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Some topological operators Via grills

A.A. Nasef[a], A. A. Azzam[b*]

[a] *Department of Physics and Engineering Mathematics , Faculty of Engineering, Kafr El-Sheikh University, Kafr El-Sheikh, Egypt.*
[b] *Department of Mathematics, Faculty of Science, Assuit University, New Valley, Egypt.*

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Abstract. In this paper, we define and study two operators Φ^s and Ψ^s with grill. Characterization and basic properties of these operators are obtained. Also, we generalize a grill topological spaces via topology τ^s induced from operators Φ^s and Ψ^s.

Keywords: Grill, topological spaces, Φ^s_G , Ψ^s_G-operators and τ^s_G.

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1. Introduction

The concept of grill topological spaces depended on the two operators are Φ and Ψ. The first to introduce this concept by Choquet [3] in 1947. This concept is similar to the concept of ideals, nets and filters. Some theories and characteristics of the topological spaces with grill on both sets and functions has been studied in [1, 2, 4, 5, 12]. Also for the investigation of many topological notions similar compactifications, proximity spaces and extension problems of different kinds [See 7, 8, 9, 13]. In [11] Roy and Mukherjee introduce grill topological space τ_G, some other characterizations and also the relationship between τ and τ_G. Our purpose in this paper, is to define and study new operators Φ^s and Ψ^s with grill. Characterization and basic properties of these operators are obtained. Also, we generalize a grill topological spaces via topology τ^s induced from operators Φ^s and Ψ^s.

*Corresponding author.
E-mail address: azzam0911@gmail.com (A. A. Azzam).

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Definition 1.1 A subset A of topological space (X,τ) is called semi-open set [10] if $A \subseteq Cl(Int(A))$ where $A \subseteq X$ and it's called semi-closed set [6] if $Int(Cl(A)) \subseteq A$, for $A \subseteq X$. $sCl(A)$ symbol expresses the intersection of all semi-closed sets of (X,τ) containing A and it is called the semi-closure of A.

The family of all semi-open (resp. semi-closed) sets of (X,τ) is denoted by $SO(X,\tau)$ (resp. $SC(X,\tau)$).

2. Preliminaries

While taking our idea in this research, some symbols such as X, $Int(A)$ and $Cl(A)$ are using to mean X carries topology τ, interior and closure of a set A in (X,τ) whenever $A \subseteq X$. Also, $P(X)$ will be written to mean the power set of X. A collection G of nonempty subsets of X is named a grill ([3]) if
(i) $\phi \notin G$,
(ii) $A \subseteq B \subseteq X$ and $A \in G \Rightarrow B \in G$,
(iii) $A, B \subseteq X$ and $A \cup B \in G \Rightarrow A \in G$ or $B \in G$.
For instance, let R be the set of all real numbers. Consider a subset $G = \{A \subseteq R : m(A) \neq 0\}$, where $m(A)$ denotes the Lebesgue measure of A, then G is a grill.

Remark 1 *(1)-The minimal grill is* $G = \{X\}$ *in the space* (X,τ).
(2)-The maximum grill is $G = P(X)\backslash\{\phi\}$ *in any topology* τ *on the space* X.

Definition 2.1 (see [11]) Let (X,τ,G) be a grill topological space. An operator $\Phi : P(X) \rightarrow P(X)$ is defined as follows: $\Phi(A) = \Phi_G(A,\tau) = \{x \in X : A \cap U \in G$ for every open set U containing $x\}$ for each $A \in P(X)$. The mapping Φ is called the operator associated with the grill G and the topology τ.

Definition 2.2 (see [11]) Let G be a grill topological space (X,τ). Then we define a map $\Psi : P(X) \rightarrow P(X)$ by $\Psi(A) = A \cup \Phi(A)$ for all $A \in P(X)$. The map Ψ is a kuratowski closure axioms. Corresponding to a grill G on a topological space (X,τ), there exists a unique topology τ_G on X given by $\tau_G = \{U \subseteq X : \Psi(X \setminus U) = X \setminus U\}$, where for any $A \subseteq X, \Psi(A) = A \cup \Phi(A) = \tau_G - Cl(A)$. For any grill G on a topological space (X,τ), $\tau \subseteq \tau_G$. If (X,τ) is a topological space with a grill G on X, then we call it a grill topological space and denote it by (X,τ,G).

