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Deflection of buckled annular porous plate

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Abstract: In this research, deflection of buckled an annular saturated porous plate under uniformly distributed radial pressure load for clamped-clamped boundary conditions has been studied. The pores are saturated by fluid and properties of the plate are variable continually in the thickness direction. Governing equations are obtained by classical plate theory and Sanders nonlinear strain-displacement relation. Shooting method is used for numerical solution of nonlinear ordinary differential equations. For verifying present solution, obtained results may be compared by solid circular plate.

Keywords: Annular plate, deflection, Shooting method porous materials, Radial ratio

1- Introduction

porous materials are consisting of solid matrix and fluid inside the pores of matrix which can be liquid or gas. Porous materials Can be discovered in nature like wood, rock and soil. Artificial Porous materials are like metals, foams, ceramic and polymers, and they are used in several industrials such as aerospace and building [14]. The Porous plate was investigated and studied for many years by many researchers. The first author that studied porous plates was Biot[1]. Buckling of porous slab saturated with fluid studied by him. He showed that with change pore compressibility, critical buckling load also changes. A porous beam with simply supported boundary conditions was investigated by Magnucki and Stasiewicz [2]. They show porosity coefficient is proportional to buckling load. Magnucki et al. [3] studied bending and buckling rectangular porous plates with variable properties in the direction of thickness under in-plane compression and transverse deformation theory. The Analytic solution in terms of Fourier series for the large deflection of the functionally graded plate and shallow shells under transversal mechanical load and temperature field was studied by Woo and Meguid [20]. Abrate[21] investigated that the natural frequencies of the functionally graded plates are proportionate to those of homogenous isotropic plates. They Also obtained similar results for buckling load and static deflection of the FG plate. Javaheri and Eslami [22,23] showed a closed-form solution for buckling temperature of rectangular FG plates. The Plate was under different types of thermal loads and based on classical plate theory and higher-order shear deformation. Also, they studied Buckling of FG plates under in-planecompression base on CPT [24]. Nonlinear bending and axisymmetric thermal buckling and post-buckling of functionally graded annular studied by Aghelinejad et al. [25]. They used the von-Karman plate theory and shooting method. They investigated bending and post-buckling behavior of FG annular plates. Sepahi et al. [26] obtained thermal buckling and post-buckling of the functionally graded annular plate which properties of plate graded in the radial direction. They used FSDT for gained the equations. Khorshidvand et al. [27] studied buckling analysis of a circular functionally graded plate with surface-bounded piezoelectric layers based on first order shear deformation theory. Thermal buckling analysis of annular/circular microplates made from functionally graded Graphene reinforced porous nanocomposite investigated by Arshid et al. [28] They located microstructure on Pasternak elastic foundation. Generalized differential quadrature method (GDQM) was used to solve the governing equations.E. Arshid et al. [29] studied Static and Dynamic analysis of FG-GNPs reinforced Porous Nanocomposite Annular Micro-Plates. They analyzed bending, buckling and free vibration of micro-scaled functionally graded graphene nanoplates reinforced porous nanocomposite annular plate located on the bi-parameter elastic foundation exposed to hygo-thermo-mechanical loads. Porosity-dependent vibration analysis of FG microplates embedded by polymeric nanocomposite were studied by E. Arshid and et al. [30].

2. Governing equations

an annular plate like fig a with inner radius R1 and outer radius R2 and thickness h is supposed. The plate is created from porous materials. Its pores are saturated with fluid. Cylindrical coordination is set in the middle of the plane and z-axis is in the thickness directions and z is equal $-\frac{h}{2}\ to + \frac{h}{2}$. There are three cases for pores distribution. Plate properties are variable continually in the thickness direction. Three cases of pore dispensation are considered along the thickness direction [2,9,10,14]. For the first case, nonlinear symmetric dispensation is considered. The middle plane of the plate is symmetry plane and moduli of elasticity, which are related to pore distributions, is obtained as follow and relation between E, G and z are [14]:

$E(z)=E0[1-e1\cos(\frac{\pi z}{h})]$ (1)

$G(z)=G0[1-e1\cos(\frac{\pi z}{h})]$ (2)

