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The Minkowski's and Young type determinantal inequalities for certain accretive-dissipative matrices

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Abstract. In this note, we investigate the Minkowski's and Young type determinantal inequalities for accretive-dissipative matrices $S = A + iB$ satisfying $0 < B < A$. Our results improve some recent ones in the literature.

Keywords: Complex matrix, Accretive-dissipative matrix, Minkowski's determinantal inequality, Young type determinantal inequality.

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1. Introduction and preliminaries

For fixed $n \geq 1$, let $\mathbb{M}_n(\mathbb{C})$ be the set of all complex $n \times n$ matrices. We denote by I_n the identity of $\mathbb{M}_n(\mathbb{C})$. For any $S \in \mathbb{M}_n(\mathbb{C})$, S^* stands for the conjugate transpose of S. We say S is positive definite (positive semidefinite) if $S = S^*$ and $x^*Ax > 0$ ($x^*Ax \geq 0$, respectively) for all nonzero $x \in \mathbb{C}^n$. It is known that every $S \in \mathbb{M}_n(\mathbb{C})$ has a unique Toeplitz decomposition of the form $S = A + iB$ with $A = A^*$ and $B = B^*$. In case A and B are both positive definite, S is called accretive-dissipative.

For each $A \in \mathbb{M}_n(\mathbb{C})$, let $\{s_j(A)\}_{j=1}^n$ be the decreasing sequence of singular values of $|A| = (AA^*)^{\frac{1}{2}}$. Given any $A, B \in \mathbb{M}_n(\mathbb{C})$, Garg and Aujla [1] showed that

$$\prod_{j=1}^{k} s_j(|A+B|^r) \leq \prod_{j=1}^{k} s_j(I_n + |A|^r) \prod_{j=1}^{k} s_j(I_n + |B|^r)$$

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and

$$\prod_{j=1}^{k} s_j\big(I_n + f(|A+B|)\big) \leq \prod_{j=1}^{k} s_j\big(I_n + f(|A|)\big) \prod_{j=1}^{k} s_j\big(I_n + f(|B|)\big)$$

for every $1 \leq r \leq 2$, $1 \leq k \leq n$ and operator concave function $f : [0,\infty) \to [0,\infty)$. If A and B are positive semidefinite, $r = 1$ and $f(X) = X$ for any $X \in \mathbb{M}_n(\mathbb{C})$, these inequalities imply

$$\prod_{j=1}^{k} s_j(A+B) \leq \prod_{j=1}^{k} s_j(I_n + A) \prod_{j=1}^{k} s_j(I_n + B)$$

and

$$\prod_{j=1}^{k} s_j(I_n + A + B) \leq \prod_{j=1}^{k} s_j(I_n + A) \prod_{j=1}^{k} s_j(I_n + B).$$

In particular, in the case $k = n$, we get

$$\det(A+B) \leq \det(I_n + A)\det(I_n + B) \tag{1}$$

and

$$\det(I_n + A + B) \leq \det(I_n + A)\det(I_n + B). \tag{2}$$

Given any accretive-dissipative matrices $S, T \in \mathbb{M}_n(\mathbb{C})$, Kittaneh and Sakkijha [5] computed

$$|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} \leq \sqrt{2}|\det(S+T)|^{\frac{1}{n}} \tag{3}$$

and for any $0 < \alpha < 1$,

$$|\det S|^{\alpha}|\det T|^{1-\alpha} \leq 2^{\frac{n}{2}}\Big|\det\big(\alpha S + (1-\alpha)T\big)\Big|. \tag{4}$$

Proposition 1.1 [4, Lemma 6] Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be positive semidefinite. Then

$$|\det(A+iB)| \leq \det(A+B) \leq 2^{\frac{n}{2}}|\det(A+iB)|.$$

The following results are also proved in [6].

Proposition 1.2 [6, Theorem 2.11] Let $S, T \in \mathbb{M}_n(\mathbb{C})$ be accretive-dissipative. Then

$$|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} \leq 2\sqrt{2}|\det(I_n + S)|^{\frac{1}{n}}|\det(I_n + T)|^{\frac{1}{n}} \tag{5}$$

and

$$|\det(\alpha I_n + S)|^{\frac{1}{n}} + \Big|\det\big((1-\alpha)I_n + T\big)\Big|^{\frac{1}{n}} \leq 2\sqrt{2}|\det(I_n + S)|^{\frac{1}{n}}|\det(I_n + T)|^{\frac{1}{n}} \tag{6}$$

for every $0 \leq \alpha \leq 1$.

