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On the topological equivalence of some generalized metric spaces

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Abstract. The aim of this paper is to establish the equivalence between the concepts of an S-metric space and a cone S-metric space using some topological approaches. We introduce a new notion of a TVS-cone S-metric space using some facts about topological vector spaces. We see that the known results on cone S-metric spaces (or N-cone metric spaces) can be directly obtained from the studies on S-metric spaces.

Keywords: S-metric, cone S-metric, N-cone metric.

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1. Introduction

The study of cone metric spaces was started with the paper [10]. Since then, various studies have been obtained on cone metric spaces. But, using the topological aspects and some different approaches, it was proved that the notions of a metric space and a cone metric space are equivalent (for example, see [4, 5, 13, 14] for more details).

Recently, S-metric spaces have been introduced as a generalization of metric spaces in [25]. Many fixed-point results have been extensively studied since then using various approaches (see [15, 17–29]). The relationships between a metric and an S-metric were given with some counter examples (see [11, 12, 21]). Then, Dhamodharan and Krishnakumar introduced a new generalized metric space called as a cone S-metric space [2]. This metric space is also called as N-cone metric space by Malviya and Fisher in [16]. Some well-known fixed-point results were generalized on both cone S-metric and N-cone metric spaces (for example, [2, 6, 16]).

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In the present work, we show the topological equivalence of an S-metric space and a cone S-metric space. To do this, we introduce a new notion called as a TVS-cone S-metric space as a generalization of both metric and cone S-metric (or N-cone metric) spaces. In Section 2, we recall some necessary definitions and lemmas in the sequel. In Section 3, we present a notion of a TVS-cone S-metric space and establish the equivalence between new this space and a cone S-metric space. Also, we see that some known theorems studied on cone S-metric spaces (or N-cone metric spaces) can be directly obtained from the studies on S-metric spaces. In Section 4, we investigate the relationships between an S-metric space and a cone S-metric space in view of their topological properties. In Section 5, we give a brief account of review about the obtained results and draw a diagram which shows the relations among some known generalized metric spaces.

2. Preliminaries

In this section, we recall some necessary notions and results related to cone, S-metric and cone S-metric (or N-cone metric).

Definition 2.1 [25] Let X be a nonempty set and $\mathcal{S} : X \times X \times X \to [0, \infty)$ be a function satisfying the following conditions for all $u, v, z, a \in X$:

(1) $\mathcal{S}(u, v, z) \geqslant 0$,
(2) $\mathcal{S}(u, v, z) = 0$ if and only if $u = v = z$,
(3) $\mathcal{S}(u, v, z) \leqslant \mathcal{S}(u, u, a) + \mathcal{S}(v, v, a) + \mathcal{S}(z, z, a)$.

Then the function $\mathcal{S}$ is called an S-metric on X and the pair $(X, \mathcal{S})$ is called an S-metric space.

Definition 2.2 [25] Let $(X, \mathcal{S})$ be an S-metric space and $\{u_n\}$ be a sequence in this space.

(1) A sequence $\{u_n\} \subset X$ converges to $u \in X$ if $\mathcal{S}(u_n, u_n, u) \to 0$ as $n \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n \geqslant n_0$ we have $\mathcal{S}(u_n, u_n, u) < \varepsilon$.
(2) A sequence $\{u_n\} \subset X$ is a Cauchy sequence if $\mathcal{S}(u_n, u_n, u_m) \to 0$ as $n, m \to \infty$, that is, for each $\varepsilon > 0$, there exists $n_0 \in \mathbb{N}$ such that for all $n, m \geqslant n_0$ we have $\mathcal{S}(u_n, u_n, u_m) < \varepsilon$.
(3) The S-metric space $(X, \mathcal{S})$ is complete if every Cauchy sequence is a convergent sequence.

