Journal of Optimization in Soft Computing (JOSC)
Vol. 3, Issue 3, pp: (1-7), Autumn-2025
Journal homepage: https://sanad.iau.ir/journal/josc

Research paper

Synchronization of an Extended Hyperchaotic Chen System with unknown parameters by Leveraging Adaptive Control

Hadi Soheili Rad[1] and Elahe Moradi[2*]

1. Department of Electrical Engineering, YI.C., Islamic Azad University, Tehran, Iran.
2. Department of Electrical Engineering, YI.C., Islamic Azad University, Tehran, Iran.

Article Info

Article History:
Received: 2025/07/07
Revised: 2025/09/09
Accepted: 2025/09/21

DOI:10.82553/josc.2025.202507071211340

Keywords:
Adaptive Control, Chaos, Synchronization, Hyperchaotic Systems, Extended Chen System.

**Corresponding Author's Email Address: Elahe.Moradi@iau.ac.ir*

Abstract

In this study, an extended hyperchaotic Chen system is introduced. This system exhibits a significantly higher level of dynamical complexity compared to many conventional nonlinear systems. Such complexity broadens its potential for applications in secure communication systems and contributes to enhanced protection of sensitive information in networked environments. However, the system's extreme sensitivity to parameter variations and initial conditions increases the risk of decoding errors. To address this challenge, the synchronization problem is investigated under the assumption of unknown parameters in the primary system. The presented adaptive control scheme effectively estimates these unknown parameters and ensures reliable synchronization between master and slave systems, improving the system's performance in practical scenarios. The stability of the proposed control strategy is established using Lyapunov theory, and simulation results substantiate its performance and reliability. Additionally, the hyperchaotic behavior of the proposed system is validated by calculating its Lyapunov exponents and performing dynamical analysis in MATLAB.

1. Introduction

The widespread applications of chaotic phenomena across various scientific and engineering disciplines, including secure communications, nonlinear circuits, chemical reactions, power electronics, and laser systems, have significantly increased the interest in studying their intrinsic properties and control methods. One of the main reasons for the growing attention to chaotic systems is their unique characteristics, such as high sensitivity to slight changes in initial conditions, broad frequency spectra, and the existence of strange attractors [1-3].

In recent years, extensive research has been carried out on the stability and control of chaotic systems. The stability of real-world systems, depending on their applications, is generally categorized into two types: asymptotic stability and finite-time stability [4-7]. A wide range of methods has been developed to achieve stabilization and control of chaotic and hyperchaotic dynamics in such systems [8-11].

In previous studies, chaotic systems have been broadly categorized based on their dynamical complexity into three main types: low-dimensional chaos, high-dimensional (hyperchaotic) systems, and spatiotemporal chaos. One of the simplest and widely accepted definitions found in the literature [12-14] states that "a nonlinear system possessing at least two positive Lyapunov exponents is referred to as hyperchaotic." The presence of two or more positive Lyapunov exponents is considered the most fundamental characteristic that distinguishes hyperchaotic dynamics from conventional low-dimensional chaos in nonlinear systems. The first and simplest hyperchaotic system was introduced by Rössler in 1979, which described the behavior of a chemical reaction [15]. Since then, various hyperchaotic systems have been developed and presented by researchers across different scientific disciplines [16-18].

With the increasing application of chaotic and hyperchaotic phenomena across various scientific and industrial domains, a wide range of control strategies have been developed to manage these complex dynamical behaviors. Common approaches include the backstepping method [19], linear and velocity-based feedback controls [20], sliding mode control [21, 22], H_∞ control techniques [23], adaptive observers [24], and fuzzy-adaptive control methods [25].

In [26], external disturbances and parameter uncertainties were not considered in the study of two hyperchaotic systems, and all system parameters were assumed to be known and precisely defined. Similarly, [27] employed a sliding mode control method to stabilize the hyperchaotic Rössler system under bounded disturbances with known limits and nonlinear inputs. However, the Rössler system's parameters were entirely known, and no uncertainties were incorporated into the system dynamics.

