Journal of Mechanical Research and Application (JMRA),Vol. 12 No.3, 1401(2022),40-45

Buckling of Nonhomogeneous Cylindrical Shells Under Torsion Using First Order Shear Deformation Theory

M.M. Najafizadeh, P. Khazaeinejad

Department of Mechanical Engineering, Arak Branch, Islamic Azad University, Arak, Iran; m-najafizadeh@iau-arak.ac.ir

Abstract

The buckling analysis of a nonhomogeneous cylindrical shell under torsion is studied in the present paper. The shell has nonhomogeneity properties which are assumed as a suitable function through the thickness direction. The stability equations are derived by the adjacent equilibrium criterion method based on the first order shear deformation theory. These equations are employed to analyze the buckling behavior and obtaining the critical torsional loads. A detailed numerical study is carried out to bring out the effects of nonhomogeneity parameter, transverse shear, and aspect ratio on the critical torsional loads. Validity of the present analysis was checked by comparing the results with those are available in the literature.

Keywords: Buckling, Nonhomogeneous cylindrical shell, Torsion, First order shear deformation theory

Introduction

Cylindrical shells with various material properties are frequently analyzed in order to economize on the amount of material used, to lighten of the shells, and to increase the strength of shells. It is known that by carefully choosing this parameter, a significant increase in stiffness, buckling and vibration capacities of the shell may be obtained. Many investigations have been reported for buckling problem of cylindrical shells (e.g. references [1-5]). Another case of buckling problem is those written for buckling behavior of orthotropic, composite and nonhomogeneous shells under various mechanical and thermal loads (e.g. references [6-10]). There are also many papers dealing with the buckling of functionally graded cylindrical shells which their material properties vary in the thickness direction (e.g. references [11-22]). From the above-mentioned references, it is evident that a few studies have focused on the buckling behavior of nonhomogeneous cylindrical shells. Recently, nonhomogeneous shell structures have found wide applications in aerospace, automotive and marine industrials. The nonhomogeneity of materials can be created due to various problems such as production techniques, radiation effect, and thermal polishing processes. The properties of these materials vary as a piecewise continuous or continuous functions of position in the body. By carefully choosing the nonhomogeneous properties, we can decrease the geometry dimensions and weights of the shell structures.

This work presents the torsional buckling analysis of nonhomogeneous circular long cylindrical shells based on the first order shear deformation theory. Using the adjacent equilibrium criterion method, the governing stability equations in terms of force and moment resultants are derived. The closed-form solution is applied to help understand the buckling behavior of nonhomogeneous shell. The shell is assumed to be long enough and the influence of boundary conditions on the critical loads is little. The effects of the shell characteristics variations and nonhomogeneity parameter on the smallest torsional buckling loads are discussed in detail.

Shell Stability Equations

A cylindrical shell of mean radius R, thickness h, and length L, is considered with the cylindrical coordinates (x, θ, z) as shown in Figure 1. The material constitution is varied gradually by the following function [10]

$$P(z) = P_0(1 + k\phi(z)) \tag{1}$$

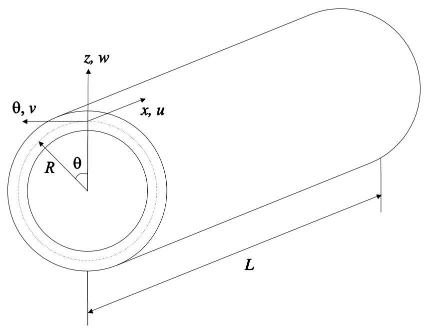

Figure 1. Schematic view of the problem studied

where P is the corresponding properties of nonhomogeneous cylindrical shell which can be substituted by modulus of elasticity E and the Poisson's ratio ν, P_0 denotes a material property of homogeneous shell, k is the nonhomogeneity parameter satisfying $0 \le k < 1$, and $\phi(z)$ is continuous function of the variation of material property. For simplicity, the Poisson's ratio is held to be constant. The following expressions including the non-linear terms are considered for kinematic relations

$$\varepsilon_x^0 = u_{0,x} + \frac{1}{2} w_{0,x}^2$$

$$\varepsilon_\theta^0 = \frac{v_{0,\theta} + w_0}{R} + \frac{1}{2}\left(\frac{v_0 - w_{0,\theta}}{R}\right)^2$$

$$\gamma_{x\theta}^0 = \frac{u_{0,\theta}}{R} + v_{0,x} + \frac{w_{0,x} w_{0,\theta} - v_0 w_{0,x}}{R}$$

