

Some fixed point results of T -contractions on partially ordered cone metric spaces under c -distances

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Abstract. The main purpose of this article is to state some fixed point results of T -contractions on partially ordered cone metric spaces under c -distances using two ways; directed and indirected ways. Some notes and corollaries are also added to demonstrate the applicability of main results.

Keywords: T -contraction, partially ordered cone metric space, c -distance, fixed point.

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1. Introduction and preliminaries

In 1996, Kada et al. [10] proposed the following concept of w -distances on metric spaces to improve Ekland's variational principle and Takahashi's non-convex minimization theorem.

Definition 1.1 Assume (X, d) is a metric space and $p : X \times X \rightarrow [0, +\infty)$ is a function satisfying the following conditions for all $x, y, z \in X$:

- (w_1) $p(x, z) \leq p(x, y) + p(y, z)$;
- (w_2) p is lower semi-continuous in its second variable;
- (w_3) for each $\varepsilon > 0$, there exists $\delta > 0$ such that $p(z, x) \leq \delta$ and $p(z, y) \leq \delta$ imply that $d(x, y) \leq \varepsilon$.

Then p is named a w -distance on X .

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Note that each metric is a w -distance, but the converse is not correct. Also, it should be mentioned that a w -distance has two important differences from a usual metric. $w(x, y) = 0$ is not equivalent to $x = y$ and a w -distance is not necessarily symmetric. For these, it is enough to take $w(x, y) = y$ for any $x, y \in [0, +\infty)$. For other examples, convergent properties of w -distances and various fixed point results regarding this distance, see [10, 23, 27] and their references. In 2011, Cho et al. [6] defined a cone version of the w -distance which is called a c -distance and proved several fixed point theorems in ordered cone metric spaces in which cone metric spaces are introduced by Huang and Zhang [9] and partially ordered metric spaces are stated by Ran and Reurings [24].

Definition 1.2 Assume E is a real Banach space and θ denote the zero element in E . A subset P of E is a cone if the followings are held:

- (a) P is closed, non-empty and $P \neq \{\theta\}$;
- (b) $a, b \in [0, +\infty)$ and $x, y \in P$ imply that $ax + by \in P$;
- (c) $x \in P$ and $-x \in P$ imply $x = \theta$.

Given a $P \subset E$, we define a partial order \preceq with respect to P by $x \preceq y$ iff $y - x \in P$. We write $x \prec y$ if $x \preceq y$ and $x \neq y$. Also, we take $x \ll y$ iff $y - x \in \text{int}P$, $\text{int}P$ is interior of P . If $\text{int}P \neq \emptyset$, the cone P is called solid.

Definition 1.3 [9, Huang and Zhang, 2007] Assume $X \neq \emptyset$ and E is a real Banach space equipped with the partial ordering \preceq with respect to the cone $P \subset E$. A mapping $d : X \times X \rightarrow P$ is called a cone metric on X if, for all $x, y, z \in X$, the following conditions are held:

- (d₁) $d(x, y) = \theta$ iff $x = y$;
- (d₂) $d(x, y) = d(y, x)$;
- (d₃) $d(x, z) \preceq d(x, y) + d(y, z)$.

In this manner, (X, d) is called a cone metric space.

Definition 1.4 [6, Cho et al., 2011] Assume (X, d) is a cone metric space. A function $q : X \times X \rightarrow P$ is called a c -distance on X if, for all $x, y, z \in X$, the following properties hold:

- (q₁) $q(x, z) \preceq q(x, y) + q(y, z)$;
- (q₂) if $q(x, y_n) \preceq u$ for some $u = u_x$ and all $n \geq 1$, then $q(x, y) \preceq u$ when $\{y_n\}$ is a sequence in X converging to a $y \in X$;
- (q₃) for all $c \in \text{int}P$, there is $e \in E$ with $\theta \ll e$ so that $q(z, x) \ll e$ and $q(z, y) \ll e$ induce $d(x, y) \ll c$.

Note that each cone metric is a c -distance, but the converse is not correct. Also, it should be mentioned that a c -distance has two important differences from a cone metric. $q(x, y) = \theta$ is not equivalent to $x = y$ and a c -distance is not necessarily symmetric.