Chaotic signals can be used as carriers for message transmission. Since the carrier signal must be accurately reconstructed at the receiver to ensure proper decoding of the message, the synchronization of chaotic systems has received significant attention. Notable contributions in this area include the works of Pecora and Carroll [28]. In [29], the synchronization problem of two non-identical hyperchaotic systems, Rössler and Chen, was investigated using the sliding mode control approach under nonlinear control inputs. However, this work did not consider critical issues such as system uncertainties, external disturbances, and unknown parameters. The authors of [30] proposed a model-free deep reinforcement learning method to synchronize a target chaotic system with a reference system. Without requiring prior knowledge of system dynamics, the agent learns optimal perturbation strategies through interaction. The method, validated on several standard chaotic systems including Lorenz, Rössler, Chua, and the logistic map, demonstrates high effectiveness in synchronization control.

In this study, the synchronization problem of an extended hyperchaotic Chen system with uncertain parameters is investigated using an adaptive control approach. Unlike previous works that often assume known system parameters, this research addresses a challenging scenario where parameter values are inherently unknown. The proposed methodology develops a robust adaptive control mechanism capable of accurately estimating these unknown parameters while ensuring complete synchronization between master and slave systems. This approach fills a significant gap in the literature by providing a systematic framework to handle parameter uncertainties in complex hyperchaotic systems, thereby enhancing the applicability of chaos-based secure communication techniques.

The structure of the paper is organized as follows: Section 2 presents the system model and the proposed control framework. Section 3 provides numerical simulation results obtained using MATLAB. Finally, Section 4 concludes the study.

2. Model Description

Chaotic system synchronization has found widespread applications across various scientific and engineering domains. In general, synchronization refers to the process in which the state trajectories of two chaotic systems oscillate identically and in unison. In this framework, one system is considered as the master system, while the other serves as the slave system. The primary objective is to design an appropriate control signal that, when applied to the slave system, ensures its dynamical behavior closely matches that of the master. The state-space equations of the master system are presented in Eq. (1).

$$\begin{cases} \dot{x}_m = a(y_m - x_m) + w_m \\ \dot{y}_m = dx_m + cy_m - kx_m z_m^2 \\ \dot{z}_m = x_m y_m - bz_m \\ \dot{w}_m = y_m z_m + rw_m \end{cases} \tag{1}$$

In Eq. (1), $x_m, y_m, z_m, w_m \in \mathbb{R}$ represent the state variables of the master system, while a, b, c, d, r, k are constant, but unknown parameters of the master system. The state-space equations of the slave system are presented in Eq. (2).

$$\begin{cases} \dot{x}_s = a_1(t)(y_s - x_s) + w_s + U_1(t) \\ \dot{y}_s = d_1(t)x_s + c_1(t)y_s - k_1(t)x_s z_s^2 + U_2(t) \\ \dot{z}_s = x_s y_s - b_1(t)z_s + U_3(t) \\ \dot{w}_s = y_s z_s + r_1(t)w_s + U_4(t) \end{cases} \tag{2}$$

In Eq. (2), $x_s, y_s, z_s, w_s \in \mathbb{R}$ denote the state variables of the slave system, and $a_1(t), b_1(t), c_1(t), d_1(t), r_1(t), k_1(t)$ represent the time-varying parameters that are to be identified through the adaptive rule. Moreover, $U_1(t), U_2(t), U_3(t), U_4(t)$ are the control signals.

In this study, an indirect adaptive control approach is employed to estimate the unknown parameters of the master system and to achieve synchronization between the master and slave systems [31, 32].

Theorem 1. If the error between the state variables and parameters of the master and slave systems is defined as in Eq. (3), then the conditions necessary for designing an adaptive control law to ensure synchronization between the two systems are established.

$$\begin{cases} e_1 = x_m - x_s \\ e_2 = y_m - y_s \\ e_3 = z_m - z_s \\ e_4 = w_m - w_s \end{cases}, \begin{cases} e_a = a_1(t) - a,\ e_b = b_1(t) - b \\ e_c = c_1(t) - c,\ e_d = d_1(t) - d \\ e_k = k_1(t) - k,\ e_r = r_1(t) - r \end{cases} \quad (3)$$

