New Approach for Approximation of Dispersivity in Porous Media

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Abstract

This paper presents a simple yet powerful original approach for approximating dispersivity in porous media. The key to this approach is both use of least-squares criterion and image processing technique. Two dimensional transparent models, including horizontal and vertical configurations, under different combinations of grain sizes and pore velocities, experimented. Concentration data were obtained on a 0.5×0.5 mm grid, which represents approximately 1.8×10^5 measuring points. Estimates of the longitudinal and transverse dispersivity are obtained for each experiment in three time step, and discussed. Acquired results revealed that presented procedure successfully determine dispersivity coefficients and the maximum error of estimation is 4 to 7%. Furthermore, calculated disperssivity varied from 0.001 to 0.003 m corresponding to grain sizes that agree with literature.

Keywords: dispersity; dispersion; porous media; least-squares criterion; image processing.

1. Introduction

The problem of solute dispersion during underground water movement has attracted interest from the early of this century. The intensive size of natural resources and the large production of wastes in modern society pose a threat to the groundwater quality. In order to understand the behavior of a contaminant in groundwater resources, advection and dispersion as well as adsorption are some of the key factors and play an important role in spreading of a contaminant. Numerous theoretical and experimental studies have been carried out to investigate the mechanisms of solute movement in porous media considering Krupp and Elrick, 1968; Huang et al., 1995, Ersahin et al., 2002; K. Inoue et al. 2009; Aysegul Ozgenc Aksoy, M. S. Guney 2010 [1-5]. These studies indicated that local transverse dispersion is a key factor in the smoothing of concentration fluctuations and controlling the rate of dilution of conservative solutes. In heterogeneous media, local transverse dispersion transfers longitudinal spreading of solute plumes to effective mixing, which is particularly important for transport of compounds that react upon mixing. For equilibrium reactions and instantaneous irreversible reactions, it can be shown that the length of such plumes is inversely

proportional to the transverse dispersivity. Transverse dispersion also controls the dissolution of NAPL pools and the mass transfer of volatile compounds through the capillary fringe [6, 7].

Considering experimental demands and other limitations, such as slow and costly measurements methods, more researchers have been searching for non-destructive and non-intrusive techniques that image processing is one of these. Recently, image analysis techniques have been attended to quantify the behavior of solute. Application of image processing has demonstrated that a time series of digitized images, reflecting the movement of solute, could be used to monitor solute transport in porous media as well as to estimate transport parameters such as dispersion coefficient, dispersivity and adsorption factors. Image analysis offers a non-intrusive approach to characterize solute transport and cost reduction related to labor and sampling of other conservative methods [8].

This paper aimed to use computer vision as a non-contact measurement technique for the less error and more feasibility. Therefore, present research consists of three steps: (1) development of physical models and preparing digitized images in different conditions and times; (2) image processing of recorded pictures and provide of concentration matrixes; (3) lest square parameter fitting in

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analytical solution and point of dispersivity coefficients.

2. Theory

2. 1. Governing equations

The equation governing transport in groundwater is a statement of the law of the conservation of mass. It is assumed that the porous medium is homogenous, isotropic, and saturated; it is further assumed that the flow is steady state and that Darcy's law applies. The differential equation for simulating groundwater in two dimensions is written as [9]:

$$\frac{\partial}{\partial x}\left(T_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(T_y \frac{\partial h}{\partial y}\right) = S \frac{\partial h}{\partial t} + W \qquad (1)$$

Where T_i is transmissity in i direction (L^2/T), b aquifer thickness (L), S storage coefficient, W source or sink term (L/T) and h hydraulic head (L). The governing flow equation must be solved before the transport equation can be solved. The governing transport equation in two dimensions is written as [10]:

$$\frac{\partial}{\partial x}\left(D_x \frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D_y \frac{\partial C}{\partial y}\right) - \frac{\partial}{\partial x}(Cv_x) - \frac{\partial}{\partial y}(Cv_y) - \frac{C_o W}{nb} = \frac{\partial C}{\partial t} \qquad (2)$$

Where C is concentration of solute (M/L^3), v_x and v_y seepage velocities (L/T), C_o initial concentration (M/L^3), W source or sink term, and D_x and D_y are longitudinal and transverse dispersion coefficient (L^2/T), respectively.

