Journal of Linear and Topological Algebra
Vol. 12*, No.* 04*,* 2023*,* 235- 242
DOR: 20.1001.1.22520201.2023.12.04.1.5
DOI: 10.30495/JLTA.2023.1992240.1577

Solvability of infinite systems of differential equations of general order in the sequence space bv_∞

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Received 24 July 2023; Revised 5 November 2023; Accepted 5 December 2023.
Communicated by Hamidreza Rahimi

Abstract. We introduce the Hausdorff measure of noncompactness in the sequence space bv_∞ and investigate the existence of solution of infinite systems of differential equations with respect to Hausdorff measure of noncompactness. Finally, we present an example to defend of theorem of existential.

Keywords: Green function, differential equation, measure of noncompactness, Meir-Keeler condensing operator, sequence space.

2010 AMS Subject Classification: 47H10, 46L07.

1. Introduction and preliminaries

Infinite systems define numerous real world questions which can be encountered in the theory of nervous nets, the theory of separation of polymer and so on (see [9, 11, 15, 19, 20, 24–26]). Also, Kuratowski [17] defined the measure of noncompactness (MNC) which used for solving the infinite systems of differential equations. In addition, you can observe the further usages the MNC in kinds of integral equations and differential equations ([1, 3, 5, 10, 12, 13, 16, 21–23]). Recently, Banaś and Lecko [7] proved some existence results for infinite systems of differential equations in the classical Banach spaces ℓ_1 (absolutely summable series), c (all convergent) and c_0, (null sequences).

In this work, we study the solution of the following differential equations of order $n \geqslant 2$:

$$v_i^{(n)} + f_i\big(\gamma, v(\gamma)\big) = 0$$

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Print ISSN: 2252-0201
Online ISSN: 2345-5934

http://jlta.ctb.iau.ir

with the boundary conditions given by $v_i(\iota) = v_i'(\iota) = v_i'(\iota) = \ldots = v_i^{(n-2)}(\iota) = 0$ and $v_i(\vartheta) = 0$, where $v(\gamma) = \big(v_i(\gamma)\big)_{i=1}^{\infty}$, $(\gamma \in [\iota, \vartheta])$.

In this part, a few auxiliary facts are represented, that we can use in our paper. Let E be a Banach space with the zero element θ, in addition, the elements x and r respectively are indicated in the center and radius of the closed ball $B(x, r)$ in E. Let $\emptyset \neq \mathfrak{M}_E \subseteq E$ the family of all bounded and $\emptyset \neq \mathfrak{N}_E \subseteq E$ subfamily of all relatively compact sets. The symbols Conv(A) and $\bar{A}$ for the non-empty subsets convex and closure A in E, respectively.

Definition 1.1 [2] The mapping $\tilde{\mu} : \mathfrak{M}_\Gamma \to \mathbb{R}_+ = [0, +\infty)$ is measure of noncompactness (MNC) in Γ if for all $\mathcal{R}, \mathcal{Y}_1, \mathcal{Y}_2 \in \mathfrak{M}_\Gamma$, we have

(i) The family $\emptyset \neq \ker \tilde{\mu} = \{\mathcal{R} \in \mathfrak{M}_\Gamma : \tilde{\mu}(\mathcal{R}) = 0\} \subseteq \mathfrak{N}_\Gamma$;
(ii) If $\mathcal{Y}_1 \subset \mathcal{Y}_2$, then $\tilde{\mu}(\mathcal{Y}_1) \leqslant \tilde{\mu}(\mathcal{Y}_2)$;
(iii) $\tilde{\mu}(\overline{\mathcal{R}}) = \tilde{\mu}(\mathcal{R}) = \tilde{\mu}(\text{Conv}\mathcal{R})$;
(iv) for all $0 \leqslant \jmath \leqslant 1$, $\tilde{\mu}(\jmath\mathcal{Y}_1 + (1-\jmath)\mathcal{Y}_2) \leqslant \jmath\tilde{\mu}(\mathcal{Y}_1) + (1-\jmath)\tilde{\mu}(\mathcal{Y}_2)$;
(v) If $\overline{\mathcal{R}_n} = \mathcal{R}_n$ in $\mathfrak{M}_\Gamma$ and $\mathcal{R}_{n+1} \subset \mathcal{R}_n$ for all $n \in \mathbb{N}$ and $\lim\limits_{n\to\infty} \tilde{\mu}(\mathcal{R}_n) = 0$, then $\emptyset \neq \mathcal{R}_\infty = \bigcap\limits_{n=1}^{\infty} \mathcal{R}_n$.