Lemma 2.3 [25] Let $(X, \mathcal{S})$ be an S-metric space and $u, v \in X$. Then we have

$$\mathcal{S}(u, u, v) = \mathcal{S}(v, v, u).$$

Definition 2.4 [25] Let $(X, \mathcal{S})$ be an S-metric space. For $r > 0$ and $u \in X$, the open ball $B_S(u, r)$ defined as follows:

$$B_S(u, r) = \{v \in X : \mathcal{S}(v, v, u) < r\}.$$

Definition 2.5 [10] Let E be a real Banach space and K be a subset of E. K is called a cone if and only if

(1) K is closed, nonempty and $K \neq \{0\}$,

(2) If $a, b \in \mathbb{R}$ with $a, b \geqslant 0$ and $u, v \in K$, then $au + bv \in K$,
(3) If $u \in K$ and $-u \in K$ then $u = 0$.

Then the pair (E, K) is called a cone space. Given a cone $K \subset E$, a partial ordering $\precsim$ with respect to K is defined by $u \precsim v$ if and only if $v - u \in K$. It was written $u \prec v$ to indicate that $u \precsim v$ but $u \neq v$. Also $u \ll v$ stands for $v - u \in intK$ where $intK$ denotes the interior of K [10].

Lemma 2.6 [14] Let (E, K) be a cone space with $u \in K$ and $v \in intK$. Then one can find $n \in \mathbb{N}$ such that $u \ll nv$.

Lemma 2.7 [14] Let $v \in intK$. If $u \geqslant v$ for all u then $u \in intK$.

Lemma 2.8 [14] Let (E, K) be a cone space. If $u \leqslant v \ll z$ then $u \ll z$.

Definition 2.9 [2] Suppose that E is a real Banach space, K is a cone in E with $intK \neq \emptyset$ and $\precsim$ is partial ordering with respect to K. Let X be a nonempty set and a function $\mathcal{S} : X \times X \times X \to E$ satisfies the following conditions

(1) $0 \precsim \mathcal{S}(u, v, z)$,
(2) $\mathcal{S}(u, v, z) = 0$ if and only if $u = v = z$,
(3) $\mathcal{S}(u, v, z) \precsim \mathcal{S}(u, u, a) + \mathcal{S}(v, v, a) + \mathcal{S}(z, z, a)$.

Then the function $\mathcal{S}$ is called a cone S-metric on X and the pair $(X, \mathcal{S})$ is called a cone S-metric space.

We note that the notion of a cone S-metric is also called as an N-cone metric in [16].

Lemma 2.10 [2] Let $(X, \mathcal{S})$ be a cone S-metric space. Then we get

$$\mathcal{S}(u, u, v) = \mathcal{S}(v, v, u).$$

Definition 2.11 [6] Let $(X, \mathcal{S})$ be a cone S-metric space, each cone S-metric $\mathcal{S}$ on X generates a topology τ_S on X whose base is the family of the open balls $B_S(u, c)$ defined as $B_S(u, c) = \{v \in X : \mathcal{S}(v, v, u) \ll c\}$ for $c \in E$ with $0 \ll c$ and for all $u \in X$.

3. TVS-cone S-metric spaces

Let E be a Hausdorff topological vector space (briefly TVS) with its zero vector θ_E. A nonempty and closed subset K of E is called a (convex) cone if $K + K \subseteq K$, $\lambda K \subseteq K$ for $\lambda \geqslant 0$ and $K \cap (-K) = \{\theta_E\}$. Also assume that the cone K has a nonempty interior $intK$. For a given cone $K \subseteq E$, a partial ordering $\precsim_K$ with respect to K is defined by

$$u \precsim_K v \Longleftrightarrow v - u \in K.$$

$u \prec_K v$ stands for $u \precsim_K v$ and $u \neq v$. Also $u \ll v$ stands for $v - u \in intK$ where $intK$ denotes the interior of K [4, 13].