Equation.2 can only be solved analytically under the most simplifying conditions where velocities, dispersion coefficients are constant, and source terms are simple functions. One of the first two dimensional analytical models was that developed by Wilson and Miller (1978). It is one of the simplest to use and can account for lateral and transverse dispersion, adsorption, and first-order decay in uniform flow field. Concentration C at any point in the x-y plane can be predicted by solving Equation.2 for an instantaneous spike source or for continuous injection. Velocity in the y direction is assumed to be zero, and the x axis is oriented in the direction of flow. Tracer is assumed to be injected uniformly throughout the vertical axis. The solution becomes [10]:

$$C(x,y,t) = \frac{m'}{4\pi n t \sqrt{D_x D_y}} exp\left[-\frac{((x-x_o)-vt)^2}{4D_x t} - \frac{(y-y_o)^2}{4D_y t} - \lambda t\right] \qquad (3)$$

Where m' is injected contaminant mass per vertical unit (M/L), t time since start of injection (T), n the porosity, v the seepage velocity, λ the decay coefficient ($1/T$). When analyzing the effect of adsorption, the retardation coefficient, R, is used to redefine v, D_x, and D_y for the sorbing materials defined as $v_R = v/R$ and $D_{xR} = D_x/R$ and $D_{yR} = D_y/R$, correspondingly [10]. When the coordinate system is aligned with the mean velocity vector, the longitudinal (D_L) and transverse (D_T) hydrodynamic dispersion coefficients are related to the pore fluid velocity through the following equations:

$$D_L = \alpha_L |V| + D'_m \qquad (4)$$
$$D_T = \alpha_T |V| + D'_m \qquad (5)$$

Where D'_m is the effective molecular diffusion coefficient of the solute in the porous medium, $|V|$ the magnitude of the velocity vector, and α_L and α_T are the longitudinal and transverse components of the dispersivity tensor,

respectively. The longitudinal and transverse dispersivity are considered to be characteristic properties of a region of a porous medium, and typically, they are conveniently treated as independent of the pore fluid velocity and Pe´clet number [11, 12]. The Pe´clet number is defined as $Pe = VL_P/D'_m$ Where V is the mean pore velocity and L_P is the pore length.

2. 2. Image processing

Image processing (IP) refers to the use of variations in brightness, or intensity, within a digital image as a means of finding features of interest. The fundamental assumption is that similar features will have similar brightness [13, 14]. A specific brightness value is thus representative of a particular feature and can be used to make quantitative observations. This procedure provides an accurate and efficient way to obtain concentration fields with good temporal and spatial details to study patterns of plume progress in porous environments. Normally, IP includes the following stage: (1) photography and preparing digitized images in a favored format; (2) constructing a standard curve that relates concentration and optical density; and (3) converting the optical density to concentration for each pixel, based on the standard curve [15].

2. 3. Least-Squares method

The goal of Least-Squares Estimation (LSE) is to find a good estimation of parameters that fit a function, *f(x)*, of a set of data, $x_1...x_n$. The LSE method consists of two variables, P and S. P is an independent variable but S depends on variable P. This process estimates the values for points A_{11} and A_{oo} for a function $L[X,Y] = A_{11}X + A_{oo}Y$, such that $L[X,Y]$ is a best-fit area. This best-fit area passes through N points (P_o, S_o) to (P_{N-1}, S_{N-1}) and minimizes the sum of squared distances between each point *(Pi, Si)*, where $i = 0, ..., N-1$. This process is performed to ensure that each point set *(Pi, Si)* is as close as possible to the best-fit area.

