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Soft regular generalized b-closed sets in soft topological spaces

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Abstract. The main purpose of this paper is to introduce and study new classes of soft closed sets like soft regular generalized b-closed sets in soft topological spaces (briefly soft rgb-closed set) Moreover, soft $rg\alpha$-closed, soft gpr-closed, soft gb-closed, soft gsp-closed, soft $g\alpha$-closed, soft $g\alpha b$-closed, and soft sgb-closed sets in soft topological spaces are introduced in this paper and we investigate the relations between soft rgb-closed set and the associated soft sets. Also, the concept of soft semi-regularization of soft topology is introduced and studied their some properties. We introduce these concepts which are defined over an initial universe with a fixed set of parameters.

Keywords: Soft set, soft g-closed, soft rg-closed, soft b-closed.

2010 AMS Subject Classification: 54A10, 54C10, 54A05.

1. Introduction

In 1999, D. Molodtsov [12] introduced the concept of soft set theory to solve complicated problems in economics, engineering, and environment. He has established the fundamental results of this new theory and successfully applied the soft set theory into several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. Soft set theory has a wider application and its progress is very rapid in different fields. Levine [10] introduced generalized closed sets in general topology. Kannan [9] introduced soft generlized closed and open sets in soft

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topological spaces which are defined over an initial universe with a fixed set of parameters. He studied their some properties. After then Yuksel et al. [16] studied behavior relative to soft subspaces of soft generalized closed sets and continued investigating the properties of soft generalized closed and open sets. They established their several properties in a soft compact (soft Lindel of, soft countably compact, soft regular, soft normal) space. Muhammad Shabir and Munazza Naz [13] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. Soft semi-open sets and its properties were introduced and studied by Bin Chen[4]. The main purpose of this paper is to introduce and study new classes of soft closed sets like soft rgb-closed, soft $rg\alpha$-closed, soft gpr-closed, soft gb-closed, soft gsp-closed, soft $g\alpha$-closed, soft $g\alpha b$-closed, and soft sgb-closed sets in soft topological spaces. Moreover, the concept of soft semi-regularization of soft topology is introduced and studied their some properties. Let (F, A) be a soft set over U, the soft closure of (F, A) and soft interior of (F, A) will be denoted by $cl^S(F, A)$ and $int^S(F, A)$ respectively, union of all soft b-open sets over U contained in (F, A) is called soft b-interior of (F, A) and it is denoted by $bint^S(F, A)$, the intersection of all soft b-closed sets over U containing (F, A) is called soft b-closure of (F, A) and it is denoted by $bcl^S(F, A)$.

2. Preliminaries

We now begin by recalling some definitions and then we shall give some of the basic consequences of our definitions, further results in this area are given in [3.10, 3.21, 3.24, 3.26, 3.29, 3.30, 4.3, 4.8].

Definition 2.1 ([12], [15]) A pair (F, A) is called a soft set (over U) where F is a mapping $F : A \longrightarrow P(U)$. In other words, the soft set is a parameterized family of subsets of the set U. Every set $F(e)$, $e \in A$, from this family may be considered as the set of e-elements of the soft set (F, A), or as the set of e-approximate elements of the soft set. Clearly, a soft set is not a set. For two soft sets (F, A) and (G, B) over the common universe U, we say that (F, A) is a soft subset of (G, B) if $A \subseteq B$ and for all $e \in A$, $F(e)$ and $G(e)$ are identical approximations. We write $(F, A)\tilde{\subseteq}(G, B)$. (F, A) is said to be a soft superset of (G, B), if (G, B) is a soft subset of (F, A). Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A). A soft set (F, A) over U is called a null soft set, denoted by $\Phi = (\phi, \phi)$, if, $F(e) = \phi$ for all $e \in A$. Similarly, it is called universal soft set, denoted by (U, E), if, $F(e) = U$, for all $e \in A$. The collection of soft sets (F, A) over a universe U and the parameter set A is a family of soft sets denoted by $SS(U_A)$.

Definition 2.2 ([11]) The union of two soft sets (F, A) and (G, B) over U is the soft set (H, C), where $C = A \cup B$ and

$$H(e) = \begin{cases} F(e) & e \in A - B \\ G(e) & e \in B - A \\ F(e) \cup G(e) & e \in A \cap B \end{cases}, \tag{1}$$

for all $e \in C$. We write $(F, A)\tilde{\cup}(G, B) = (H, C)$. [5] The intersection (H, C) of (F, A) and (G, B) over U, denoted $(F, A)\tilde{\cap}(G, B)$, is defined as $C = A \cap B$, and $H(e) = F(e) \cap G(e)$ for all $e \in C$.