Let Y be a locally convex Hausdorff TVS with its zero vector θ, K be a proper, closed and convex cone in Y with $intK \neq \emptyset$, $e \in intK$ and $\precsim_K$ be a partial ordering with respect to K. The nonlinear scalarization function $\xi_e : Y \to \mathbb{R}$ is defined by

$$\xi_e(v) = \inf \{r \in \mathbb{R} : v \in re - K\},$$

for all $v \in Y$ (see [1, 3, 7–9] for more details).

We recall the following lemma given in [1, 3, 7–9].

Lemma 3.1 For each $r \in \mathbb{R}$ and $v \in Y$, the following statements are satisfied:

(1) $\xi_e(v) \leqslant r$ if and only if $v \in re - K$,
(2) $\xi_e(v) > r$ if and only if $v \notin re - K$,
(3) $\xi_e(v) \geqslant r$ if and only if $v \notin re - intK$,
(4) $\xi_e(v) < r$ if and only if $v \in re - intK$,
(5) $\xi_e(.)$ is positively homogeneous and continuous on Y,
(6) If $v_1 \in v_2 + K$ then $\xi_e(v_2) \leqslant \xi_e(v_1)$,
(7) $\xi_e(v_1 + v_2) \leqslant \xi_e(v_1) + \xi_e(v_2)$ for all $v_1, v_2 \in Y$.

Now we introduce the notion of a TVS-cone S-metric space.

Definition 3.2 Let X be a nonempty set, Y be a Hausdorff TVS ordered by a cone K and $\mathcal{S} : X \times X \times X \to Y$ be a vector-valued function. If the following conditions hold

(1) $\theta \precsim_K \mathcal{S}(u, v, z)$,
(2) $\mathcal{S}(u, v, z) = \theta$ if and only if $u = v = z$,
(3) $\mathcal{S}(u, v, z) \precsim_K \mathcal{S}(u, u, a) + \mathcal{S}(v, v, a) + \mathcal{S}(z, z, a)$

for all $u, v, z, a \in X$, then the function $\mathcal{S}$ is called a TVS-cone S-metric and the pair $(X, \mathcal{S})$ is called a TVS-cone S-metric space.

Remark 1 *A cone S-metric space is a special case of a TVS-cone S-metric space.*

Theorem 3.3 Let $(X, \mathcal{S})$ be a TVS-cone S-metric space such that the cone K has nonempty interior and $e \in intK$. Then the function $\mathcal{S}^S : X \times X \times X \to [0, \infty)$ defined by $\mathcal{S}^S = \xi_e \circ \mathcal{S}$ is an S-metric.

Proof. Using the condition (1) given in Definition 3.2 and Lemma 3.1, we get $\mathcal{S}^S(u, v, z) \geqslant 0$ for all $u, v, z \in X$. From the condition (2) given in Definition 3.2 and Lemma 3.1, we obtain the following cases:
Case 1: If $u = v = z$, then we have $\mathcal{S}^S(u, v, z) = \xi_e \circ \mathcal{S}(u, v, z) = \xi_e(\theta) = 0$.
Case 2: If $\mathcal{S}^S(u, v, z) = 0$, then we have

$$\xi_e \circ \mathcal{S}(u, v, z) = 0 \Rightarrow \mathcal{S}(u, v, z) \in K \cap (-K) = \{\theta\} \Rightarrow u = v = z.$$

If we apply the condition (3) given in Definition 3.2 together with the conditions (6) and (7) given in Lemma 3.1, then we obtain

$$\begin{aligned}
\mathcal{S}^S(u, v, z) &= \xi_e \circ \mathcal{S}(u, v, z) \\
&\leqslant \xi_e \left(\mathcal{S}(u, u, a) + \mathcal{S}(v, v, a) + \mathcal{S}(z, z, a)\right) \\
&\leqslant \xi_e \left(\mathcal{S}(u, u, a) + \mathcal{S}(v, v, a)\right) + \xi_e \left(\mathcal{S}(z, z, a)\right) \\
&\leqslant \xi_e \left(\mathcal{S}(u, u, a)\right) + \xi_e \left(\mathcal{S}(v, v, a)\right) + \xi_e \left(\mathcal{S}(z, z, a)\right) \\
&= \mathcal{S}^S(u, u, a) + \mathcal{S}^S(v, v, a) + \mathcal{S}^S(z, z, a)
\end{aligned}$$

for all $u, v, z, a \in X$. Therefore, $\mathcal{S}^S$ is an S-metric. ■

Remark 2 *Let $(X, \mathcal{S})$ be a cone S-metric space. Then the function $\mathcal{S}^S : X \times X \times X \to [0, \infty)$ defined by $\mathcal{S}^S = \xi_e \circ \mathcal{S}$ is an S-metric.*