$$\gamma_{xz}^0 = u_1 + w_{0,x}, \qquad \gamma_{\theta z}^0 = v_1 + \frac{w_{0,\theta}}{R}$$

$$\gamma_{\theta z}^0 = v_1 + \frac{w_{0,\theta}}{R}$$

$$\kappa_x = u_{1,x}, \qquad \kappa_\theta = \frac{v_{1,\theta}}{R}, \qquad \kappa_{x\theta} = \frac{u_{1,\theta}}{R} + v_{1,x} \tag{2}$$

where u_0 (x,θ), v_0 (x,θ) and w_0 (x,θ) are the middle surface displacements, u_1 (x,θ) and v_1 (x,θ) describe the rotations about θ- and x-axes, respectively, and ",x" and ",θ" denote partial derivatives respect to x and θ, respectively. By substituting the stress-strain relations into the definitions of force and moment resultants, the following constitutive equations are obtained

$$\begin{aligned}
N_x &= A_{11}\varepsilon_x^0 + B_{11}\kappa_x + A_{12}\varepsilon_\theta^0 + B_{12}\kappa_\theta \\
M_x &= B_{11}\varepsilon_x^0 + D_{11}\kappa_x + B_{12}\varepsilon_\theta^0 + D_{12}\kappa_\theta \\
N_\theta &= A_{12}\varepsilon_x^0 + B_{12}\kappa_x + A_{11}\varepsilon_\theta^0 + B_{11}\kappa_\theta \\
M_\theta &= B_{12}\varepsilon_x^0 + D_{12}\kappa_x + B_{11}\varepsilon_\theta^0 + D_{11}\kappa_\theta \\
N_{x\theta} &= A_{22}\gamma_{x\theta}^0 + B_{22}\kappa_{x\theta} \\
M_{x\theta} &= B_{22}\gamma_{x\theta}^0 + D_{22}\kappa_{x\theta} \\
Q_x &= A_{22}\gamma_{xz}^0 \\
Q_\theta &= A_{22}\gamma_{\theta z}^0
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
(A_{11}, B_{11}, D_{11}) &= \int_{-h/2}^{h/2} \frac{E(z)}{1-\nu^2}(1, z, z^2)\, dz \\
(A_{12}, B_{12}, D_{12}) &= \int_{-h/2}^{h/2} \frac{\nu E(z)}{1-\nu^2}(1, z, z^2)\, dz \\
(A_{22}, B_{22}, D_{22}) &= \int_{-h/2}^{h/2} \frac{E(z)}{2+2\nu}(1, z, z^2)\, dz
\end{aligned} \tag{4}$$

By employing the variational approach, the three nonlinear equations of equilibrium can be derived. The adjacent equilibrium criterion method [2] is used to establish the stability equations. If we assume that the equilibrium state of an FG cylindrical shell under mechanical load is defined in terms of displacement components u^0, v^0, and w^0, the displacement components of a neighboring stable state differ by u^1, v^1, and w^1, with respect to the equilibrium position. Thus, the Donnell stability equations for FG cylindrical shells under torsion can be obtained as follows:

$$
\begin{aligned}
&RN^1_{x,x}+N^1_{x\theta,x}=0\\
&RN^1_{x\theta,x}+N^1_{\theta,\theta}+N^0_{x\theta}w^1_{0,x}=0\\
&RM^1_{x,x}+M^1_{x\theta,x}-RQ^1_x=0\\
&RM^1_{x\theta,x}+M^1_{\theta,\theta}-RQ^1_\theta=0\\
&R^2Q^1_{x,x}+RQ^1_{\theta,\theta}+R^2N^0_xw^1_{,xx}\\
&\qquad+2RN^0_{x\theta}w^1_{0,x\theta}-RN^0_{x\theta}v^1_{0,x}=0
\end{aligned}
\tag{5}
$$

Here, $N_{x\theta}=\tau/h$, where τ is the torsional loads. For torsional loading the displacement function can be expressed as

$$w=A\sin(m_1x-n\theta) \tag{6}$$

where $m_1=m\pi R/L$, m is the half-wave number in the direction of the x-axis and n is the wave number in the direction of the θ-axis. However, the assumed displacement function satisfies the resulting equations, but does not satisfy any of the commonly used boundary conditions at the cylinder ends. This expression may be used to study the buckling behavior of nonhomogeneous long cylindrical shells that the influence of boundary conditions on the critical loads is little [2]. By substituting expression (6) in the governing equations (5), a function is obtained for buckling loads of nonhomogeneous cylindrical shells and any values of wave numbers.