Definition 2.3 ([18]) The soft set $(F, A) \in SS(U_A)$ is called a soft point in (U, A), denoted by e_F, if for the element $e \in A$, $F(e) \neq \phi$ and $F(e') = \phi$ for all $e' \in A - \{e\}$. The soft point e_F is said to be in the soft set (G, A), denoted by $e_F \tilde{\in} (G, A)$, if for the element $e \in A$ and $F(e) \subseteq G(e)$.

Definition 2.4 ([13]) The difference (H, E) of two soft sets (F, E) and (G, E) over U, denoted by $(F, E) \backslash (G, E)$, is defined as $H(e) = F(e) \backslash G(e)$ for all $e \in E$.

Definition 2.5 ([13]) Let (F, A) be a soft set over X. The complement of (F, A) with respect to the universal soft set (X, E), denoted by $(F, A)^c$, is defined as (F^c, D), where $D = E \backslash \{e \in A | F(e) = X\} = \{e \in A | F(e) = X\}^c$, and for all $e \in D$,

$$F^c(e) = \begin{cases} X \backslash F(e) & e \in A, \\ X & \text{otherwise.} \end{cases}$$

Proposition 2.6 ([13]) Let (F, E) and (G, E) be the soft sets over X. Then

(1) $((F, E) \tilde{\cup} (G, E))^c = (F, E)^c \tilde{\cap} (G, E)^c$.
(2) $((F, E) \tilde{\cap} (G, E))^c = (F, E)^c \tilde{\cup} (G, E)^c$.

Definition 2.7 ([13]) Let τ be the collection of soft sets over X. Then τ is called a soft topology on X if τ satisfies the following axioms:

(i) Φ, (X, E) belong to τ.
(ii) The union of any number of soft sets in τ belongs to τ.
(iii) The intersection of any two soft sets in τ belongs to τ.

The triplet(X, E, τ) is called a soft topological space over X. The members of τ are called soft open sets in X and complements of them are called soft closed sets in X.

Definition 2.8 ([7]) The soft closure of (F, A) is the intersection of all soft closed sets containing (F, A).(i.e) The smallest soft closed set containing (F, A) and is denoted by $cl^S(F, A)$. The soft interior of (F, A) is the union of all soft open set is contained in (F, A) and is denoted by $int^S(F, A)$. Similarly , we define soft regular closure, soft α-closure, soft pre$-$closure, soft semi closure, soft b-closure and soft semi preopen closure of the soft set (F, A) of a soft topological space X and are denoted by $rcl^S(F, A)$, $\alpha cl^S(F, A)$, $pcl^S(F, A)$, $scl^S(F, A)$, $bcl^S(F, A)$ and $spcl^S(F, A)$ respectively. The family of all soft α-open (resp. soft semi$-$open, soft preopen, soft semi$-$preopen, soft b-open, soft regular open) sets in a soft topological space (X, τ, E) is denoted by τ^α (resp. $SSO(X, \tau)$, $SPO(X, \tau)$, $SSPO(X, \tau)$, $SBO(X, \tau)$, $SRO(X, \tau)$). The complement of the soft $\alpha-$open , soft semi$-$open, soft preopen, soft semi$-$preopen, soft $b-$open, soft regular open are their respective soft $\alpha-$closed, soft semi$-$closed, soft preclosed, soft semi-preclosed, soft b-closed, soft regular closed.

Definition 2.9 A soft subset (F, A) in a soft topological space (X, E, τ) is called

(1) soft generalized closed (briefly soft g-closed) [8] in X if $cl^S(F, A) \tilde{\subseteq} (G, B)$ whenever $(F, A) \tilde{\subseteq} (G, B)$ and (G, B) is soft open in X .
(2) soft semi open [4] if $(F, A) \tilde{\subseteq} cl^S(int^S(F, A))$.
(3) soft regular open[3] if $(F, A) = int^S(cl^S(F, A))$.
(4) soft $\alpha-$open [1] if $(F, A) \tilde{\subseteq} int^S(cl^S(int^S(F, A)))$.
(5) soft $b-$open [2] if $(F, A) \tilde{\subseteq} cl^S(int^S(F, A)) \tilde{\cup} int^S(cl^S(F, A))$.
(6) soft semi preopen or (soft $\beta-$open)[3] if $(F, A) \tilde{\subseteq} cl^S(int^S(cl^S(F, A)))$.
(7) soft pre-open set [3] if $(F, A) \tilde{\subseteq} int^S(cl^S(F, A))$.