By defining the adaptation laws for parameter estimation and control signals as given in Eq. (4), the synchronization error of the state variables converges to zero, and the parameter estimation errors remain bounded.

$$\begin{cases} U_1(t) = \big(K_1 - a_1(t)\big)e_1 + a_1(t)e_2 + e_4 \\ U_2(t) = d_1(t)e_1 + \big(K_2 + c_1(t)\big)e_2 + k_1(t)(x_s z_s^2 - x_m z_m^2) \\ U_3(t) = -x_s y_s + \big(K_3 - b_1(t)\big)e_3 \\ U_4(t) = -y_s z_s + (r_1(t) + K_4)e_4 \end{cases} \quad (4)$$

$$\begin{cases} \dot{a}_1(t) = e_1(y_m - x_m) \\ \dot{b}_1(t) = -z_m e_3 \\ \dot{c}_1(t) = y_m e_2 \\ \dot{d}_1(t) = x_m e_2 \\ \dot{k}_1(t) = -x_m z_m^2 e_2 \\ \dot{r}_1(t) = w_m e_4 \end{cases}$$

In Eq. (4), the constants $K_1, K_2, K_3, and\ K_4 > 0$ are assumed to be positive definite.

Proof: Based on the error definition given in Eq. (3), the error system dynamics are formulated as shown in Eq. (5).

$$\begin{cases} \dot{e}_1 = a(y_m - x_m) + w_m - a_1(t)(y_s - x_s) - w_s - U_1(t) \\ \dot{e}_2 = dx_m + cy_m - kx_m z_m^2 - d_1(t)x_s - c_1(t)y_s + k_1(t)x_s Z_s^2 - U_2(t) \\ \dot{e}_3 = x_m y_m - bz_m - x_s y_s + b_1(t)z_s - U_3(t) \\ \dot{e}_4 = y_m z_m + rw_m - y_s z_s - r_1(t)w_s - U_4(t) \end{cases} \quad (5)$$

The Lyapunov function is selected as given in Eq. (6).

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_k^2 + e_r^2) \quad (6)$$

By substituting the values from Eq. (4), the derivative of the Lyapunov function is obtained as shown in Eq. (7).

$$\dot{V} = -K_1 e_1^2 - K_2 e_2^2 - K_3 e_3^2 - K_4 e_4^2 \quad (7)$$

By selecting positive constants $K_1, K_2, K_3,$ and $K_4 > 0$, the derivative of the Lyapunov function $\dot{V}$ becomes negative semi-definite. According to Lyapunov's theorem, given that V is positive definite and $\dot{V}$ is negative semi-definite, the synchronization errors of the state variables (e_1, e_2, e_3, e_4) and the parameter estimation errors $(e_a, e_b, e_c, e_d, e_k, e_r)$ are guaranteed to be bounded.

Theorem 2: If the function $g(t)$ is uniformly continuous ($\dot{g}(t)$ exists and is bounded) and if $\lim_{t\to\infty}\int_0^t g(\tau)d\tau$ the exists, then it follows that $\lim_{t\to\infty} g(t) = 0$. By choosing $g(t) = \dot{V}$, we obtain:

$$\dot{g}(t) = \ddot{V} = -2(K_1 e_1 \dot{e}_1 + K_2 e_2 \dot{e}_2 + K_3 e_3 \dot{e}_3 + K_4 e_4 \dot{e}_4) \quad (8)$$

Based on Lyapunov's theorem, it has been demonstrated that the synchronization errors of the state variables (e_1, e_2, e_3, e_4) are bounded, and the parameters $K_1, K_2, K_3,$ and $K_4 > 0$ are chosen as positive and bounded constants. Moreover, by substituting the values from Eq. (4) into Eq. (5), the error system dynamics can be expressed as Eq. (9):

$$\begin{cases} \dot{e}_1 = (a - a_1(t))(y_m - x_m) - K_1 e_1 \\ \dot{e}_2 = \big(d - d_1(t)\big)x_m + \big(c - c_1(t)\big)y_m - \big(k - k_1(t)\big)x_m z_m^2 - K_2 e_2 \\ \dot{e}_3 = -\big(b - b_1(t)\big)z_m - K_3 e_3 \\ \dot{e}_4 = (r - r_1(t)w_m - K_4 e_4 \end{cases} \quad (9)$$

