



Mechanical and Thermal Analysis of FGPPM in Hollow Cylinder under symmetric and Non-axisymmetric loads

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Abstract

This lecture review examines the analytical investigation of mechanical and thermal stresses in hollow cylinders made of functionally graded porous piezoelectric materials (FGPPMs). The study synthesizes findings from two research papers that developed direct solution methodologies for radially symmetric (1D) and non-axisymmetric (2D) loading scenarios. The research addresses multi-physics coupling between elastic deformation, piezoelectric effects, porous pressure interactions, and thermal expansion in materials with power law property variations. The analytical framework employs complex Fourier series expansion and successfully overcomes limitations of traditional potential function methods. Key findings reveal that material grading significantly influences stress distributions, with critical power law indices determining transition behaviors in hoop stress patterns. The compressibility coefficient (B) and pore volume fraction (ϕ) substantially modify both mechanical and electrical responses. Temperature distributions, displacement patterns, and electric potential variations demonstrate strong dependence on material grading parameters. The studies validate analytical solutions through comparison with finite volume numerical methods, showing excellent agreement for temperature, radial stress, and circumferential stress distributions. The developed methodology provides engineers with powerful tools for optimizing FGPPM structures in applications including medical ultrasonic devices, nondestructive testing equipment, underwater acoustic systems, and pressure vessel technology. This work represents a

significant advancement in smart materials analysis and establishes new paradigms for multi-physics modeling of advanced engineering materials. Future research directions include dynamic analysis extensions and experimental validation of theoretical predictions [1,2].

Keyword: FGM; Piezoelectric; porothermoelasticity; Hollow cylinder.

1. Introduction

This lecture review examines the comprehensive analytical framework for studying mechanical and thermal stresses in hollow cylinders made of functionally graded porous piezoelectric materials (FGPPMs), based on two related research papers by Jabbari et al. The work represents a significant advancement in the field of smart material analysis, extending traditional approaches to incorporate the complex interactions between piezoelectric effects, poroelasticity, and functionally graded material properties.

1.1 Functionally Graded Materials (FGMs)

Functionally graded materials are engineered composites where material properties vary continuously and gradually from one surface to another. The combination of ceramic and metallic components allows these materials to withstand extreme thermal conditions while maintaining structural integrity. The governing equations for temperature and stress distributions become coordinate-dependent due to position-dependent material properties.

1.2 Piezoelectric Materials

Piezoelectric materials exhibit coupled electro-mechanical behavior, making them ideal for sensor and actuator applications in smart composite systems. The integration of functionally graded concepts with piezoelectric materials has led to the development of functionally graded piezoelectric materials (FGPMs).

1.3 Porous Piezoelectric Materials

Porous piezoelectric ceramics have gained importance in various applications including medical ultrasonic devices, nondestructive testing, underwater acoustics, and low-

frequency hydrophones. The introduction of porosity affects the material's mechanical, thermal, and electrical properties.

2 Mathematical Framework [1,2]

2.1 Heat Conduction Problem

Both studies solve the steady-state heat conduction equation for FGPPMs:

$$\text{1D Case (Radially Symmetric):} \quad \frac{\partial^2 T}{\partial r^2} + \left(\frac{k'(r)}{k(r)} + \frac{1}{r} \right) \frac{\partial T}{\partial r} = 0 \quad (1)$$

$$\text{2D Case (Non-axisymmetric):} \quad \frac{\partial^2 T}{\partial r^2} + \left(\frac{k'(r)}{k(r)} + \frac{1}{r} \right) \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0 \quad (2)$$

Where the thermal conductivity follows a power law: $K = k_0 r^{m_3}$ (3)

2.2 Stress Analysis

2.2.1 The stress-strain relations for FGPPM incorporate:

- Elastic deformation
- Piezoelectric coupling
- Porous pressure effects
- Thermal expansion