$e1=1-(\frac{E_1}{E_0})=1-(\frac{G_1}{G_0})$ (3)

e1 is known as the porosity coefficient of the plate(0<e1<1), E1 and E0 are young moduli at the middle of plane(z=0) and the upper and lower surfaces of the plate($z=\pm\frac{h}{2}$), respectively. G1 is shear moduli at (z=0) and G0 is shear moduli at ($z=\pm\frac{h}{2}$). Ej=2Gj (1+ ν), j=0,1, is the relation between elastic and shear moduli and ν is Poisson's ratio that invariant supposed in the thickness direction of the plate. In the case of nonsymmetric pore dispensation, moduli of elasticity are defined [14]:

$E(z)=E0\{1-e1\cos[(\frac{\pi}{2h})(z+\frac{h}{2})]\}$ (4)

$G(z)=G0\{1-e1\cos[(\frac{\pi}{2h})(z+\frac{h}{2})]\}$ (5)

E1 and E0 are young moduli at lower ($z=\frac{-h}{2}$) and the upper ($z=\frac{+h}{2}$) surfaces of the plate, respectively. G1 and G0 are shear moduli at the upper and lower surfaces of the plate, respectively. Fig.2 shows asymmetric pore distribution for variation shear modulus along thickness direction.

In monotonous and moduli of elasticity are expressed [14]:

$E(z)=E0(1-e1)$ (6)

$G(z)=G0(1-e1)$ (7)

2.2. stress-strain relationship

Biot [1] linear poroelasticity theory has two specifications [14]:

1.increasing the pore pressure leads to dilation of pore.

2.compression of the pores makes increment the pressure of the pores.

Stress-strain relations for elastic porous material is expressed [14]:

$$\sigma ij=2G\varepsilon ii+\left(\frac{2G\nu_u}{1-2\nu_u}\right)\varepsilon kk\delta ij-\alpha p\delta ij \qquad (8)$$

$$p=M(\xi-\alpha\varepsilon kk) \qquad (9)$$

$$M=\frac{2G(\nu_u-\nu)}{\alpha^2(1-2\nu_u)(1-2\nu)} \qquad (10)$$

$$\nu u=\frac{\nu+\frac{\alpha B(1-2\nu)}{3}}{1-\frac{\alpha B(1-2\nu)}{3}} \qquad (11)$$

p is called pore flowing pressure; M is biot's moduli; νu is unemptied of Poisson's ratio ($0<\nu u<0.5$); α is biot's coefficient ($0<\alpha<1$); B is Skempton pore pressure coefficient; ξ is variation of fluid volume; εkk is volumetric strain. By shortening Eq. (8) to plane stress in cylindrical coordinate and under undrained condition ($\xi=0$), it will be obtained [10,14]:

$$\sigma r=A1\varepsilon r+B1\,\varepsilon\theta \qquad (12)$$

$$\sigma\theta=A1\,\varepsilon\theta+B1\,\varepsilon r \qquad (13)$$

$$p=M(-\alpha\varepsilon kk) \qquad (14)$$

A1 and B1 are invariable amounts in terms of the constant $\overline{C1}$ and $\overline{C2}$ [14]:

$$\overline{C1}=2\left[1+\frac{\nu_u}{1-2\nu_u}+\frac{\nu_u-\nu}{(1-2\nu_u)(1-2\nu)}\right]G(z) \qquad (15)$$

$$\overline{C2}=C1-2G(z) \qquad (16)$$

$$A1=\left(\frac{2}{1-\nu_u{}^2}\right)\left[1+\nu u+\frac{(\nu_u-\nu)(1+\nu_u)}{1-2\nu}\left(1-\frac{\overline{C2}}{\overline{C1}}\right)\right]G(z) \qquad (17)$$

$$B1=\left(\frac{2}{1-\nu_u{}^2}\right)\left[(1+\nu u)\,\nu u+\frac{(\nu_u-\nu)(1+\nu_u)}{1-2\nu}\left(1-\frac{\overline{C2}}{\overline{C1}}\right)\right]G(z) \qquad (18)$$

2.3. relation between strain and displacement:

according to classical plate theory and reference [۳۲], Strain relations for distance the middle plane from z are explained [14]:

$$\varepsilon r=\bar{\varepsilon}_r+zkr \qquad (19)$$

$$\varepsilon\theta=\bar{\varepsilon}_\theta+zk\theta \qquad (20)$$

that $\bar{\varepsilon}_r$ and $\bar{\varepsilon}_\theta$ called engineering strain in the middle plane and kr , kθ are curvatures.