Lemma 1.5 [6] Assume (X, d) is a cone metric space, q is a c -distance on X , $\{x_n\}$ and $\{y_n\}$ are sequences in X and $x, y, z \in X$. Also, suppose $\{u_n\}$ and $\{v_n\}$ are two sequences in P converging to θ . Then

- (qp₁) If $q(x_n, y) \preceq u_n$ and $q(x_n, z) \preceq v_n$ for $n \in \mathbb{N}$, then $y = z$. Specially, if $q(x, y) = \theta$ and $q(x, z) = \theta$, then $y = z$.
- (qp₂) If $q(x_n, y_n) \preceq u_n$ and $q(x_n, z) \preceq v_n$ for $n \in \mathbb{N}$, $\{y_n\}$ converges to z .
- (qp₃) If $q(x_n, x_m) \preceq u_n$ for $m > n$, $\{x_n\}$ is a Cauchy sequence in X .
- (qp₄) If $q(y, x_n) \preceq u_n$ for $n \in \mathbb{N}$, $\{x_n\}$ is a Cauchy sequence in X .

For other examples, convergent properties of c -distances and some fixed point results regarding this distance, see [2, 20–22] and their references. Moreover, there have been defined several types of weak distances in various metric spaces by many researchers that some of them can be found in [3, 7, 8, 12, 13, 16, 26] and references therein.

On the other hand, Chi [5, 2009] defined the concept of a T -contraction. After that, many authors applied this concept to prove some well-known fixed point theorems in [14, 15, 17–19, 22] and their reference.

Definition 1.6 Assume (X, d) is a metric space and $f, T : X \rightarrow X$ are two mappings. Then f is called a T -contraction if there is $\alpha \in [0, 1)$ so that

$$d(Tfx, Tf y) \leq \alpha d(Tx, Ty) \quad (1)$$

for all $x, y \in X$.

It is clear that if T is an identity mapping, then T -contraction and Banach contraction will be equal. To prove the existence of a fixed point for T -contraction mapping, we need two conditions for T , which are defined below.

Definition 1.7 [5, 15] Assume (X, d) is a (cone) metric space and $T : X \rightarrow X$ is a mapping. Then T is called

- (i) sequentially convergent if $\{Tx_n\}$ is convergent for every sequence $\{x_n\}$, then $\{x_n\}$ is convergent;
- (ii) continuous if $\lim_{n \rightarrow \infty} x_n = x$ implies that $\lim_{n \rightarrow \infty} Tx_n = Tx$ for all $\{x_n\}$ in X .

But, in 2012, Aydi et al. [1] proved that the fixed point results of T -contractions are equivalent to former fixed point results (also, see [4]). The same is done for a w -distance (c -distance) in [11, 27] and references therein.

Proposition 1.8 [1, 4, 11, 27] Assume (X, d) is a complete metric space and p is a w -distance on X . Also, suppose T is a continuous, injective and sequentially convergent mapping on X . Presume $d^* : X \times X \rightarrow \mathbb{R}$ and $p^* : X \times X \rightarrow [0, +\infty)$ are defined by

$$d^*(x, y) = d(Tx, Ty) \quad \text{and} \quad p^*(x, y) = p(Tx, Ty) \quad (2)$$

for all $x, y \in X$, respectively. Then d^* is a complete metric and p^* is a w -distance.

Note that Proposition 1.8 is held for both cone metric spaces and partially metric spaces and combination of both spaces, named a partially ordered cone metric space.

2. Main results

The following result is the first main theorem of this paper that shows the existence of fixed point for a T -contraction mapping of Chatterjea type on a cone metric space under a c -distance.