Proposition 1.3 [6, Theorem 2.12] Let $S, T \in \mathbb{M}_n(\mathbb{C})$ be accretive-dissipative. Then, for every $0 < \alpha < 1$,

$$|\det S|^{\alpha}|\det T|^{1-\alpha} \leq 2^{\frac{3n}{2}}|\det(I_n + \alpha S)||\det(I_n + (1-\alpha)T)| \tag{7}$$

and

$$\left|\det(I_n + S)\right|^{\alpha}\left|\det(I_n + T)\right|^{1-\alpha} \leq 2^{\frac{3n}{2}}\left|\det(I_n + \alpha S)\right|\left|\det\left(I_n + (1-\alpha)T\right)\right|. \tag{8}$$

Proposition 1.4 [2, property 2] Let $A \in \mathbb{M}_n(\mathbb{C})$ be accretive-dissipative. Then, there exists a unique square root R of A that belongs to accretive-dissipative. If $R = S + iT$ is the Toeplitz decomposition of R, then $0 < T < R$.

Throughout the paper, we consider specific accretive-dissipative matrices $S = A + iB$ with $0 < B < A$. We denote by $\mathcal{R}_n^{++}$ the set of such matrices, that is,

$$\mathcal{R}_n^{++} = \{S \in \mathbb{M}_n(\mathbb{C}) : S = A + iB \text{ with } 0 < B < A\}.$$

Our initial motivation for considering $\mathcal{R}_n^{++}$ comes from Proposition 1.4 which says every accretive-dissipative matrix $T \in \mathbb{M}_n(\mathbb{C})$ has a unique square root $S = T^{\frac{1}{2}} = A + iB$ with $0 < B < A$. Note that the converse of this simply holds: if $S = A + iB$ is accretive-dissipative with $0 < B < A$, then S^2 is accretive-dissipative. Consequently, $\mathcal{R}_n^{++}$ coincides with the set of all matrices $S \in \mathbb{M}_n(\mathbb{C})$ such that both S and S^2 are accretive-dissipative.

The aim of this paper is to investigate some known determinantal inequalities for elements of $\mathcal{R}_n^{++}$. We obtain specific Minkowski's and Young type determinantal inequalities in Sections 2 and 3 for such matrices. Moreover, we show by some easy examples that Theorem 2.2 (Theorem 3.2) substaintially improve the upper bounds of (3) and (6) (of (4) and (8), respectively).

2. The Minkowski's determinantal inequalities

In this section, we investigate the Minkowski's determinantal inequality for elements of $\mathcal{R}_n^{++}$. Let us first recall a known results.

Lemma 2.1 [3, Corollary 7.8.21] Let $A, B \in \mathbb{M}_n(\mathbb{C})$ be positive definite. Then

$$(\det A)^{\frac{1}{n}} + (\det B)^{\frac{1}{n}} \leq \left(\det(A + B)\right)^{\frac{1}{n}}. \tag{9}$$

Remark 1 *Let A and B be two positive definite and Hermition matrices. Then, by [3, Theorem 7.7.3], $B < A$ implies $\left(A^{-\frac{1}{2}}\right)^* BA^{-\frac{1}{2}} < \left(A^{-\frac{1}{2}}\right)^* AA^{-\frac{1}{2}}$ and so, $A^{-\frac{1}{2}}BA^{-\frac{1}{2}} < I_n$. Therefore, all eigenvalues of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$ are positive and less than 1. We will use this fact in the proof of Theorem 2.2 below.*

Theorem 2.2 Let $S, T \in \mathcal{R}_n^{++}$ with the Toeplitz decompositions $S = A + iB$ and $T = C + iD$. Suppose that $\{\beta_j\}_{j=1}^n$ and $\{\gamma_j\}_{j=1}^n$ are the sets of eigenvalues of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$

and $C^{-\frac{1}{2}}DC^{-\frac{1}{2}}$, respectively. Then

$$|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} \leq \sqrt{1+p^2}\ \left(\det(A+C)\right)^{\frac{1}{n}},$$

where $p := \max_{1\leq j\leq n}\{\beta_j, \gamma_j\}$.