(8) soft regular generalized closed (briefly soft $rg-$closed)[17] in a soft topological space (X, E, τ) if $cl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}SRO(X, \tau)$.

(9) soft pre generalized closed (briefly soft $pg-$closed)[14] in a soft topological space (X, E, τ) if $pcl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}SPO(X, \tau)$.

Remark 1 *([6]) The Cardinality of $SS(U_A)$ is given by $n(SS(U_A)) = 2^{n(U)\times n(A)}$. That means, if $U = \{c_1, c_2, c_3, c_4\}$ and $A = \{e_1, e_2\}$, then $n(SS(U_A)) = 2^{4\times 2} = 256$.*

Lemma 2.10 ([2]) In a soft topological space we have the following:

(i) Every soft regular open set is soft open.
(ii) Every soft open set is soft $\alpha-$open.
(iii) Every soft $\alpha-$open set is both soft semi$-$open and soft pre$-$open.
(iv) Every soft semi$-$open set and every soft pre-open set is soft $\beta-$open.

Theorem 2.11 ([2]) In a soft topological space X, every soft b-open set is soft β-open set.

Theorem 2.12 ([2]) In a soft topological space X

(i) Every soft $p-$open set is soft $b-$open set.
(ii) Every soft semi$-$open set is soft $b-$open set.

Remark 2 *By [(2.10), (2.11) and (2.12)] we consider that for any soft set (F, A) in a soft topological space (X, τ, E). Then (F, A) satisfies the following:*

(i) $spcl^S(F, A)\tilde{\subseteq}bcl^S(F, A)\tilde{\subseteq}scl^S(F, A)\tilde{\subseteq}\alpha cl^S(F, A)\tilde{\subseteq}cl^S(F, A)\tilde{\subseteq}rcl^S(F, A)$.
(ii) $spcl^S(F, A)\tilde{\subseteq}bcl^S(F, A)\tilde{\subseteq}pcl^S(F, A)\tilde{\subseteq}\alpha cl^S(F, A)\tilde{\subseteq}cl^S(F, A)\tilde{\subseteq}rcl^S(F, A)$.

3. Soft regular generalized *b*-closed sets

First we shall define a modification of soft $g-$closed sets.

Definition 3.1 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft regular generalized $\alpha-$closed (briefly soft $rg\alpha-$closed) if $\alpha cl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}SRO(X, \tau)$.

Definition 3.2 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft generalized preregular closed (briefly soft $gpr-$closed) if $pcl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}SRO(X, \tau)$.

Definition 3.3 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft generalized $b-$closed (briefly soft $gb-$closed) if $bcl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}\tau$.

Definition 3.4 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft generalized semi pre$-$closed (briefly soft $gsp-$closed) if $spcl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}\tau$.

Definition 3.5 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft generalized $\alpha-$closed (briefly soft $g\alpha-$closed) if $\alpha cl^S(F, A)\tilde{\subseteq}(G, B)$ whenever $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}\tau^{\alpha}$.

Definition 3.6 Let (X, τ, E) be a soft topological space. A subset (F, A) of X is said to be soft generalized $\alpha b-$closed (briefly soft $g\alpha b-$closed) if $bcl^S(F, A\tilde{\subseteq}(G, B)$ whenever

$(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\in}\tau^{\alpha}$.

Definition 3.7 Let (X,τ,E) be a soft topological space. A subset (F,A) of X is said to be soft semi generalized $b-$closed (briefly soft $sgb-$closed) if $bcl^S(F,A)\tilde{\subseteq}(G,B)$ whenever $(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\in}SSO(X,\tau)$.

Definition 3.8 Let (X,τ,E) be a soft topological space. A subset (F,A) of X is said to be soft regular generalized $b-$closed (briefly soft $rgb-$closed) if $bcl^S(F,A)\tilde{\subseteq}(G,B)$ whenever $(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\in}SRO(X,\tau)$.

