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Buckling of Stiffened Thin-Walled Cylindrical Shells under Axial Compression with Symmetrical Imperfections

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Abstract

This study aimed to investigate the effects of stiffeners on buckling of thin cylindrical shells under uniform axial compression. To this end, more than 300 finite element models of stiffened cylindrical shells were prepared. The variables considered are shell thickness, number, dimension and the location of the vertical and horizontal stiffeners as well as circular symmetrical imperfections. Results show that the stiffeners can increase buckling of the stiffened cylindrical shells under axial compression. It is also shown that buckling of the cylindrical shells is susceptible to some circular imperfection patterns. In this context, buckling graph of the models are compared with each other; obviously, the stiffened shells with more stiffeners have upper buckling graph in force - displacement curves.

Key words:Cylindrical shell, Thin-walled, Stiffener, Buckling, Axial compression

1. Introduction

Cylindrical thin-walled shells are common components of industries, especially in pressure vessels, liquid storage tanks and silos. During the past decades, several cylindrical shells were damaged due to the extreme loads such as tornados, explosions and earthquakes.

Performance of shell structures during pas events showed that shell buckling is the most common failure mode of thin-walled cylindrical shells. Although there is not a unique borderline between thin and thick cylindrical shells, thin shells usually have the radius to thickness ratio of 100 to 2000 [1]. Shell Buckling depends on several parameters such as geometric specifications, loading condition and imperfections [2]. Shariati et al. [3] presented experimental and numerical investigations on the effects of length, sector angle and boundary conditions on the buckling and postbucklingbehavior of cylindrical shells. In order to increase the shell strength one can use horizontal and vertical stiffeners that can be used alone or by a combination of both. Generally stiffener sections are in T, L and Z Figures. Stiffened cylindrical shells subjected to axial compression can fail in one of three modes: local shell buckling, global instability or local stiffener buckling [4]. When the stiffeners are more widely spaced, local shell instability will proceed global collapse in axially compressed cylindrical shells. Stringers, the longitudinal stiffeners, are most effective in increasing the axial bending strength of cylindrical shells and their optimization has resulted in closely spaced stiffening, which generates general instability failure [5]. But rings, latitudinal stiffeners, can show more influence on shells strength against internal pressures. More detailed experimental and theoretical studies have been conducted by Singer et al. [6-13] for the buckling of stringer stiffened cylindrical shells including effects such as load eccentricity, stiffener eccentricity and geometry, boundary conditions, prebuckling deformations and geometric imperfections. In 1993, Shen, et al. [14] investigated the buckling and postbucklingbehavior of perfect and imperfect, stringer and ring stiffened cylindrical shells of finite length subject to combined loading of external pressure and axial compression. In 2003, Park and Park [15] developed an efficient nonlinear finite element method that covers both initial deformations and initial stresses of general distribution in calculating the ultimate strength of ring-stiffened cylinders. Also early works were done by KrasovskyandKostyrko (2007) [16]. They tested the buckling capacity of inside and outside stiffened shells at simply supported and clamped edges.

This project aimed to access the effects of the stiffeners on buckling of the cylindrical shells under uniform axial compression and also the circular symmetrical imperfections influences are checked.

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2. Modeling Assumptions

In this context, stiffened shells have been modeled by ANSYS and have been analyzed under axial compression. Here is the some of the assumptions:

1.Nonlinear Static analysis with large displacements and clamped edges are considered here.

2.Yieldstress and elastic modules are respectively assumed to be 240 MPa and E=2.1*10^{11} $(\frac{N}{m^2})$.

3.Nonlinear slope are considered as 2% of the linear slope of the steel stress-strain graph.

4.Four node Shell181 elements (Fig. 1) with three degrees of freedom at each node were used to model stiffened cylindrical shells. The elements were capable of considering material nonlinearity and large deformations. A bilinear elastic-plastic model was considered for modeling material properties of cylindrical shell.

3. Numerical analysis

Buckling of ring-stiffened and stringer-stiffened cylindrical shells by moving the stiffeners.In this part, two cylindrical shells with radius of 2.5 & 5m and the height of 10m were modeled. For each of them ring stiffeners and stringer stiffeners with the 10 cm width in the distance of 0.5, 1 & 2 meter were considered. The axial buckling loads of the stiffened cylindrical shells with different thicknesses are presented in Figs. s2 and 3

s: Stiffeners width(cm)

l: Distance of the stiffeners(m)

r: Shell radius(m)

t: Shell thickness(m)

T: Loading steps

Pcr: Axial critical buckling load(Newton)

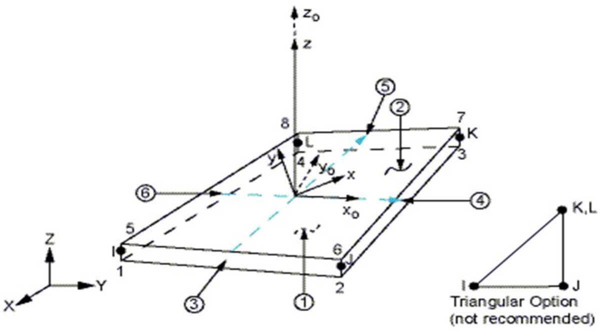


Fig.1. Specification of shell elements

Table 1. Axial buckling loads of the ring- stiffened cylindrical shells under axial compression(r=2.5&5 m)

