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Derivation of Equation of Motion for the Pillow-Shape Seismic Base Isolation System

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Abstract

Rolling-based seismic isolation systems, in which rollers of circular or non-circular section are used, are less expensive and easier to manufacture. However, this type of isolation suffers from either lack of restoring or re-centering capability, or weakness against uplift forces. To resolve the first shortcoming the use of elliptical as well as pillow-shape rolling parts has been suggested, and for the second one, using some uplift restrainers has been proposed. In this paper a kind of base isolating device, called Pillow-Shape Seismic Base Isolation System (PSBIS) is introduced which has both re-centering and uplift-restrained capabilities. The paper is concentrated on derivation of equation governing the motion of PSBIS device. To derive this equation, since the behavior of the device is highly nonlinear the use of Lagrange equation of motion is the most appropriate approach. For this purpose, considering the rotation angle of the pillow-shape roller as the generalized coordinate, the horizontal and vertical components of the pillow-shape roller acceleration have been formulated and the kinetic and potential energy terms as well as the virtual work of the non-conservative, resulted from rolling resistance and seismic forces, have been developed to be substituted in the basic Lagrange equation. To verify the derived equation of motion a simplified sample of the PSBIS device was built and used experimentally, and also the motion of the pillow-shape parts was traced by drawing them in Auto-CAD software. Results show the validity of the derived equation of motion .

Keywords: Rolling Resistance, Lagrange Equation of Motion, Differential equation

1. Introduction

Among various seismic base isolating devices, proposed by researchers so far, those which are based on using rollers, either of circular or non-circular section are less expensive and easier to manufacture. To make benefit from these two advantages, some researchers have worked on developing this type of isolators since early 80s. Arnold [1] published a paper entitled ‘rolling with the punch of earthquakes-base isolation is used in two New-Zealand office buildings. Later, Pham [2] proposed a base-isolation design using spherically-ended rollers and telescopic shock absorbers to be used for upgrading of the seismic strength of 16 valve damping resistors in the HVDC transmission network of the Electricity Corporation of New Zealand. It should be noted that due to relatively small width of the proposed spherically-ended rollers stress concentration is very likely at either edge of the rollers, however, Pham has not addressed this phenomenon in his study.

Jangid and his colleagues and also Londhe and Jangid [4], investigated the effectiveness of elliptical rolling rods isolation system between the base and the foundation of structure. They derived equations governing the motion of a multi-storey shear type building supported on the elliptical rolling rods. The obtained dynamic response of the system to both harmonic and real earthquake ground motions by integrating the incremental equations along with an iterative technique. They also conducted a parametric study to investigate the effects of important parameters on the effectiveness of elliptical rolling rods, including the fundamental time period of the superstructure, the coefficient of rolling friction and the eccentricity of the elliptical rolling rods and the frequency content of ground motion. They showed that the elliptical rolling rods are quite effective in reducing the dynamic response of the system without undergoing the large displacements or living the large residual base displacement.

Xiangyun and Qianfeng [5] presented the equation of motion for base isolation system using roller bearings with restoring property. They assumed a base isolated rigid body system to be subjected to steady harmonic excitation and analyzed the response, and suggested the optimum stiffness of the isolated layer of the multistory masonry building with different damping ratios.

Zhang and his colleagues [6] studied the seismic responses of multi-storey shear type buildings isolated by rolling rods. Through taking the characteristics of pure

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rolling into account based on the sliding isolation, the equation of rolling isolation was proposed. The responses of the rolling isolation structure were compared with the corresponding fixed base structure in order to investigate the effectiveness of the rolling rods. It was found that when the rolling friction coefficient is larger than 0.15 cm, there is no need to the stoppers to limit the lateral motion .

Hosseini and Soroor [7] proposed orthogonal pairs of rollers on concave beds (OPRCB) as a base isolation system for short- to mid-rise buildings, which does not need high technology for manufacturing and is not costly, contrary to other existing systems like lead-rubber bearing or friction pendulum bearing systems. In their proposed system each isolator consists of two Orthogonal Pairs of Rollers on Concave Beds (OPRCB). Rolling rods installed in two orthogonal directions make possible the movement of the superstructure in all horizontal directions. The concave beds, in addition to giving the system both restoring and re-centering capabilities, make the force–displacement behavior of the isolators to be of hardening type. It should be noted that in spite of the advantages of the OPRCB isolators they are weak against uplift.