2.2.2 The constitutive relations are expressed as: [1,2]

$$\begin{aligned} \sigma_{rr} &= C_{11}^* \varepsilon_{rr} + C_{12}^* \varepsilon_{\theta\theta} + e_{21} E_r - Z_1 T(r, \theta) \\ \sigma_{\theta\theta} &= C_{12}^* \varepsilon_{rr} + C_{22}^* \varepsilon_{\theta\theta} + e_{22} E_r - Z_2 T(r, \theta) \\ \sigma_{r\theta} &= 2 C_{66}^* \varepsilon_{r\theta} - e_{26} E_\theta \\ D_{rr} &= e_{21} \varepsilon_{rr} + e_{22} \varepsilon_{\theta\theta} - \varepsilon_{22} E_r + g_{21} T(r, \theta) \\ D_{\theta\theta} &= 2 e_{26} \varepsilon_{r\theta} - \varepsilon_{21} E_\theta + g_{22} T(r, \theta) \end{aligned} \quad (4)$$

Where C_{ij}^* represents modified elastic constants.

2.2.3 Porous Effects

The porous pressure under undrained conditions is given by: $p = -M \gamma (\varepsilon_{rr} + \varepsilon_{\theta\theta})$

Where:

- M is Biot's modulus
- γ is Biot's coefficient of effective stress
- B is the compressibility coefficient (Skempton coefficient)
- ϕ is the pore volume fraction

3. Solution Methodology

3.1- Direct Method Approach

Both studies employ a direct method rather than potential function approaches, providing several advantages:

- Mathematical capability to handle complex boundary conditions
- No limitations on boundary condition types
- Superior handling of complicated mathematical functions

3.2. Material Property Assumptions [1,2]

Material properties (except Poisson's ratio) are expressed as power law functions:

$$\begin{aligned} \alpha_r &= \alpha_{01} r^{m_1} & C_{ij}^* &= \bar{C}_{ij} r^{m_4} & K &= k_0 r^{m_3} & \alpha_\theta &= \alpha_{02} r^{m_2} \\ e_{2i} &= \bar{e}_{2i} r^{m_5} & \varepsilon_{22} &= \bar{\varepsilon}_{22} r^{m_6} & g_{2i} &= \bar{g}_{2i} r^{m_7} \end{aligned} \quad (6)$$

For simplification, the studies assume: $m_1 = m_2 = m_7$ and $m_4 = m_5 = m_6$.

4. Solution Techniques

4.1 1-D: Radially Symmetric Case:

- General solutions assumed as $u_n^g(r), v_n^g(r)$ and $\phi_n^g(r)$
- Characteristic equations solved to obtain eigenvalues.
- Particular solutions incorporated for thermal loading.
- Four boundary conditions determine unknown coefficients.

4.2 2-D: Non-axisymmetric Case:

- Complex Fourier series expansion: $T(r, \theta) = \sum_{n=-\infty}^{\infty} T_n(r) e^{in\theta}$
- System reduces to coupled ordinary differential equations
- Six-order polynomial characteristic equations for eigenvalues
- Six boundary conditions for coefficient determination

5.Key Results and Observations

5.1 Temperature Distribution

- Power law index significantly affects temperature distribution
- Higher power law index (m) results in decreased temperature
- Non-axisymmetric cases show harmonic patterns

5.2 Displacement Patterns

- Radial displacement magnitude decreases with increasing power index
- Circumferential displacements follow boundary condition patterns
- Electric potential shows similar trends to mechanical displacements

5.3 Stress Distributions

- Radial stress: Increases with power law index.
- Hoop stress: Shows complex behavior depending on b/a ratio.
 - For $m < 1$: Decreases with radius (similar to isotropic materials).
 - For $m > 1$: Increases with radius due to higher outer layer stiffness.
 - Critical value $m \approx 1$ for $b/a = 1.2$ where hoop stress remains approximately constant.

5.4 Porous Effects

The studies demonstrate that:

- Compressibility coefficient (B) significantly affects stress distributions
- Pore volume fraction (ϕ) influences both mechanical and electrical responses
- Porous pressure coupling modifies traditional stress patterns

5.5 Validation and Verification

The non-axisymmetric study includes comparison with finite volume method results from literature, showing good agreement for:

- Temperature distributions
- Radial stress patterns
- Circumferential stress patterns

6. Applications and Significance

6.1 Practical Applications

- Medical ultrasonic devices
- Nondestructive testing equipment
- Underwater acoustic systems
- Low-frequency hydrophones
- Smart composite structures
- Pressure vessel technology