Theorem 2.1 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, presume $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, suppose that there are $\alpha, \beta, \gamma : X \rightarrow [0, 1)$ so that the following conditions are held:

- (t₁) $\alpha(fx) \leq \alpha(x)$, $\beta(fx) \leq \beta(x)$ and $\gamma(fx) \leq \gamma(x)$ for all $x \in X$;
- (t₂) $(\alpha + 2\beta + 2\gamma)(x) < 1$ for all $x \in X$;
- (t₃) for all comparable $x, y \in X$,

$$q(Tfx, Tf y) \preceq \alpha(x)q(Tx, Ty) + \beta(x)q(Tx, Tf y) + \gamma(x)q(Ty, Tf x), \quad (3)$$

$$q(Tfy, Tf x) \preceq \alpha(x)q(Ty, Tx) + \beta(x)q(Tfy, Tx) + \gamma(x)q(Tfx, Ty). \quad (4)$$

If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Proof. We will prove this theorem via two ways; directed and indirected methods.

Directed method. If $fx_0 = x_0$, then x_0 is a fixed point of f and the proof ends. Now, presume $fx_0 \neq x_0$. As f is nondecreasing with respect to \sqsubseteq and $x_0 \sqsubseteq fx_0$, we have by induction that

$$x_0 \sqsubseteq x_1 = fx_0 \sqsubseteq \cdots \sqsubseteq x^n = f^n x_0 \sqsubseteq \cdots$$

in which $x_n = fx_{n-1} = f^n x_0$ for all $n \in \mathbb{N}$. Setting $x = x_n$ and $y = x_{n-1}$ in (3), we obtain

$$\begin{aligned} q(Tx_{n+1}, Tx_n) &= q(Tfx_n, Tf x_{n-1}) \\ &\preceq \alpha(x_n)q(Tx_n, Tx_{n-1}) + \beta(x_n)q(Tx_n, Tx_n) \\ &\quad + \gamma(x_n)q(Tx_{n-1}, Tx_{n+1}) \\ &\preceq \alpha(fx_{n-1})q(Tx_n, Tx_{n-1}) \\ &\quad + \beta(fx_{n-1})[q(Tx_n, Tx_{n+1}) + q(Tx_{n+1}, Tx_n)] \\ &\quad + \gamma(fx_{n-1})[q(Tx_{n-1}, Tx_n) + q(Tx_n, Tx_{n+1})] \\ &\preceq \alpha(x_{n-1})q(Tx_n, Tx_{n-1}) + (\beta + \gamma)(x_{n-1})q(Tx_n, Tx_{n+1}) \\ &\quad + \beta(x_{n-1})q(Tx_{n+1}, Tx_n) + \gamma(x_{n-1})q(Tx_{n-1}, Tx_n) \\ &\quad \vdots \\ &\preceq \alpha(x_0)q(Tx_n, Tx_{n-1}) + (\beta + \gamma)(x_0)q(Tx_n, Tx_{n+1}) \\ &\quad + \beta(x_0)q(Tx_{n+1}, Tx_n) + \gamma(x_0)q(Tx_{n-1}, Tx_n). \end{aligned} \quad (5)$$

Similarly, setting $x = x_n$ and $y = x_{n-1}$ in (4), we have

$$\begin{aligned} q(Tx_n, Tx_{n+1}) &\preceq \alpha(x_0)q(Tx_{n-1}, Tx_n) + \beta(x_0)q(Tx_n, Tx_{n+1}) \\ &\quad + (\beta + \gamma)(x_0)q(Tx_{n+1}, Tx_n) + \gamma(x_0)q(Tx_n, Tx_{n-1}). \end{aligned} \quad (6)$$

Adding up (5) and (6), we obtain

$$\begin{aligned} q(Tx_{n+1}, Tx_n) + q(Tx_n, Tx_{n+1}) &\preceq (\alpha + \gamma)(x_0)[q(Tx_n, Tx_{n-1}) + q(Tx_{n-1}, Tx_n)] \\ &\quad + (2\beta + \gamma)(x_0)[q(Tx_{n+1}, Tx_n) + q(Tx_n, Tx_{n+1})]. \end{aligned}$$