3. Materials and methods

As shown in Figure.1, all experiments are carried out in transparent Plexiglas models, filled with glass beads, in two configurations: **i-** vertical box with 650mm length, 130mm height and 25mm thickness; **ii-** horizontal box with 450mm length, 450mm width and 25mm thickness. Combinations of three grain size (d_{50} = 0.4, 1.8 and 3 mm, relatively uniform particle) and five seepage velocities (4, 9, 17, 31, 64 and 76 m/d) were experimented in transparent models. Details of experiments listed in table.1. In each experiment, the trace liquid was injected into the transparent porous media and the subsequent plum pattern of mixing was recorded by Kodak (using a canon Power-Shot SX120IS digital Kodak system with the resolution up to 10,000 dpi). After that, according to flowchart offered in figure.2, the image analysis process involves several steps that details can be found in Robert

A. Schincariol et al. (1993) [14]. Rhodamine used as a tracer, which is non-degrading, non-sobbing, rather inert, and non-reactive. To prevent lighting non-uniformity, the system was placed in dark-room, and at begins of each experiment, adequate distilled water was transmitted through columns to improve packing homogeneity.

The concentration maps are extremely detailed with a spatial resolution of 0.5×0.5mm over the entire area of plume. Tracer mass, M, was estimated as following:

$$M_{Tracer} = \left(\sum C_i \right) \times n \times B \times \alpha \qquad (6)$$

Where C_i is the concentration value, n materiel porosity, B the thickness of medium and α the area of each pixel. The errors of analysis (e_m) was determined by Equation.7:

$$e_m = \frac{\left| M_{Tracer} - M_{Tracer}^{real} \right|}{M_{Tracer}^{real}} \times 100\% \qquad (7)$$
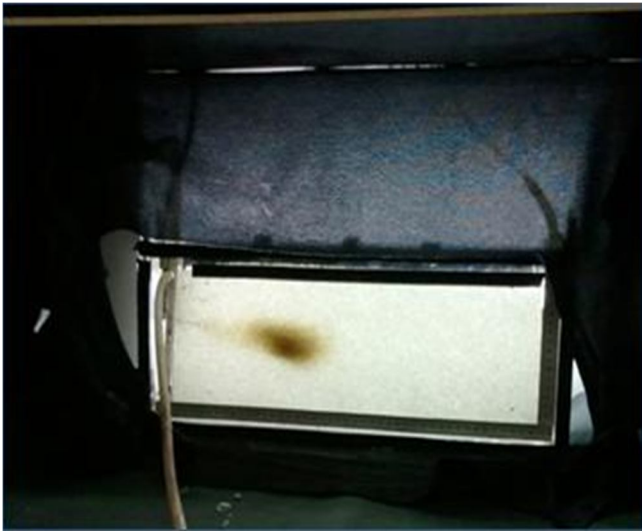
where M_{Tracer}^{real} is the injected mass of tracer.

(a)

(b)

(c)

(d)

Figure1. Pictures of physical models; (a), (b) dark-room set-up, (c) vertical box in dark-room, (d) horizontal box configuration

Table1. Experimental design and details in present study

Injection Type	Status	Grain size (mm)	Test N.O.					
			V= 4m/d	V= 9m/d	V= 17m/d	V= 31m/d	V= 64m/d	V= 76m/d
Pulls	Vertical model	0.5-0.7	1	2	3	4	5	6
		1.7 – 2.4	7	8	9	10	11	12
		2.4 – 3.4	13	14	15	16	17	18
	Horizontal model	0.5-0.7	19	20	21	-	-	-
		1.7 – 2.4	-	22	23	24	-	-
		2.4 – 3.4	-	25	26	27	-	-
Continuous	Vertical model	0.5-0.7	-	28	29	-	-	-
		1.7 – 2.4	-	30	31	-	-	-
		2.4 – 3.4	-	32	33	-	-	-
	Horizontal model	0.5-0.7	-	34	35	-	-	-
		1.7 – 2.4	-	36	37	-	-	-
		2.4 – 3.4	-	38	39	-	-	-

Inserting the images of original experiments	Inserting the background (base) image
Transform the RGB images to intensity images	Transform the RGB base image to intensity image
Image registration with standard image	Image registration with standard image
Standardize the optical intensity of image	Standardize the optical intensity of base image
Cropping the images	Cropping the base image