Definition 1.2 [6] Let (Y, d) be a metric space and $\mathcal{P} \in \mathfrak{M}_Y$. The Kuratowski MNC $\tilde{\mu}(\mathcal{P})$ is defined by

$$\tilde{\mu}(\mathcal{P}) = \inf\Big\{\varepsilon > 0 : \mathcal{P} \subset \bigcup_{\kappa=1}^{m} K_\kappa, K_\kappa \subset Y, \text{diam}(K_\kappa) < \varepsilon \ (\kappa = 1, \ldots, m);\ m \in \mathbb{N}\Big\},$$

where $\text{diam}(K_\kappa) = \sup\{d(o, \tau) : o, \tau \in K_\kappa\}$.

The Hausdorff MNC $\tilde{\mu}(\mathcal{P})$ is

$$\tilde{\mu}(\mathcal{P}) = \inf\Big\{\varepsilon > 0 : \mathcal{P} \subset \bigcup_{\kappa=1}^{m} D(z_\kappa, r_\kappa), z_\kappa \in Y, r_\kappa < \varepsilon \ (\kappa = 1, \ldots, m);\ m \in \mathbb{N}\Big\}.$$

Lemma 1.3 [6] Suppose that (Y, d) is a metric space and $\mathcal{P}, \mathcal{P}_1, \mathcal{P}_2 \in \mathfrak{M}_Y$. Then

(i) $\tilde{\mu}(\mathcal{P}) = 0$ iff $\mathcal{P}$ is totally bounded,
(ii) $\mathcal{P}_1 \subset \mathcal{P}_2$ implies that $\tilde{\mu}(\mathcal{P}_1) \leqslant \tilde{\mu}(\mathcal{P}_2)$,
(iii) $\tilde{\mu}(\overline{\mathcal{P}}) = \tilde{\mu}(\mathcal{P})$,
(iv) $\tilde{\mu}(\mathcal{P}_1 \cup \mathcal{P}_2) = \max\{\tilde{\mu}(\mathcal{P}_1), \tilde{\mu}(\mathcal{P}_2)\}$.

Definition 1.4 [4] Let Γ be a Banach space and $\emptyset \neq \mathfrak{G} \subset \Gamma$. Also, suppose that $\tilde{\mu}$ is an arbitrary MNC on Γ. The operator $Q : \mathfrak{G} \to \mathfrak{G}$ is Meir–Keeler condensing operator if for each $\varepsilon > 0$, there exists $\delta > 0$ such that $\varepsilon \leqslant \tilde{\mu}(\mathcal{U}) < \varepsilon + \delta$ implies that $\tilde{\mu}(Q(\mathcal{U})) < \varepsilon$ for all bounded $\mathcal{U} \subseteq \mathfrak{G}$.

Theorem 1.5 [4] Let $\emptyset \neq \mathfrak{D} = \overline{\mathfrak{D}} \subseteq \Gamma$ be bounded and convex, $\tilde{\mu}$ be a MNC on Γ and $Q : \mathfrak{D} \to \mathfrak{D}$ be a continuous Meir–Keeler condensing operator. Then Q has a fixed point.

Let $K = [0, s]$ and Γ is a Banach space. Then $C(K, \Gamma)$ is Banach space with norm

$$\|x\|_{C(K,\Gamma)} := \sup\{\|x(\rho)\| : \rho \in K\},\ x \in C(K, \Gamma).$$

Proposition 1.6 [6] Let $\Upsilon \subseteq C(K,\Gamma)$ be bounded and equicontinuous. Then $\tilde{\mu}(\Upsilon(.))$ is continuous on K, $\tilde{\mu}(\Upsilon) = \sup_{\xi \in K} \tilde{\mu}(\Upsilon(\xi))$ and

$$\tilde{\mu}\big(\int_0^{\xi} \Upsilon(\zeta)d\zeta\big) \leqslant \int_0^{\xi} \tilde{\mu}(\Upsilon(\zeta))d\zeta.$$

2. Sequence space bv_∞.

Baser et al. [8] defined the following sequence space

$$bv_\infty = \{\nu = (\nu_k) \in \omega : \sup_{k\in\mathbb{N}} |\nu_k - \nu_{k-1}| < \infty\},$$

by norm $\|\nu\|_{bv_\infty} = \sup_{k\in\mathbb{N}} |\nu_k - \nu_{k-1}|$, where ω is space of all complex valued sequences.