Example **3.9** Let the set of students under consideration be $X = \{a_1,a_2,a_3\}$. Let $E = \{$ pleasing personality (e_1); conduct (e_2); good result (e_3); sincerity $(e_4)\}$ be the set of parameters framed to choose the best student. Suppose that the soft set (F,A) describing the Mr. X opinion to choose the best student of an academic year was defined by $A=\{e_1,e_2\}$ and $F(e_1)=\{a_1\}$, $F(e_2)=\{a_1,a_2,a_3\}$

In addition, we assume that the best student in the opinion of another teacher, say Mr. Y, is described by the soft set (G,B), where
$B=\{e_1,e_3,e_4\}$
$G(e_1)=\{a_2,a_3\}$, $G(e_3)=\{a_1,a_2,a_3\}$, $G(e_4)=\{a_1,a_2,a_3\}$
Consider that:
$\tau = \{\Phi,(U,E),(F,A),(G,B)\}$.The subset (G,B) in a soft topological space (X,τ,E) such that:
$\alpha cl^S(G,B) = pcl^S(G,B) = bcl^S(G,B) = scl^S(G,B) = (G,B)$. Hence (G,B) is a soft rgb-closed, soft $rg\alpha$-closed, soft gpr-closed, soft gb-closed, soft gsp-closed, soft $g\alpha$-closed, soft $g\alpha b$-closed and soft sgb-closed set in X.

Theorem 3.10 Every soft closed set is soft rgb-closed.

Proof. Let (F,A) be any soft closed set in soft topological space X such that $(F,A)\tilde{\subseteq}(G,B)$, where $(G,B)\tilde{\in}SRO(X,\tau)$. Since (F,A) is soft closed, thus $cl^S(F,A)=(F,A)$. But $bcl^S(D,C)\tilde{\subseteq}cl^S(D,C)$, for any soft set (D,C) in X. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft rgb-closed set in X. ∎

Theorem 3.11 Every soft $b-$closed set is soft $rgb-$closed.

Proof. Let (F,A) be any soft $b-$closed set in soft topological space X such that $(F,A)\tilde{\subseteq}(G,B)$, where $(G,B)\tilde{\in}SRO(X,\tau)$. Since (F,A) is soft $b-$closed, thus $bcl^S(F,A)=(F,A)$. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ∎

Theorem 3.12 Every soft $\alpha-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $\alpha-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since (F,A) is soft $\alpha-$closed, thus $\alpha cl^S(F,A) = (F,A)$. But $bcl^S(D,C)\tilde{\subseteq}\alpha cl^S(D,C)$, for any soft set (D,C) in X. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ∎

Theorem 3.13 Every soft semi$-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft semi$-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since (F,A) is soft semiclosed, thus $scl^S(F,A)=(F,A)$. But $bcl^S(D,C)\tilde{\subseteq}scl^S(D,C)$, for any soft set (D,C) in X. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ∎

Theorem 3.14 Every soft pre$-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft pre$-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since (F,A) is soft pre$-$closed, thus $pcl^S(F,A) = (F,A)$. But

$bcl^S(D,C)\tilde{\subseteq}pcl^S(D,C)$, for any soft set (D,C) in X. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.15 Every soft $g-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $g-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since each soft regular open set is soft open and (F,A) is soft $g-$closed, then $cl^S(F,A)\tilde{\subseteq}(G,B)$. But $bcl^S(D,C)\tilde{\subseteq}pcl^S(D,C)$, for any soft set (D,C) in X. Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.16 Every soft $pg-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $pg-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since each soft regular open set is soft pre$-$open set and (F,A) is soft $pg-$closed, then $pcl^S(F,A)\tilde{\subseteq}(G,B)$. But $bcl^S(D,C)\tilde{\subseteq}pcl^S(D,C)$, for any soft set (D,C) in X, thus $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.17 Every soft $gb-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $gb-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since each soft regular open set is soft open and (F,A) is soft $gb-$closed, then $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.18 Every soft gsp-closed set is soft rgb-closed set.

Proof. Let (F,A) be any soft $gsp-$closed set in in X and (G,B) be any soft regular open set containing (F,A). Since each soft regular open set is soft open and (F,A) is soft $gsp-$closed, then $spcl^S(F,A)\tilde{\subseteq}(G,B)$. But $spcl^S(F,A) = (F,A)\tilde{\cup}int^S(cl^S(int^S(F,A)))$. This implies that $(F,A)\tilde{\cup}int^S(cl^S(int^S(F,A)))\tilde{\subseteq}(G,B)$. Moreover,

$$int^S(cl^S(int^S(F,A)))\tilde{\subseteq}cl^S(int^S(F,A))\tilde{\cap}int^S(cl^S(F,A))\tilde{\subseteq}int^S(cl^S(F,A)).$$