6.2 Theoretical Contributions

- First comprehensive treatment of FGPPM hollow cylinders
- Integration of multiple physical phenomena (thermal, mechanical, electrical, porous)
- Development of direct solution methodology for complex boundary conditions
- Parametric studies revealing effects of material grading and porosity

7. Limitations and Future Directions

7.1 Current Limitations

- Simplified material property relationships (power law assumptions)
- Limited to steady-state thermal conditions
- Assumed undrained porous conditions
- Simplified porous structure model

7.2 Future Research Directions

- Dynamic analysis of FGPPM structures
- More complex material property variations
- Coupled fluid-structure interactions in porous media

- Experimental validation of theoretical predictions
- Extension to other geometries and loading conditions

8.Result and Discussion

8.1 Radially symmetric loads on FGPPMs [1]

Figure (1) shows the variations of the temperature along the radial direction for different values of the power law index. The figure shows that as the power law index m increases, the temperature decreased. The radial stresses are plotted along the radial direction and shown in Figs. (3). The magnitude of the radial stress is increased as m is increased. The hoop stress along the radius decreases for $m < 1$ (similar to thick cylinders made of isotropic materials), due to the acting internal pressure and zero external pressure. For $m > 1$, the hoop stress increases as the radius increases, since the modulus of elasticity is an increasing function of the radius. Physically, this means that the outer layers of the cylinder are biased to maintain the stress due to their higher stiffness. There is a limiting value for m , where the hoop stress remains almost a constant along the radius. For low values of the ratio b/a , such as $b/a = 1.2$, this value of m is about 1. For higher b/a ratios, this condition is achieved for higher values of m . For example, for $b/a = 5$, this condition is reached for $m = 1.5$. Figure (5) shows the plot of the radial electric potential along the radius. The magnitude of the radial electric potential is decreased as the power index m is increased. Figure (6) also shows the same plot of the electrical displacements for different values of the pore volume fraction (ϕ).

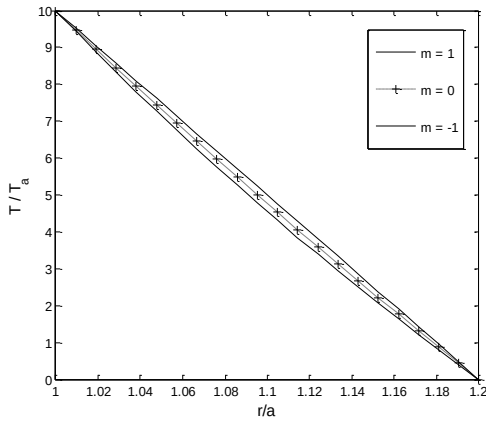


Fig. 1 Temperature distribution in the cross section of cylindrical

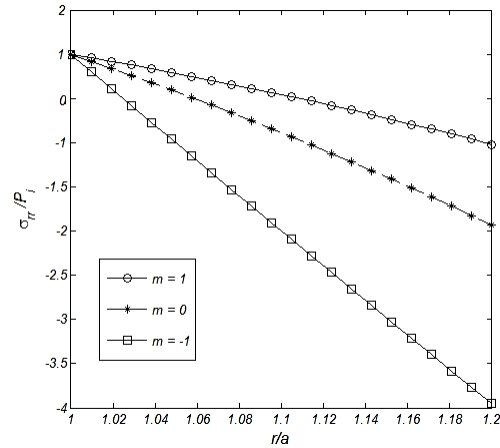


Fig. 3. Radial distribution of radial stress

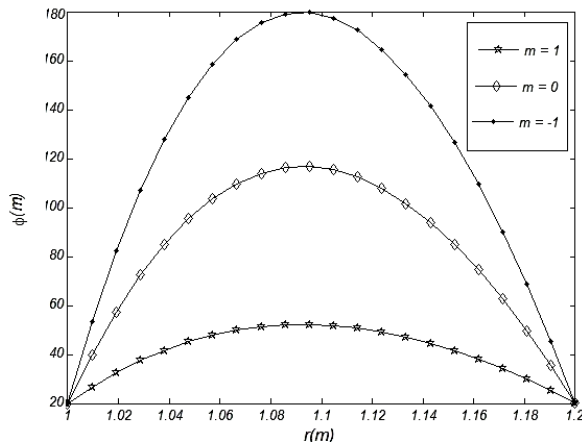
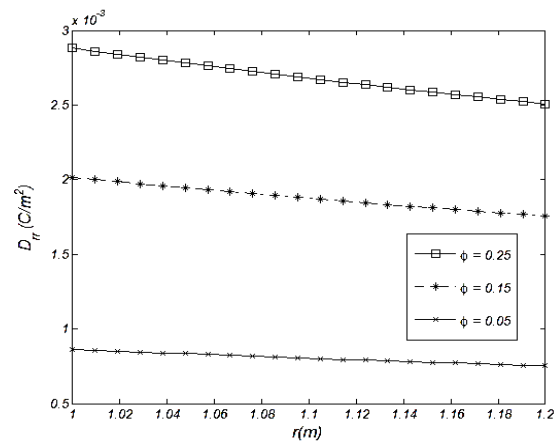