Buckling of Ring-Stiffened shells(10^6 N)											
Ring-Stiffened shell (r=2.5)						Ring-Stiffened shell (r=5)					
r	t	r/t	Pcr(l=2,s=10)	Pcr(l=1,s=10)	Pcr(l=0.5,s=10)	r	t	r/t	Pcr(l=2,s=10)	Pcr(l=1,s=10)	Pcr(l=0.5,s=10)
2.5	0.03	83	138.45632	132.67104	140.05216	5.0	0.03	167	254.92416	256.71456	261.26656
2.5	0.025	100	111.14608	110.67008	114.79472	5.0	0.025	200	208.32928	209.64192	213.288
2.5	0.022	114	96.26528	92.4872	99.96016	5.0	0.022	227	179.33504	181.4688	184.94016
2.5	0.02	125	86.09536	86.34288	90.41632	5.0	0.02	250	165.1453	166.0166	170.6733
2.5	0.018	139	78.13408	79.3224	82.18896	5.0	0.018	278	134.6	135.13056	148.11712
2.5	0.015	167	63.9578	65.15328	68.1144	5.0	0.015	333	103.0896	113.15328	115.20928
2.5	0.012	208	49.61984	50.2672	53.46912	5.0	0.012	417	94.34016	94.28224	100.86144
2.5	0.01	250	41.21008	41.81536	44.45952	5.0	0.01	500	77.16096	76.77856	82.79872
2.5	0.008	313	32.93232	34.11248	36.41168	5.0	0.008	625	54.34976	53.79264	64.44096
2.5	0.006	417	25.5648	25.28384	27.40784	5.0	0.006	833	25.732544	39.36	46.13952

Table 2. Axial buckling loads of the stringer- stiffened cylindrical shells under axial compression(r=2.5&5 m)

Buckling of Stringer-Stiffened shells(10^6 N)											
Stringer-Stiffened shell (r=2.5)						Stringer-Stiffened shell (r=5.0)					
r	t	r/t	Pcr(l=2,s=10)	Pcr(l=1,s=10)	Pcr(l=0.5,s=10)	r	t	r/t	Pcr(l=2,s=10)	Pcr(l=1,s=10)	Pcr(l=0.5,s=10)
2.5	0.03	83	134.71912	143.26464	156.17568	5.0	0.03	167	261.78064	273.95392	297.48032
2.5	0.025	100	111.60576	117.42624	128.06048	5.0	0.025	200	213.28352	223.30144	243.21664
2.5	0.022	114	98.58696	101.85088	111.50528	5.0	0.022	227	181.48648	193.65728	212.4
2.5	0.02	125	86.73384	92.33552	100.50848	5.0	0.02	250	167.33632	178.24928	191.1616
2.5	0.018	139	77.84096	82.47584	89.87776	5.0	0.018	278	146.15904	158.30496	170.01472
2.5	0.015	167	63.25688	67.32928	74.03424	5.0	0.015	333	126.16032	133.23904	147.80544
2.5	0.012	208	50.23928	53.4672	58.53952	5.0	0.012	417	97.58656	103.63552	113.1104
2.5	0.01	250	41.39	44.24832	48.35936	5.0	0.01	500	83.23184	85.64288	93.34336
2.5	0.008	313	33.743616	35.27536	38.66592	5.0	0.008	625	54.486576	67.9296	72.60352
2.5	0.006	417	23.102936	26.151584	28.9976	5.0	0.006	833	39.789312	50.490336	56.781888

Table 1 shows that when the distance of the ring stiffeners reduces, the shell buckling strength increases but it doesn't have much effect unless the stiffeners are too close to each other. But in stringer-stiffened shells the more stiffeners used, the more buckling strength is expected because the section area increases(see Table 2). Also buckling figures are mentioned which indicate that number of the stiffeners doesn’t have any effects on linear and nonlinear buckling slopes(see Figs. 2 and 3).

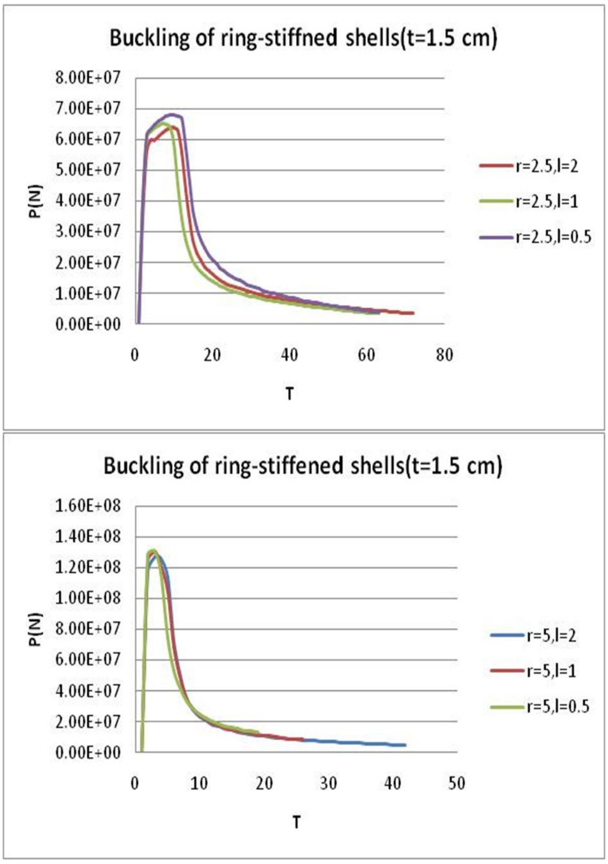


Fig. 2. Buckling of the ring-stiffened cylindrical shells by changing the number of stiffeners(t= 1.5 cm)