Subtraction of a background image

Median smoothing on a 3x3 window

Computing an optical density vs. concentration standard curve

Using a standard curve to convert optical density values to concentration

Plotting the concentration contours

Figure2. The algorithm of image processing extracted from Schincariol et al. (1993) and McNeil et al. (2006) [13, 14]

4. Results and discussion
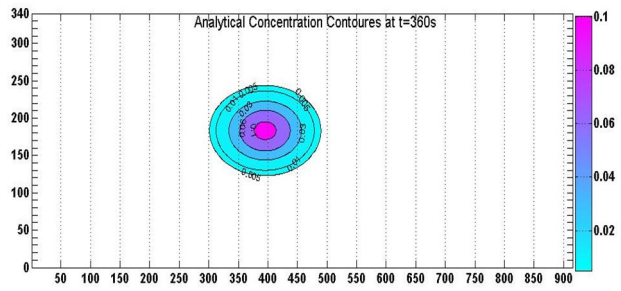
4. 1. Pulse injection

For an instantaneous spike injection, a comparison of the IP results with the exact solve from analytical solution fitted by LSE is offered in figure.3. Just as seen, the results of the IP for the plum transfer were close to those of the analytical solution. It is clear that this approach could be used to estimation of dispersivity and plum development in transparent porous media.
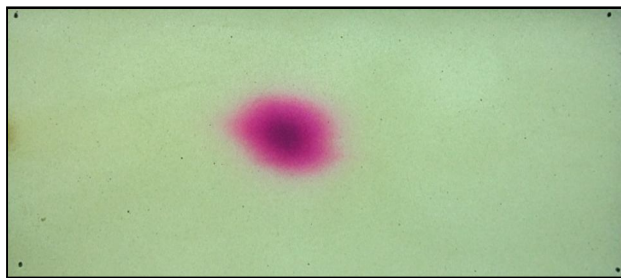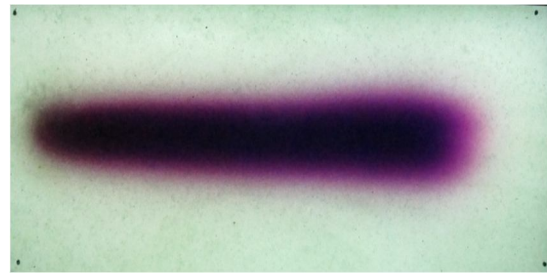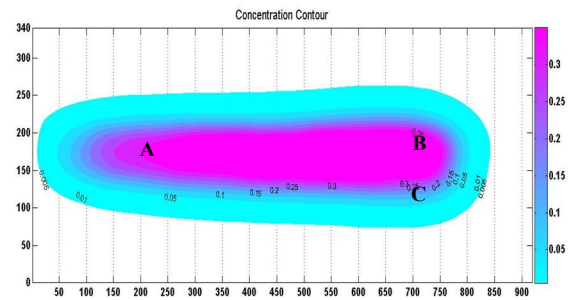

(a)
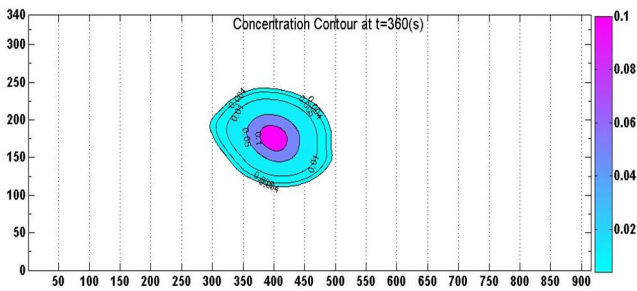
(b)

(c)

Figure3. Correlation of the IP and analytical solution for pulse injection; (a) Picture of experiment, (b) Image proccesing output, (c) Analytical solution with best fitting by LSE

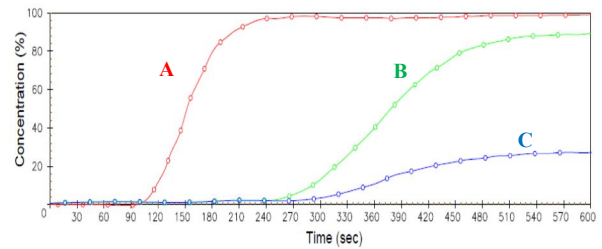### 4. 2. Continuous injection

Similar to pervious section, in figure.4, a comparison of the IP results with the analytical solution for continuous injection is presented. In analytical solution, parameters best fitted by LSE. Furthermore, the concentration variations versus time extracted in the points of A, B, and C, where time-history was plotted in figure 4.c.