Fig. 5. Electric potential in the cross section of cylindrical

Fig. (6) Radial electrical displacement in the cross section of cylinder for different values of the compressibility coefficient (ϕ).

8.2 Non-axisymmetric loads on FGPPMs [2]

Figure (7) shows the temperature distribution in the wall thickness along the radius and circumferential directions. The effect of the power-law index on the temperature distribution in the also shown in Figure (8). Figure (9) show the radial thermal stresses in the cross section of the cylinder. It is interesting to see that all components of stresses follow a harmonic pattern on the outside surface. The radial and shear stresses are zero at the insider surface, due to the assumed boundary conditions. The effect of the power-law index on the distribution of the radial thermal

stresses is shown in Figure (10). This figure is the plot of stresses versus r/a at $\theta = \frac{\pi}{3}$. It is shown

that as m increases, the radial thermal stresses are increased.

Figures (11) show the radial mechanical stresses in the cross section of the cylinder respectively where the pore compressibility coefficient (B) is changed the other parameters are fixed. Figures (12) show these stresses based on the pore volume fraction (ϕ) is pore volume per total volume..

The stress distributions are shown in Figures (13) Stress patterns in the inside and outside surfaces follow harmonic patterns. The given harmonic boundary conditions for at $r=a$ have general influence on the pattern of the stress distributions in the cylinders cross section, as seen from Figures 13. The effect of the power law index on the distribution of the radial mechanical stresses is shown in Figure (14). This figure is the plot of stresses versus r/a at $\theta = \frac{\pi}{3}$. It is shown

that as m increases, the radial mechanical stresses increase.