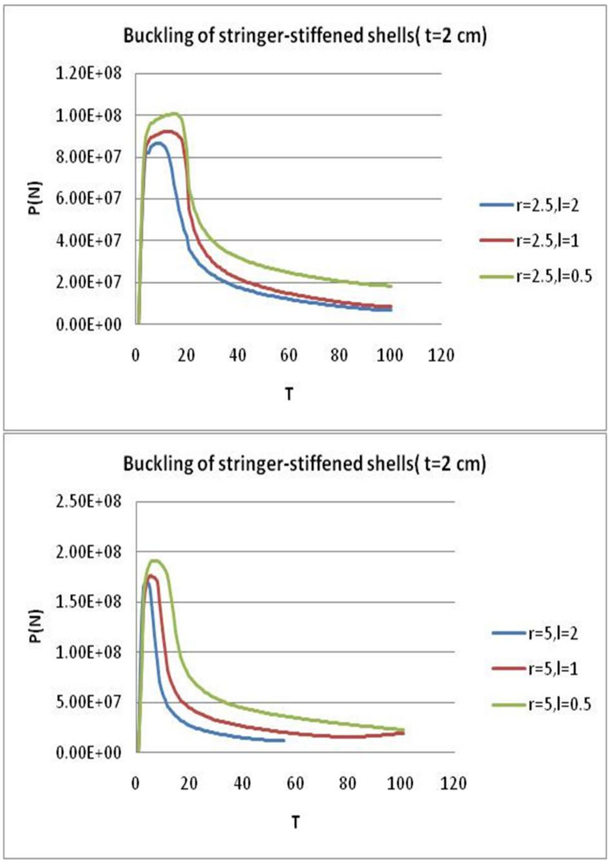


Fig. 3. Buckling of the Stringer-stiffened cylindrical shells by changing the number of stiffeners(t= 2.0 cm)

3.1. Shell buckling by changing the stiffeners width

In order to investigate the effects of the stiffeners width on the buckling of the shells, a cylindrical shell with radius of 2.5 and the height of 10m and thickness 2cm was modeled. The stiffeners with the distance of 0.5 m were considered. The axial buckling loads of the ring and stringer stiffened cylindrical shells with different width are presented in Fig. 4.

Fig. 4 shows that increasing stiffeners' width doesn't have enough effect on the buckling strength of the ring-stiffened shells duo to uniform axial compression, So that the width of 5 cm to 20 cm only 8 percent of buckling resistance increased. On the other hand, stiffeners width doesn't have any effects on linear and nonlinear buckling slope (see Fig. 5).

In stringer-stiffened shells, the stiffeners' width addition increases the axial load buckling strength (see Fig. 4). As the stiffeners' width increases, more destruction appears in stringer-stiffened shells because the postbuckling zone slope becomes less. As a result more instability is observed(see Figs. 6 and 7).

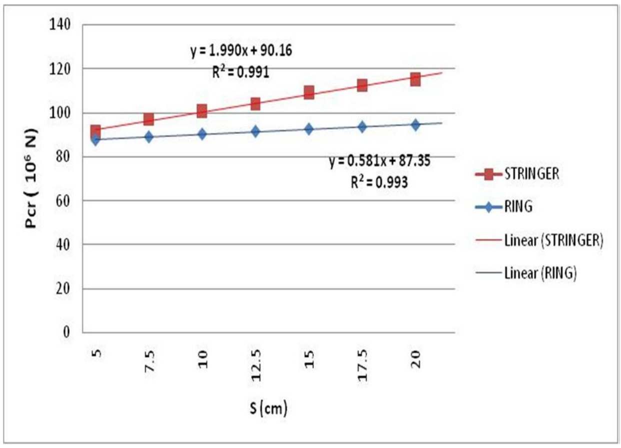


Fig. 4. Buckling loads of the ring- stiffened and stringer-stiffened cylindrical shells with different width

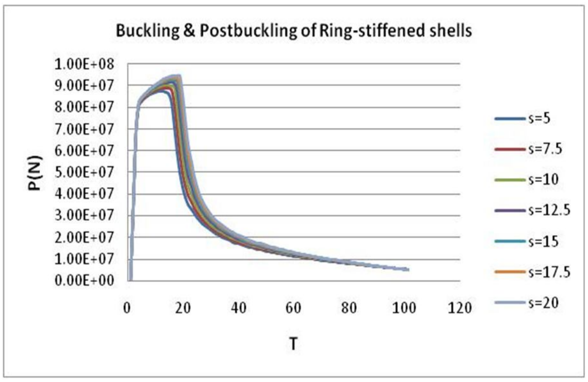


Fig. 5. Buckling and postbuckling of the ring- stiffened cylindrical shells with different width