(a)

(b)

(c)

Figure4. Correlation of the IP and analytical solution for continuous injection; (a) Picture of experiment, (b) Image procceing, (c) Time history of solute concentration at A, B and C

The final outputs of this section underline the potential of image processing to identify the dispersion coefficient and the dispersivity. As seen in figure.5, the close match between the observed and least square results would indicate that the test data can be closely approximated by the model. To demonstrate the efficacy of this new approach, the errors of analysis (e_m) was determined by Equation.7 that the maximum error ranged of 4 to 7%. Most of this error was due to the velocity non-uniformity, background subtraction and other correction techniques that were used to correct for lighting non-consistencies inherent in the original data.

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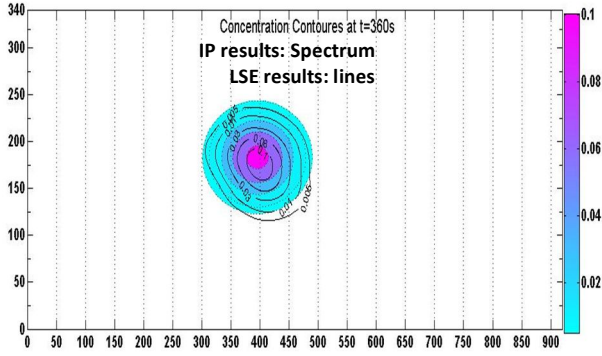
Figure5. Close match between the IP and LSE results

4. 3. Dispersion and dispersivity values

A comprehensive comparative study of the image processing model results for transverse dispersion coefficients with other models indicated that image processing could successfully predict the dispersion coefficients over a broad range of Pe´clet numbers. Analysis of the dispersion records in porous media is validated proffered approach to prediction of longitudinal and transverse dispersion coefficients. The proposed approach is significantly more practical and flexible and easy to use than previous expensive methods and they cover the entire range of groundwater circumstances. The longitudinal and transverse dispersion coefficients are estimated through an inverse analysis that involves matching the LSE analytical simulation results to the concentration matrixes of the IP. As shown in figure.6, the data are plotted as the transverse dispersion coefficient divided by the molecular diffusion coefficient (D_T/D_m) versus Pe´clet number. The model results are in good agreement with Branko Bijeljic and de Josselin studies [16].
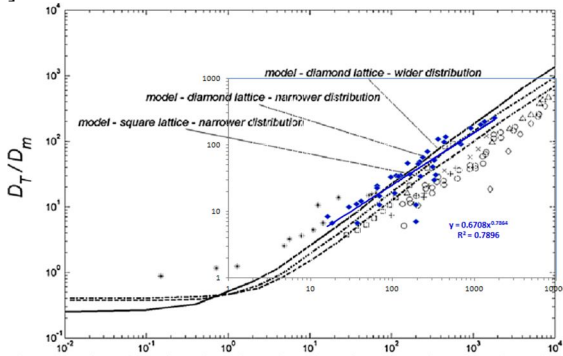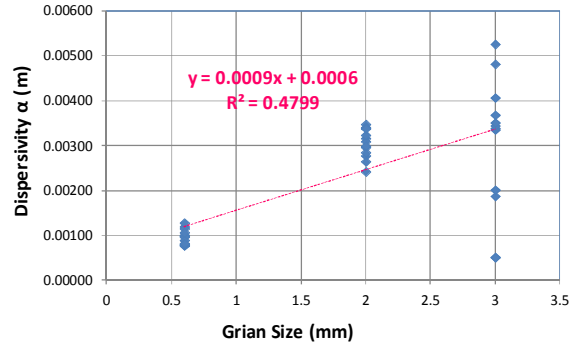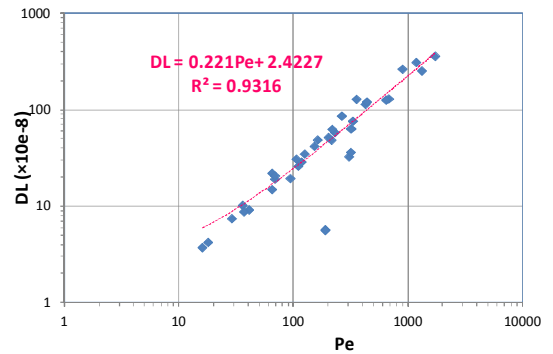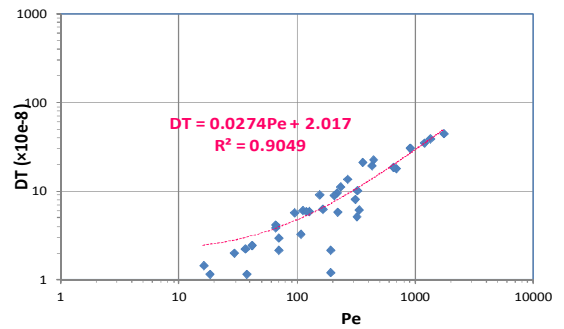