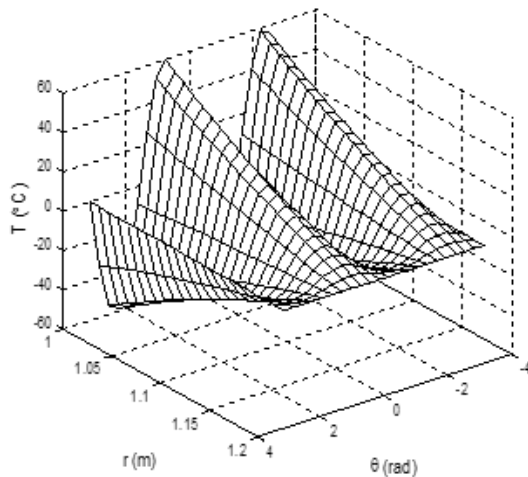


Fig.(7) Temperature distribution in the cross section of cylindrical

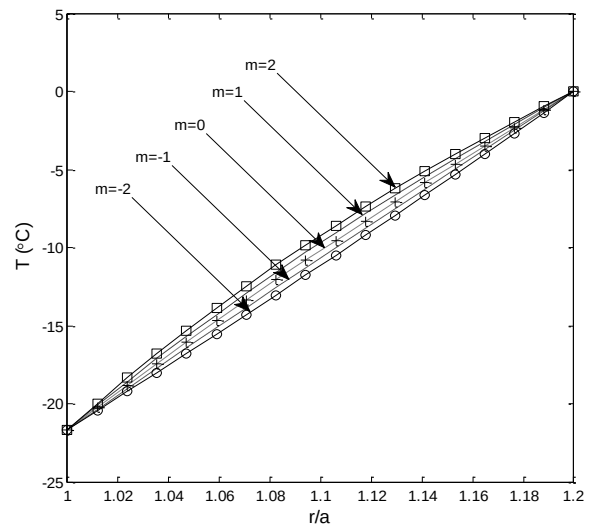


Fig.(8) Temperature distribution of radial at $\theta = \pi/3$

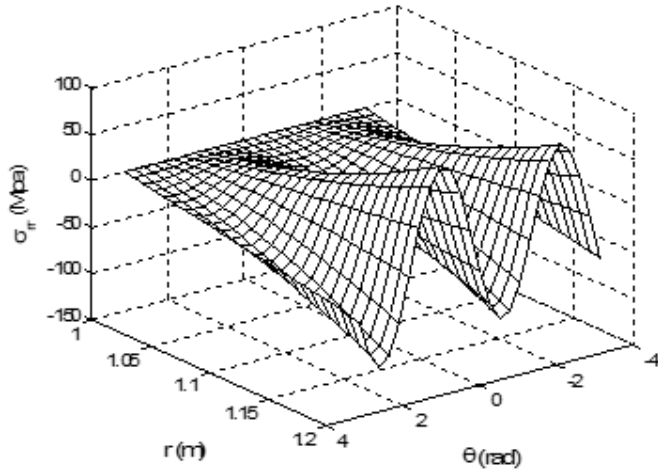


Fig.(9) Radial thermal stress in the cross section of cylindrical

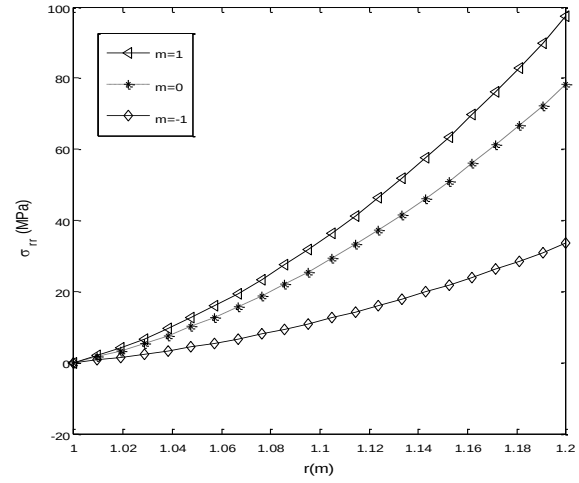
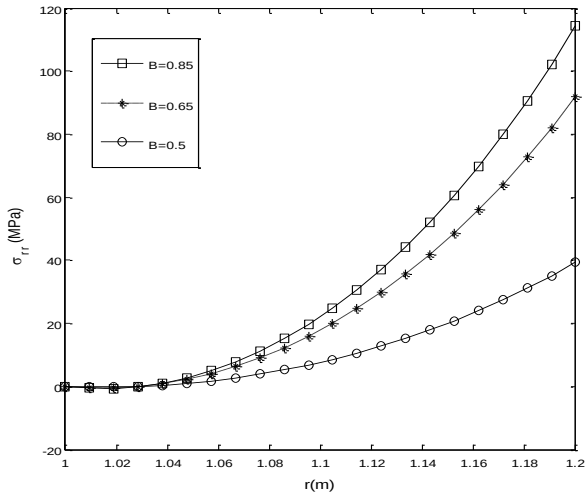
Fig.(10) Radial distribution of radial thermal stress σ_{rr} at $\theta = \pi/3$ 

Fig.(11)

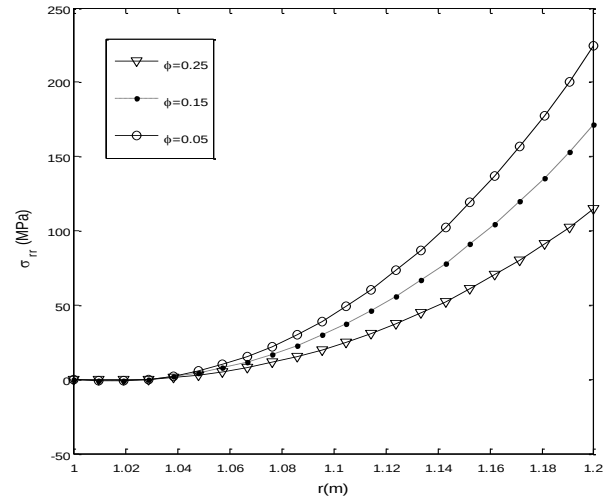
Fig.(11) Radial thermal stress in the cross section of cylindrical based on the compressibility coefficient (B) changing.