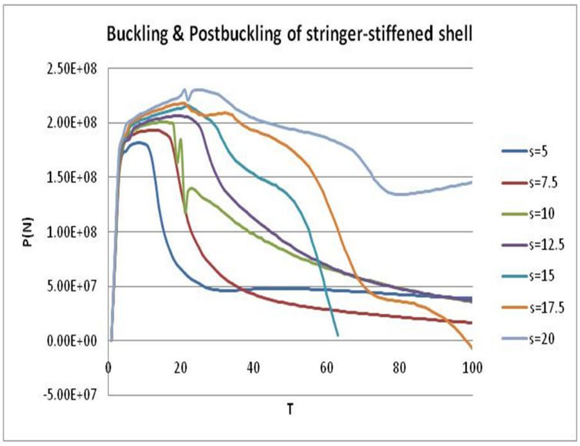


Fig. 6. Buckling and postbuckling of the stringer- stiffened cylindrical shells with different width

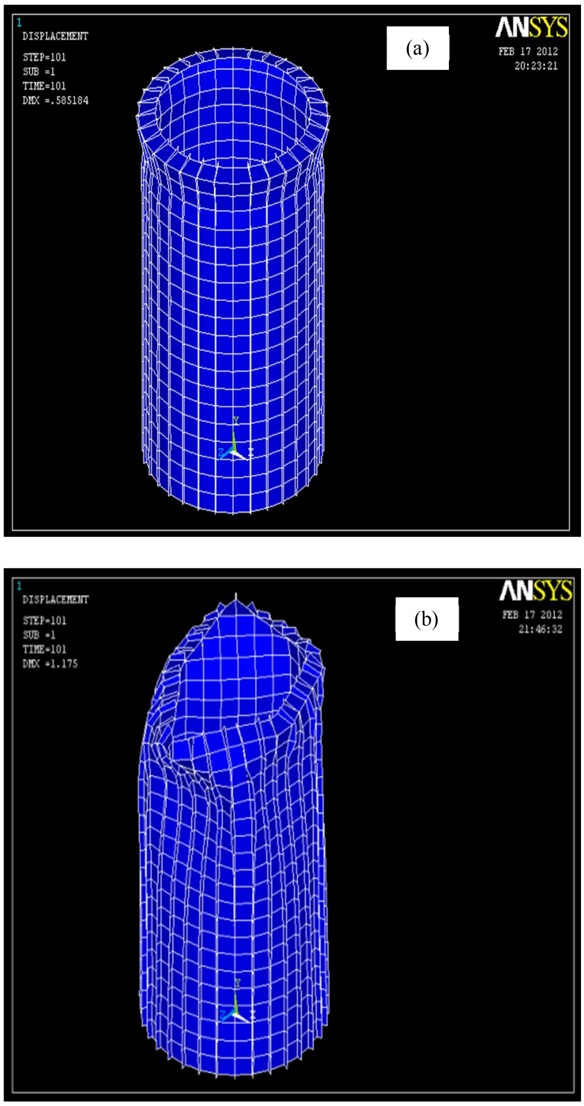


Fig. 7. (a) Buckling of the stringer- stiffened cylindrical shell with 5 cm width. (b) Buckling of the stringer- stiffened cylindrical shell with 15 cm width.

3.2. Buckling of internal ring-stiffened and stringer-stiffened cylindrical shells

In order to compare the internal and external ring-stiffened and stringer-stiffened cylindrical shells, two cylindrical shells with radius of 2.5 & 5m and the height of 10m were modeled. For each of them, stiffeners with the 10 cm width in the distance of 1 meter were considered. The axial buckling loads of internal and external stiffened cylindrical shells with different thicknesses are presented in Table 3. It indicates that using ring-stiffeners inside or outside the cylindrical shells doesn't change the axial buckling load. But in stringer-stiffened cylindrical shells using internal stiffeners have more buckling strength under axial compression against outside ones (see Table 4).

Table 3. Buckling load of internal and external ring-stiffened cylindrical shells under axial compression(r=2.5&5 m)

Buckling of Ring-Stiffened shells(10^6 N)									
r	t	r/t	inside	outside	r	t	r/t	inside	outside
			Pcr(l=1,s=10)	Pcr(l=1,s=10)				Pcr(l=1,s=10)	Pcr(l=1,s=10)
2.5	0.03	83	133.0696	132.67104	5.0	0.03	167	257.1869	256.7146
2.5	0.025	100	110.88608	110.67008	5.0	0.03	200	209.7222	209.6419
2.5	0.022	114	95.38768	92.4872	5.0	0.02	227	181.5302	181.4688
2.5	0.02	125	86.37504	86.34288	5.0	0.02	250	167.4589	166.0166
2.5	0.018	139	78.36768	79.3224	5.0	0.02	278	134.8426	135.1306
2.5	0.015	167	63.43312	65.15328	5.0	0.02	333	111.4304	113.1533
2.5	0.012	208	49.91184	50.2672	5.0	0.01	417	93.56192	94.28224
2.5	0.01	250	41.70736	41.81536	5.0	0.01	500	76.94336	76.77856
2.5	0.008	313	33.95696	34.11248	5.0	0.01	625	54.10336	53.79264
2.5	0.006	417	25.38384	25.28384	5.0	0.01	833	39.72288	39.36