According to sanders assumption and in terms of displacement component [14,۳۳]:

$$\bar{\varepsilon}_r=\frac{dU}{dr}+\frac{1}{2}(\frac{dW}{dr})2 \quad (21)$$

$$\bar{\varepsilon}_\theta=\frac{U}{r} \quad (22)$$

$$kr=-\frac{d^2W}{d^2r} \quad (23)$$

$$k\theta=-\frac{1}{r}\frac{dW}{dr} \quad (24)$$

That W and U are transverse and radial displacement components of the middle of the plate in the z and r-Axes orientations.

2.4. obtain the governing equilibrium equation of plate

Governing equilibrium equations of an annular plate under uniform compressive radial load with axisymmetric conditions and use principle of minimum total potential energy are shown [10,14]:

$$\frac{dNr}{dr}+\frac{(Nr-N\theta)}{dr}=0 \quad (25)$$

$$\frac{1}{r}\frac{d}{dr}[r\,Nr\frac{dW}{dr}]-\frac{1}{r}\frac{dM\theta}{dr}+\frac{2}{r}\frac{dMr}{dr}+\frac{d^2Mr}{d^2r}=0 \quad (26)$$

N defined force:

$$\begin{bmatrix}Nr\\N\theta\end{bmatrix}=\int_{-h/2}^{+h/2}\begin{bmatrix}\sigma r\\\sigma\theta\end{bmatrix}dz \quad (27)$$

M defined moment:

$$\begin{bmatrix}Mr\\M\theta\end{bmatrix}=\int_{-h/2}^{+h/2}\begin{bmatrix}\sigma r\\\sigma\theta\end{bmatrix}zdz \quad (28)$$

By substitution of equations (12,13) and (19,20) in equations (27, 28):

$$\begin{Bmatrix}N_r\\N_\theta\end{Bmatrix}=\begin{bmatrix}A_2 & B_2\\B_2 & A_2\end{bmatrix}\begin{Bmatrix}\varepsilon_r\\\varepsilon_\theta\end{Bmatrix}+\begin{bmatrix}A_3 & B_3\\B_3 & A_3\end{bmatrix}\begin{Bmatrix}K_r\\K_\theta\end{Bmatrix} \quad (29)$$

$\begin{Bmatrix} M_r \\ M_\theta \end{Bmatrix}=\begin{bmatrix} A_3 & B_3 \\ B_3 & A_3 \end{bmatrix}\begin{Bmatrix} \varepsilon_r \\ \varepsilon_\theta \end{Bmatrix} + \begin{bmatrix} A_4 & B_4 \\ B_4 & A_4 \end{bmatrix}\begin{Bmatrix} K_r \\ K_\theta \end{Bmatrix}$ (30)

(A2, A3, A4) $=\int_{-h/2}^{+h/2}$ A1(1, z, z2)dz (31)

(B2, B3, B4)= $\int_{-h/2}^{+h/2}$ B1(1, z,z2)dz (32)

By substituting equations Eq. (21,22,23,24) and Eq. (29,30) in equations (25,26) the governing equilibrium equations are obtained:

$A2(\frac{d^2U}{dr^2}+\frac{1}{r}\frac{dU}{dr}-\frac{U}{r^2}+(\frac{d^2W}{dr^2})(\frac{dW}{dr})+\frac{1}{2r}(\frac{dW}{dr})2+A3(-\frac{d^3W}{dr^3}-\frac{1}{r}\frac{d^2W}{dr^2}+\frac{1}{r^2}\frac{dW}{dr})+B2(-\frac{1}{2r}(\frac{dW}{dr})2)=0$ (33)