Proof. We may compute

$$\begin{aligned}
|\det S|^{\frac{1}{n}} &= |\det(A+iB)|^{\frac{1}{n}} \\
&= \left|\det\left(A^{\frac{1}{2}}(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})A^{\frac{1}{2}}\right)\right|^{\frac{1}{n}} \\
&= |(\det A)^{\frac{1}{2}}\det(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})(\det A)^{\frac{1}{2}}|^{\frac{1}{n}} \\
&= |\det(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})\det A|^{\frac{1}{n}} \\
&= |\det(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})|^{\frac{1}{n}}(\det A)^{\frac{1}{n}} \\
&= \big(\prod_{j=1}^{n}|1+i\beta_j|\ \big)^{\frac{1}{n}}(\det A)^{\frac{1}{n}} \\
&= \big(\prod_{j=1}^{n}\sqrt{1+\beta_j^2}\ \big)^{\frac{1}{n}}(\det A)^{\frac{1}{n}}.
\end{aligned}$$

So, for $\beta_{\max} := \max_{1\leq j\leq n}\{\beta_j\}$, we get

$$|\det S|^{\frac{1}{n}} \leq \sqrt{1+\beta_{\max}^2}(\det A)^{\frac{1}{n}}. \tag{10}$$

An analogous computation also gives

$$|\det T|^{\frac{1}{n}} \leq \sqrt{1+\gamma_{\max}^2}(\det C)^{\frac{1}{n}}, \tag{11}$$

where $\gamma_{\max} := \max_{1\leq j\leq n}\{\gamma_j\}$. Now, (10) and (11) imply

$$\begin{aligned}
|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} &\leq \sqrt{1+\beta_{\max}^2}(\det A)^{\frac{1}{n}} + \sqrt{1+\gamma_{\max}^2}(\det C)^{\frac{1}{n}} \\
&\leq \sqrt{1+p^2}\left((\det A)^{\frac{1}{n}} + (\det C)^{\frac{1}{n}}\right) \\
&\leq \sqrt{1+p^2}\ \left(\det(A+C)\right)^{\frac{1}{n}} \qquad \text{(by Lemma 2.1)},
\end{aligned}$$

where ($p = \max_{1\leq j\leq n}\{\beta_j, \gamma_j\}$). This completes the proof. ■

Proposition 2.3 Let $S = A + iB \in \mathbb{M}_n(\mathbb{C})$ be an accretive-dissipative. Then

$$|\det S| \geq \left(1+\beta_{\min}^2\ \right)^{\frac{n}{2}}\det A, \tag{12}$$

where $\{\beta_j\}_{j=1}^n$ is the set of eigenvalues of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$ and $\beta_{\min} := \min_{1\leq j\leq n}\{\beta_j\}$. In particular, we have $|\det S| > \det A$.

Proof. We can write

$$\begin{aligned}
|\det S| &= |\det(A+iB)| \\
&= \left|\det\left(A^{\frac{1}{2}}(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})A^{\frac{1}{2}}\right)\right| \\
&= \left|\det\left(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)\right| \det A \\
&= \prod_{j=1}^{n} |1+i\beta_j| \det A \\
&\geq \Big(\prod_{j=1}^{n} \sqrt{1+\beta_{\min}^2}\,\Big) \det A \\
&= \left(1+\beta_{\min}^2\,\right)^{\frac{n}{2}} \det A,
\end{aligned}$$

completing the proof. ■

Note that we have always $p < 1$ in Theorem 2.2. Indeed, since $B < A$ and $D < C$, Remark *1* impleis $\beta_j < 1$ and $\gamma_j < 1$ for all $1 \leq j \leq n$, and hence $p < 1$. Moreover, Proposition 2.3 implies $\det(A+C) < \det(S+T)$, and thus Theorem 2.2 is an improvement of (3). Furthermore, Theorem 2.2 implies immediately the following generalization of (5).

Corollary 2.4 Let $S, T \in \mathcal{R}_n^{++}$. Under the condition of Theorem 2.2, we have

$$|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} \leq \sqrt{1+p^2}\,\left(\det(I_n+A)\right)^{\frac{1}{n}}\left(\det(I_n+C)\right)^{\frac{1}{n}},$$

where $p := \max_{1\leq j\leq n}\{\beta_j, \gamma_j\}$.

