhomogeneity parameter.

The equation of motion for the inhomogeneous elastic solid in the absence of body forces can be presented as [27]

$$\frac{\partial \sigma_{xy}^{(3)}}{\partial x} + \frac{\partial \sigma_{yz}^{(3)}}{\partial x} = \rho_3 \frac{\partial^2 v_3}{\partial t^2} \tag{18}$$

where μ_2 and ρ_3 are the rigidity and density in the inhomogeneous half-space. The propagation of Love waves along x-direction causing the displacement of particle along y-direction will generate only the ξ_{xy} and ξ_{yz} components and other strain components will be zero, hence stress-strain relations are

$$\left. \begin{array}{l} \sigma_{xy}^{(3)} = 2\mu_2 e^{sz} \xi_{xy}, \\ \sigma_{yz}^{(3)} = 2\mu_2 e^{sz} \xi_{yz}. \end{array} \right\} \tag{19}$$

Using Eq. (19) in equation of motion given by Eq. (18) and can be presented as,

$$\frac{\partial}{\partial x}(\mu_2 e^{sz} \frac{\partial v_3}{\partial x}) + \frac{\partial}{\partial z}(\mu_2 e^{sz} \frac{\partial v_3}{\partial z}) = \rho_3 e^{sz} \frac{\partial^2 v_3}{\partial t^2} \tag{20}$$

Consider the harmonic solution for Eq. (20) is,

$$v_3(x, z, t) = \phi_3(z) e^{ik(x - ct)} \tag{21}$$

On solving for Eq. (20), we have

$$\frac{d^2 \phi_3}{dz^2} + s \frac{d\phi_3}{dz} - k^2 m_4^2 \phi_3 = 0 \tag{22}$$

where $m_4^2 = (1 - \frac{c^2}{c_3^2})$, $c_3 = \sqrt{\frac{\mu_2}{\rho_3}}$ and $\omega = kc$, 'k' is wave number and 'c' is the phase velocity. Now, again considering the substitution for Eq. (22),

$$\phi_3(z) = U(z) e^{-\frac{s}{2}z} \tag{23}$$

Using the value of Eq. (23) in Eq. (22), we get

$$\frac{d^2 U}{dz^2} - k^2 n^2 U(z) = 0 \tag{24}$$

Solution of Eq. (23) can be written as,

$$U(z) = E e^{knz} + F e^{-knz} \tag{25}$$

where, $n^2 = (m_4^2 + \dfrac{s^2}{4k^2})$

As z-axis is taken positively downward then the solution vanishes at $z \rightarrow \infty$, hence the appropriate solution is

$$U(z) = F e^{-k\,nz} \tag{26}$$

From the Eqs. (21), (23) and (26), we have displacement components for inhomogeneous half-space,

$$v_3(x,z,t) = F e^{-\left(nk + \frac{s}{2}\right)z} e^{\iota k(x-ct)} \tag{27}$$

6   BOUNDARY CONDITIONS AND DISPERSION EQUATION

The propagation of Love waves in this presumed model must satisfy the following boundary conditions:

i.   The upper surface of fiber-reinforced layer (M_1) is stress free i.e. $\sigma_{yz}^{(1)} = 0$, at $z = -(h_1 + h_2)$

where $\sigma_{yz}^{(1)}$ is the stress component in layer (M_1).

ii.   Stress and displacement components are continuous at the interface between fiber-reinforced (M_1) and dry sandy layer (M_2), $\sigma_{yz}^{(1)} = \sigma_{yz}^{(2)}$, $v_1 = v_2$, at $z = -h_2$.

where $\sigma_{yz}^{(i)}$ and v_i are the stress–displacement relations components in layers $M_i (i = 1,2)$ along xz- direction respectively.

iii.   Stress and displacement components are continuous at the interface between dry sandy layer and inhomogeneous half-space $\sigma_{yz}^{(2)} = \sigma_{yz}^{(3)}$, $v_2 = v_3$, at $z = 0$.

where $\sigma_{yz}^{(j)}$ and v_j are the stress–displacement relation components in layer and half-space $M_j (j-2,3)$ along xz- direction respectively.