Lemma 2.1 [18] Let F be normed space and $\emptyset \neq \mathcal{G} \subseteq F$ be bounded, where F is c_0 or l_q $(1 \leqslant q < \infty)$. Also, let $P_n : F \to F$ be operator $P_n(\nu) = (\nu_0, \nu_1, \ldots, \nu_n, 0, 0, \ldots)$. Then

$$\tilde{\mu}(\mathcal{G}) = \lim_{n\to\infty} \big\{ \sup_{\nu\in\mathcal{G}} \|(I - P_n)\nu\| \big\}.$$

Theorem 2.2 Let $\emptyset \neq U \subseteq bv_\infty$ be bounded. Then the Huasdorff MNC $\tilde{\mu}$ in bv_∞ is defined by

$$\tilde{\mu}_{bv_\infty}(U) = \lim_{n\to\infty} \Big[\sup_{x\in U} \Big(\sup_{j\geqslant n} |\nu_j - \nu_{j-1}| \Big) \Big]. \tag{1}$$

Proof. Define the operator $q_n : bv_\infty \to bv_\infty$, $q_n(\nu) = (\nu_0, \nu_1, \ldots, \nu_n, 0, 0, \ldots)$ for $\nu = (\nu_1, \nu_2, ...) \in bv_\infty$. Obviously

$$U \subset q_n U + (J - q_n)U. \tag{2}$$

By (2) and the properties of $\tilde{\mu}$, we get

$$\tilde{\mu}(U) \leqslant \tilde{\mu}(q_n U) + \tilde{\mu}((J - q_n)U) = \tilde{\mu}((J - q_n)U) \leqslant diam((J - q_n)U) = \sup_{\nu\in U} \|(J - q_n)\nu\|,$$

where $\|(J - q_n)\nu\| = \sup_{j\geqslant n} |\nu_j - \nu_{j-1}|$, when n is large enough. So,

$$\tilde{\mu}(U) \leqslant \lim_{n\to\infty} \sup_{\nu\in U} \|(J - q_n)\nu\|. \tag{3}$$

Conversely, suppose that $\epsilon > 0$ and $\{z_1, z_2, ..., z_i\}$ be a $[\tilde{\mu}(U) + \epsilon]$-net of U. Then

$$U \subset \{z_1, z_2, ..., z_i\} + [\tilde{\mu}(U) + \epsilon] B(bv_\infty),$$

where $B(bv_\infty)$ is the unite ball of bv_∞. Hence,

$$\sup_{\nu\in U} \|(J - q_n)\nu\| \leqslant \sup_{1\leqslant j\leqslant i} \|(J - q_n)z_j\| + [\tilde{\mu}(U) + \epsilon],$$

which implies that

$$\lim_{n\to\infty} \sup_{\nu\in U} \|(J-q_n)\nu\| \leqslant \tilde{\mu}(U)+\epsilon. \tag{4}$$

Since ϵ is arbitrary and by (3) and (4), (1) holds. ■

3. Application

Now, we study the following infinite system in the sequence space bv_∞.