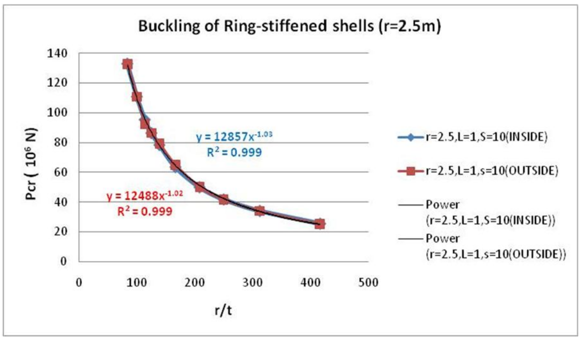


Fig. 8. Axial buckling loads of the internal and external ring- stiffened cylindrical shells under axial compression (r=2.5m)

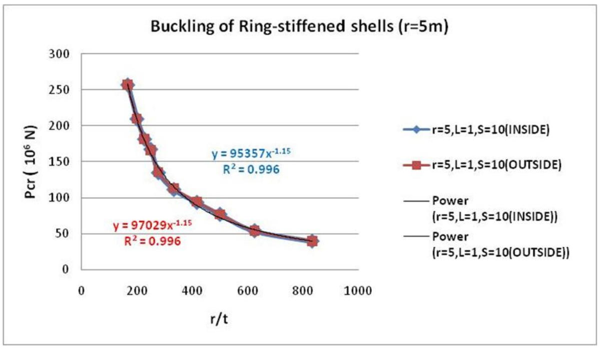


Fig. 9. Axial buckling loads of the internal and external ring- stiffened cylindrical shells under axial compression (r=5.0m)

Table 4. Buckling load of internal and external stringer-stiffened cylindrical shells under axial compression(r=2.5&5 m)

Buckling of Stringer-Stiffened shells (10^6 N)									
r	t	r/t	Inside	Outside	r	t	r/t	Inside	Outside
			Pcr(l=1,s=10)	Pcr(l=1,s=10)				Pcr(l=1,s=10)	Pcr(l=1,s=10)
2.5	0.03	83	148.45648	143.26464	5.0	0.03	167	276.05056	273.95392
2.5	0.025	100	122.17504	117.42624	5.0	0.025	200	228.63584	223.30144
2.5	0.022	114	107.8776	101.85088	5.0	0.022	227	204.537056	193.65728
2.5	0.02	125	96.948	92.33552	5.0	0.02	250	185.224	178.24928
2.5	0.018	139	86.53696	82.47584	5.0	0.018	278	168.28544	158.30496
2.5	0.015	167	72.44368	67.32928	5.0	0.015	333	136.81408	133.23904
2.5	0.012	208	57.94976	53.4672	5.0	0.012	417	105.93856	103.63552
2.5	0.01	250	48.07888	44.24832	5.0	0.01	500	93.13376	85.64288
2.5	0.008	313	38.42128	35.27536	5.0	0.008	625	73.79616	67.9296
2.5	0.006	417	28.438032	26.151584	5.0	0.006	833	55.516224	50.490336

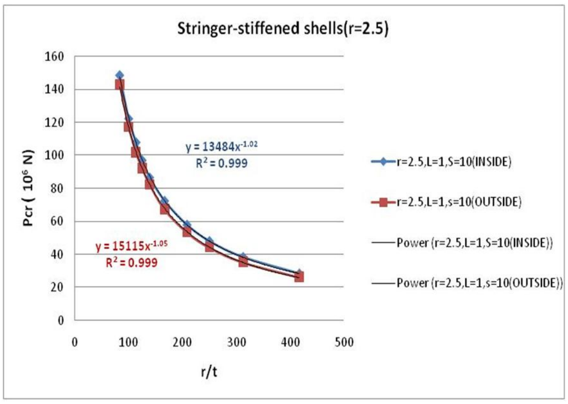


Fig. 10. Axial buckling loads of the internal and external stringer-stiffened cylindrical shells under axial compression (r=2.5m)

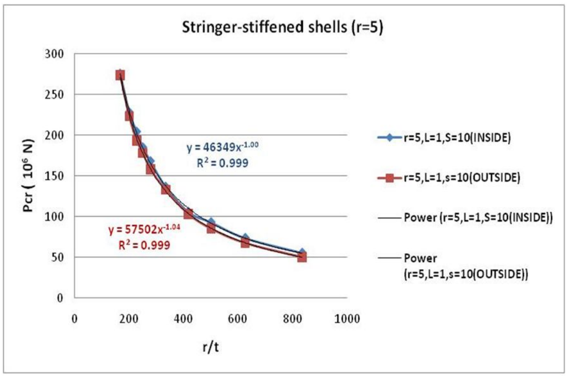


Fig. 11. Axial buckling loads of the internal and external stringer-stiffened cylindrical shells under axial compression (r=5m)

2.5, 5, 7.5 m and the height of 10m and thickness 2cm were modeled. For each of them, stiffeners with the 10 cm width in the distance of 2 meter were considered. Circular symmetrical imperfections at the height of 1, 3, 5, 7 and 9 m with different amplitude are applied. Axial buckling load of models are presented in Table 5 and Fig. 12. It is obvious that imperfections can reduce the buckling load of ring-stiffened cylindrical shells and also as the radius of the shells increases, Circular symmetrical imperfections have more effect on buckling strength. Herein W denotes the imperfection amplitude (m).