$$\begin{cases} \dot{e}_1 = e_a(y_m - x_m) - K_1 e_1 \\ \dot{e}_2 = e_d x_m + e_c y_m - e_k x_m z_m^2 - K_2 e_2 \\ \dot{e}_3 = -e_b z_m - K_3 e_3 \\ \dot{e}_4 = e_r w_m - K_4 e_4 \end{cases}$$

$$\dot{E} = AE + F, E = [e_1, e_2, e_3, e_4]^T$$

$$A = -diag(K_1, K_2, K_3, K_4),$$

$$F = \begin{bmatrix} e_a(y_m - x_m) \\ e_d x_m + e_c y_m - e_k x_m z_m^2 \\ -e_b z_m \\ e_r w_m \end{bmatrix}$$

Given that the matrix A in Eq. (9) is Hurwitz and the term F is bounded, it follows that $\dot{E}$ (the derivative vector of the state variable errors) is also bounded.

In Eq. (8), considering the existence and boundedness of all $K_i, e_i, \dot{e}_i$ for $i = 1, 2, 3, 4$, the $\dot{g}$ exists and is bounded as well. Moreover, based on Lyapunov's theorem, the boundedness of the Lyapunov function V has been established; hence:

$$\lim_{t\to\infty}\int_0^t g(\tau)d\tau = \lim_{t\to\infty}\int_0^t \dot{V}(\tau)d\tau = V(\infty) - V(0) < \infty \quad (10)$$

Since all conditions of Theorem 2 hold for the function $g(t) = \dot{V}$, it follows that:

$$\lim_{t\to\infty} g(t) = 0 \ \rightarrow\ \lim_{t\to\infty} V(\dot{t}) = 0 \ \rightarrow\ \lim_{t\to\infty} e_i(t) = 0\ , i = 1, 2, 3, 4 \quad (11)$$

Therefore, it is proven that the synchronization errors of the state variables (e_1, e_2, e_3, e_4) asymptotically approach zero, while the parameter estimation errors $(e_a, e_b, e_c, e_d, e_k, e_r)$ remain bounded.

The block diagram of this control loop is illustrated in Fig. 1.

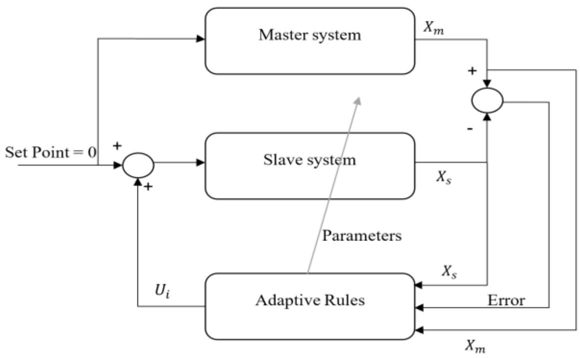

Figure 1. Identification loop of the system.

3. Simulation Result

In this section, the performance of the proposed adaptive control scheme is evaluated through numerical simulations conducted in the MATLAB environment. The simulation experiments are designed to validate the effectiveness of the control strategy introduced in Section 2, particularly in achieving synchronization between the master and slave systems and in accurately estimating the unknown system parameters.

To this end, the control signals and adaptation laws defined in Eq. (4) are applied to the slave system, whose dynamics are described in Eq. (2). The true parameter values of the master system are selected as listed in Table 1, and the corresponding initial estimates used in the adaptive mechanism are provided in Table 2.

Table 1. System parameters used in the simulation.

Parameter	Value
a	35
b	3
c	12
d	7
r	0.3
k	0.01

Table 2. Initial estimated values of the system parameters.

Parameter	Value
$a_1(0)$	30
$b_1(0)$	4
$c_1(0)$	11.5
$d_1(0)$	7.5
$r_1(0)$	0.9
$k_1(0)$	0.1

Moreover, the initial conditions for both the master and the slave systems are specified in Tables 3 and 4, respectively, to ensure a consistent simulation setup.