$$v_i^{(n)}(\gamma) + f_i(\gamma, v(\gamma)) = 0 \ (n \geqslant 2) \tag{5}$$

with the boundary conditions $v_i(\iota) = v_i^{'}(\iota) = v_i^{''}(\iota) = \ldots = v_i^{(n-2)}(\iota) = 0$ and $v_i(\vartheta) = 0$, where $v(\gamma) = (v_i(\gamma))_{i=1}^{\infty}$ $(\gamma \in [\iota, \vartheta])$. A solution of differential equation (5) is $v \in C^n(K, \mathbb{R})$ iff $v \in C(K, \mathbb{R})$ $(K = [\iota, \vartheta])$ is a solution of the following infinite system of integral equation

$$v_i(\gamma) = \int_\iota^\vartheta G(\gamma, \sigma) f_i(\sigma, v(\sigma)) d\sigma,$$

where $f_i \in C(K \times \mathbb{R}^\infty, \mathbb{R})$ and the Green's function associated to (5) is (Duffy [14])

$$G(\gamma,\sigma) = \begin{cases} \frac{(\vartheta-\sigma)^{m-1}(\gamma-\iota)^{m-1}-(\vartheta-\iota)^{m-1}(\gamma-\sigma)^{m-1}}{(\vartheta-\iota)^{m-1}(m-1)!} & (\iota \leqslant \sigma \leqslant \gamma \leqslant \vartheta) \\ \frac{(\vartheta-\sigma)^{m-1}(\gamma-\iota)^{m-1}}{(\vartheta-\iota)^{m-1}(m-1)!} & (\iota \leqslant \gamma \leqslant \sigma \leqslant \vartheta) \end{cases}.$$

Then, we have

$$v_i^{'}(\gamma) = \int_\iota^\vartheta \frac{\partial}{\partial\gamma} G(\gamma,\sigma) f_i(\sigma, v(\sigma)) d\sigma, \ldots, v_i^{(m-2)}(\gamma) = \int_\iota^\vartheta \frac{\partial^{m-2}}{\partial\gamma^{m-2}} G(\gamma,\sigma) f_i(\sigma, v(\sigma)) d\sigma.$$

Also, $G(\gamma, \sigma) \leqslant \frac{2(\vartheta-\iota)^{m-1}}{(m-1)!}$ for all $(\gamma, \sigma) \in K^2$. Here, we consider assumption:
Let $f_i \in C(K \times \mathbb{R}^\infty, \mathbb{R})$, $(i \in \mathbb{N})$ be functions. The operator $f : K \times bv_\infty \to bv_\infty$ is defined by $(\gamma, v) \to (fv)(\gamma) = (f_1(\gamma, v), f_2(\gamma, v), f_3(\gamma, v), \ldots)$, the family of functions $((fv)(\gamma))_{\gamma\in K}$ are equicontinuous at each point of the space bv_∞. Also,

$$|f_k(\sigma, v) - f_{k-1}(\sigma, v)| \leqslant p_k(\sigma) + q_k(\sigma)|v_k - v_{k-1}|,$$

where $p_k, q_k : [\iota, \vartheta] \to \mathbb{R}_+$ are continuous, the sequence $\{p_k(\sigma)\}_{k=1}^\infty$ convergence uniformly to zero on $K = [\iota, \vartheta]$ and the sequence $\{q_k(\sigma)\}_{k=1}^\infty$ is equibounded on K. Let

$$P = \sup_{\sigma\in K, k\in\mathbb{N}} \{p_k(\sigma)\} \ \text{ and } \ Q = \sup_{\sigma\in K, k\in\mathbb{N}} \{q_k(\sigma)\}.$$

Theorem 3.1 By having the condition above and $\dfrac{2Q(\vartheta-\iota)^m}{(m-1)!} \leqslant 1$, (5) admits at least one solution $v(\gamma) = (v_i(\gamma))_{i=1}^\infty \in bv_\infty$ for all $\gamma \in K$.

Proof. The real number $M_1 > 0$ can be chosen such that $\sup_{k\in\mathbb{N}} \Big| v_k(\sigma) - v_{k-1}(\sigma) \Big| \leqslant M_1 < \infty$ and