For the given values of shell parameters, the values of the longitudinal and circumferential wave numbers are chosen by an optimization program to give the smallest value of buckling load.

Results and Discussion

In this section, numerical results are given for critical torsional loads of nonhomogeneous thin cylindrical shells which are long enough. The stainless steel is used as homogeneous material with E_0=200 GPa and ν_0=0.3. Nonhomogeneous function for variations of cylindrical shell is considered as a quadratic function with $\phi(z)=(z/h)^2$. Figure 2 illustrates the through-the-thickness distribution of the Young's modulus for different values of k. Since the assumed displacement function only satisfies the stability equations and does not satisfy any of the commonly used boundary conditions at the cylinder ends, the presented results can be applicable when the cylindrical shell is long enough and the influence of boundary conditions on the critical loads is less. First, the results for a homogeneous cylindrical shell comprised of alumina (E=380 GPa, ν=0.3) are validated and then the results are given for nonhomogeneous cylindrical shells. The comparison study with the results of Ref. [22] which is based on the classical shell theory is given in Table 1.

Table 2 gives the critical torsional loads for nonhomogeneous cylindrical shells. The numbers in the bracket are the longitudinal and circumferential wave numbers, respectively. A good agreement can be seen between the results.

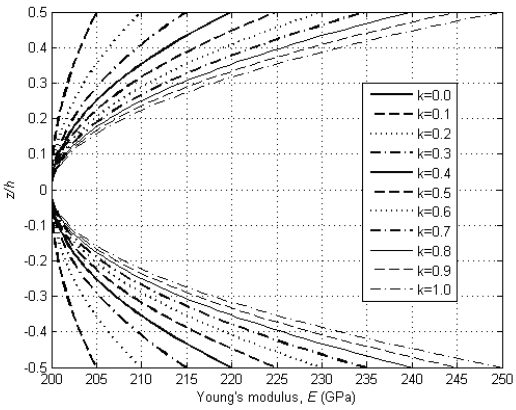

Figure 2. Through-the-thickness distribution of nonhomogeneity property for Young's modulus

L/R	R/h	m	n	Ref. [22]	Present
10	10	1	2	4811.700	4773.523
	50	1	2	345.632	361.334
	100	1	3	174.421	176.514
	500	1	4	19.525	20.036
40	10	2	1	1801.223	1860.974
	50	1	1	172.447	180.624
	100	1	1	105.162	113.352
	500	1	2	9.721	9.978
100	10	4	1	1761.193	1789.934

	50	2	1	154.194	158.390
	100	1	1	60.845	61.372
	500	1	1	7.570	8.097

Table1- Validation of critical torsional loads (MPa) for homogeneous cylindrical shell with E=380 GPa

When the nonhomogeneity parameter k is increased, values of the critical torsional loads are also increased, but the wavenumbers have not changed. Results show that the values of the critical loads are very sensitive according to the wavenumbers. When the thickness ratio R/h is increased, the value of the critical loads is decreased, but it seems that the variations of the circumferential wavenumbers are dependent on various parameters.

k	*R*/*h*			
	10	50	100	500
	(2,1)	(1,1)	(1,1)	(1,2)
0.0	948.025[a]	90.764[a]	55.352[a]	5.116[a]
	979.460[b]	95.065[b]	59.658[b]	5.252[b]
0.1	960.006[a]	91.835[a]	55.892[a]	5.184[a]
	991.678[b]	96.172[b]	60.234[b]	5.321[b]
0.3	983.968[a]	93.978[a]	56.972[a]	5.319[a]
	1016.116[b]	98.387[b]	61.386[b]	5.458[b]
0.5	1007.930[a]	96.120[a]	58.052[a]	5.454[a]
	1064.990[b]	100.599[b]	62.538[b]	5.594[b]
0.7	1031.892[a]	98.262[a]	59.132[a]	5.589[a]
	1064.990[b]	102.814[b]	63.688[b]	5.732[b]
0.9	1055.854[a]	100.405[a]	60.212[a]	5.724[a]
	1089.426[b]	105.028[b]	64.840[b]	5.871[b]
[a]Ref. [23]; [b]Present				

Table 2- The critical torsional loads (MPa) for nonhomogeneous cylindrical shell with L/R=40