Using the above boundary conditions in Eqs. (9), (16) and (24), we get following equations:

$$
\left.
\begin{aligned}
&A(Q - m_1 R)e^{\iota k M (h_1 + h_2)} + B(Q - m_2 R)e^{-\iota k M (h_1 + h_2)} = 0 \\
&A(Q - m_1 R)e^{\iota k (M_0 + M)h_2} + B(Q - m_2 R)e^{\iota k (M_0 - M)h_2} + C\frac{\mu}{\mu_T \eta}e^{\iota k\, m_3 h_2} - D\frac{\mu}{\mu_T \eta}e^{\iota k\, m_3 h_2} \\
&A\,e^{\iota k (M_0 + M)h_2} + B\,e^{\iota k (M_0 - M)h_2} - C\,e^{\iota k\, m_3 h_2} - D\,e^{-\iota k\, m_3 h_2} = 0 \\
&-m_3 C + m_3 D + F\frac{\mu_2 \eta}{\mu}\left(n_2 + \frac{s}{2k}\right) = 0 \\
&C + D - F = 0
\end{aligned}
\right\} \tag{28}
$$

Eliminating arbitrary constants *A, B, C, D* and *E* from above equation, we get dispersion relation,

$$\frac{m_3 \mu \tan[k\, h_1\, M]}{\eta \mu_T \left(MR + \iota(Q - M_0 R)\tan[k\, h_1\, M]\right)} = \frac{m_3 \mu + (s_1 + n_2)\eta \mu_2 \tan[k\, h_2 m_3]}{(s_1 + n_2)\eta \mu_2 - m_3 \mu \tan[k\, h_2\, m_3]} \tag{29}$$

Eq. (29) is the desired dispersion relation for the propagation of Love wave in dry sandy layer constrained between fiber-reinforced medium and inhomogeneous half-space.

7   PARTICULAR CASES

If $\mu_T \rightarrow \mu_1, \mu_L \rightarrow \mu_1$ then $P \rightarrow \mu_1, Q \rightarrow 0, R \rightarrow \mu_1$, Then Eq. (29) becomes

$$\frac{m_3\mu\tan[kh_1M\ ]}{\eta\mu_1^2(M-\iota M_0)\tan[kh_1M\ ]}=\frac{m_3\mu+(s_1+n_2)\eta\mu_2\tan[kh_2m_3]}{(s_1+n_2)\eta\mu_2-m_3\mu\tan[kh_2m_3]} \tag{30}$$

Eq. (30) is the dispersion relation of a Love wave in dry sandy layer sandwiched between isotropic layer and inhomogeneous half-space.

In the absence of sandiness from sandwiched layer and homogeneous half-space i.e. $\eta=1$, $s=0$. Then Eq. (29) can be represented as:

$$\frac{m_3\mu\tan[kh_1M\ ]}{\mu_T(MR+\iota(Q-M_0R))\tan[kh_1M\ ]}=\frac{m_3\mu+n_2\mu_2\tan[kh_2m_3]}{n_2\mu_2-m_3\mu\tan[kh_2m_3]} \tag{31}$$

Eq. (31) is the dispersion relation of a Love waves in isotropic layer clamped between fibre-reinforced layer and isotropic half-space.

When the $\mu_T=\mu_1=\mu_L$, $P\to\mu_1,Q\to0,R\to\mu_1,a_1=a_3=0$, $\eta=1$ and $s=0$, i.e. each medium M_1,M_2 and M_3 are isotropic. Then Eq. (29) reduces to

$$\frac{m_3\mu\tan[kh_1M\ ]}{\mu_1^2(M-i\ M_0)\tan[kh_1M\ ]}=\frac{m_3\mu+m_4\mu_2\tan[kh_2m_3]}{m_4\mu_2-m_3\mu\tan[kh_2m_3]} \tag{32}$$

Eq. (32) is the dispersion relation of a Love waves in isotropic layer fastened between two isotropic mediums.

8   NUMERICAL RESULTS AND DISCUSSION

The methodologies explained in the present study compose a mathematical model for Love wave propagation in dry sandy layer sandwiched between fiber-reinforced and inhomogeneous semi-infinite mediums. The role of the obtained structure in examining the propagation characteristic of mediums may be discussed through numerical examples and presented graphically. For numerical computation motives, we have considered the following values of elastic constants provided in Table 1. [29] [30]

Table 1
Values of rigidity and density for fiber reinforced layer, dry sandy layer and inhomogeneous half-space.