***Example* 2.3** (see[11]) Let τ denote the cofinite topology on a uncountable set X and let G be the grill of all uncountable subset of X. Then it is clearly $\tau \setminus \{\phi\} \subseteq G$. We show that τ_G is the countable topology which is denoted by τ_{co} on X. If $V \in \tau_G$, then $V = U \setminus A$ where $U \in \tau$ and $A \notin G$ implies that $(X \setminus U)$ is finite and A is countable. Now $X \setminus V = X \cap (X \setminus V) = X \cap (X \setminus (U \cap (X \setminus A))) = X \cap ((X \setminus U) \cup A) = (X \setminus U) \cup A$ which is countable and hence $V \in \tau_{co}$. On the other hand if $V \in \tau_{co}$ implies that $X \setminus V = A \notin G$ and hence $V = X \setminus A$, where $X \in \tau$ and $A \notin G$ so $V \in \tau_G$. Thus $\tau_G = \tau_{co}$.

Lemma 2.4 (see[11]) For a grill topological space (X,τ,G), $\tau \subseteq B(G,\tau) \subseteq \tau_G$, where $B(G,\tau) = \{V \backslash A : V \in \tau and A \notin G\}$ is an open base for τ_G.

***Example* 2.5** (see[1])Let (X,τ) be a topological space satisfying conditions of a grill. If G contains all power set of X except null-set, it is obvious that $\tau_G = \tau$. Since for any τ_G-basic open set $V = X - A$ with $U \in \tau$ and $A \notin G$, we have $A = \phi$, so that $V = U \in \tau$. Hence by Lemma 2.5 we have in this case $\tau = B(G,\tau) = \tau_G$.

3. New Operators Via Grills

In this section, we define and study new operators Φ^s and Ψ^s with grill. Characterizations and basic properties of these operators are obtained. Also, we generalize a grill topological spaces via topology τ^s induced from operators Φ^s and Ψ^s.

Definition 3.1 Let (X, τ) be a topological space and G be a grill on X. We define a mapping $\Phi^s : P(X) \to P(X)$, denoted by Φ^s_G for $A \in P(X)$ (simply $\Phi^s(A)$), called the operator associated with G and τ which is defined by $\Phi^s_G(A) = \{x \in X : U_x \cap A \in G, \forall U_x \in SO(X, \tau)\}$ $\forall A \in P(X)$.

Theorem 3.2 Let (X, τ, G) be a topological space with grill on X. Then, for every $A, B \subseteq X$, the next properties are true:
(a) If $A \subseteq B$ then $\Phi^s(A) \subseteq \Phi^s(B)$,
(b) $\Phi^s(A \cup B) = \Phi^s(A) \cup \Phi^s(B)$,
(c) $\Phi^s(\Phi^s(A)) \subseteq \Phi^s(A) = sCl(\Phi^s(A)) \subseteq sCl(A)$
(d) If $A \notin G$, then $\Phi^s(A) = \phi$.