However, $spcl^S(D,C)\tilde{\subseteq}bcl^S(D,C)\tilde{\subseteq}scl^S(D,C)$, for any soft set (D,C) in X. Since $(G,B)\tilde{\in}SRO(X,\tau)$ and $(F,A)\tilde{\subseteq}(G,B)$, then $int^S(cl^S(G,B)) = (G,B)$ and $scl^S(F,A)\tilde{\subseteq}scl^S(G,B)$. In another side, we have

$$(F,A)\tilde{\cup}int^S(cl^S(F,A)) = scl^S(F,A)\tilde{\subseteq}scl^S(G,B) = (G,B)\tilde{\cup}int^S(cl^S(G,B)) = (G,B),$$

thus $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.19 Every soft $g\alpha-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $g\alpha-$closed set in X and (G,B) be any soft regular open set containing (F,A) . Since each soft regular open set is soft $\alpha-$open and (F,A) is soft $g\alpha-$closed, then $\alpha cl^S(F,A)\tilde{\subseteq}(G,B)$. But $bcl^S(D,C)\tilde{\subseteq}\alpha cl^S(D,C)$, for any soft set (D,C) in X, thus $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.20 Every soft $g\alpha b-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $g\alpha b-$closed set in X and (G,B) be any soft regular open set containing (F,A) . Since each soft regular open set is soft $\alpha-$open and (F,A) is soft $g\alpha b-$closed, then $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.21 Every soft $sgb-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $sgb-$closed set in X and (G,B) be any soft regular open set containing (F,A) . Since each soft regular open set is soft semi$-$open and (F,A) is soft $sgb-$closed, then $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Remark 3 *The converse of above theorems need not be true in general. The following example supports our claim.*

Example 3.22 Let $X=\{s_1,s_2\}$, $E=\{e_1,e_2\}$, $A=\{e_1\}$, $F(e_1)=\{s_1\}$, $G(e_1)=X$ and $\tau=\{\Phi,(X,E),(F,A),(G,A)\}=\{\Phi,\{(e_1,X),(e_2,X)\},\{(e_1,s_1)\},\{(e_1,X)\}\}$. Then (X,τ,E) is soft topological space and $n(SS(X_E))=2^{2\times 2}=16$. Hence there are 16 soft sets over X can be considered as following:
$K_1=\Phi, K_2=(X,E), K_3=(F,A), K_4=(G,A), K_5=\{(e_1,\{s_2\})\}, K_6=\{(e_1,\{s_1\}),(e_2,X)\}$
$K_7=\{(e_1,\{s_2\}),(e_2,X)\}, K_8=\{(e_1,\{s_2\}),(e_2,\{s_2\})\}, K_9=\{(e_1,\{s_1\}),(e_2,\{s_1\})\}$
$K_{10}=\{(e_2,X)\}, K_{11}=\{(e_2,s_1)\}, K_{12}=\{(e_2,\{s_2\})\}, K_{13}=\{(e_1,X),(e_2,\{s_1\})\}$,
$K_{14}=\{(e_1,X),(e_2,\{s_2\})\}, K_{15}=\{(e_1,\{s_2\}),(e_2,\{s_1\})\}, K_{16}=\{(e_1,\{s_2\}),(e_2,\{s_2\})\}$.
Moreover, K_1,K_2,K_7 and K_{10} are all soft closed subset over X. Clearly for the soft sub set $K_3, K_3\tilde{\subseteq}K_3$,
$K_3\tilde{\in}\tau\tilde{\subseteq}\tau^\alpha\tilde{\subseteq}SSO(X,\tau)$ and $K_3\tilde{\in}\tau\tilde{\subseteq}SSPO(X,\tau)$ but $cl^S(K_3)=bcl^S(K_3)=spcl^S(K_3)=pcl^S(K_3)=\alpha cl^S(K_3)=K_2$ is not contained in K_3 . Moreover, $SRO(X,\tau)=\{\Phi,K_2\}$. Then K_3 is soft $rgb-$closed set, but is not soft $gb-$closed set, soft $gsp-$closed, soft $g\alpha-$closed, soft $g\alpha b-$closed, soft $sgb-$closed set, soft $g-$closed, and soft $pg-$closed.

