



Research paper

Calculation of conductivity limit for fixed temperature limited plates using panel numerical method

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Abstract

The dimensionless Nusselt number is one of the important parameters in convection heat transfer, which can be considered as a measure of the heat transferred from the body through convection. In heat transfer, the Nusselt number indicates the rate of displacement heat transfer to conductive heat transfer. When heat transfer occurs only through conduction, it is called conduction limit, and its calculation is particularly important in the industry. This research uses a panel numerical method to calculate the conductivity limit for two-sided and one-side active plates for square, circular, and triangular cases. Finally, by comparing all the results, the performance of the panel numerical method for calculating the conductivity limit was confirmed.

1. Introduction

In the case where convection heat dissipation is considered, the Nusselt number is expressed as a function of the dimensionless numbers of Reynolds, Grashof and Prandtl, as well as the aspect ratio and the characteristic length of the object [1]:

$$Nu_L = f(Re_L, Gr_L, Pr, AR, L) \quad (1)$$

If the Reynolds number and Grashof tend to zero, the heat transfer from the body to the surrounding environment takes place only through conduction:

$$Nu_L = f(AR, L) \quad (2)$$

In this case, the value of the Nusselt number is called the conduction limit. Two methods can be used to calculate the conductivity limit of an isothermal object. In the first method, the temperature distribution around the object is calculated according to the solution of the Laplace equation; Then, the conductivity limit value is obtained by obtaining the temperature gradient on the boundary of the object. In the second method, the amount of heat transferred from the boundary of the object can be calculated without referring to the temperature distribution around it, and

according to this value, the temperature of the boundary of the object and the temperature of the surrounding environment, the conductivity limit value can be obtained [2].

According to the definition, the amount of heat transferred through conduction with the surrounding environment is as follows:

$$Q = KS(T_o - T_\infty) \quad (3)$$

In the above relation, S is called the shape coefficient of conduction. According to the definition, the value of the conduction limit will be as follows:

$$S_{\sqrt{A}}^* = \frac{S}{\sqrt{A}} = \frac{Q}{K(T_o - T_\infty)\sqrt{A}} \quad (4)$$

In fact, the panel method is used to obtain Q in the above relationship.

2. The general steps of doing the work

For a better understanding of the method, in this section, the work process is divided as follows [3]:

- Object boundary paneling:

The advantage of using the panel numerical method compared to other numerical methods,

such as finite difference, is that, instead of gridding the whole object, only the boundary of the object is gridded. Also, in the panel method, because it works on the boundary of the object, the panels are linear or surface (depending on whether the problem is two-dimensional or three-dimensional). Therefore, to calculate the conduction limit using the panel method, the boundary of the object is modeled with several panels, and according to the geometric coordinates of each panel, a control point is assigned to it to perform calculations [4].

- Picking thermal springs:

In this part, according to the panelization of the boundary of the body, a heat source is placed on each panel, the distribution of each of which is fixed on its panel. Therefore, with the content that will be explained in the following parts, the method of calculating the power of the springs is investigated, from which the amount of transferred heat can be obtained.

- Calculation of temperature distribution around a point source:

It was mentioned in the previous part that the power of thermal springs on a panel is a constant value. Therefore, a panel's differential element can be considered a point source whose temperature distribution around it satisfies the Laplace equation. In this way, solving this differential equation around a point source is easy.

- Calculation of the thermal effect of the control points due to a thermal spring:

The temperature increase of the desired control point can be calculated by calculating the thermal effect of an element on a control point and then by integrating this value on the panel.

- Calculate the temperature of the control points:

To calculate the temperature of control points, by using the law of sum of effects, the temperature of the desired control point can be obtained by summing the effects of all panels on a control point.

- Calculation of transferred heat:

In order to calculate the heat transferred from the surface of the object, the power of the thermal springs on each panel should be calculated. According to the boundary conditions of constant temperature, a system of linear equations appears; this system of linear equations can be solved with the help of the Gauss elimination method, and the power of the panels can be calculated. By calculating the heat transferred from the boundary of the object using the law of conservation of energy for the steady state, it can be said that the heat transferred from the surface of the object is equal to the algebraic sum of the heat exchanged from the boundary of the panels.

- Calculation of conductivity limit:

Having the heat transferred from the boundary of the body and knowing the temperature of the surface and the environment, the value of the conductivity limit can be calculated according to equation 4.