Medium	Rigidity ($\times10N/m^3$)	Density (kg/m^3)
Fiber reinforced	$\mu_T=2.46$, $\mu_L=5.66$	$\rho_1=7800$
Dry sandy	$\mu=6.54$	$\rho_2=3409$
Inhomogeneous half space	$\mu_1=7.10$	$\rho_3=3321$

The effects of parameters are demonstrated graphically from Figs. 2-5. Graphs are plotted between phase velocity $\left(\frac{c}{c_2}\right)$ as vertical axis and wave number (kh_2) as horizontal axis. Fig. 2(a) presents the effects of reinforced parameters (a_1^2,a_3^2) on the dimensionless phase velocity $\left(\frac{c}{c_2}\right)$ of surface type Love wave with vary in dimensionless wave number (kh_2). A slight change in reinforced parameters leads to significant variation in the phase velocity of Love wave. From figure, we have an increasing pattern in phase velocity with the increment of $\left(a_1^2\right)$ and decrement of $\left(a_3^2\right)$ reinforced parameters. Similarly, Fig. 2(b) depicts the decreasing trend in the phase velocity with increase of $\left(a_3^2\right)$ and decrease of $\left(a_1^2\right)$ reinforced parameters. From Fig. 2 (a) - (b), it is concluded that phase velocity is directly proportional to reinforced parameter $\left(a_1^2\right)$, whereas inversely proportional to $\left(a_3^2\right)$.

(a)

(b)
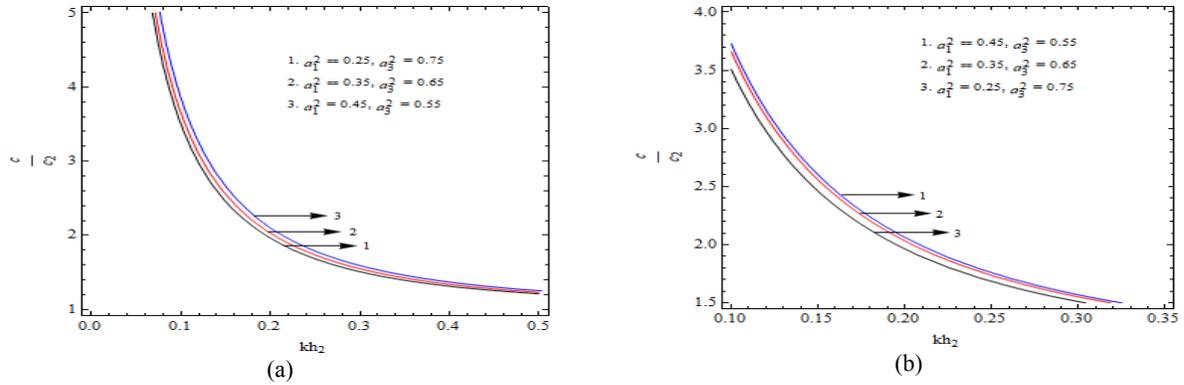
Fig.2

a) Variation of Phase velocity $\left(\dfrac{c}{c_1}\right)$ versus wave number (kh_2) for different values of reinforced parameter (a_1^2 and a_3^2). b) Variation of Phase velocity $\left(\dfrac{c}{c_1}\right)$ versus wave number (kh_2) for different values of reinforced parameter (a_1^2 and a_3^2).

Fig. 3 exhibits the variation of dimensionless phase velocity $\left(\dfrac{c}{c_2}\right)$ of surface type Love wave against the non-dimensional varying wave number $\left(kh_2\right)$ for different values of sandiness parameter ($\eta = 1$, 1.2, and 1.4). From figure, it is clear that as the property of sandwiched layer change from the isotropic to sandier medium, substantially effect on phase velocity is observed and favourable phenomena can be perceived as the sandiness increases phase velocity of Love wave also increases.

Fig. 4 shows the variation of non-dimensional phase velocity of Love wave against the wave number $\left(kh_2\right)$. The curves have been plotted for the different values of inhomogeneity s_1 $\left(=\dfrac{s}{k}\right) = 0.10$, 0.40 and 0.80. It has been noted that as the value of inhomogeneity increases phase velocity increases. It seems that all curves are emerging from same point but after certain point each curves are getting apart and significant variation on phase velocity appears. It can also be narrated from Fig.4 that until certain point $\left(kh_2 < 0.1\right)$ effect of inhomogeneity parameter on phase velocity is too marginal for different values inhomogeneity.