Table 5.Buckling load of imperfect ring-stiffened cylindrical shells under axial compression

L=2 ,s=10					
imprefect Ring-stiffened shell (imperfection between stiffners)					
t	w	w/t	r=2.5 m	r=5 m	r=7.5 m
			Pcr(10^6 N)	Pcr(10^6 N)	Pcr(10^6 N)
0.02	0.025	1.25	80.58096	119.31232	151.32
0.02	0.022	1.1	80.75536	104.344	132.972
0.02	0.02	1	81.01248	110.60192	80.14368
0.02	0.018	0.9	81.28112	76.06208	145.85424
0.02	0.015	0.75	81.7712	165.71552	160.884
0.02	0.012	0.6	82.4	165.88704	165.98448
0.02	0.01	0.5	82.82304	71.80128	194.02272
0.02	0.008	0.4	83.27776	157.77152	180.94128
0.02	0.006	0.3	83.8288	159.08384	241.008
0.02	0	0	86.09536	165.1453	241.35552

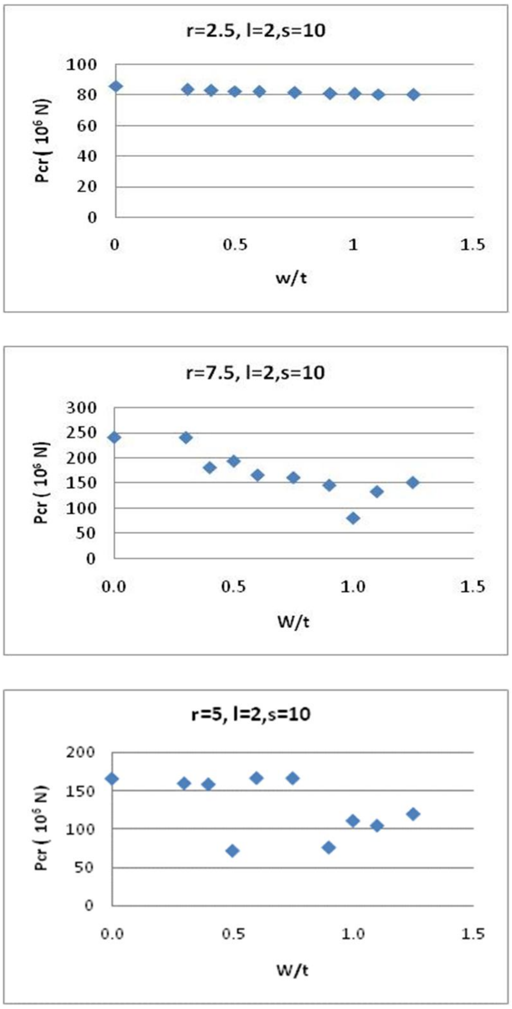


Fig. 12. Buckling load of the ring-stiffened cylindrical shells with symmetrical imperfections between stiffeners

Table 6. Buckling load of imperfect stringer-stiffened cylindrical shells under axial compression

L=1,s=10				
imperfect Stringer-stiffened shell (imperfection between stiffeners)				
t	w	w/t	r=2.5	r=5.0
			Pcr(10^6 N)	Pcr(10^6 N)
0.02	0.025	1.25	93.51296	175.6512
0.02	0.022	1.1	93.31536	175.0531
0.02	0.02	1	93.20064	179.1891
0.02	0.018	0.9	91.13728	178.609
0.02	0.015	0.75	93.05778	178.7491
0.02	0.012	0.6	92.77264	176.4141
0.02	0.01	0.5	92.30704	172.4477
0.02	0.008	0.4	92.49936	177.8227
0.02	0.006	0.3	92.4264	171.688
0.02	0	0	92.335	178.2393

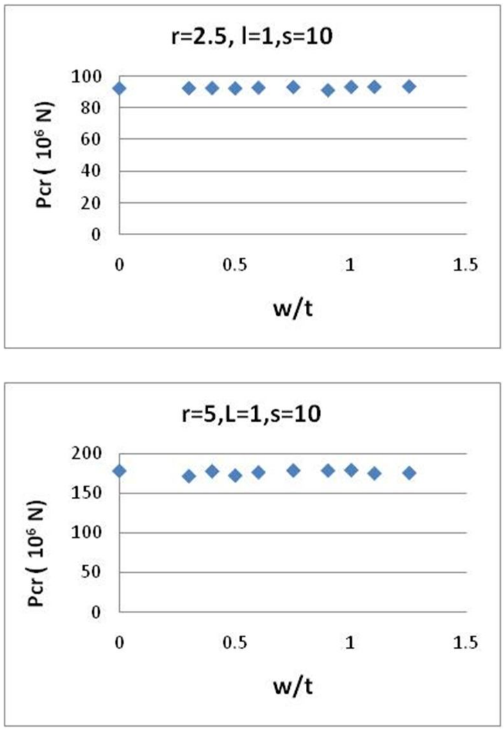


Fig. 13. Buckling load of the stringer-stiffened cylindrical shells with symmetrical imperfections between stiffeners