Proof. (a) It is clear from Definition 3.1.
(b) Firstly, we prove that $\Phi^s(A \cup B) \subseteq \Phi^s(A) \cup \Phi^s(B)$. Suppose that $x \notin \Phi^s(A) \cup \Phi^s(B)$. Then there exist U_1 and $U_2 \in SO(X, \tau)$ such that $A \cap U_1$ and $B \cap U_2 \notin G$ and hence $(A \cap U_1) \cup (B \cap U_2) \notin G$. If $U_1 \cap U_2 \in SO(X, \tau)$ and $(A \cup B) \cap (U_1 \cap U_2) \subseteq (A \cap U_1) \cup (B \cap U_2) \notin G$, so $x \notin \Phi^s(A \cup B)$. Secondly, let $x \notin \Phi^s(A \cup B)$, then there is $U \in SO(X, \tau)$ such that $(A \cup B) \cap U \notin G \Rightarrow (A \cap U) \cup (B \cap U) \notin G$. that is $x \notin \Phi^s(A) \cup \Phi^s(B)$.
(c) Let $x \notin sClA \Rightarrow x \notin A$ and $x \notin Int(Cl(A)) \Rightarrow \exists U \in SO(X, \tau)$ such that $U \cap A \notin G \Rightarrow x \notin \Phi^s(A)$. Thus $\Phi^s(A) \subseteq sCl(A)$. We prove that $sCl(\Phi^s(A)) \subseteq \Phi^s(A)$, so , let $x \in sCl(\Phi^s(A))$ and $U \in SO(X, \tau) \Rightarrow U \cap \Phi^s(A) \neq \phi$. Let $z \in U \cap \Phi^s(A)$, that is $z \in U$ and $z \in \Phi^s(A)$. Then $U \cap A \in G$ and $x \in \Phi^s(A)$. Then $sCl(\Phi^s(A)) \subseteq \Phi^s(A)$ and indeed $\Phi^s(A) \subseteq sCl(\Phi^s(A))$ so, $sCl(\Phi^s(A)) = \Phi^s(A)$.
(d) The proof is obvious. ■

Note: For any two grills G_1 and G_2 with topology τ defined on a space X and $G_1 \subseteq G_2$, then $\Phi^s_{G_1}(A) \subseteq \Phi^s_{G_2}(A)$ for any $A \subseteq X$.

Theorem 3.3 Let (X, τ, G) be a grill topological space. If $U \in SO(X, \tau)$. Then $U \cap \Phi^s(A) = U \cap \Phi^s(U \cap A)$, for $A \subseteq X$.

Proof. By Theorem 3.2, we have $U \cap \Phi^s(U \cap A) \supseteq U \cap \Phi^s(A)$, let $x \in U \cap \Phi^s(A)$ and $V \in SO(X, \tau)$. That $U \cap V \in SO(X, \tau)$ and $x \in \Phi^s(A)$ which leads to $(U \cap V) \cap A \in G$, that is $(U \cap A) \cap V \in G \Rightarrow x \in \Phi^s(U \cap A) \Rightarrow x \in U \cap \Phi^s(U \cap A)$. Thus $U \cap \Phi^s(A) = U \cap \Phi^s(U \cap A)$. ■

Theorem 3.4 If (X, τ, G) is a grill topological space with $SO(X, \tau) \setminus \{\phi\} \subseteq G$, then for $U \in SO(X, \tau), U \subseteq \Phi^s(U)$.

Proof. If $SO(X, \tau) \setminus \{\phi\} \subseteq G$, then $\Phi^s(X)$ coincide X. Let $x \notin \Phi^s(X)$ implies that exists $V \in SO(X, \tau)$ such that $V \cap X \notin G \Rightarrow V \notin G$, a contradiction. If $U = \phi$, we have $\Phi^s(U) = \phi = U$. By using Theorem 3.3, we have for any $U \in SO(X, \tau) \setminus \{\phi\}$, $U \cap \Phi^s(X) = U \cap \Phi^s(X \cap U)$, and that $U = U \cap X = U \cap \Phi^s(U)$. Thus $U \subseteq \Phi^s(U)$. ■

Lemma 3.5 For any $A, B \subseteq X$ and any grill G on a space X, $\Phi^s(A) \backslash \Phi^s(B) = \Phi^s(A \backslash B) \backslash \Phi^s(B)$.