Theorem 3.23 Every soft $rg-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $rg-$closed set in X and (G,B) be any soft regular open set containing (F,A) . Since (F,A) is soft $rg-$closed and $(G,B)\tilde{\in}SRO(X,\tau)$, then $cl^S(F,A)\tilde{\subseteq}(G,B)$. But $bcl^S(D,C)\tilde{\subseteq}cl^S(D,C)$, for any soft set (D,C) in X, thus $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.24 Every soft $gpr-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $gpr-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since $(G,B)\tilde{\in}SRO(X,\tau)$ and (F,A) is soft $gpr-$closed, then $pcl^S(F,A)\tilde{\subseteq}(G,B)$. But $bcl^S(D,C)\tilde{\subseteq}pcl^S(D,C)$, for any soft set (D,C) in X . Therefore $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Theorem 3.25 Every soft $rg\alpha-$closed set is soft $rgb-$closed set.

Proof. Let (F,A) be any soft $rg\alpha-$closed set in X and (G,B) be any soft regular open set containing (F,A). Since (F,A) is soft $rg\alpha-$closed and $(G,B)\tilde{\in}SRO(X,\tau)$, then $\alpha cl^S(F,A)\tilde{\subseteq}(G,B)$. $bcl^S(D,C)\tilde{\subseteq}\alpha cl^S(D,C)$, for any soft set (D,C) in X, thus $bcl^S(F,A)\tilde{\subseteq}(G,B)$. Hence (F,A) is soft $rgb-$closed set. ■

Remark 4 *The converse of theorems (3.23), (3.24), and (3.25) need not be true in general. The following example supports our claim.*

Example 3.26 Let $X=\{s_1,s_2\}$, $E=\{e_1,e_2\}$, $A=\{e_1\}$, $B=\{e_2\}$, $F(e_1)=\{s_1\}$, $G(e_2)=\{s_2\}$, $H(e_1)=\{s_1\}$, $H(e_2)=\{s_2\}$ and $\tau=\{\Phi,(X,E),(F,A),(G,B),(H,E)\}=\{\Phi,\{(e_1,X),(e_2,X)\}$,
$\{(e_1,s_1)\},\{(e_2,s_2)\},\{(e_1,s_1)\},\{(e_2,s_2)\}\}$. Then (X,τ,E) is soft topological space. Let $K_1=\phi, K_2=(X,E), K_3=(F,A), K_4=(G,B), K_5=(H,E)$. Then K_1, K_2, $K_6=\{(e_1,\{s_2\}),(e_2,X)\}$ $K_7=\{(e_1,X),(e_2,\{s_1\})\}, K_8=\{(e_1,\{s_2\}),(e_2,\{s_1\})\}$ are all soft closed subset over X. Clearly for the soft sub set $K_4, K_4\tilde{\subseteq}K_4, K_4\tilde{\in}SRO(X,\tau)$, but $cl^S(K_4)=pcl^S(K_4)=\alpha cl^S(K_4)=K_6$ is not contained in K_4. Moreover, we have only two soft regular open sets K_2 and K_4 containing K_4. In another side, $bcl^S(K_4)=K_4\tilde{\subseteq}K_2$,

$bcl^S(K_4) = K_4 \tilde{\subseteq} K_4$. Therefore K_4 is soft $rgb-$closed but is not soft $gpr-$closed, soft $rg-$closed set, and soft $rg\alpha-$closed set.

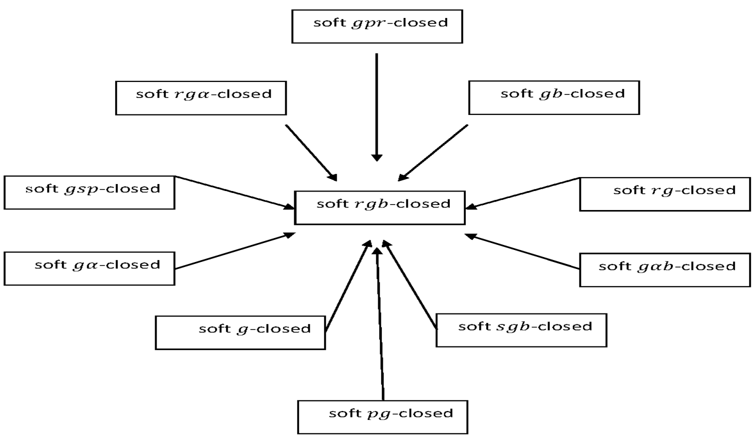
Theorem 3.27 Let (F, A) be a soft $rgb-$closed subset of a soft topological space (X, E, τ). Then $bcl^S(F, A) - (F, A)$ does not contain any non-empty soft regular closed sets.