Lu and his colleagues [8] performed an experimental verification of variable-frequency rocking bearings for near-fault seismic isolation. Mentioning that a near-fault earthquake usually possesses a long-period pulse-like component that may result in an excessive isolator displacement in a conventional isolation system, and consequently increase seismic risk or lead to an oversized isolator design, they suggested a variable-frequency rocking-type bearing (VFRB). Their proposed rocking bearing has a rocking surface with a variable curvature, and by properly selecting the geometry of the rocking surface, the isolation stiffness and frequency of the proposed rocking bearing become the functions of the bearing displacement. In order to experimentally verify its feasibility, a full-scale steel frame isolated by prototype VFRB bearings was tested by using a shaking table test. The rocking surface of the prototype bearings were defined by a six-order polynomial function, so they have a relatively higher initial stiffness followed by a softening behavior. This mechanical property enable the VFRB isolation system to effectively suppress the excessive isolator displacement induced by a pulse-like near-fault earthquake, while retains a reasonable isolation efficiency. Their test data show good agreement with the simulated ones. Moreover, their experimental results demonstrate that the VFRB-isolated frame exhibits the desired behavior in a near-fault earthquake, and thus confirms the applicability of VFRB isolators for near-fault seismic isolation.

Wang and his colleagues [9] have developed a sloped rolling-type isolation device for seismic protection of important equipment and facilities, which has multi-roller and inbuilt damping mechanisms. They have mentioned that in addition to having a good potential to control the transmitted acceleration response as a steady level, the multi-roller mechanism has better stability performance and re-centering capability than the conventional single-roller mechanism. Moreover, the inbuilt damping mechanism facilitates to suppress excessive displacement responses and to stop rolling motion after earthquakes. Based on the analytical results, they have proposed a simplified mathematical hysteretic model to represent the twin-flag hysteresis loop of the isolation device. The shaking table test results on several plane arrangements of isolated reservation cabinets and an isolated raised floor system show that the adoption of the isolation devices are effective in reducing seismic demands for the protected objects.

Foti and his colleagues [10], studied the dynamic behavior of a rolling isolation device composed of cylinders moving in between two rubber layers, from a theoretical point of view. The device reduces the seismic energy both by decoupling the motion of the structure from the base and through the viscoelastic behavior of the rubber (or neoprene). They explained the behavior and the efficacy of this device by relations obtained for the rolling friction coefficient versus the rolling velocity and for the horizontal force versus the vertical load acting on the device. They mentioned that future studies would be necessary to determine the appropriate dimensions of the elements composing the isolator that most reduce seismic vibrations in structures. However, they have not addressed the seismic behavior of their device.

Harvey and his colleagues [11,12] have proposed a double rolling isolation systems and worked on its mathematical model and experimental validation. They have mentioned that rolling isolation systems (RISs) protect fragile building contents from earthquake hazards by decoupling horizontal floor motions from the horizontal responses of the isolated object, and that the RISs in use today have displacement capacities of about 20 cm, and have suggested increasing the displacement capacity by stacking two systems. Then they have evaluated a non-linear model of the coupled dynamics of double RISs. They have derived the model through the fundamental form of Lagrange's equation and have involved then on homonymic constraints of spheres rolling between non-parallel surfaces. The derivation requires the use of two translating and rotating reference frames. They have validated their proposed model through comparisons between experimentally measured and numerically predicted time histories and peak response quantities—total acceleration and relative displacement. Finally, they have assessed the effects of the initial conditions, the mass of the isolated object, and the amplitude and period of the disturbance on the system's performance.

Following their previous studies, Harvey and his colleagues [13] examined the performances of lightly- and heavily-damped rolling isolation systems (RISs) located within earthquake-excited structures. Six steel structures of varying height and stiffness are selected so as to represent a range of potential RIS installations. Computation models of representative frames from each of the six structures are reduced through dynamic condensation and assembled with models for biaxial isotropic hysteretic behavior within each floor. A novel reduced order modeling approach is presented in this paper. The method combines a dynamic condensation of a linear-elastic frame with the inelastic-push over curve for a detailed elastic–plastic frame model and a novel bi-axial hysteretic model for the net inter-story inelastic behavior. The reduced inelastic model combines stiffness and mass matrices from the reduced linear model with the bi-axial inelastic floor model, and is subsequently fit to push-over curves from the detailed hysteretic model. The resulting reduced order model has three coordinates per floor and provides a much simpler model for simulating the floor responses of inelastic structures. The resulting inelastic structural models are isotropic in plan and uniform along the height. Suites of recorded ground motions representative of near-fault and far-field hazards are scaled and inputted into these hysteretic reduced models. The bidirectional floor responses at varying heights are then applied to experimentally-validated models of lightly- and heavily-damped RISs. Empirical cumulative distribution functions of peak isolator responses (relative displacement and total acceleration) for the two systems are compared, from which installation guidelines are presented. It that study although they have referred to rolling resistance, they have not fully addressed its calculation and have not presented any formulation for that.