3. Mathematical steps of the work method

The surface of the object must first be modeled with surface panels to calculate the conduction limit of three-dimensional objects (for example, a cube or a flat plate). Then, a control point i is added to each of these panels, which is considered in the middle of them, and a boundary point j is assigned, which represents the number of the panel. Now, if the number of panels on the surface of the object is considered to be equal to m and the area of each of them is equal to s_j , and the power per unit of the panel surface is equal to q_j/k , the temperature distribution for a differential element ds_j from panel j , according to Laplace's equation, becomes as follows [1,4]:

$$\nabla^2 \Psi_j = 0 \quad (5)$$

In this regard, Ψ_j is defined as follows:

$$\Psi_j = T - T_\infty \quad (6)$$

By solving the above equation, the thermal effect of the differential element ds_j , at the control point i , is equal to:

$$dT_{ij} = \frac{q_j}{4\pi k} \frac{ds_j}{R_{ij}} \quad (7)$$

By integrating the above equation on the surface s_j , it is possible to calculate the increase in the temperature of the control point i due to the effect of panel j :

$$\Delta T_{ij} = C_{ij} q_j \quad (8)$$

which in the above relation C_{ij} is equal to:

$$C_{ij} = \frac{1}{4\pi k} \iint_{s_j} \frac{ds_j}{R_{ij}} \quad (9)$$

According to the law of sum of effects, in order to calculate the temperature of the control point i , the sum of the effects of all panels should be obtained as follows:

$$\sum_{j=1}^m C_{ij} q_j = \sum_{j=1}^m \Delta T_{ij} \quad (10)$$

By applying the boundary condition of constant temperature $T_i = T_0$, the sum of the effects of the panels at a control point is equal to $T_0 - T_\infty$, so the above relationship becomes the following relationship:

$$\sum_{j=1}^m C_{ij} q_j = T_0 - T_\infty \quad (11)$$

With the values of T_0 and C_{ij} known to calculate the values of q_j , if the above relation is used for the

values $i=1,2,\dots,m$, a system of linear equations will appear in the form of the following relation:

$$\begin{aligned} C_{11}q_1 + C_{12}q_2 + \dots + C_{1m}q_m &= T_o - T_\infty \\ \vdots & \\ C_{m1}q_1 + C_{m2}q_2 + \dots + C_{mm}q_m &= T_o - T_\infty \end{aligned} \quad (12)$$

The above equation is shown in matrix form as follows [5]:

$$[C_{ij}] [q_j] = [B_i] \quad (13)$$

In the above equation, $i=1,\dots,m$ and $j=1,\dots,m$ and $B_i = T_o - T_\infty$.

As was said before, to calculate the values of q_j , the system of linear equations above should be solved using the Gauss elimination method.

With the power of the panels, the heat transferred from the boundary of the body according to the law of conservation of energy in a steady state becomes as follows:

$$Q = \sum_{j=1}^m q_j s_j \quad (14)$$

It is worth mentioning that with Q and $(T_o - T_\infty)$, the value of the shape coefficient of conduction can be calculated from the following equation:

$$S = \frac{Q}{K(T_o - T_\infty)} \quad (15)$$

Obviously, using the above relation, the conduction limit becomes dimensionless as follows:

$$S_{\sqrt{A}}^* = Nu_{\sqrt{A}} = \frac{S}{\sqrt{A}} \quad (16)$$

Considering that obtaining C_{ij} is of special importance for calculating the conductivity limit, the method of its calculation will be explained in the next part.

3.1. Calculation of the coefficient C_{ij}

For three-dimensional objects, the C_{ij} coefficient (according to Figure 1) is shown as follows [6]:

$$C_{ij} = \frac{1}{4\pi k} \iint_{s_j} \frac{ds_j}{R_{ij}} \quad (17)$$

where the integral can be calculated as follows:

$$\iint_{s_j} \frac{1}{R_{ij}} ds_j = \iint \frac{dxdy}{\sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}} \quad (18)$$

The calculation of this integral has been done analytically for panels that are in one plane, as shown in the figure 2; the coordinate axes are transferred so that the z-axis is always perpendicular to the plane of the panel. With this, the above integral will be in the following form:

$$\iint_{s_j} \frac{dxdy}{\sqrt{(x-x_i)^2 + (y-y_i)^2}} = \left[(x-x_i) \ln \left[(y-y_i) + \sqrt{(x-x_i)^2 + (y-y_i)^2} \right] + (y-y_i) \ln \left[(x-x_i) + \sqrt{(x-x_i)^2 + (y-y_i)^2} \right] \right]_{x_j-\frac{\omega}{2}}^{x_j+\frac{\omega}{2}} \Big|_{y_j-\frac{\gamma}{2}}^{y_j+\frac{\gamma}{2}} \quad (19)$$

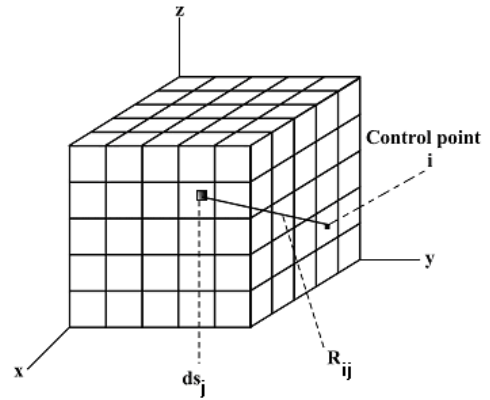


Figure 1- Paneling of a three-dimensional object

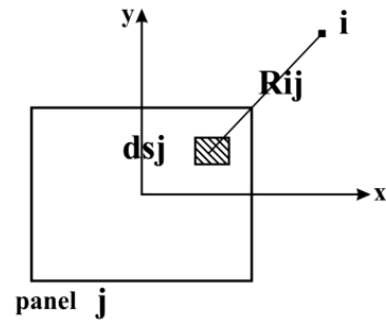


Figure 2- How to place the coordinate axes in a plane

In the relation above, ω is the length, and γ is the width of panel j . By calculating this value, the C_{ij} coefficient is obtained [7,8].