Proof. Theorem 2.2 yields

$$\begin{aligned}
|\det S|^{\frac{1}{n}} + |\det T|^{\frac{1}{n}} &\leq \sqrt{1+p^2}\,\left(\det(A+C)\right)^{\frac{1}{n}} \\
&\leq \sqrt{1+p^2}\,\left(\det(I_n+A)\right)^{\frac{1}{n}}\left(\det(I_n+C)\right)^{\frac{1}{n}} \quad \text{(by (1))},
\end{aligned}$$

as desired. ■

Corollary 2.5 (See (6)) Let $S, T \in \mathcal{R}_n^{++}$ be an in Theorem 2.2. For given $0 \leq \alpha \leq 1$, suppose that $\{\beta_j\}_{j=1}^n$ and $\{\gamma_j\}_{j=1}^n$ are the sets of eigenvalues of $(\alpha I_n+A)^{-\frac{1}{2}}B(\alpha I_n+A)^{-\frac{1}{2}}$ and $\left((1-\alpha)I_n+C\right)^{-\frac{1}{2}}D\left((1-\alpha)I_n+C\right)^{-\frac{1}{2}}$, respectively. Then

$$\left|\det\left(\alpha I_n+S\right)\right|^{\frac{1}{n}} + \left|\det\left((1-\alpha)I_n+T\right)\right|^{\frac{1}{n}} \leq \sqrt{1+p^2}\,\left(\det(I_n+A)\right)^{\frac{1}{n}}\left(\det(I_n+C)\right)^{\frac{1}{n}},$$

where $p := \max_{1\leq j\leq n}\{\beta_j, \gamma_j\}$.

Proof. Note that by replacing S and T with $\alpha I_n + S$ and $(1-\alpha)I_n + T$ respectively, Theorem 2.2 implies

$$|\det(\alpha I_n+S)|^{\frac{1}{n}} + \left|\det\left((1-\alpha)I_n+T\right)\right|^{\frac{1}{n}} \leq \sqrt{1+p^2}\,\left(\det(I_n+A+C)\right)^{\frac{1}{n}}.$$

So, we get

$$\begin{aligned}\left|\det\big(\alpha I_n+S\big)\right|^{\frac{1}{n}}+\left|\det\big((1-\alpha)I_n+T\big)\right|^{\frac{1}{n}}\\ &\leq\sqrt{1+p^2}\,\big(\det(I_n+A+C)\big)^{\frac{1}{n}}\\ &\leq\sqrt{1+p^2}\,\big(\det(I_n+A)\big)^{\frac{1}{n}}\big(\det(I_n+C)\big)^{\frac{1}{n}}\quad\text{(by (2))}.\end{aligned}$$

■

We now examine Theorem 2.2 by a small square matrix and compare it with (3).

***Example* 2.6** Let $S=A+iB$ and $T=C+iD$ be of the forms

$$S=\begin{bmatrix}7&3\\3&7\end{bmatrix}+i\begin{bmatrix}3&2\\2&3\end{bmatrix}\qquad\text{and}\qquad T=\begin{bmatrix}4&2\\2&4\end{bmatrix}+i\begin{bmatrix}2&1\\1&2\end{bmatrix}.$$

It is easy to verify that $S,T\in\mathcal{R}_n^{++}$. Then we have $|\det S|=46.0977223$, $|\det T|=15$, $|\det(S+T)|=113.137085$ and $|\det(A+C)|=96$. Also, $\{0.25,0.5\}$ and $\{0.5\}$ are the sets of eigenvalues of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$ and $C^{-\frac{1}{2}}DC^{-\frac{1}{2}}$, respectively. So, (3) says that

$$|\det S|^{\frac{1}{2}}+|\det T|^{\frac{1}{2}}\leq 15.0424124, \tag{13}$$

while Theorem 2.2 with $p=0.5$, for example, gives

$$|\det S|^{\frac{1}{2}}+|\det T|^{\frac{1}{2}}\leq\sqrt{1+(0.5)^2}\times(96)^{\frac{1}{2}}=10.9544512. \tag{14}$$

Since $|\det S|^{\frac{1}{2}}+|\det T|^{\frac{1}{2}}$ equals 10.6624769 exactly, we see that our approximation is better than that obtained by (3).

3. The Young type determinantal inequalities for $\mathcal{R}_n^{++}$

In this section, we prove a Young type determinantal inequality for elements of $\mathcal{R}_n^{++}$, which improves (4), (7) and (8).

Lemma 3.1 [3, Corollary 7.6.8] Let $A,B\in\mathbb{M}_n(\mathbb{C})$ be positive definite and $0<\alpha<1$. Then,

$$(\det A)^{\alpha}(\det B)^{1-\alpha}\leq\det\big(\alpha A+(1-\alpha)B\big).$$

Theorem 3.2 Let $S,T\in\mathcal{R}_n^{++}$ with the Toeplitz decompositions $S=A+iB$ and $T=C+iD$. Let $\{\beta_j\}_{j=1}^n$ and $\{\gamma_j\}_{j=1}^n$ be the sets of eigenvalues of $A^{-\frac{1}{2}}BA^{-\frac{1}{2}}$ and $C^{-\frac{1}{2}}DC^{-\frac{1}{2}}$, respectively. Then, for any $0<\alpha<1$,

$$|\det S|^{\alpha}|\det T|^{1-\alpha}\leq(1+p^2)^{\frac{n}{2}}\det\big(\alpha A+(1-\alpha)C\big),$$

where $p:=\max\limits_{1\leq j\leq n}\{\beta_j,\gamma_j\}$.