Table 3. Initial conditions of the master system.

Parameter	Value
$x_m(0)$	-0.1
$y_m(0)$	0.2
$z_m(0)$	-0.6
$w_m(0)$	0.4

Table 4. Initial conditions of the slave system.

Parameter	Value
$x_s(0)$	1
$y_s(0)$	0.5
$z_s(0)$	1
$w_s(0)$	-1

The adaptation gain constants are chosen as follows: $K_1 = K_3 = K_4 = 10, K_2 = 80$. These values are selected to guarantee both fast convergence of synchronization errors and robustness against parameter uncertainties. The simulation results, including time responses of the system states, synchronization errors, and parameter estimates, are graphically illustrated in Figs. 2 to 5. These results demonstrate the dynamic behavior of both the master and slave systems over time, highlighting the convergence of synchronization errors toward zero, which confirms the effectiveness of the proposed adaptive control scheme.

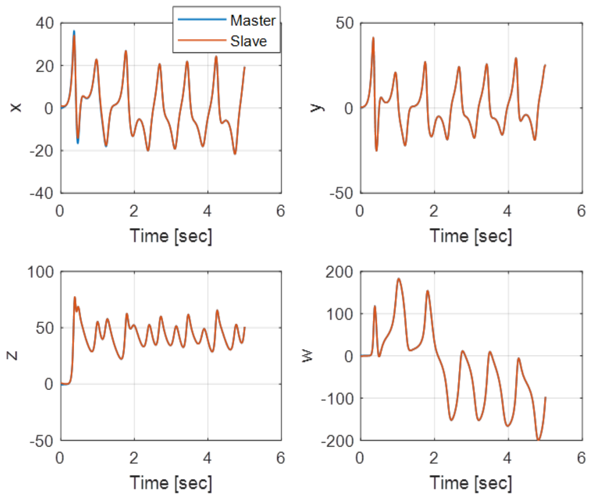

Figure 2. State trajectories of the master and slave systems.

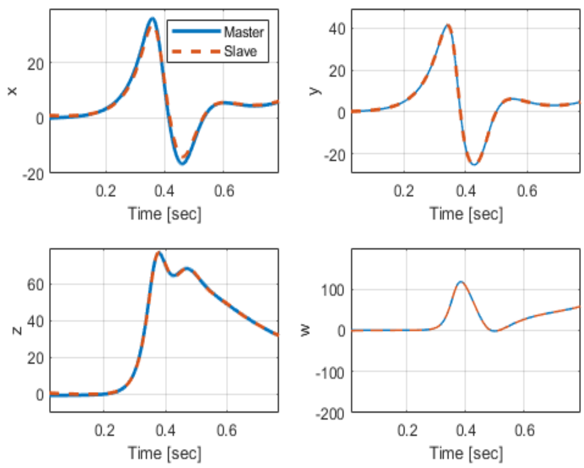

Figure 3. Enlarged view of the state trajectories of the master and slave systems.

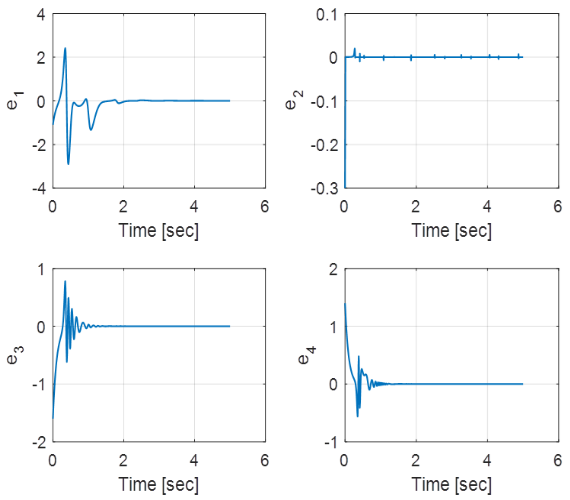

Figure 4. Synchronization errors of the state variables.