Although the aforementioned rolling-based isolating system has several advantages comparing to other types of seismic isolation, they have still some shortcomings such as stress concentration between rollers and their bed due to the relatively small size of rollers, and weakness of the device against uplift, which causes separation of the rollers from their bed, which not only invalidates the equation of motion, but also results in collision between the rollers and their bed. Very recently, Tayaran [14] has introduced a somehow new isolator, called Pillow-Shape Base Isolation System (PSBIS), by which the two mentioned shortcoming can be resolved. This paper presents the analytical part of that study.

2. General Features of the PSBIS

Figure 1 shows a general view of the PSBIS, its initial and rotated states and the forces engaged in its dynamic motion. Each pillow is made by cutting off the middle part of a cylinder of radius (r), which results in a remaining pillow-shape part of height (h).

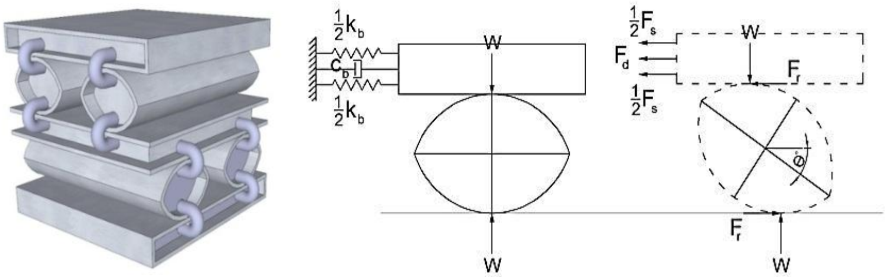

Figure 1. The general view of the PSBIS (left) and its initial and rotated states and the forces engaged in its dynamic motion (right)

The U-shape elements in Figure 1 are the uplift restrainers. As it is seen, for each set of PSBIS eight pairs of the U-shape restrainers are required. Each U-shape restrainer is equipped with two rollers, one at each end, which keep in touch the pillow-shape roller and its lower and upper plates, while letting them to move easily with respect to each other during the rolling motion as shown in Figure 2.

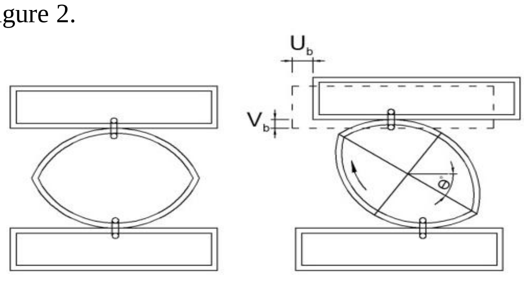

Figure 2. Motion of pillow-shape rollers in the presence of U-shape restrainers

As shown in Figure 2, the larger diameter of pillows is horizontal in the initial state, thus the vertical load and its reaction are on one vertical line, but when the pillow turns, the vertical load and the reaction are no longer on the same line, and a distance is created between them which creates a restoring moment causing the pillow to tend to return to its initial position. It can be seen in Figure 2 that rotation of the pillow roller results in both horizontal and vertical displacements of the foundation of the isolated building, indicated in the figure by, respectively, u_b and v_b. As it can be seen in Figure 2, the size of pillow rollers is relatively large comparing to other rolling isolation systems.

According to Figure 2, when the horizontal displacement of the isolated structure at base increases, the magnitude of the restoring moment increases as well, and in this way occurrence of large lateral displacements of the system is prevented. Furthermore, regarding the relatively large size of the pillow rollers, the vertical loads are transferred via a relatively long line between the rollers and their bed, and therefore, the stress concentration can be satisfactorily prevented in the system.