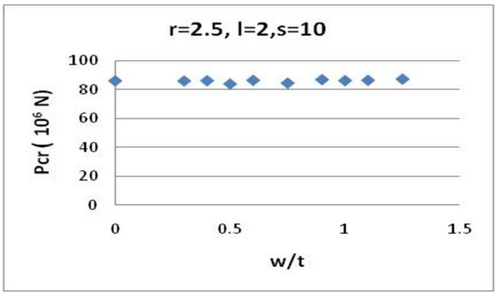
To investigate the effects of symmetrical imperfections between stiffeners on the buckling of the stringer-stiffened shells, two cylindrical shell with radius of 2.5, 5 m and the height of 10m and thickness 2cm were modeled. For each of them, stiffeners with the 10 cm width in the distance of 1 meter were considered. Symmetrical imperfections between stiffeners with different amplitude are applied. Buckling loads are presented in Table 6 and Fig. 13. Results shows that symmetrical imperfections between stiffeners doesn’t have much effect on buckling load of the stringer-stiffened cylindrical shells

3.3.2. Imperfection between stiffeners and shell

To study the effects of symmetrical imperfections between stiffeners and shell on the buckling of the ring-stiffened shells, two cylindrical shells with radius of 2.5, 5 m and the height of 10m and thickness 2cm were modeled. For each of them, stiffeners with the 10 cm width in the distance of 1 and 2 meter were considered. Symmetrical imperfections between stiffeners and shell with different amplitude are applied. Buckling load of the models are presented in Tables7 and 8. Results indicates that as the radius of the shells increases, symmetrical imperfections have more effect on buckling strength(see Fig. 14) and also the more stiffeners are used, the less sensitivity observed in buckling load of shells despite of imperfections are increased(see Fig. 15).

Table 7. Buckling load of imperfect ring-stiffened cylindrical shells under axial compression (L=2 m)

L=2 ,s=10				
imperfect Ring-stiffened shell (imperfection between stiffeners and shell)				
t	w	w/t	r=2.5 m	r=5 m
			Pcr(10^6 N)	Pcr(10^6 N)
0.02	0.025	1.25	87.25408	118.4326
0.02	0.022	1.1	86.43296	74.06336
0.02	0.02	1	86.23728	158.5053
0.02	0.018	0.9	86.91424	144.2413
0.02	0.015	0.75	84.53248	66.70208
0.02	0.012	0.6	86.43776	169.1619
0.02	0.01	0.5	83.924	64.66112
0.02	0.008	0.4	86.16688	141.5728
0.02	0.006	0.3	86.02016	65.17376
0.02	0	0	86.09536	165.1453



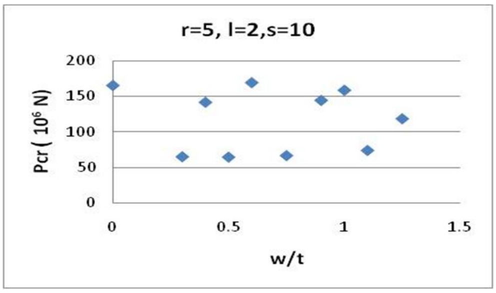
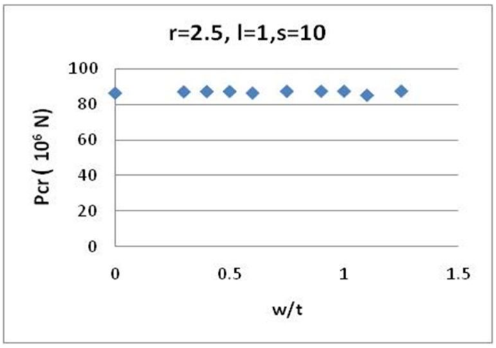


Fig. 14. Buckling load of the ring-stiffened cylindrical shells with symmetrical imperfections between stiffeners and shell

To study the effects of symmetrical imperfections between stiffeners and shell on the buckling of the stringer-stiffened shells, two cylindrical shell with radius of 2.5, 5 m and the height of 10m and thickness 2cm were modeled. For each of them, stiffeners with the 10 cm width in the distance of 2 meter were considered. Circular symmetrical imperfections between stiffeners and shell at the height of 1, 3, 5, 7 and 9 m with different amplitude are applied. Buckling load of the models is presented in Table 9. Results show that imperfections between stiffeners and shell can reduce the buckling load of stringer-stiffened cylindrical shells and also as the radius of the shells increases, symmetrical imperfections have more effect on buckling strength(see Fig. 16).