Figure6. Comparison of our findings with results reported by Pe Bijeljic, B., M. J. Blunt, 2007 [16]
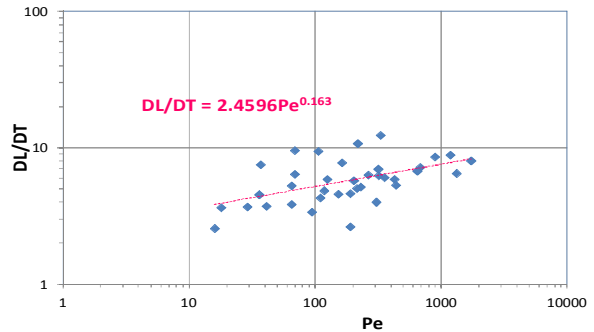

(a)

(b)

(c)

(d)

Figure7. Dispersion and dispersivity values

Furthermore, as shown in figure.7.a, dependency of the longitudinal dispersivities, α_L, on the grain size, is observed and the longitudinal dispersivity values appear within the limits of the mean grain size. It may be noting that the maximum pore velocities in these experiments were rather high, about 76m/day, which is one order of magnitude higher than typical groundwater velocities. The linear relationship between seepage velocity and transverse dispersion coefficient holds for Peˊclet numbers greater than 100. These are in the order of values reported by others [17, 18, 19].

As shown in figure.7.d, the ratio of the longitudinal to transverse dispersion coefficients (D_L/D_T) was varied with *Pe*. The ratio between the longitudinal and the transverse dispersion coefficients varies with the dispersion regime and in the advection dominated regime, it is not appropriate to take D_T to be one order of magnitude less than D_L.

Finally, as shown in figure.6 and 7, in the lower Pe regime, where only molecular diffusion affects transport, D_L was equal to D_T, but the narrower pore size distributions have a higher D_T/D_m ratio. Furthermore, the contribution of effective molecular diffusion to transverse dispersion cannot be neglected at typical ground water flow velocities less than 10m/day.

5. Conclusions

Sampling and concentration measurement of solute for the determination of plum distribution and dispersivitis is time consuming, laborious and expensive. The proposed approach provides accurate concentration distributions measuring method in porous materials with minimal equipment and computing needs. Our findings suggest the technique is non-intrusive, does not disturb plume dynamics, and provides detailed concentration data over the whole plume in any time. At begin of experiments, we did not appreciate the requirements for rigorous lighting control. When care is taken with the photographic setup to control room lighting and a broad range of concentrations are run for the optical density-concentration calibration curve, the maximum error reduced. The maximum eligible error in result of IP model is %5 for small grain size, %4 for middle grain size and %7 for large grain size. Most of this error was due to the velocity non-uniformity, porous media non-homogeneity, background subtraction and other correction techniques that were used to correct for the lighting non-uniformities inherent in the original images.

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