Using the ideas of [2, 16], we give the following definition.

Definition 3.4 Let $(X,\mathcal{S})$ be a TVS-cone S-metric space, Y be a Hausdorff TVS ordered by a cone K, $u \in X$ and $\{u_n\}$ be a sequence in X.

(1) $\{u_n\}$ converges to u if and only if $\mathcal{S}(u_n,u_n,u) \to \theta$ as $n \to \infty$, that is, for every $\theta << c$, $c \in Y$ there exists $n_0 \in \mathbb{N}$ such that $\mathcal{S}(u_n,u_n,u) \ll c$ for all $n \geqslant n_0$. It is denoted by $\lim\limits_{n\to\infty} u_n = u$.

(2) $\{u_n\}$ is a Cauchy sequence if $\mathcal{S}(u_n,u_n,u_m) \to \theta$ as $n,m \to \infty$, that is, for every $\theta << c$, $c \in Y$ there exists $n_0 \in \mathbb{N}$ such that $\mathcal{S}(u_n,u_n,u_m) \ll c$ for all $n,m \geqslant n_0$.

(3) $(X,\mathcal{S})$ is complete if every Cauchy sequence in X is convergent.

Theorem 3.5 Let $(X,\mathcal{S})$ be a TVS-cone S-metric space, $u \in X$, $\{u_n\}$ be a sequence in X and $\mathcal{S}^S$ be defined as in Theorem 3.3. Then the following statements hold:

(1) If $\{u_n\}$ converges to u in $(X,\mathcal{S})$, then $\{u_n\}$ converges to u in $(X,\mathcal{S}^S)$.

(2) If $\{u_n\}$ is a Cauchy sequence in $(X,\mathcal{S})$, then $\{u_n\}$ is a Cauchy sequence in $(X,\mathcal{S}^S)$.

(3) If $(X,\mathcal{S})$ is complete, then $(X,\mathcal{S}^S)$ is complete.

Proof. (1) Let $\varepsilon > 0$ be given. Using Lemma 3.1 and Theorem 3.3, if $\{u_n\}$ converges to u in $(X,\mathcal{S})$, then there exists $n_0 \in \mathbb{N}$ such that

$$\mathcal{S}(u_n,u_n,u) \ll \varepsilon e \Longleftrightarrow \mathcal{S}^S(u_n,u_n,u) = \xi_e \circ \mathcal{S}(u_n,u_n,u) < \varepsilon,$$

for all $n \geqslant n_0$ since $e \in intK$. Therefore, the condition (1) holds.

(2) Let $\{u_n\}$ be a Cauchy sequence in $(X,\mathcal{S})$. Then there exists $n_0 \in \mathbb{N}$ such that

$$\mathcal{S}(u_n,u_n,u_m) \ll \varepsilon e \Longleftrightarrow \mathcal{S}^S(u_n,u_n,u_m) < \varepsilon,$$

for all $n,m \geqslant n_0$. Hence, $\{u_n\}$ is a Cauchy sequence in $(X,\mathcal{S}^S)$.

(3) From the conditions (1) and (2), the condition (3) holds. ■

Theorem 3.6 Let $(X,\mathcal{S})$ be a complete TVS-cone S-metric space and the self-mapping $T : X \to X$ satisfies the condition $\mathcal{S}(Tu,Tu,Tv) \precsim_K h\mathcal{S}(u,u,v)$ for all $u,v \in X$ and some $h \in [0,1)$. Then T has a unique fixed point in X.

