الاستیسیته با مواد FGM در مختصات استوانه ای

روابط سازگاری در مختصات استوانه ای دو بعدی[1]:

$$\frac{\partial e_{\theta r}}{\partial r}-\frac{1}{r}\frac{\partial e_{r}}{\partial \theta}+\frac{e_{\theta r}+e_{r\theta}}{r}-\frac{\partial \varphi_{z}}{\partial r}=0 \quad (1\text{-}1\text{-}1)$$

$$\frac{\partial e_{\theta}}{\partial r}-\frac{1}{r}\frac{\partial e_{r\theta}}{\partial \theta}+\frac{e_{\theta}-e_{r}}{r}-\frac{1}{r}\frac{\partial \varphi_{z}}{\partial \theta}=0 \quad (2\text{-}1\text{-}1)$$

$$\frac{\partial m_{\theta z}}{\partial r}-\frac{1}{r}\frac{\partial m_{rz}}{\partial \theta}+\frac{1}{r}m_{\theta z}=0 \quad (3\text{-}1\text{-}1)$$

خواص مواد FGM دو بعدی[2]:

$$\begin{aligned}
\lambda(r,\theta)&=\lambda_{\circ} r^{m_1}e^{m_2\theta}\\
\mu(r,\theta)&=\mu_{\circ} r^{m_1}e^{m_2\theta}\\
\kappa(r,\theta)&=\kappa_{\circ} r^{m_1}e^{m_2\theta}\\
\gamma(r,\theta)&=\gamma_{\circ} r^{m_1}e^{m_2\theta}
\end{aligned} \quad (2\text{-}1)$$

روابط کرنش – تنش [3]:

$$\begin{aligned}
m_{rz}&=\gamma\frac{\partial \varphi_{z}}{\partial r}\rightarrow\frac{\partial \varphi_{z}}{\partial r}=\frac{1}{\gamma}m_{rz}\rightarrow\frac{\partial \varphi_{z}}{\partial r}=\frac{1}{\gamma_{\circ} r^{m_1}e^{m_2\theta}}m_{rz}\\
m_{\theta z}&=\gamma\frac{1}{r}\frac{\partial \varphi_{z}}{\partial \theta}\rightarrow\frac{1}{r}\frac{\partial \varphi_{z}}{\partial \theta}=\frac{1}{\gamma}m_{\theta z}\rightarrow\frac{1}{r}\frac{\partial \varphi_{z}}{\partial \theta}=\frac{1}{\gamma_{\circ} r^{m_1}e^{m_2\theta}}m_{\theta z}\\
e_{r}&=\frac{(\kappa+\lambda+2\mu)}{(\kappa+2\mu)(\kappa+2\lambda+2\mu)}\sigma_{r}-\frac{\lambda}{(\kappa+2\mu)(\kappa+2\lambda+2\mu)}\sigma_{\theta}\\
e_{\theta}&=\frac{-\lambda}{(\kappa+2\mu)(\kappa+2\lambda+2\mu)}\sigma_{r}+\frac{(\kappa+\lambda+2\mu)}{(\kappa+2\mu)(\kappa+2\lambda+2\mu)}\sigma_{\theta}\\
e_{r\theta}&=\frac{(\kappa+\mu)}{\kappa(\kappa+2\mu)}\tau_{r\theta}-\frac{\mu}{\kappa(\kappa+2\mu)}\tau_{\theta r}\\
e_{\theta r}&=-\frac{\mu}{\kappa(\kappa+2\mu)}\tau_{r\theta}+\frac{(\kappa+\mu)}{\kappa(\kappa+2\mu)}\tau_{\theta r}
\end{aligned} \quad (3\text{-}1)$$

با جایگذاری روابط (1-2) در روابط(1-3) داریم:

$$
\begin{aligned}
e_r &= \frac{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + \lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\sigma_r \\
&- \frac{\lambda_{\circ} r^{m_1} e^{m_2\theta}}{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\sigma_\theta \\
e_\theta &= \frac{-\lambda_{\circ} r^{m_1} e^{m_2\theta}}{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)\left(\kappa + 2\lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\sigma_r \\
&+ \frac{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + \lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)\left(\kappa + 2\lambda_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\sigma_\theta \\
e_{r\theta} &= \frac{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + \mu_{\circ} r^{m_1} e^{m_2\theta}\right)}{\kappa_{\circ} r^{m_1} e^{m_2\theta}\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\tau_{r\theta} - \frac{\mu_{\circ} r^{m_1} e^{m_2\theta}}{\kappa_{\circ} r^{m_1} e^{m_2\theta}\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\tau_{\theta r} \\
e_{\theta r} &= -\frac{\mu_{\circ} r^{m_1} e^{m_2\theta}}{\kappa_{\circ} r^{m_1} e^{m_2\theta}\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\tau_{r\theta} + \frac{\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + \mu_{\circ} r^{m_1} e^{m_2\theta}\right)}{\kappa_{\circ} r^{m_1} e^{m_2\theta}\left(\kappa_{\circ} r^{m_1} e^{m_2\theta} + 2\mu_{\circ} r^{m_1} e^{m_2\theta}\right)}\tau_{\theta r}
\end{aligned}
\tag{4–1}
$$