3. Rotatıon-Dısplacement Relatıonshıp In PSBIS Devıce

It is well-known that in a circle with radius (r) the perimeter equals $2\pi r$ and the distance traveled (Δ) due to a rotation θ radians is equal to θr.

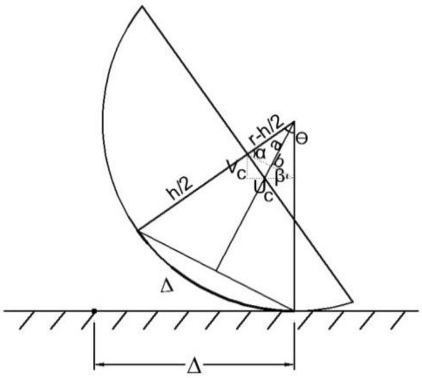

Figure 3: One half of the PSBIS's pillow rotated θ radians

Based on Figure 3, and the mentioned relation, one can write:

$$\sin\frac{\theta}{2} = \frac{b}{r-\frac{h}{2}} \quad \rightarrow \quad b = \left(r-\frac{h}{2}\right)\sin\frac{\theta}{2} \tag{1}$$

$$\cos\frac{\theta}{2} = \frac{a}{r-\frac{h}{2}} \quad \rightarrow \quad a = \left(r-\frac{h}{2}\right)\cos\frac{\theta}{2} \tag{2}$$

and because of the isosceles triangle, it can be written:

$$\alpha = \frac{\pi}{2} - \frac{\theta}{2} \tag{3}$$

$$\beta = \frac{\pi}{2} - \alpha = \frac{\theta}{2} \tag{4}$$

Therefore, vertical and horizontal displacements of the centroid of the pillow (v_c and u_c) can be obtained by the following equations:

$$\sin\beta = \frac{v_c}{2b} \quad \rightarrow \quad v_c = 2b\sin\frac{\theta}{2} = (2r-h)\sin^2\frac{\theta}{2} \tag{5}$$

$$\cos\beta = \frac{u_c{}'}{2b} \quad \rightarrow \quad u'_c = 2b\cos\frac{\theta}{2} = 2\left(r-\frac{h}{2}\right)\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$\rightarrow u'_c = \left(r-\frac{h}{2}\right)\sin\theta \tag{6}$$

$$u_c = \Delta - u'_c = \theta r - \left(r-\frac{h}{2}\right)\sin\theta = \theta r - \left(r-\frac{h}{2}\right)\sin\theta \tag{7}$$

It is clear that vertical and horizontal displacements (v_b and u_b) of the column's base, isolated by PSBIS, is twice as displacements of the pillow's centroid, which are:

$$v_b = 2v_c = (4r-2h)\sin^2\frac{\theta}{2} \tag{8}$$

$$u_b = 2u_c = 2\theta r - (2r-h)\sin\theta \tag{9}$$

Calculating the first and second time derivatives of the vertical and horizontal displacements results in:

$$\dot{v}_b = 2\dot{v}_c = 2\left(r-\frac{h}{2}\right)\dot{\theta}\sin\theta \tag{10}$$

$$\ddot{v}_b = 2\ddot{v}_c = 2\left(r-\frac{h}{2}\right)(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \tag{11}$$

$$\dot{u}_b = 2\dot{u}_c = 2r\dot{\theta} - 2\left(r-\frac{h}{2}\right)\dot{\theta}\cos\theta \tag{12}$$

$$\ddot{u}_b = 2\ddot{u}_c = 2r\ddot{\theta} - 2\left(r-\frac{h}{2}\right)(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) \tag{13}$$

The velocity and acceleration values, obtained by Equations (10) to (13), will be used in derivation of the PSBIS device equation of motion as explained later in the paper.

4. Verification of Rotation-Displacement Relationship

To verify the derived equation of motion for the column's base isolated by PSBIS device two approaches were used. In the first approach, a pair of pillow shape plates of 16 mm thickness, 400 mm height and curvature radius of 300 mm were built, and the motion of a vertical rectangular plate on the pair of pillow shape plates were delicately traced with rolling the pillow shape plate in a direction in vertical plane, and plotted on a large gridded paper, as shown in Figure 4. The traced path of the vertical plate (shown in black in the Figure) was scanned and exported to EXCEL program as a curve. In the second approach the pair of pillows shape plates as well as the rectangular plate with the same setting and geometry as the first approach, were drawn using AutoCAD software, and by applying incremental rotations, the horizontal and vertical displacements of the rectangular plate were extracted and combined as one curve, as shown in Figure 5.