Proof. We can write

$$\begin{aligned}
|\det S| &= |\det(A+iB)| \\
&= \left|\det\left(A^{\frac{1}{2}}(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}})A^{\frac{1}{2}}\right)\right| \\
&= |(\det A)^{\frac{1}{2}}\det\left(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)(\det A)^{\frac{1}{2}}| \\
&= |\det\left(I_n + iA^{-\frac{1}{2}}BA^{-\frac{1}{2}}\right)|\det A \\
&= \Big(\prod_{j=1}^{n} |1+i\beta_j|\Big)\det A \\
&= \Big(\prod_{j=1}^{n} \sqrt{1+\beta_j^2}\Big)\det A.
\end{aligned}$$

Similarly, we get $|\det T| = \big(\prod_{j=1}^{n}\sqrt{1+\gamma_j^2}\big)\det C$. Therefore, defining $p = \max_{1\le j\le n}\{\beta_j, \gamma_j\}$, we conclude that

$$\begin{aligned}
|\det S|^{\alpha}|\det T|^{1-\alpha} &= \Big(\prod_{j=1}^{n}\sqrt{1+\beta_j^2}\Big)^{\alpha}(\det A)^{\alpha}\Big(\prod_{j=1}^{n}\sqrt{1+\gamma_j^2}\Big)^{1-\alpha}(\det C)^{1-\alpha} \\
&\le \Big(\prod_{j=1}^{n}\sqrt{1+p^2}\Big)(\det A)^{\alpha}(\det C)^{1-\alpha} \\
&\le (1+p^2)^{\frac{n}{2}}\det\big(\alpha A + (1-\alpha)C\big) \qquad \text{(by Lemma 3.1)},
\end{aligned}$$

completing the proof. ■

Corollary 3.3 Let $S, T \in \mathcal{R}_n^{++}$ with the Toeplitz decompositions $S = A + iB$ and $T = C + iD$. If $\{\beta_j\}_{j=1}^n$ and $\{\gamma_j\}_{j=1}^n$ are the sets of eigenvalues of $(I_n+A)^{-\frac{1}{2}}B(I_n+A)^{-\frac{1}{2}}$ and $(I_n+C)^{-\frac{1}{2}}D(I_n+C)^{-\frac{1}{2}}$, respectively, then

$$|\det(I_n+S)|^{\alpha}|\det(I_n+T)|^{1-\alpha} \le (1+p^2)^{\frac{n}{2}}\det\big(I_n + \alpha A + (1-\alpha)C\big),$$

where $p := \max_{1\le j\le n}\{\beta_j, \gamma_j\}$.

Proof. Statement follows immediately from Theorem 3.2 by replacing S and T with $I_n + S$ and $I_n + T$, respectively. ■

Observe that using Proposition 2.3 and the fact $p < 1$ (Remark *1*), we see that Theorem 3.2 generalizes (4). Moreover, by (1) and (2), we have

$$\det\big(\alpha A + (1-\alpha)C\big) \le \det(I_n + \alpha A)\det\big(I_n + (1-\alpha)C\big)$$

and

$$\det\big(I_n + \alpha A + (1-\alpha)C\big) \le \det(I_n + \alpha A)\det\big(I_n + (1-\alpha)C\big)$$

for $0 < \alpha < 1$, and hence, Theorem 3.2 and Corollary 3.3 improve (7) and (8), respectively.

***Example* 3.4** Consider the matrices S and T of Example 2.6 and let $\alpha = 0.6$. We may compute $|\det(\alpha A + (1-\alpha)C)| = 26.88$ and $|\det(\alpha S + (1-\alpha)T)| = 31.4859969$. Then (4) gives $|\det S|^{\alpha}|\det T|^{1-\alpha} \leq 62.9719938$, while Theorem 3.2 for $p = 0.5$ implies

$$|\det S|^{\alpha}|\det T|^{1-\alpha} \leq (1 + (0.5)^2) \times 26.88 = 33.6.$$

Since $|\det S|^{\alpha}|\det T|^{1-\alpha} = 29.4199115$ exactly, Theorem 3.2 gives a much more better upper bound comparing with that obtained by (4).

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