Proof. Using Theorem 3.3 and Theorem 3.5, we obtain that $(X,\mathcal{S}^S)$ is a complete S-metric space. From Lemma 3.1, we get

$$\mathcal{S}(Tu,Tu,Tv) \precsim_K h\mathcal{S}(u,u,v) \Longrightarrow \mathcal{S}^S(Tu,Tu,Tv) \leqslant h\mathcal{S}^S(u,u,v)$$

for all $u,v \in X$. Then the proof is easily seen from Theorem 3.1 on page 263 in [25]. ■

Remark 3 (1) *Theorem 3.6, Theorem* 3.1 (*on page 263 in [25]*) *and Theorem* 2.1 (*on page 239 in [2]*) *are equivalent.*

(2) *By the similar arguments used in the proof of Theorem 3.6, we obtain the following relations*:

(i) *Theorem* 2.5 (*on page 242 in [2]*) *and Theorem* 4 (*on page 244 in [19]*) *are equivalent.*

(ii) *Theorem* 2.3 (*on page 240 in [2]*) *and Theorem* 3 (*on page 240 in [19]*) *are equivalent.*

(iii) *Theorem* 2.1 (*on page 7 in [16]*) *and Corollary* 2.19 (*on page 122 in [24]*) *are equivalent.*

(iv) *Theorem* 2.1 (*on page 35 in [6]*) *and Theorem* 3.1 (*on page 263 in [25]*) *are equivalent.*

(v) Theorem 2.2 (*on page 35 in [6]*) *and Corollary* 2.8 (*on page 118 in [24]*) *are equivalent.*

(vi) Theorem 2.3 (*on page 36 in [6]*) *and Corollary* 2.15 (*on page 121 in [24]*) *are equivalent.*

4. Topological equivalence of S-metric and cone S-metric spaces

In the following theorem, we give the topological equivalence of an S-metric and a cone S-metric space.

Theorem 4.1 Let E be a Banach space ordered by a cone K with nonempty interior, X be a nonempty set and $\mathcal{S} : X \times X \times X \to K$ be a cone S-metric on X. Then there exists an S-metric $\mathcal{S}^*$ on X generating the same topology as $\mathcal{S}$.

Proof. Let $a \in (0,1)$ and $e \in intK$. Put $h = \frac{1}{a}$ and define the function $\Theta : X \times X \times X \to [0,\infty)$ as

$$\Theta(u,v,z) = \begin{cases} h^{\min\{\alpha : \mathcal{S}(u,v,z) \ll h^{\alpha} e\}} & \text{if } \mathcal{S}(u,v,z) \neq 0 \\ 0 & \text{if } \mathcal{S}(u,v,z) = 0 \end{cases}, \tag{1}$$

where $\alpha \in \mathbb{Z}$. It can be easily checked that $\Theta(u,u,v) = \Theta(v,v,u)$ and

$$\Theta(u,v,z) = 0 \iff u = v = z.$$

Now we define the function $\mathcal{S}^* : X \times X \times X \to [0,\infty)$ by

$$\mathcal{S}^*(u,v,z) = \inf\left\{ \sum_{i=1}^{n-2} \Theta(u_i, u_{i+1}, u_{i+2}) : u_1 = u, \dots, u_{n-2} = u, u_{n-1} = v, u_n = z \right\}. \tag{2}$$

From the definitions (1) and (2), we have $\mathcal{S}^*(u,v,z) \geqslant 0$ and

$$\mathcal{S}^*(u,v,z) = 0 \iff u = v = z.$$

We show that the triangle inequality is satisfied by the function $\mathcal{S}^*$. For $\varepsilon > 0$, we prove

$$\mathcal{S}^*(u,v,z) \leqslant \mathcal{S}^*(u,u,a) + \mathcal{S}^*(v,v,a) + \mathcal{S}^*(z,z,a) + \varepsilon.$$