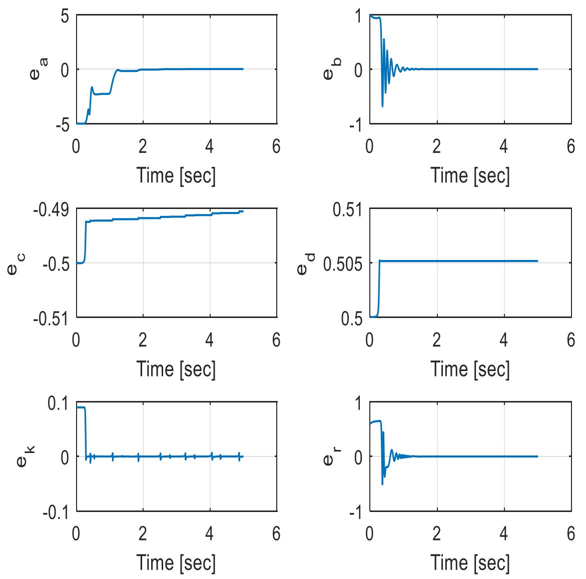

Figure 5. Parameter estimation errors.

As shown in Fig. 6, the control signals exhibit a consistent and stable dynamical behavior that reflects the effectiveness of the proposed control strategy. Their temporal patterns indicate appropriate responsiveness to system states and disturbances, confirming that the control mechanism successfully regulates the system dynamics to achieve the desired synchronization performance.

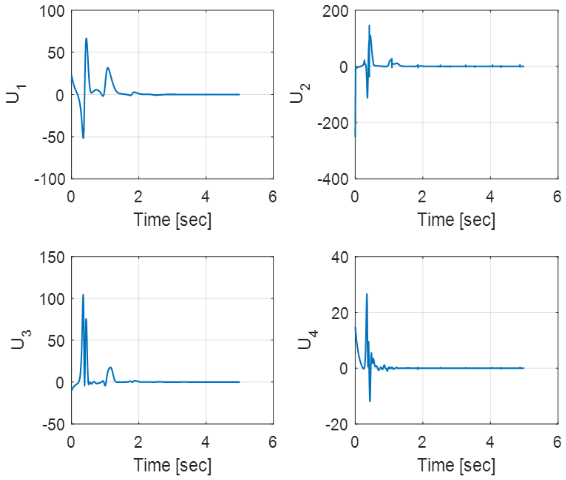

Figure 6. Control input signal.

As observed from the simulation outputs, the synchronization errors of the state variables (e_1, e_2, e_3, e_4) asymptotically converge to zero over time, confirming the successful synchronization of the slave system with the master. Additionally, the estimation errors associated with the unknown parameters remain bounded throughout the simulation period. These findings verify the theoretical analysis and highlight the robustness and effectiveness of the proposed adaptive control approach in handling complex hyperchaotic dynamics with unknown parameters.

4. Conclusion

In this paper, an extended hyperchaotic Chen system with unknown parameters was investigated, and a robust indirect adaptive control strategy was developed to achieve complete synchronization between the master and slave systems. The proposed scheme not only estimates the unknown parameters with high accuracy but also compensates for dynamic variations by leveraging an adaptive mechanism rooted in Lyapunov stability theory. Rigorous mathematical analysis confirms that the synchronization errors asymptotically converge to zero, while the parameter estimation errors remain bounded. The practical effectiveness and reliability of the control strategy were further validated through comprehensive MATLAB simulations, which demonstrated the system's ability to maintain synchronization despite parameter uncertainties.

References

[1] Z. Jing, D. Xu, Y. Chang, L. Chen, "Bifurcations, chaos, and system collapse in a three-node power system, " Electrical Power and Energy Systems, 25, 443-461, (2003), https://doi.org/10.1016/S0142-0615(02)00130-8. https://doi.org/10.1016/S0960-0779(02)00373-9.