$(\frac{A_2}{A_4})[\frac{dU}{dr}+\frac{1}{2}(\frac{dW}{dr})2](\frac{d^2W}{dr^2}) + (\frac{B_2}{A_4})[(\frac{U}{r})(\frac{d^2W}{dr^2})+(\frac{A_2}{A_4})(\frac{U}{r})(\frac{1}{r}\frac{dW}{dr})+(\frac{B_2}{A_4})[\frac{dU}{dr}+\frac{1}{2}(\frac{dW}{dr})2](\frac{1}{r}\frac{dW}{dr}) + (\frac{B_3}{A_4})(-\frac{3}{r}\frac{d^2W}{dr^2}\frac{dW}{dr})+(\frac{A_3}{A_4})[-\frac{1}{r^2}(\frac{dW}{dr})2-(\frac{d^2W}{dr^2})2]+(\frac{B_2A_3}{A_4A_2})(\frac{1}{r}\frac{d^2W}{dr^2}\frac{dW}{dr})=$

$(1-\frac{A_3A_3}{A_4A_2})(\frac{d^4W}{dr^4}+\frac{2}{r}\frac{d^3W}{dr^3}-\frac{1}{r^2}\frac{d^2W}{dr^2}+\frac{1}{r^3}\frac{dW}{dr})$ (34)

Boundary conditions:

Clamped-clamped:

r =R1 clamped

w=0, $\frac{dW}{dr}$ =0, u=0 (35)

r=R2 clamped

w=0, $\frac{dW}{dr}$=0, Nr = - P

3.Numerical method

In this article, governing differential equations is a boundary value problem (BVP) and to solve it, the shooting method is used. The governing differential equations of the problem is two coupled differential equations that are written instead of

Six first-order differential equations. For easement Eq. (42) are written like as follow: [14,17,18,25,٣٣,3٤]

$(\frac{dY}{dX})$=H (x, Y) (36)

B0Y(c)=b0 (37)

B1Y (1) =b1 (38)

That:

$$Y= (y1, y2, y3, y4, y5, y6) =\left(w, \frac{dW}{dX}, \frac{d^2W}{dX^2}, \frac{d^3W}{dx^3}, u, \frac{du}{dX}\right) \qquad (39)$$

$$H\ (x, Y)=(y2,y3,y4,\omega,y6,\mu)=\left(\frac{dW}{dX}, \frac{d^2W}{dX^2}, \frac{d^3W}{dx^3}, \frac{d^4W}{dx^4}, \frac{du}{dX}, \frac{d^2u}{dX^2}\right) \qquad (40)$$

$$\omega = -\frac{2}{x}\,y4 + \frac{1}{x^2}\,y3 - \frac{1}{x^3}\,y2+ \{f4(y6+\frac{1}{2}\,y22)\ y3+f2(\frac{1}{x}\,y5\ y3) + f4\ (\frac{1}{x}\,y5\ y2) + f2(y6 + \frac{1}{2}\,y22)\ (\frac{1}{x}\,y2) + (f2f3 - 3f5)\ (\frac{1}{x}\,y3\ y2) - f1(\frac{1}{x^2}\,y22 + y32)\}/(1-f1f3)$$

$$\mu = -\frac{1}{x}\,y6 + \frac{1}{x^2}\,y5 - y3y2 - \frac{1}{2x}\,y22 + f3(y4 + \frac{1}{x}\,y3 - \frac{1}{x^2}\,y2) + f6\ (\frac{1}{2x}y22) \qquad (41)$$

clamped-clamped:

$$B0=\begin{bmatrix} 1\ 0\ 0\ 0\ 0\ 0 \\ 0\ 1\ 0\ 0\ 0\ 0 \\ 0\ 0\ 0\ 0\ 1\ 0 \end{bmatrix} \qquad b0=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (42)$$

$$B1=\left[\begin{matrix}1\\0\\0\end{matrix}\middle|\begin{matrix}0\\1\\\left(\frac{y_2}{2}\right)-\left(\frac{f_8}{x}\right)\end{matrix}\middle|\begin{matrix}0\\0\\-f_3\end{matrix}\middle|\begin{matrix}0\\0\\0\end{matrix}\middle|\begin{matrix}0\\0\\\frac{f_6}{x}\end{matrix}\middle|\begin{matrix}0\\0\\1\end{matrix}\right] \qquad b1=\begin{bmatrix} 0 \\ 0 \\ -\lambda \end{bmatrix} \qquad (43)$$

Initial value problem is expressed:

$$\left(\frac{dZ}{dX}\right)=H\ (x, Z) \qquad (44)$$

$$Z(c)= \{z1 ,z2 , z3, z4, z5, z6\} =I^* \qquad (45)$$

4.numerical results

Deflection of buckled a saturated porous annular plate in this paper is researched. Effect of porosity, pore dispensation and boundary condition and radial ratio are investigated.