By the definition (2), there exists $u_1 = u, \dots, u_{n-1} = u, u_n = a$ with

$$\sum \Theta(u_i, u_i, u_{i+1}) \leqslant \mathcal{S}^*(u,u,a) + \frac{\varepsilon}{3},$$

$v_1 = v, \dots, v_{n-1} = v, v_n = a$ with

$$\sum \Theta(v_i, v_i, v_{i+1}) \leqslant \mathcal{S}^*(v,v,a) + \frac{\varepsilon}{3}$$

and $z_1 = z, \dots, z_{n-1} = z, z_n = a$ with

$$\sum \Theta(z_i, z_i, z_{i+1}) \leqslant \mathcal{S}^*(z,z,a) + \frac{\varepsilon}{3}.$$

Therefore, we get

$$\begin{aligned}\mathcal{S}^*(u,v,z) &\leqslant \sum \Theta(u_i,u_i,u_{i+1}) + \sum \Theta(v_i,v_i,v_{i+1}) + \sum \Theta(z_i,z_i,z_{i+1}) \\ &\leqslant \mathcal{S}^*(u,u,a) + \mathcal{S}^*(v,v,a) + \mathcal{S}^*(z,z,a) + \varepsilon,\end{aligned}$$

that is, $\mathcal{S}^*$ is an S-metric.

Now we show that each $B_S(u,c)$ contains some $B_{S^*}(u,r)$. Let us consider the open ball $B_{S^*}(u,r)$ for $u \in X$ and $r \in [0,\infty)$. It can be found $\alpha \in \mathbb{Z}$ such that $h^\alpha < r$. We put $c \ll h^\alpha e$. If $\mathcal{S}(u,u,v) \ll c$ then $\Theta(u,u,v) \leqslant h^\alpha < r$ and $\mathcal{S}^*(u,u,v) \leqslant \Theta(u,u,v) < r$, for each $v \in X$. Then we get

$$B_S(u,c) \subseteq B_{S^*}(u,r). \tag{3}$$

Conversely, let us consider the open ball $B_S(u,c)$ for $u \in X$ and $c \in E$. For each $u,v \in X$ and $r \in [0,\infty)$ if $\mathcal{S}^*(u,u,v) < r$ then we can find $u_1 = u, \ldots, u_{n-1} = u, u_n = v$ with

$$\sum \Theta(u_i,u_i,u_{i+1}) < r.$$

However for each $i < n$, we have $\mathcal{S}(u_i,u_i,u_{i+1}) \ll \Theta(u_i,u_i,u_{i+1})e$ and so

$$\mathcal{S}(u,u,v) \leqslant \sum_{i=1}^{n-1} \Theta(u_i,u_i,u_{i+1})e \leqslant re.$$

If we choose r satisfying $re \ll c$, then we have $\mathcal{S}(u,u,v) \ll c$ and

$$B_{S^*}(u,r) \subseteq B_S(u,c). \tag{4}$$

Therefore, from the inequalities (3) and (4), $\mathcal{S}^*$ induces the same topology as the cone S-metric topology of S. ■

5. Conclusion

We have defined the concept of a TVS-cone S-metric space as a generalization of a cone S-metric space. We have established the equivalence between the notions of an S-metric space and a TVS-cone S-metric space (resp. cone S-metric space) and presented some related results. Also it is shown the topological equivalence of these spaces. On the other hand, complex valued S-metric spaces are a special class of cone S-metric spaces. But it is important to study some fixed-point results in complex valued S-metric spaces since some contractions have a product and quotient (see [17, 28] for more details).

From the known (see [2, 4, 5, 10–14, 16, 21] for more details) and obtained results, we get the following diagram:

$$\begin{array}{ccc} \text{metric spaces} & \Longleftrightarrow & \text{cone metric spaces} \\ \Downarrow & & \Downarrow \\ S\text{-metric spaces} & \Longleftrightarrow & \text{cone } S\text{-metric spaces} = N\text{-cone metric spaces} \end{array}$$

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