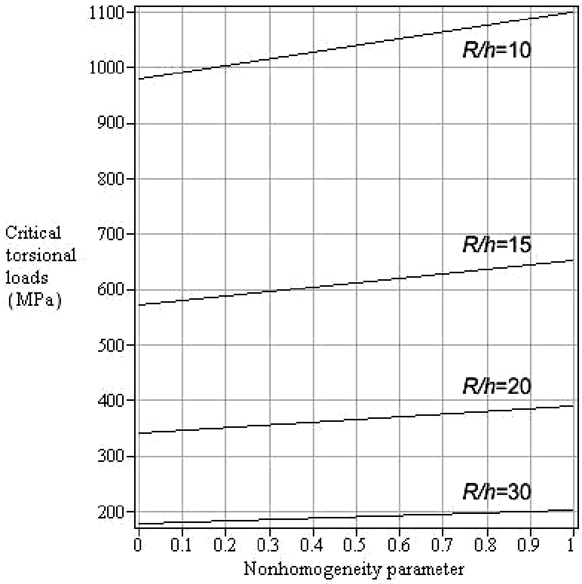

Figure 3. Effect of the nonhomogeneity parameter and thickness ratio on the critical torsional load of shell

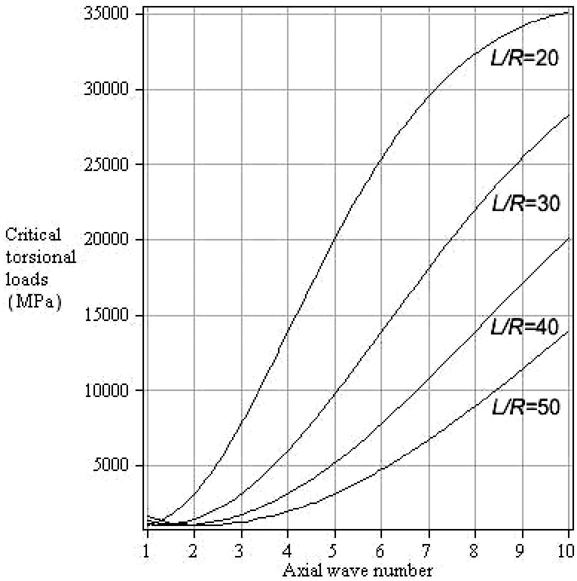

Figure 4. Effect of the axial wave number and aspect ratio on the critical torsional load of shell

It is found that for k=0.1, the critical torsional load computed with R/h=10 differs from that computed with k=0.9 by more than 13%. The difference is smaller for higher values of L/R and R/h ratios. It is evident that results are close to each other for thin and long shells, but are quite different for thick shells. As the aspect ratio L/R is increased, the longitudinal wavenumber is increased in some cases. Figure 3 shows the variations of nonhomogeneity parameter on critical torsional loads for various values of thickness ratio R/h. The variations of the critical torsional load of nonhomogeneous cylindrical shells versus the axial wave number, and aspect ratio L/R under torsional loading are illustrated in Figure 4.

It is observed that the torsional load is decreased by the increase of the thickness ratio R/h as same as for the increase of the aspect ratio L/R. The critical torsional loads attain minimum values for lower values of axial wave number.

Conclusions

The buckling of nonhomogeneous cylindrical shells subjected to torsional load is studied in this paper. The material properties of nonhomogeneous shell are varied along the thickness coordinate. Adjacent equilibrium criterion method is employed to determine the critical torsional loads. The results show that the values of the critical torsional loads are affected by the compositional profile variations, nonhomogeneity parameter and the variations of the shell geometry.

The following is concluded:

i. For all cases, when the nonhomogeneous profile changes as a quadratic function, the critical torsional loads changes linearly with respect to nonhomogeneity parameter.
ii. When the thickness and aspect ratios are increased, the values of the critical torsional loads are decreased, but the wavenumbers have an irregular increase.
iii. Depending on the shell geometry, the longitudinal wavenumbers corresponding to lowest values of the critical torsional load can be greater than one.

Acknowledgments

This work is supported by a Research Grant entitled "Torsional Buckling of Nonhomogeneous Cylindrical Shells" from Islamic Azad University-Arak Branch. The authors gratefully acknowledge the financial support of this work.