Setting

$$u_n = q(Tx_{n+1}, Tx_n) + q(Tx_n, Tx_{n+1}),$$

we have

$$u_n \preceq (\alpha + \gamma)(x_0)u_{n-1} + (2\beta + \gamma)(x_0)u_n.$$

Thus, we have $u_n \preceq \lambda u_{n-1}$ in which, by (t2),

$$\lambda = \frac{(\alpha + \gamma)(x_0)}{1 - (2\beta + \gamma)(x_0)} < 1.$$

Following this process, we have $u_n \preceq \lambda^n u_0$ for all $n \in \mathbb{N}$. Thus,

$$q(Tx_n, Tx_{n+1}) \preceq u_n \preceq \lambda^n [q(Tx_1, Tx_0) + q(Tx_0, Tx_1)]. \quad (7)$$

Assume $m > n$ for $m, n \in \mathbb{N}$. It follows from (7) and $\lambda \in [0, 1)$ that

$$q(Tx_n, Tx_m) \preceq \frac{\lambda^n}{1 - \lambda} [q(Tx_1, Tx_0) + q(Tx_0, Tx_1)].$$

Using Lemma 1.5, $\{Tx_n\}$ is a Cauchy sequence on X . As X is complete, $\{Tx_n\}$ is a convergent sequence. Since T is injective, continuous and sequentially convergent, we conclude that there exists a $x' \in X$ so that $x_n \rightarrow x'$ as $n \rightarrow \infty$. Since f is continuous, $fx_n \rightarrow fx'$ as $n \rightarrow \infty$ and $Tfx_n \rightarrow Tfx'$. Because of the uniqueness of limit, we have $Tfx' = Tx'$. It follows from the injectivity of T that $fx' = x'$; that is, x' is a fixed point of f . Now, suppose that $fz = z$. Then, (3) implies that

$$\begin{aligned} q(Tz, Tz) &= q(Tfz, Tfz) \\ &\preceq \alpha(z)q(Tz, Tz) + \beta(z)q(Tz, Tfz) + \gamma(z)q(Tz, Tfz) \\ &= (\alpha + \beta + \gamma)(z)q(Tz, Tz). \end{aligned}$$

Since

$$(\alpha + \beta + \gamma)(z) < (\alpha + 2\beta + 2\gamma)(z) < 1,$$

we have $q(Tz, Tz) = \theta$. This completes the proof.

Indirected method. Applying Proposition 3.1 of Karimizad's work [11] and Proposition 1.8, we conclude that $d^* : X \times X \rightarrow \mathbb{R}$ and $q^* : X \times X \rightarrow [0, +\infty)$ defined by

$$\begin{cases} d^*(x, y) = d(Tx, Ty), \\ q^*(x, y) = q(Tx, Ty), \end{cases}$$

for all $x, y \in X$ are a complete metric and a c -distance, respectively. In this case, we reach Theorem 3.1 of [21] with notations d^* and q^* , and thus, the resident of the proof follows the proof of Theorem 3.1 of [21]. \blacksquare

Note that if T is an identity mapping, we have the same Theorem 3.1 of [21].

Corollary 2.2 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, suppose $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, presume there are $\alpha, \beta, \gamma > 0$ so that the following conditions hold:

- (t₁) $\alpha + 2\beta + 2\gamma < 1$;
- (t₂) for all comparable $x, y \in X$,

$$\begin{aligned} q(Tfx, Tf y) &\preceq \alpha q(Tx, Ty) + \beta q(Tx, Tf y) + \gamma q(Ty, Tf x), \\ q(Tf y, Tf x) &\preceq \alpha q(Ty, Tx) + \beta q(Tf y, Tx) + \gamma q(Tf x, Ty). \end{aligned}$$

If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Proof. It is enough to take $\alpha(x) = \alpha$, $\beta(x) = \beta$ and $\gamma(x) = \gamma$ in Theorem 2.1. \blacksquare

Again if T is an identity mapping, we have the same Corollary 3.1 of [21].

Next result is the second main theorem of this paper that shows the existence of fixed point for a T -contraction mapping of Kannan type on a cone metric space under a c -distance.