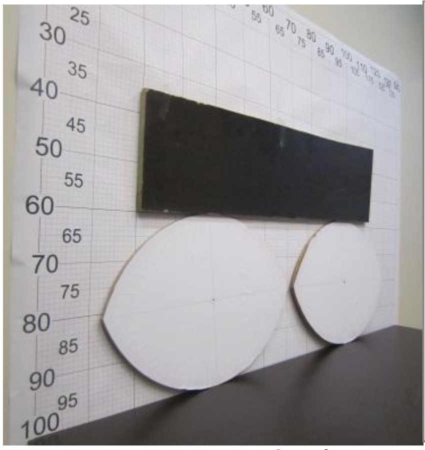

Figure 4: Practical verification

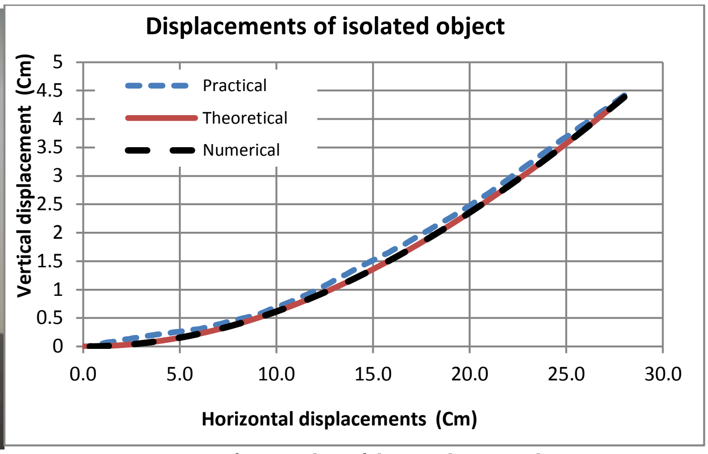

Figure 5: Verification plots of the sample PSBIS device

Figure 5 the plots obtained by the two approaches (practical and numerical) are compared with the plot obtained by theoretical formulas. The little difference between plots obtained from theoretical/numerical and practical approaches is due to the deficiencies in building-up process of the sample PSBIS.

5. Free Vibration Equation of Motion of PSBIS Device

Forces contributing in the equation of motion of SDOF system, isolated by a PSBIS device, include spring force, F_s, and damping force, F_d, due to the stiffness, k_b, and damping, c_b, (see Figure 1) as well as rolling resistance forces, F_r, at bottom and top of rollers, and the weight of the SDOF and its base, $w = m_b g$. As shown in Figure 1, spring and damping forces act during the horizontal movement of the base, u_b, and the system weight, w, act during the vertical movement of the base, v_b, while the rolling resistance forces, F_r, act at lower and upper points of the pillow roller's perimeter and their movement is half of the base movement. Due to this fact the governing equation of motion of the PSBIS cannot be determined using Newton's second law since the forces engaged in the motion of the system have different points of action with different movements.

On this basis, the energy method was used for derivation of equation of motion for this system. In this study Lagrange equation of motion was used (Clough and Penzien [15], which is based on kinetic and potential energies and their corresponding relative derivatives, and also the variation of non-conservative forces. To develop this equation for the SDOF system, isolated by a PSBIS device, by considering θ, the angle of rotation of the pillow roller as shown in Figure 1, as the main generalized coordinate, one can write:

$$F_s = k u_b \tag{14}$$

$$F_d = c \ddot{u}_b \tag{15}$$

$$T = \frac{1}{2} m_b \left(\dot{u}_b{}^2 + \dot{v}_b{}^2\right) = 2 m_b \dot{\theta}^2 \left(\frac{h^2}{4} + (2r^2 - rh)(1 - \cos\theta)\right) \tag{16}$$

$$\frac{\partial T}{\partial \theta} = 2 m_b \dot{\theta}^2 (2r^2 - rh) \sin\theta \tag{17}$$

$$\frac{\partial T}{\partial \dot{\theta}} = 4 m_b \dot{\theta} \left(\frac{h^2}{4} + (2r^2 - rh)(1 - \cos\theta)\right) \tag{18}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\theta}}\right) = m_b \ddot{\theta}\left(h^2 + 4(2r^2 - rh)(1 - \cos\theta)\right) + 4 m_b \dot{\theta}^2 (2r^2 - rh) \sin\theta \tag{19}$$