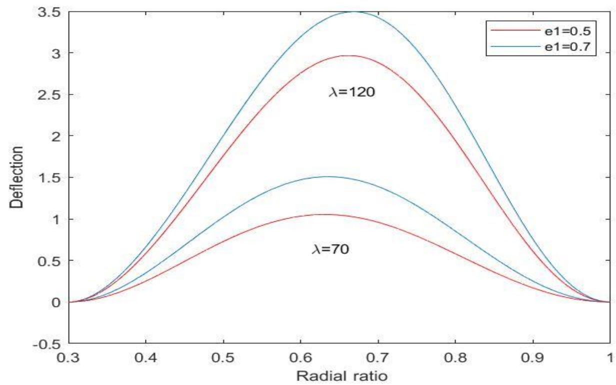
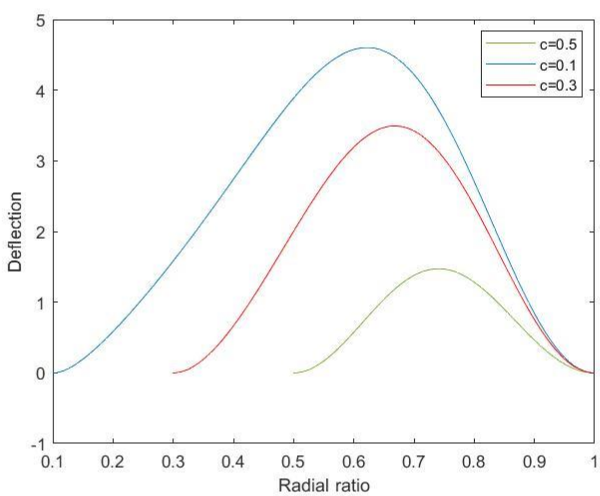


Fig1. Deflection of porous annular plate after buckled for clamped-clamped boundary conditions and for e1=0.5,0.7 (B=0, $\nu = 0.3, c = 0.3$)

Figure.1 is drawn under different boundary conditions and for e1=0.5,0.7 coefficients and for $\lambda = 70,100$.Figure explained the value of deflection and its proportional load for each point of the plate. For this, they can be called post-buckling configuration [14]. Fig 10 presented variations deflection in clamped-clamped boundary conditions for e1=0.5 porosity coefficient.it can be understood that increasing the load causes increase of the deflection. Figure.1shows increasing the porosity coefficient, increase the deflection and reducing the post-buckling strength.



2. variation of radial ratio for deflection of clamped-clamped symmetric pore distribution (e1=0.7, B=0, $\nu = 0.3$, lambda=120)

In Fig 2. investigated effect of variation radial ratio for deflection under the clamped-clamped boundary conditions for e1=0.7 and symmetric pore distribution is selected. The figure expresses with increase the radial ratio, deflection of the annular plate decreasing and also the post-buckling strength of the annular plate increases. It can be found maximum deflection occurred at the middle of the two edges in clamped-clamped boundary condition.

conclusions

In this paper, Deflection of buckled an annular plate under uniform radial pressure was investigated. Mechanical properties plate is variable in the thickness orientation. Three pore distributions are considered such as Symmetric, nonsymmetric and monotonous. Governing equilibrium is obtained from classical plate theory and sanders assumption. In this study boundary condition is considered clamped-clamped. Shooting method is used for solving BVP. The effects of porosity, radial ratio on deflection was investigated. The conclusion of this study description as follows:

This study described with increase the porosity coefficient, Deflection increase, then with decreasing the porosity coefficient, Deflection decrease. As the porosity factor increment, the deflection of the annular plate increases like the circular plate. Maximum deflection in the clamped-clamped boundary conditions occurred in the middle of two edges. Increase the radial ratio, decrease the deflection.

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