Theorem 2.3 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, presume $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, suppose that there are $\alpha, \beta, \gamma : X \rightarrow [0, 1)$ so that the following conditions are held:

- (t₁) $\alpha(fx) \leq \alpha(x)$, $\beta(fx) \leq \beta(x)$ and $\gamma(fx) \leq \gamma(x)$ for all $x \in X$;
- (t₂) $(\alpha + \beta + \gamma)(x) < 1$ for all $x \in X$;
- (t₃) for all $x, y \in X$ with $x \sqsubseteq y$,

$$q(Tfx, Tf y) \leq \alpha(x)q(Tx, Ty) + \beta(x)q(Tx, Tf y) + \gamma(x)q(Ty, Tf x). \quad (8)$$

If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Proof. The proof of Theorem 2.1 shows that it is not wise to prove such theorems in a directed way while we can obtain them from an indirected way with a short proposition. Hence, Applying Proposition 3.1 of Karimizad's work [11] and Proposition 1.8, we conclude that $d^* : X \times X \rightarrow \mathbb{R}$ and $q^* : X \times X \rightarrow [0, +\infty)$ defined by

$$\begin{cases} d^*(x, y) = d(Tx, Ty), \\ q^*(x, y) = q(Tx, Ty), \end{cases}$$

for all $x, y \in X$ are a complete metric and a c -distance, respectively. In this case, we reach Theorem 3.1 of [25] with notations d^* and q^* , and so, the resident of the proof follows the proof of Theorem 3.1 of [25]. \blacksquare

Corollary 2.4 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, suppose $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, presume there are $\alpha, \beta, \gamma > 0$ so that the following conditions hold:

- (t₁) $\alpha + \beta + \gamma < 1$;
- (t₂) for all $x, y \in X$ with $x \sqsubseteq y$,

$$q(Tfx, Tfy) \preceq \alpha q(Tx, Ty) + \beta q(Tx, Tfx) + \gamma q(Ty, Tfy) \quad (9)$$

If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Proof. It is enough to take $\alpha(x) = \alpha$, $\beta(x) = \beta$ and $\gamma(x) = \gamma$ in Theorem 2.3. \blacksquare

Note that if T is an identity mapping in Theorem 2.3 and Corollary 2.4, we have the same Theorem 3.1 of [25] and Theorem 3.1 of [6], respectively. Moreover, if we take $\beta(x) = \gamma(x) = 0$ and $\beta = \gamma = 0$, we can state well-known contraction, named Banach type.

Theorem 2.5 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, presume $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, suppose that there is $\alpha : X \rightarrow [0, 1)$ so that

$$\alpha(fx) \leq \alpha(x)$$

for all $x \in X$ and

$$q(Tfx, Tfy) \preceq \alpha(x)q(Tx, Ty) \quad (10)$$

for all $x, y \in X$ with $x \sqsubseteq y$. If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Corollary 2.6 Assume (X, \sqsubseteq, d) is a complete partially ordered cone metric space and q is a c -distance on X . Also, suppose $f, T : X \rightarrow X$ are two mapping so that T is injective, continuous and sequentially convergent and f is continuous and nondecreasing respect to \sqsubseteq . Moreover, presume there is $\alpha \in [0, 1)$ so that

$$q(Tfx, Tfy) \preceq \alpha q(Tx, Ty) \quad (11)$$

for all $x, y \in X$ with $x \sqsubseteq y$. If there exists $x_0 \in X$ so that $x_0 \sqsubseteq fx_0$, then f has a fixed point. Moreover, if $fz = z$, then $q(Tz, Tz) = \theta$.

Note that if T is an identity mapping in Theorem 2.5 and Corollary 2.6, we have the same Theorem 3.2 of [11], respectively. It should be noted that if we take $E = \mathbb{R}$ and $P = [0, +\infty)$, we can obtain Theorems 2.1-2.3-2.5 and Corollaries 2.2-2.4-2.6 in metric spaces under a w -distance.

Note that all examples in references of this paper can be arranged by the main theorems and their corresponding corollaries to show the existence of fixed points of T -contractions on a partially ordered cone metric space under a c -distance.

3. Conclusion

In this paper, we proved some famous fixed point theorems of T -contraction on partially ordered cone metric spaces under c -distances using two methods; directed and indirected

ways. Thus, our theorems and corollaries unify, extend and generalize well-known comparable results of fixed point theory in cone metric spaces under c -distances. Moreover, we gave some examples and remarks to show the importance of obtained results.

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