Proof. From Theorem 3.2 we have that $\Phi^s(A) = \Phi^s(A \backslash B) \cup \Phi^s(A \cap B) = \Phi^s((A \backslash B) \cup (A \cap B)) \subseteq \Phi^s(A \backslash B) \cup \Phi^s(B)$. Thus $\Phi^s(A) \backslash \Phi^s(B) \subseteq \Phi^s(A \backslash B) \backslash \Phi^s(B)$. Also, $\Phi^s(A \backslash B) \subseteq$

$\Phi^s(A)$ implies $\Phi^s(A\backslash B)\backslash\Phi^s(B) \subseteq \Phi^s(A)\backslash\Phi^s(B)$. Hence the proof is completed. ■

Corollary 3.6 Let (X, τ, G) be a grill topological space. Suppose that $A, B \subseteq X$ with $B \notin G$. Then $\Phi^s(A \cup B) = \Phi^s(A) = \Phi^s(A\backslash B)$.

Proof. From Theorem 3.2(b) we see $\Phi^s(A \cup B) = \Phi^s(A) \cup \Phi^s(B) = \Phi^s(A)$. By (a) from Theorem 3.2, we can see $\Phi^s(A\backslash B) \subseteq \Phi^s(A)$. Also, from Lemma 3.5, $\Phi^s(A)\backslash\Phi^s(B) \subseteq \Phi^s(A\backslash B)$, then $\Phi^s(A) \subseteq \Phi^s(A\backslash B)$. Therefore $\Phi^s(A) = \Phi^s(A\backslash B)$.

■

Corollary 3.7 For any (X, τ, G) a grill topological space, we have $\Phi^s_G(A) \subseteq \Phi_G(A)$ for $A \subseteq X$ and $\Phi^s(A) \subseteq A$. that is $\Phi^s(A) \subseteq A \subseteq \Phi_G(A)$

Definition 3.8 Let (X, τ, G) a grill topological space. An operator $\Psi^s_G : P(X) \to P(X)$ is defined as $\Psi^s_G(A) = \{x \in X : \exists\ U_x \in SO(X, \tau)$ such that $U\backslash A \notin G\}$, for any $A \subseteq X$ and notes that $\Psi^s_G(A) = X\backslash\Phi^s(X\backslash A)$ or $\Psi^s(A) = A \cup \Phi^s(A)$.

Theorem 3.9 The operator Ψ^s satisfies Kuratowskis closure axioms.

Proof. From Theorem 3.2, we have $\Phi^s(\phi) = \phi$, and its clear that $A \subseteq \Psi^s(A), \forall A \subseteq X$. Then $\Psi^s(A \cup B) = (A \cup B) \cup \Phi^s(A \cup B) = \Psi^s(A) \cup \Psi^s(B)$. For any $A \subseteq X, \Psi^s(\Psi^s(A)) = \Psi^s(A \cup \Phi^s(A)) = A \cup \Phi^s(A) \cup \Phi^s(A \cup \Phi^s(A)) = A \cup \Phi^s(A) \cup \Phi^s(A) \cup \Phi^s(\Phi^s(A)) = A \cup \Phi^s(A) = \Psi^s(A)$. ■

Theorem 3.10 Let (X, τ) be a topological space and G be a grill on X. Then the following properties hold:
(a) If $A, B \in P(X)$, then $\Psi^s(A \cap B) = \Psi^s(A) \cap \Psi^s(B)$,
(b) $\Psi^s(A) = X\backslash\Phi^s(X)$ if $A \notin G$,
(c) If $A, B \subseteq X$ and $(A\backslash B) \cup (B\backslash A) \notin G$, then $\Psi^s(A) = \Psi^s(B)$,
(d) If $A \subseteq X$, then $\Psi^s(A)$ is semi-clopen in (X, τ),
(e) $\Psi^s(A) \subseteq \Psi^s(B)$ whenever $A \subseteq B$,
(f) $\Psi^s(A) \subseteq \Psi^s(\Psi^s(A))$ at $A \subseteq X$,
(g) If $A \subseteq X$ and $Z \notin G$, then $\Psi^s(A\backslash Z) = \Psi^s(A)$,
(h) If $A \subseteq X$ and $Z \notin G$, then $\Psi^s(A \cup Z) = \Psi^s(A)$.