References

1- Donnell, L.H., "Stability of thin-walled tubes under torsion", 1934, NACA Tr-479.
2- Brush, D.O., and Almorth, B.O., "Buckling of bars, plates, and shells", New York-McGraw-Hill Book Company, 1975.
3- Shen, H.-S., and Chen, T.-Y., "A boundary layer theory for the buckling of thin cylindrical shells under external pressure", 1988, *Applied Mathematics and Mechanics*, Vol. 9, pp. 557-571.
4- Thornton, E.A., "Thermal buckling of plates and shells", 1993, *Applied Mechanics Review*, Vol. 46(10), pp. 485-506.
5- Eslami, M.R., and Ziaii, A.R., "Thermoelastic buckling of thin cylindrical shells based on improved stability equations", 1996, *Journal of Thermal Stresses*, Vol. 19(4), pp. 299-315.
6- Kardomateas, G.A., and Philobos, M.S., "Buckling of thick orthotropic cylindrical shells under combined external pressure and axial compression", 1995, *AIAA Journal*, Vol. 33(10), pp. 1946-1953.
7- Shen, H.-S., "Post-buckling analysis of imperfect stiffened laminated cylindrical shells under combined external pressure and axial compression", 1997, *Computers and Structures*, Vol. 63, pp. 335-348.
8- Thangaratnam, R., "Thermal buckling of laminated composite shells", 1990, *AIAA Journal*, Vol. 28(5), pp. 859-860.
9- Eslami, M.R., "Buckling of composite cylindrical shells under mechanical and thermal loads", 1999, *Journal of Thermal Stresses*, Vol. 22(6), pp. 527-545.
10- Sofiyev, A.H., Yücel, K., Avcar, M., and Zerin, Z., "The dynamic stability of orthotropic cylindrical shells with non-homogeneous material properties under axial compressive load varying as a parabolic function of time", 2006, *Journal of Reinforced Plastics and Composites*, Vol. 25(18), pp. 1877-1886.
11- Reddy, J.N., and Chin, CD., "Thermoelastical analysis of functionally graded cylinders and plates", 1998, *Journal of Thermal Stresses*, Vol. 21, pp. 593-626.
12- Ng, T.Y., Lam, K.Y., Liew, K.M., and Reddy, J.N., "Dynamic stability analysis of functionally graded cylindrical shells under periodic axial loading", 2001, *International*

Journal of Solids and Structures, Vol. 38, pp. 1295-1309.

13- Shahsiah, R., and Eslami, M.R., "Functionally graded cylindrical shell thermal instability based on improved Donnell equation", 2003, *AIAA Journal*, Vol. 41(9), pp. 1819-1824.

14- Woo, J., and Meguid, S.A., "Thermomechanical postbuckling analysis of functionally graded plates and shallow cylindrical shells", 2003, *Acta Mechanica*, Vol. 165, pp. 99-115.

15- Shen, H.S., "Post-buckling analysis of pressure loaded functionally graded cylindrical shells in thermal environments", 2003, *Engineering Structures*, Vol. 25, pp. 487-497.

16- Sofiyev, A.H., and Schnack, E., "The stability of functionally graded cylindrical shells under linearly increasing dynamic torsional loading", 2004, *Engineering Structures*, Vol. 26, pp. 1321-1331.

17- Shen, H.S., and Noda N., 2005, "Post-buckling of FGM cylindrical shell under combined axial and radial mechanical loads in thermal environments", 2004, *International Journal of Solids and Structures*, Vol. 42, pp. 4641-4662.

18- Narimani, R., Karami Khorramabadi, M., and Khazaeinejad, P., "Mechanical buckling of functionally graded cylindrical shells based on the first order shear deformation theory", *ASME Pressure Vessels and Piping Division Conference*, July 22-26, 2007, San Antonio, Texas.

19- Sofiyev, A.H., "Vibration and stability of composite cylindrical shells containing a FG layer subjected to various loads", 2007, *Structural Engineering and Mechanics*, Vol. 27(3), pp. 365-391.

20- Khazaeinejad, P., Najafizadeh, M.M., Jenabi, J., and Isvandzibaei, M.R., "On the buckling of functionally graded cylindrical shells under combined external pressure and axial compression", 2009, *ASME Journal of Pressure Vessel and Technology*, Vol. 132(6).

21- Khazaeinejad, P., and Najafizadeh, M.M., "Mechanical buckling of cylindrical shells with varying material properties", 2010, *Proceedings of the IMechE, Part C: Journal of Mechanical Engineering Science*, Vol. 224 (C8), pp. 1551-1557.

22- Karami Khorramabadi, M., and Khazaeinejad, P., "On the stability of functionally graded cylindrical shells under torsion", *17th Annual (International) Conference on Mechanical Engineering*, May 19-21, 2009, University of Tehran, Iran.