For panels not on the same plane, panel j is divided into m parts and the distance of each part to control point i is calculated using geometric relations. Finally, the coefficient C_{ij} is approximated with a sum in the following form:

$$C_{ij} = \frac{1}{4\pi k} \sum_{k=1}^m \frac{\Delta s_k}{R_{ik}} \quad (20)$$

To calculate the coefficient C_{ij} , the following points should be taken into account [9]:

- 1- To calculate the effect of a panel on itself, it must be calculated analytically to eliminate the effect of individual points.
- 2- The length to width ratio of the panels should be as close as possible to a square panel.
- 3- The average distance value can be used to calculate C_{ij} .

The next part will discuss the results obtained using the above method for the pages examined in this research.

4. Result and Discussion

- Two-sided active square plates

To calculate the conductivity limit of square plates like the one shown in Figure 3, we panelize the surface of the plate [10].

As shown in figure 3, the panels are selected as square because the plate is square and according to the point mentioned in the previous section. Because the screen is active on both sides, the back is also paneled similarly.

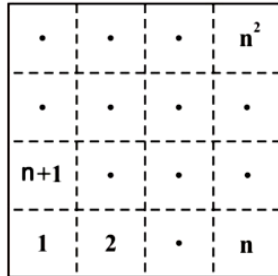


Figure 3- How to panel the square plates of the figure

By using the method explained in the previous section, the computer program related to the page of the two active sides is written, the results of which can be seen in figure 4.

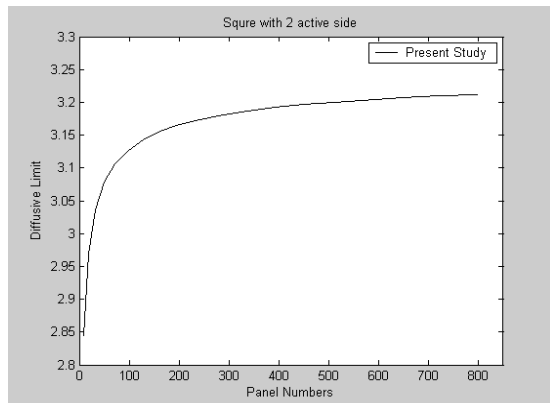


Figure 4 - The results obtained for the square plate with two active sides

As shown in the diagram above, the conduction limit tends to constant value with the increase in the number of panels. In the diagram above, it can be seen that the value of this number is equal to 3.2118.

Using a numerical method, Yovanovich [11,12] obtained the conductivity limit value for a square plate with two active sides with a small thickness. The value obtained using his method is 3.205.

The error between the values obtained from these two methods is around 0.2%, which is small.

- One-side active square plates

The working method in this part is the same as the previous part, with the difference that only one side of the square surface is paneled here.

With the increase in the number of panels, the conduction limit tends to be constant. This number equals 3.5422 for the square plates of one active side.

- Two-sided active circular plates

To calculate the conduction limit of circular plates like the one shown in figure 5, we panelize the surface of the plate.

In this part, the working method is the same as explained in the previous sections. The difference is that here, because the panels are not rectangular, it is not possible to use equation 19 for the panels on one page. Instead, equation 20 is used to examine the effect of panels on one page.

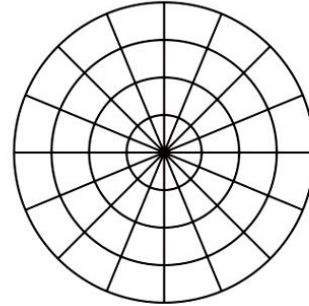


Figure 5- How to panelize the circular plates of the figure

The limit value obtained using this method tends to be 3.2053 by increasing the number of panels.

- One-side active circular plates

Here, the conduction limit tends to the number 3.5330 with increased panels.

- Two-sided active triangular plates

To calculate the conduction limit of triangular plates like the one shown in Figure 6, we panelize the surface of the plate.

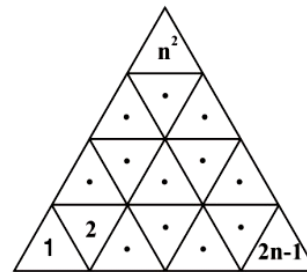


Figure 6- How to panel the triangular plates of the figure

As explained for the circular plates, relation 19 cannot be used and relation 20 must be used.

the conductivity limit tends to 3.3305 with the increase of panels.

- One-side active triangular plates

Here, the conductivity limit tends to 3.71 with the increase of panels.

5. Conclusion

As it can be seen, the values obtained for triangular plates are higher than those for the other two cases, but their difference is not great. Therefore, one value can be used as a conduction limit for all plates (by accepting a small error value). In addition, this difference is so small that it does not have much effect on the calculation of the Nusselt number.

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