Fig. 5 shows the effect of thickness ratio on the propagation of Love wave in the presence reinforced, sandiness and inhomogeneity parameter. Curves have been plotted for different values of thickness ratio $H\left(=\dfrac{h_1}{h_2}\right) = 0.20$, 0.22, and 0.24. Decreasing Pattern of thickness ratio is found on the phase velocity for greater values of wave number $\left(kh_2 < 0.4\right)$ after that, narrow variation is observed. Result depicts that for the provided ratio $\left(\dfrac{h_1}{h_2}\right)$, phase velocity is decreasing as wave number is increasing.
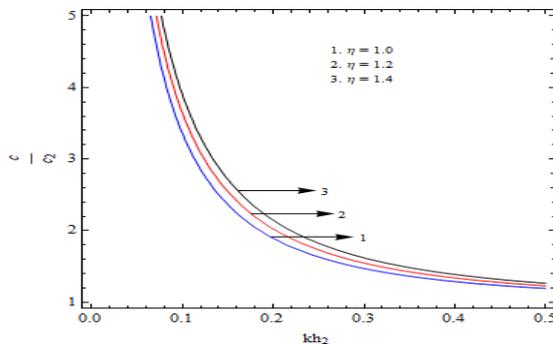
Fig.3

Variation of Phase velocity $\left(\dfrac{c}{c_2}\right)$ against wave number (kh_2) for different values of sandiness parameter (η).
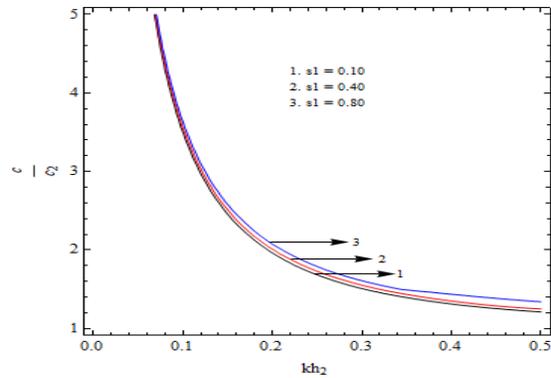
Fig.4

Variation of Phase velocity $(\dfrac{c}{c_2})$ against wave number (kh_2) for different values of inhomogeneity (s_1).
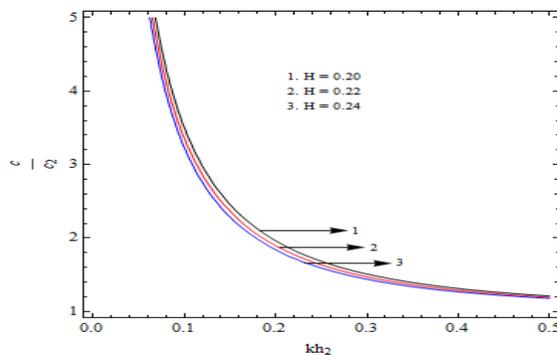
Fig.5

Variation of Phase velocity $(\dfrac{c}{c_2})$ versus wave number (kh_2) for different values of thickness ratio (H).

9   CONCLUSIONS

In this paper, efforts have made to study the dispersion of Love wave propagation in sandy layer sandwiched between fibre-reinforced layer and inhomogeneous elastic half-space. Displacement components for layers and half-space have been derived by using the method of separation of variables. Dispersion relation is derived in closed form. Numerical computations for dispersion relation are performed and graphs are plotted between phase velocity and wave number to show the effects of reinforcement, sandiness, inhomogeneity parameters and thickness ratio. Discussions were carried out in details and described graphically. From the graphs, significant effects of parameters on the phase velocity of Love wave are observed. Phase velocity decreases as wave number increases. Favouring and disfavouring effects of reinforced parameters (a_1^2) and (a_3^2) have been noticed on phase velocity of Love wave. Effect on phase velocity of sandiness (η) and inhomogeneity (s) parameters have same effect as these parameters increase phase velocity also increases. Thickness ratio (H) has reverse effect on the phase velocity of Love wave with respect to parameters mentioned earlier. From graphs, it can also be concluded that pattern of the curves behave the same for different values of parameters. As the present analytic study is based on layered structure, these outcomes would be helpful to research worker in the domain of Earth sciences, geophysics, engineering geology etc., also the results from present study may helpful in many applications. Therefore, we came to the point from this study that outcomes could be the bridge between the theoretical study and real world application.

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