$$V = \frac{1}{2} K u_b^2 + m_b g v_b \tag{20}$$

$$= 2K\left(r^2\theta^2 + \left(r - \frac{h}{2}\right)^2 \sin^2\theta - 2r\theta\left(r - \frac{h}{2}\right)\sin\theta\right) + 4 m_b g \left(r - \frac{h}{2}\right) \sin^2\frac{\theta}{2} \tag{21}$$

$$\frac{\partial V}{\partial \theta} = 2K\left(2r^2\theta + \left(r - \frac{h}{2}\right)^2 \sin 2\theta - 2r\left(r - \frac{h}{2}\right)\sin\theta - 2r\theta\left(r - \frac{h}{2}\right)\cos\theta\right) + 2 m_b g \left(r - \frac{h}{2}\right)\sin\theta \tag{22}$$

$$\delta W_{nc} = -F_r \delta u_b - C \dot{u}_b \delta u_c = \sum Q_i \, \delta q_i \tag{23}$$

As long as the pillow rolls in clockwise direction, the resulting force couple of the rolling resistance forces acts in counterclockwise direction, and vice versa. To take this fact into account the real direction of the force couple resulting from the rolling resistance forces the sign function, sign ($\dot{\theta}$), was used in calculation of the variation of the virtual work done by non-conservative forces as:

$$\delta W_{nc} = -\,\mathrm{sign}\,(\dot{\theta})\, F_r \delta u_c - C\dot{U}_b \delta u_c \tag{24}$$

$$\delta u_b = 2\delta u_c = 2r\delta\theta - 2\left(r - \frac{h}{2}\right)\delta\theta\cos\theta \tag{25}$$

$$\delta W_{nc} = -\left(\mathrm{sign}\,(\dot{\theta})F_r + C(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta\right) * \left(r - \left(r - \frac{h}{2}\right)\cos\theta\right)\delta\theta \tag{26}$$

$$F_r = \frac{m_b g\, b}{r} \tag{27}$$

$$\delta W_{nc} = \left(\ \mathrm{sign}\,(\dot{\theta})\frac{m_b\, g\, b}{r} + C(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta\right) * \left(r - \left(r - \frac{h}{2}\right)\cos\theta\right)\delta\theta \tag{28}$$

$$Q = -\left(\ \mathrm{sign}(\dot{\theta})\frac{m_b\, g\, b}{r} + C(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta)\right) * \left(r - \left(r - \frac{h}{2}\right)\cos\theta\right) \tag{29}$$

substituting the above values in Lagrange equation of motion

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot{\theta}}\right) - \frac{\partial T}{\partial\theta} + \frac{\partial V}{\partial\theta} = Q \tag{30}$$

results in the following equation of motion for the system:

$$\begin{aligned} &m_b\ddot{\theta}(h^2 + 4(2r^2 - rh)(1 - \cos\theta)) + 4m_b\dot{\theta}^2(2r^2 - rh)\sin\theta - 2m_b\dot{\theta}^2(2r^2 - rh)\sin\theta \\ &+ 2K\left(2r^2\theta + \left(r - \frac{h}{2}\right)^2\sin 2\theta - 2r\left(r - \frac{h}{2}\right)\sin\theta - 2r\theta\left(r - \frac{h}{2}\right)\cos\theta\right) + 2m_b g\left(r - \frac{h}{2}\right)\sin\theta \\ &- \left(\ \mathrm{sign}(\dot{\theta})\frac{m_b\, g\, b}{r} + C(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta)\right) * \left(r - \left(r - \frac{h}{2}\right)\cos\theta\right) = 0 \end{aligned} \tag{31}$$

In Equation (31) the parameter b is the coefficient of rolling resistance which its value depends on the materials of the engaged surfaces.