$$\begin{aligned}
\|(v)(\gamma)\|_{bv_\infty} &= \sup_{k\in\mathbb{N}} \Big| v_k(\gamma) - v_{k-1}(\gamma) \Big| \\
&= \sup_{k\in\mathbb{N}} \Big| \int_\iota^\vartheta G(\gamma,\sigma)\big[f_k\big(\sigma, v(\sigma)\big) - f_{k-1}\big(\sigma, v(\sigma)\big)\big] \Big| d\sigma \\
&\leqslant \sup_{k\in\mathbb{N}} \int_\iota^\vartheta \Big| G(\gamma,\sigma) \Big| \Big| \big[f_k\big(\sigma, v(\sigma)\big) - f_{k-1}\big(\sigma, v(\sigma)\big)\big] \Big| d\sigma \\
&\leqslant \sup_{k\in\mathbb{N}} \int_\iota^\vartheta \Big| G(\gamma,\sigma) \Big| \Big\{ p_k(\sigma) + q_k(\sigma) \Big| v_k(\sigma) - v_{k-1}(\sigma) \Big| \Big\} d\sigma \\
&\leqslant \frac{2(\vartheta-\iota)^m}{(m-1)!} \Big(P + Q \sup_{k\in\mathbb{N}} \Big| v_k(\sigma) - v_{k-1}(\sigma) \Big| \Big) \\
&\leqslant \frac{2(\vartheta-\iota)^m}{(m-1)!} \Big(P + QM_1 \Big) \\
&= r.
\end{aligned}$$

Let

$$v^0(\gamma) = \Big(v_i^0(\gamma) \Big)_{i=1}^\infty,$$

where $v_i^0(\gamma) = 0$ for all $\gamma \in K$. Also, let $\bar{D} = \bar{D}\big(v^0(\gamma), r\big)$, is closed. Obviously, $\emptyset \neq \bar{D} = D \subseteq bv_\infty$ is convex and bounded. We define the operator $T = (T_i)_{i=1}^\infty$ on $C(K, \bar{D})$ by

$$(Tv)(\gamma) = \{(T_i v)(\gamma)\} = \Big\{ \int_\iota^\vartheta G(\gamma,\sigma) f_i\big(\sigma, v(\sigma)\big) d\sigma \Big\}$$

for all $\gamma \in K$, where $v(\gamma) = (v_i(\gamma))_{i=1}^\infty \in \bar{D}$ and $v_i(\gamma) \in \mathbb{R}$. Now, we can easily show that

$$\begin{aligned}
\|(Tv)(\gamma)\|_{bv_\infty} &= \sup_{k\in\mathbb{N}} \Big| (T_k v)(\gamma) - (T_{k-1} v)(\gamma) \Big| \\
&= \sup_{k\in\mathbb{N}} \Big| \int_\iota^\vartheta G(\gamma,\sigma)\big[f_k\big(\sigma, v(\sigma)\big) - f_{k-1}\big(\sigma, v(\sigma)\big)\big] \Big| d\sigma \\
&\leqslant r \\
&< \infty.
\end{aligned}$$

Therefore, $(Tv)(\gamma) \in bv_\infty$. Moreover, $(T_i v)(\gamma)$ satisfies boundary conditions given by

$$(T_i v)(\iota) = (T_i v)'(\iota) = ... = (T_i v)^{(n-2)}(\iota) = (T_i v)(\vartheta) = 0.$$

Since

$$\Big\| (Tv)(\gamma) - v^0(\gamma) \Big\|_{bv_\infty} \leqslant r,$$

$T:\bar{D}\to\bar{D}$. Also, by theorem's hypothesis, T is continuous on $C(K,\bar{D})$. Now, we show that T is a Meir–Keeler condensing operator. Hence,

$$\begin{aligned}
\tilde{\mu}\big(T(\bar{D})\big) &= \lim_{n_1\longrightarrow\infty}\Big[\sup_{v(\gamma)\in\bar{D}}\Big\{\sup_{k\geqslant n_1}\Big|\int_{\iota}^{\vartheta}G(\gamma,\sigma)\Big(f_k\big(\sigma,v(\sigma)\big)-f_{k-1}\big(\sigma,v(\sigma)\big)\Big)d\sigma\Big|\Big\}\Big]\\
&\leqslant \lim_{n_1\longrightarrow\infty}\Big[\sup_{v(\gamma)\in\bar{D}}\Big\{\sup_{k\geqslant n_1}\int_{\iota}^{\vartheta}\Big|G(\gamma,\sigma)\Big|\Big|f_k\big(\sigma,v(\sigma)\big)-f_{k-1}\big(\sigma,v(\sigma)\big)\Big|d\sigma\Big\}\Big]\\
&\leqslant \lim_{n_1\longrightarrow\infty}\Big[\sup_{v(\gamma)\in\bar{D}}\sup_{k\geqslant n_1}\int_{\iota}^{\vartheta}\Big|G(\gamma,\sigma)\Big|\Big\{p_k(\sigma)+q_k(\sigma)|v_k(\sigma)-v_{k-1}(\sigma)|\Big\}d\sigma\Big]\\
&\leqslant \frac{2(\vartheta-\iota)^m}{(m-1)!}\Big(Q\tilde{\mu}(\bar{D})\Big).
\end{aligned}$$