[2] Z.-M. Ge, G.-H. Yang, "Hyperchaos of four state autonomous system with three positive Lyapunov exponents," Physics Letters A, 373 (3), 349-353, 2009, https://doi.org/10.1016/j.physleta.2008.11.046.
[3] R. Riahi, "Using Soft Computing and Chaos Theory in investigating the Deformed Stadium," Journal of Optimization in Soft Computing, vol. 4, no. 2, Sep. 2024, p. 8. sanad.iau.ir, https://doi.org/10.82553/josc.2024.140304031123494.
[4] E. Moradi, "Finite time stabilization of time-delay nonlinear systems with uncertainty and time-varying delay, " Journal of Control, 14(2), 79–87, (2020), https://doi.org/10.29252/joc.14.2.79.
[5] E. Moradi, M. R. Jahed-Motlagh, and M. B. Yazdi, "Delay-Dependent Finite-Time Stabilization of Uncertain Switched Time-Delay Systems with Norm-Bounded Disturbance," IETE Journal of Research, 64(2), 195–208, (2017), https://doi.org/10.1080/03772063.2017.1351898.
[6] N. Maleki and E. Moradi, "Robust H∞ Control of DC Motor in the Presence of Input Delay and Disturbance by the Predictor-Based Method," Complexity, 2022(1), (2022), https://doi.org/10.1155/2022/1902166.
[7] J. Li, J. Zheng, "Finite-time synchronization of different dimensional chaotic systems with uncertain parameters and external disturbances," Scientific Reports, (2022), https://doi.org/10.1038/s41598-022-19659-7.
[8] W. Abbasi, Y-C. Liu, "Robust and resilient stabilization and tracking control for chaotic dynamical systems with uncertainties," International Journal of Dynamics and Control, (2021), https://doi.org/10.1007/s40435-021-00782-8.
[9] H. Su, R. Luo, J. Fu, M. Huang, "Fixed time control and synchronization of a class of uncertain chaotic systems with disturbances via passive control method," Mathematics and Computers in Simulation, 198, 474-493, (2022), https://doi.org/10.1016/j.matcom.2022.03.010.
[10] J.M. Munoz-Pacheco, C. Volos, F.E. Serrano, S. Jafari, J. Kengne, K. Rajagopal, "Stabilization and Synchronization of a Complex Hidden Attractor Chaotic System by Backstepping Technique," Entropy, 23, 921, 1-22, (2021), https://doi.org/10.3390/e23070921.
[11] Ashish, M. Sajid, "Stabilization in chaotic maps using hybrid chaos control procedure," Heliyon, 10, 1-16, (2024), https://doi.org/10.1016/j.heliyon.2024.e23984.
[12] Z. Liu, R. Guo, "Stabilization of a Class of Complex Chaotic Systems by the Dynamic Feedback Control," Complexity, 1-10, (2020), https://doi.org/10.1155/2020/4938149.
[13] W. Wu, Z. Chen, Z. Yuan, "The evolution of a novel four-dimensional autonomous system: Among 3-torus, limit cycle, 2-torus, chaos and hyperchaos," Chaos, Solitons and Fractals, 39 2340–2356, (2009), https://doi.org/10.1016/j.chaos.2007.07.016.
[14] Q. Jia, "Hyperchaos generated from the Lorenz chaotic system and its control," Physics Letters A, 366, 217-222, (2007), https://doi.org/10.1016/j.physleta.2007.02.024.
[15] O.E. ROSSLER, "AN EQUATION FOR HYPERCHAOS," PHYSICS LETTERS, 71A(2,3), 155-157, (1979), AN EQUATION FOR HYPERCHAOS.
[16] L. M. Tam, J. H. Chen, H. K. Chen, W. M. S Tou, "Generation of hyperchaos from the Chen–Lee system via sinusoidal perturbation," Chaos, Solitons and Fractals, 38, 826–839, (2008), https://doi.org/doi:10.1016/j.chaos.2007.01.039.
[17] K.-Y. Shao, A. Feng, T.-T. Wang, "Fixed-Time Sliding Mode Synchronization of Uncertain Fractional-Order Hyperchaotic Systems by Using a Novel Non-Singleton-Interval Type-2 Probabilistic Fuzzy Neural Network," Fractal Fract, 7, 247, 1-16, (2023), https://doi.org/10.3390/fractalfract7030247.
[18] K. Benkouider, A. Sambas, T. Bonny, W.A. Nassan, I.A.R. Moghrabi, I.M. Sulaiman, B.A. Hassan, M. Mamat, "A comprehensive study of the novel 4D hyperchaotic system with self-exited multistability and application in the voice encryption," Scientific Reports, 14, (2024), https://doi.org/10.1038/s41598-024-63779-1.
[19] Z. Bai, S. Li, H. Liu, X. Zhang, "Adaptive Fuzzy Backstepping Control of Fractional-Order Chaotic System Synchronization Using Event-Triggered Mechanism and Disturbance Observer," Fractal Fract, 6, 714, (2022), https://doi.org/10.3390/fractalfract6120714.
[20] F.-Q. Dou, J.-A. Sun, W.-S. Duan, K.-P. Lu, "Controlling hyperchaos in the new hyperchaotic system," Communications in Nonlinear Science and Numerical Simulation, 14, 552-559, (2009), https://doi.org/10.1016/j.cnsns.2007.10.009.
[21] H. Zhang, "An Integral Sliding Mode Control of Uncertain Chaotic Systems via Disturbance Observer," 2021, 1-11, (2021), https://doi.org/10.1155/2021/6628116.
[22] H. Tian, M. Zhao, J. Liu, Q. Wang, X. Yu, Z. Wang, "Dynamic Analysis and Sliding Mode Synchronization Control of Chaotic Systems with Conditional Symmetric Fractional-Order Memristors," Fractal Fract. 8, 307, (2024), https://doi.org/10.3390/fractalfract8060307.
[23] M. Zhao, J. Wang, "H_∞ control of a chaotic finance system in the presence of external disturbance and input time-delay," 233, 320-327, (2014), http://dx.doi.org/10.1016/j.amc.2013.12.085.
[24] E. Ozpolat, A. Gulten, "Synchronization and Application of a Novel Hyperchaotic System Based on Adaptive Observers," Appl. Sci. 2024, 14, 1311, 1-20, (2024), https://doi.org/10.3390/app14031311.
[25] Y. Ai, Z. Feng, H. Wang, "Fixed-Time Adaptive Fuzzy Anti-Synchronization Control of Hyperchaotic Rossler System Based on Backstepping Method," Int. J. Fuzzy Syst, 25(6), 2501–2513, (2023), https://doi.org/10.1007/s40815-023-01536-8.
[26] H.-T. Yau, J.-J. Yan, "Chaos synchronization of different chaotic systems subjected to input nonlinearity," Applied Mathematics and Computation, 197, 775–788, (2008), https://doi.org/10.1016/j.amc.2007.08.014.