6. Equations of Motion of the Sdof System Isolated by Psbis Device under Earthquake Excitation

To derive the equation of motion of the SDOF system isolated by PSBIS device subjected to earthquake excitation the ground acceleration vector {EQ(t)}was consiereded with two horizontal and vertical components $\ddot{u}_g$and $\ddot{v}_g$as:

$$\{EQ(t)\} = \begin{Bmatrix} \ddot{u}_g\ (t) \\ \ddot{v}_g\ (t) \end{Bmatrix} \tag{32}$$

And the variation of virtual work done by non-conservative forces is modified as:

$$\begin{aligned} \delta W_{nc} &= \left[\left(-\mathrm{sign}\,(\dot{\theta})\frac{m_b g\, b}{r} - C\dot{u}_b - 2m_b\ddot{u}_g\right)\left(r - \left(r - \frac{h}{2}\right)\cos\theta\right) - (4\, m_b\ddot{v}_g\left(r - \frac{h}{2}\right)\sin\frac{\theta}{2}\cos\frac{\theta}{2})\right]\delta\theta \\ &= \sum Q_i\,\delta q_i \end{aligned} \tag{34}$$

$$\begin{aligned} Q = &\left(-\,\mathrm{sign}(\dot{\theta})\frac{m_b\, g\, b}{r} - C\left(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta\right) - 2m_b\ddot{u}_g\right)\left(r - \left(r - \frac{h}{2}\right)\cos\theta\right) \\ &- (4\, m_b\ddot{v}_g\left(r - \frac{h}{2}\right)\sin\frac{\theta}{2}\cos\frac{\theta}{2}) \end{aligned} \tag{35}$$

The final form of the Lagrange equations of motion under earthquake excitation by PSBIS is as follows:

$$\begin{aligned} &m_b\ddot{\theta}(h^2 + 4(2r^2 - rh)(1 - \cos\theta)) + 4m_b\dot{\theta}^2(2r^2 - rh)\sin\theta - 2m_b\dot{\theta}^2(2r^2 - rh)\sin\theta \\ &+ 2K\left(2r^2\theta + \left(r - \frac{h}{2}\right)^2\sin 2\theta - 2r\left(r - \frac{h}{2}\right)\sin\theta - 2r\theta\left(r - \frac{h}{2}\right)\cos\theta\right) + 2m_b g\left(r - \frac{h}{2}\right)\sin\theta \\ &- \left[-\mathrm{sign}(\dot{\theta})\frac{m_b\, g\, b}{r} - C\left(2r\dot{\theta} - 2\left(r - \frac{h}{2}\right)\dot{\theta}\cos\theta\right) - 2m_b\ddot{U}_g\right] * \left(r - \left(r - \frac{h}{2}\right)\cos\theta\right) \\ &- \left[\left(-4\, m_b\ddot{V}_g\left(r - \frac{h}{2}\right)\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)\right] = 0 \end{aligned} \tag{36}$$

This equation, which is a highly nonlinear equation and does not have any close-form solution, can be solved by various numerical techniques.

7. Check the Conditions Of Non-Sliding Motion Of Rollers During Seismic Excitations

One of the important points which should be taken into consideration in the study of a roller's motion, is the condition in which the roller can roll without sliding, since if sliding happens the rolling equation of motion of the system will not be valid anymore. Figure 7 illustrates the forces engaged in the static equilibrium of a pillow between its bed and the corresponding column's base, on which the condition for the rolling without sliding can be obtained.

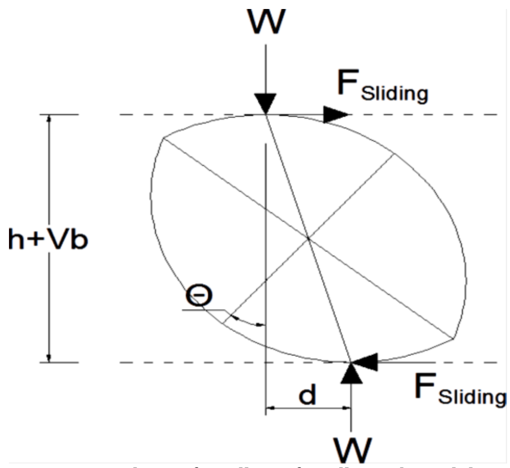

Figure 7.Conditions for rolling of a roller without sliding

Based on Figure 7 one can write:

$$\sum M_o = 0 \rightarrow W\,d = \mu\,W\,(h + v_b) \qquad (49)$$

$$W \times 2\left(r - \frac{h}{2}\right) \mathrm{Sin}\,(\theta) = \mu\,W\left(h + 4\left(r - \frac{h}{2}\right)\mathrm{Sin}^2\frac{\theta}{2}\right) \qquad (37)$$

$$2\left(r - \frac{h}{2}\right) \mathrm{Sin}\,(\theta) = \mu\left(h + 4\left(r - \frac{h}{2}\right)\mathrm{Sin}^2\frac{\theta}{2}\right) \qquad (38)$$

$$\mu = \frac{\left(r - \frac{h}{2}\right)\mathrm{Sin}\,(\theta)}{r - \left(r - \frac{h}{2}\right)\mathrm{Cos}\,\theta} \qquad (39)$$