Fig.(12)

Fig.(12) Radial thermal stress in the cross section of cylindrical based on the pore volume fraction (ϕ) changing.

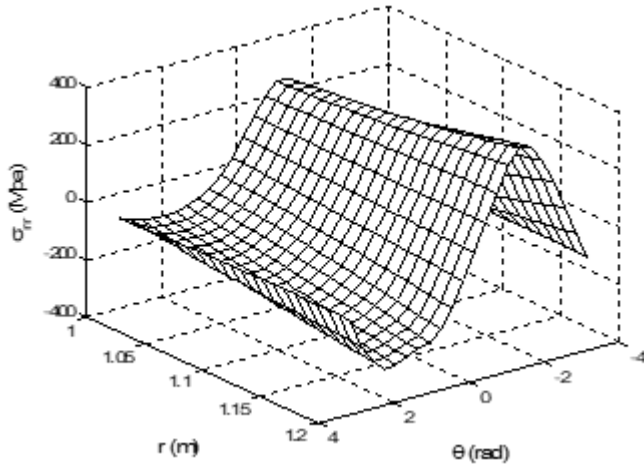
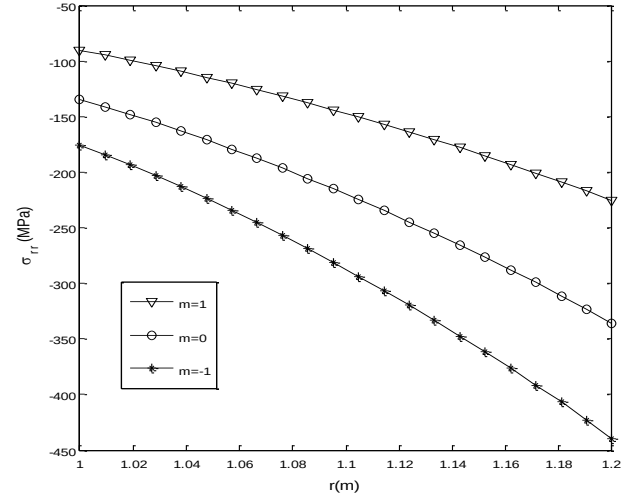


Fig.(13) Radial mechanical stress in the cross section of cylindrical

Fig.(14) Radial distribution of radial mechanical stress σ_{rr} at $\theta = \pi/3$

9. Conclusions

In the present work an attempt is made to study the problem of general solution for the thermal and mechanical stresses in a thick FGPPM hollow cylinder where the one-dimensional (radially symmetric) two-dimensional (non-axisymmetric) steady-state loads are implied. The method of solution is based on the direct method and uses the power series, rather than the potential function method. The advantage of this method is its mathematical power to handle both simple and complicated mathematical function for the thermal and mechanical stresses boundary conditions. The potential function method is capable of handling the complicated mathematical functions as boundary condition. The proposed method does not have the mathematical limitations to handle the general types of boundary conditions, which are usually occurred in the potential function method. The direct method approach successfully handles complex boundary conditions that cannot be addressed by traditional potential function methods.

10.Key findings include:

10.1 Material Grading Effects: Power law index significantly influences stress distributions, with critical values affecting transition behavior.

10.2 Porous Coupling: Pore volume fraction and compressibility coefficient substantially modify mechanical and electrical responses.

10.3 Multi-physics Integration: Successful coupling of thermal, mechanical, electrical, and porous effects in a unified analytical framework.

10.4 Practical Relevance: Results applicable to design of smart material structures in various engineering applications. The work establishes a foundation for further research in functionally graded porous piezoelectric materials and demonstrates the effectiveness of direct analytical methods for complex multi-physics problems.

References

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Biography:

Mohsen Meshkini holds a PhD degree; he received his MS degrees in Mechanical Engineering from Islamic Azad University, South Tehran Branch, Tehran, Iran in 2011 and a PhD degree in the same field of study from Sharif University of Technology, International Campus (SUTIC), Kish, Iran in 2017.