Proof. Let $(G, B) \tilde{\in} SRC(X, \tau)$ such that $(G, B) \tilde{\subseteq} bcl^S(F, A) - (F, A)$. Since each soft regular closed set is soft closed set. So $X - (G, B)$ is soft open, $(F, A) \tilde{\subseteq} X - (G, B)$ and (F, A) is soft $rgb-$closed, it follows that $bcl^S(F, A) \tilde{\subseteq} X - (G, B)$ and thus $(G, B) \tilde{\subseteq} X - bcl^S(F, A)$. This implies that $(G, B) \tilde{\subseteq} (X - bcl^S(F, A)) \tilde{\cap} (bcl^S(F, A) - (F, A)) = \Phi$. ■

Corollary 3.28 Let (F, A) be a soft $rgb-$closed set. Then (F, A) is soft $b-$closed if and only if $bcl^S(F, A) - (F, A)$ is soft regular closed.

Proof. Let (F, A) be a soft $rgb-$closed set. If (F, A) is soft $b-$closed, then we have $bcl^S(F, A) - (F, A) = \Phi$ which is soft regular closed set. Conversely, let $bcl^S(F, A) - (F, A)$ be soft regular closed.Then, by Theorem 3.29, $bcl^S(F, A) - (F, A)$ does not contain any non-empty soft regular closed subset and since $bcl^S(F, A) - (F, A)$ is soft regular closed subset of itself, then $bcl^S(F, A) - (F, A) = \Phi$. This implies that $(F, A) = bcl^S(F, A)$ and so (F, A) is soft $b-$closed set. ■

Remark 5 *By the above results we have the following diagram:*



(A⟶B means A imply B)

4. Soft Semi-Regularization of Soft Topology

In this section, we introduced soft semi-regularization spaces and study some their properties.

Definition 4.1 Let (X, E, τ) be a soft topological space, then the family of soft regular open sets forms a base for a smaller soft topology τ_s on X called the soft semi-regularization of τ.

Remark 6 *It is clearly for any soft topological space (X, E, τ) we have: $SSO(X, \tau^\alpha) = SSO(X, \tau)$. The following remark is very useful in the sequel.*

Theorem 4.2 If $(F, A)\tilde{\in}SSO(X, \tau)$, then $\tau^\alpha - cl^S(F, A) = \tau - cl^S(F, A) = \tau_s - cl^S(F, A)$.

Proof. We need only to show that $\tau_s - cl^S(F, A)\tilde{\subseteq}\tau^\alpha - cl^S(F, A)$ for $(F, A)\tilde{\in}SSO(X, \tau)$. Let x be a soft point such that $x\tilde{\notin}\tau^\alpha - cl^S(F, A)$. Then there exists a $(G, B)\tilde{\in}\tau^\alpha$ such that $x\tilde{\in}(G, B)$ and $(F, A)\tilde{\cap}(G, B) = \Phi$. This implies that $\tau - int^S(G, B)\tilde{\cap}\tau - int^S(F, A) = \Phi$ and $\tau - cl^S(\tau - int^S(G, B))\tilde{\cap}\tau - int^S(F, A) = \Phi$. Consequently, $\tau - int^S(\tau - cl^S(\tau - int^S(G, B)))\tilde{\cap}\tau - int^S(F, A) = \Phi$ and $\tau - int^S(\tau - cl^S(\tau - int^S(G, B)))\tilde{\cap}\tau - cl^S(\tau - int^S(F, A)) = \Phi$. Since $(F, A)\tilde{\in}SSO(X, \tau)$, $(F, A)\tilde{\subseteq}\tau - cl^S(\tau - int^S(F, A))$. This implies that $\tau - int^S(\tau - cl^S(\tau - int^S(G, B)))\tilde{\cap}(F, A) = \Phi$. Since $(G, B)\tilde{\in}\tau^\alpha$, $x\tilde{\in}\tau - int^S(\tau - cl^S(\tau - int^S(G, B)))$. Hence $x\tilde{\notin}\tau_s - cl^S(F, A)$ and the proof is complete. ■

Corollary 4.3 Let (X, τ, E) be a soft topological space, then $\tau_s = (\tau^\alpha)_s$.