Hence the required condition for rolling without sliding is:

$$\tan(\theta) < \frac{\mu(r - \left(r - \frac{h}{2}\right))}{r - \frac{h}{2}} \qquad (40)$$

The condition stated by Inequality Relation (40) should be met in any response calculation process of the systems isolated by PSBIS devices.

6. Conclusions

Comparison of experimental, numerical, and theoretical results of this study proves the validity of the derived equation of motion for the proposed base isolation system. This equation can be used for seismic response calculation of buildings isolated by the proposed system. To show the efficiency of this seismic isolation technique in reduction of seismic response of building structures, more equations should be developed. However, it should be noted that due to geometric nonlinearity of the governing equation of the proposed isolators, creating the matrix form for the system of equations governing the motion of isolated multi-story buildings needs extensive mathematical elaborations.

References

[1] Arnold, C. (1983). Rolling with the punch of earthquakes-base isolation is used in two New-Zealand office buildings. Architecture the AIA Journal, 72(9), 74-76.

[2] Pham, L. T. (1988). A base-isolation design using spherically-ended rollers and telescopic shock absorbers. Bulletin of the New Zealand National Society for Earthquake Engineering, 21(2), 135-139.

[3] Jangid, R. S., & Londhe, Y. B. (1998). Effectiveness of elliptical rolling rods for base isolation. Journal of Structural Engineering, 124(4), 469-472.

[4] Londhe, Y. B., & Jangid, R. S. (1999). Dynamic response of structures supported on elliptical rolling rods. Journal of Structural Mechanics and Earthquake Engineering, JSCE 16(1): 1s–10s.

[5] Xiangyun, H., & Qianfeng, W. Q. F. D. Y. (1999).Theoretic Analysis on Base Isolation System Using Roller Bearings. Journal of South China Construction University, 2, 005.

[6] Zhang, X. Z., Han, A. H., & Deng, Z. C. (2007).Dynamic analysis of base isolation by free rolling rods [J]. Journal of Shenyang University of Technology,3, 000.

[7] Hosseini, M. and Soroor, A. (2012) Using Orthogonal Pairs of Rollers on Concave Beds (OPRCB) as a base isolation system—part i: analytical, experimental and numerical studies of OPRCB isolators, The Structural Design of Tall and Special Buildings, 20(8), 928-950

[8] Lu, L. Y., Lee, T. Y., & Hsu, C. C. Experimental Verification of Variable-Frequency Rocking Bearings for Near-fault Seismic Isolation.

[9] Wang, S. J., Yu, C. H., Hsiao, J. Y., Hwang, J. S., Chang, K. C., & Tsai, M. S. (2012). Development of Sloped Rolling-Type Isolation Devices for Seismic Protection of Important Equipment and Facilities Paper Title Line.In The 15th World Conference on Earthquake Engineering

[10] Foti, D., Catalan Goni, A., &Vacca, S. (2013). On the dynamic response of rolling base isolation systems. Structural Control and Health Monitoring, 20(4), 639-648.

[11] Harvey P.S, Wiebe R, Gavin H.P (2013) On the Chaotic Response of a Nonlinear Rolling Isolation System, Physica D: Nonlinear Phenomena, 256, 36-42.

[12] Harvey, P. S., & Gavin, H. P. (2014). Double rolling isolation systems: A mathematical model and experimental validation. International Journal of Non-Linear Mechanics, 61, 80-92.

[13] Harvey, P. S., & Gavin, H. P. (2015). Assessment of a rolling isolation system using reduced order structural models. Engineering Structures, 99, 708-725.

[14] Tayaran A. (2015) Pillow-type Base Isolation System and Its Effect in Seismic Response Reduction of Buildings, M.Sc. Thesis, Islamic Azad University South of Tehran Branch, Tehran, Iran.

[15] Clough, R. W., & Penzien, J. (1993). Dynamics of structures. 1993. Copyright of Applied Mechanics & Materials.