Therefore, we obtain

$$\tilde{\mu}\big(T(\bar{D})\leqslant \frac{2(\vartheta-\iota)^m}{(m-1)!}\Big(Q\tilde{\mu}(\bar{D})\Big)<\varepsilon\Longrightarrow\tilde{\mu}(\bar{D})<\frac{(m-1)!\varepsilon}{2Q(\vartheta-\iota)^n}.$$

Now, assume

$$\delta=\frac{(m-1)!-2Q(\vartheta-\iota)^n}{2Q(\vartheta-\iota)^n}\varepsilon.$$

Theorem 1.5 grantees that T has a fixed point in $\bar{D}$. ■

***Example* 3.2** Consider the equation

$$v_k^{(5)}(\sigma)+\frac{s^2}{\sqrt{3}}\sum_{j=k}^{k+1}\Big(\frac{1}{(j+1)(j+2)(j+3)}\Big)+\sin(\sigma+5)\cos\big(v_k(\sigma)+1\big)=0, \tag{6}$$

where $\sigma\in[0,1]$. Note that (6) is a special case of (5) when

$$\begin{aligned}
f_k\big(\sigma,v(\sigma)\big) &= \frac{\sigma^2}{\sqrt{3}}\sum_{j=k}^{k+1}\Big(\frac{1}{(j+1)(j+2)(j+3)}\Big)+\sin(\sigma+5)\cos\big(v_k(\sigma)+1\big),\\
p_k(\sigma) &= \Big(\frac{6\sigma^2}{\sqrt{3}}\Big)\frac{1}{k(k+1)(k+3)(k+4)} \quad\text{and}\quad q_k(\sigma)=1.
\end{aligned}$$

Also, we have $P=(\frac{3\vartheta^2}{20\sqrt{3}})$ and $Q=1$. Now, for $\sigma\in[0,1]$ and $v(\gamma)=\big(v_i(\gamma)\big)_{i=1}^{\infty}\in bv_\infty$, we have

$$\begin{aligned}
&\Big|f_k\big(\sigma, v(\sigma)\big) - f_{k-1}\big(\sigma, v(\sigma)\big)\Big| \\
&\leqslant \frac{\sigma^2}{\sqrt{3}}\Big(\Big|\sum_{i=k}^{k+1}\big(\frac{1}{(j+1)(j+2)(j+3)}\big) - \sum_{j=k-1}^{k}\big(\frac{1}{(j+1)(j+2)(j+3)}\big)\Big|\Big) \\
&\quad + \Big|\sin(\sigma+5)\Big(\cos\big(v_k(\sigma)+1\big) - \cos\big(v_{k-1}(\sigma)+1\big)\Big)\Big| \\
&\leqslant \frac{\sigma^2}{\sqrt{3}}\Big|\frac{1}{(k+2)(k+3)(k+4)} - \frac{1}{k(k+1)(k+2)}\Big| \\
&\quad + \Big|\cos\big(v_k(\sigma)+1\big) - \cos\big(v_{k-1}(\sigma)+1\big)\Big| \\
&\leqslant \frac{\sigma^2}{\sqrt{3}}\Big|\frac{6}{k(k+1)(k+3)(k+4)}\Big| + \Big|v_k(\sigma) - v_{k-1}(\sigma)\Big|.
\end{aligned}$$

Also, $\lim_{k\to\infty}\{p_k(\sigma)\}_{k=1}^{\infty}$ converges uniformly to zero on K and $\{q_k(\sigma)\}_{k=1}^{\infty}$ equibounded on interval K. Hence, Theorem 3.1 implies that problem (6) has at least one solution in bv_∞.

Acknowledgments

The authors would like to thank the anonymous referee for his/her comments that helped us improve this article.

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