Proof. Since every soft regular closed set precisely soft semi-open sets, it follows from Remark 1 and Proposition 4.2 that $SRO(X, \tau) = SRO(X, \tau^\alpha)$. That means $SRC(X, \tau) = SRC(X, \tau^\alpha)$. This implies $\tau_s = (\tau^\alpha)_s$. ■

Corollary 4.4 If (F, A) is a soft subset of a soft topological space (X, τ, E), then
(a) $\tau^\alpha - int^S(\tau^\alpha - cl^S(F, A)) = \tau - int^S(\tau - cl^S(F, A))$.
(b) $\tau^\alpha - cl^S(\tau^\alpha - int^S(\tau^\alpha - cl^S(F, A))) = \tau - cl^S(\tau - int^S(\tau - cl^S(F, A)))$.
(c) $\tau - cl^S(\tau - int^S(\tau - cl^S(F, A)))\tilde{\subseteq}\tau^\alpha - cl^S(F, A)$.

Proof. (a) From Remark 1, it follows that $SSC(X, \tau^\alpha) = SSC(X, \tau)$ so that $\tau^\alpha - cl^S(F, A)\tilde{\in}SSC(X, \tau)$. By proposition 4.2 $\tau^\alpha - int^S(G, B) = \tau - int^S(G, B)$ for each $(G, B)\tilde{\in}SSC(X, \tau)$ so that $\tau^\alpha - int^S(\tau^\alpha - cl^S(F, A)) = \tau - int^S(\tau^\alpha - cl^S(F, A))$. Since $\tau - int^S(\tau^\alpha - cl^S(F, A)) = \tau - int^S(\tau - cl^S(F, A))$ we conclude that $\tau^\alpha - int^S(\tau^\alpha - cl^S(F, A)) = \tau - int^S(\tau - cl^S(F, A))$.
(b) This follows from (a) and proposition 4.2.
(c) This is an immediate consequence of (b). ■

Lemma 4.5 If (F, A) is a soft subset of a soft topological space (X, τ, E), then $\tau^\alpha - int^S(\tau^\alpha - cl^S(F, A)) = int^S(cl^S(F, A))$.

Proof. This follows from Corollary 4.4. ■

Lemma 4.6 Let (F, A) be a soft subset of a soft topological space (X, τ, E). Then $(F, A)\tilde{\in}SRO(X, \tau)$ if and only if $(F, A)\tilde{\in}SRO(X, \tau^\alpha)$.

Proof. This is an immediate consequence of Lemma 4.5. ■

Theorem 4.7 A soft subset (F, A) of a soft topological space (X, τ, E) is soft $rg\alpha-$closed in (X, τ, E) if and only if (F, A) is soft $rg-$closed in the soft topological space (X, τ^α, E).

Proof. *Necessity*. Suppose that (F, A) is $rg\alpha-$closed in (X, τ, E). Let $(F, A)\tilde{\subseteq}(G, B)$ and $(G, B)\tilde{\in}SRO(X, \tau^\alpha)$. Let us refer to $\alpha cl^S(F, A)$ in (X, τ^α, E) by $\alpha^\tau cl^S(F, A)$.

Then by Lemma 4.6, $(G,B)\tilde{\in}SRO(X,\tau)$ and we have $\alpha^{\tau}cl^{S}(F,A)=\alpha cl^{S}(F,A)\tilde{\subseteq}(G,B)$. Therefore, (F,A) is $rg-$closed in (X,τ^{α},E).

Sufficiency. Suppose that (F,A) is $rg-$closed in (X,τ^{α},E). $(F,A)\tilde{\subseteq}(G,B)$ and $(G,B)\tilde{\in}SRO(X,\tau)$. By Lemma 4.6, $(G,B)\tilde{\in}SRO(X,\tau^{\alpha})$ and hence $\alpha cl^{S}(F,A) = \alpha^{\tau}cl^{S}(F,A)\tilde{\subseteq}(G,B)$. Therefore, (F,A) is $rg\alpha-$closed in (X,τ,E). ■

Concluding remarks

In this paper, we introduce the concepts of soft $rgb-$closed, set soft $rg\alpha-$closed, soft $gpr-$closed, soft $gb-$closed, soft $gsp-$closed, soft $g\alpha-$closed, soft $g\alpha b-$closed, and soft $sgb-$closed sets in soft topological spaces and some of their properties are studied. We also introduce the concept of soft semi$-$regularization of soft topology and have established several interesting properties. Finally, we hope that this paper is just a beginning of new classes of functions, it will be necessary to carry out more theoretical research to investigate the relations between them.

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