[27] H.-T. Yau, J.-J. Yan, "Robust controlling hyperchaos of the Rossler system subject to input nonlinearities by using sliding mode control," Chaos, Solitons and Fractals, 33, 1767–1776, (2007), https://doi.org/10.1016/j.chaos.2006.03.016.

[28] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems, "Physical Review Letters," 64(8), 821–824, (1990), https://doi.org/10.1103/physrevlett.64.821.

[29] C.-L. Kuo, C.-C. Wang, N.-S. Pai, "Design of Variable Structure Synchronization Controller for Two Different Hyperchaotic Systems Containing Nonlinear Inputs," Journal of Applied Sciences, 9(14), 2635-2639, (2009), https://doi.org/10.3923/jas.2009.2635.2639.

[30] H. Cheng, H. Li, Q. Dai, and J. Yang, "A deep reinforcement learning method to control chaos synchronization between two identical chaotic systems," Chaos Solitons & Fractals, 174, p. 113809, (2023), https://doi.org/10.1016/j.chaos.2023.113809.

[31] H. Soheili Rad, E. Moradi "Investigation of Stability and Performance of the Extended Hyperchaotic Chen System", Journal of Novel Researches on Smart Power Systems, vol. 14, no.1, pp. 1-8, June 2025, http://jeps.dezful.iau.ir/article-1-526-fa.html.

[32] M. M. El-Dessoky, N. Almohammadi, E. Alzahrani "Control and adaptive modified function projective synchronization of different hyperchaotic dynamical systems", AIMS Mathematics, vol. 8, no. 10, pp. 23621–23634, 2023, https://doi.org/10.3934/math.20231201.