Table 8. Buckling load of imperfect ring-stiffened cylindrical shells under axial compression (L=1 m)

L=1 ,s=10				
imperfect Ring-stiffened shell (imperfection between shell and stiffeners)				
t	w	w/t	r=2.5 m	r=5 m
			Pcr(10^6 N)	Pcr(10^6 N)
0.02	0.025	1.25	87.45888	175.0345
0.02	0.022	1.1	85.16496	74.07616
0.02	0.02	1	87.38496	71.71328
0.02	0.018	0.9	87.37488	175.1715
0.02	0.015	0.75	87.35216	174.5091
0.02	0.012	0.6	86.38608	173.3654
0.02	0.01	0.5	87.26144	132.304
0.02	0.008	0.4	87.18832	130.7414
0.02	0.006	0.3	87.08592	167.3037
0.02	0	0	86.34288	166.0166



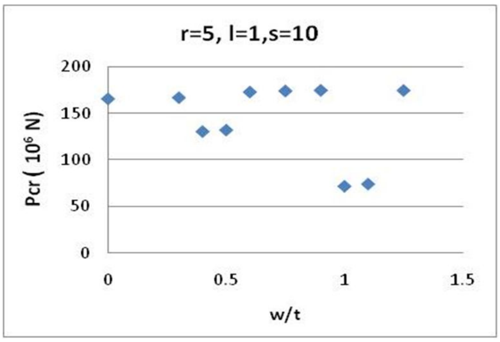


Fig. 15. Buckling load of the ring-stiffened cylindrical shells with symmetrical imperfections between stiffeners and shell

4. Conclusion

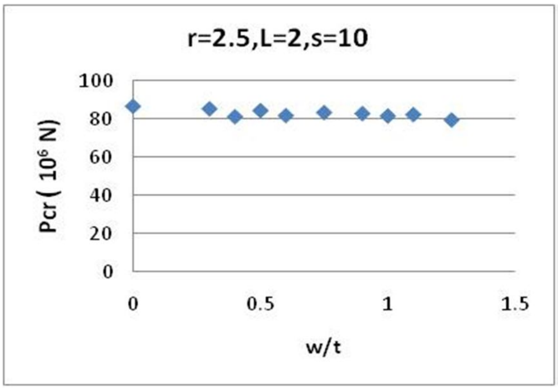
Nonlinear static analyses were carried out to investigate the effects of ring and stringer stiffeners with symmetrical imperfections on axial buckling of cylindrical shells. Results show that:

When the distance of the ring stiffeners reduces, the shell buckling strength increases but it doesn't have much effect unless the stiffeners are too close to each other. However, in stringer-stiffened shells the more stiffeners is used, the more buckling strength is observed.

Increasing stiffeners' width doesn't have enough effect on the buckling strength of the ring-stiffened shells duo to uniform axial compression but in stringer-stiffened shells, the stiffeners' width addition increases the axial load buckling strength.

Table 9. Buckling load of imperfect stringer-stiffened cylindrical shells under axial compression

L=2 ,s=10				
Imperfect Stringer-stiffened shell (imperfection in 1,3,5,7,9m)				
t	w	w/t	r=2.5	r=5.0
			Pcr(10^6 N)	Pcr(10^6 N)
0.02	0.025	1.25	79.70416	156.0523
0.02	0.022	1.1	82.46176	139.9853
0.02	0.02	1	81.77848	152.1182
0.02	0.018	0.9	83.00688	140.4074
0.02	0.015	0.75	83.5296	147.8731
0.02	0.012	0.6	81.98632	130.6566
0.02	0.01	0.5	84.5136	167.2094
0.02	0.008	0.4	81.37184	166.824
0.02	0.006	0.3	85.436	161.8142
0.02	0	0	86.73384	167.3363



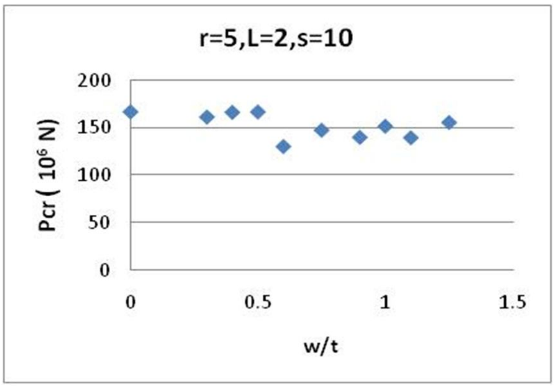


Fig. 16. Buckling load of the stringer-stiffened cylindrical shells with symmetrical imperfections between stiffeners and shell

Using ring-stiffeners inside or outside the cylindrical shells doesn't change the axial buckling load. But in stringer-stiffened cylindrical shells using internal stiffeners have more buckling strength under axial compression against outside ones.

Imperfections between stiffeners can reduce the buckling load of ring-stiffened cylindrical shells as the radius of the shells increases. Also symmetrical imperfections between stiffeners do not have much effect on buckling load of the stringer-stiffened cylindrical shells.

As the radius of the shells increases, symmetrical imperfections between ring stiffeners and shell have more effects on buckling strength but they reduce when the more stiffeners are used. In stringer-stiffened shells, imperfections between stiffeners and shells are effective when the shell radius increases.

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