Proof. (a) From $\Psi^s(A \cap B) = (A \cap B) \cup \Phi^s(A \cap B) = A \cup \Phi^s(A \cup B) \cap$
$[B \cup \Phi^s(A \cup B)] \subseteq (A \cup \Phi^s(A)) \cap (B \cup \Phi^s(B))$
$= \Psi^s(A) \cap \Psi^s(B)$. Similarly, we can prove that $= \Psi^s(A) \cap \Psi^s(B) \subseteq \Psi^s(A \cap B)$,
(b) From Definition 3.8. and Theorem 3.2,
(c) It is obvious from Corollary 3.6,
(d) It is clear from Definition 3.8,
(e) Since $A \subseteq B$, implies that $\Psi^s(A) \subseteq \Psi^s(B)$ from Theorem 3.2, then $\Psi^s(A) \subseteq \Psi^s(B)$,
(f) From Definition 3.8,
(g) Since $A \subseteq X$ and $Z \notin G$, then $A\backslash Z \subseteq A$ and $\Phi^s(Z) = \phi$ implies to $\Psi^s(A\backslash Z) = \Psi^s(A)$,
(h) The proof is similar to (g). ■

Note: For any topology τ on the space X with grill G on X, $\Psi^s(A) \subseteq \Psi(A)$, for every $A \subseteq X$.

4. Topology Induced from Grill in Terms of Ψ^s and Φ^s- Operators

Definition 4.1 It is noticeable that a grill G on a space X which carries topology τ generates a unique topology on X depends on Φ^s and Ψ^s operators symbolized by τ^s_G

and defined by $\tau_G^s = \{U \subseteq X : \Psi^s(X\backslash U) = (X\backslash U)\}$, for $A \subseteq X, \Psi^s(A) = A \cup \Phi^s(A) = \tau_G^s - ClA$.

Theorem 4.2 (a) Let G_1 and G_2 be satisfied the conditions of grills on a space X and $G_1 \subseteq G_2$, then $\tau_{G_2}^s \subseteq \tau_{G_1}^s$.
(b) A subset B of X is closed in the space (X, τ_G^s), if G is a grill on a space X and B not belong to G.
(c) $\Phi^s(A)$ is τ_G^s-Closed $\forall\ A \subseteq X$ and G is a grill on X.

Proof. (a) Presumably that $U \in \tau_{G_2}^s \Rightarrow \tau_{G_2}^s - Cl(X\backslash U) = (X\backslash U) = \Psi^s(X\backslash U)$ and this leads to $X\backslash U = (X\backslash U) \cup \Phi^s(X\backslash U) \Rightarrow \Psi_{G_2}^s(X\backslash U) \subseteq X\backslash U \Rightarrow \Psi_{G_1}^s(X\backslash U) \subseteq (X\backslash U)$ (by Theorem 3.2(a))$\Rightarrow U \in \tau_{G_1}^s$.
(b) From Theorem 3.2(d), $B \notin G \Rightarrow \Phi^s(B) = \phi$, and then $\tau_G^s - Cl(B) = \Psi^s(B) = B \cup \Phi^s(B) = B$. This proving that B is τ_G^s-Closed.
(c) $\Psi^s(\Phi^s(A)) = \Phi^s(A) \cup \Phi^s(\Phi^s(A)) = \Phi^s(A)$, by Theorem 3.2 $\Rightarrow \Phi^s(A)$ is τ_G^s- Closed. ■

Theorem 4.3 Let (X, τ, G) be a grill topological space. Then $\beta(\tau_G^s) = \{U\backslash A : U \in SO(X, \tau), A \notin G\}$ is called open base for τ_G^s.

Proof. Let $V \in \tau_G^s$ and $x \in V$. Then $(X\backslash V)$ is τ_G^s-semi-closed, so $\Psi^s(X\backslash V) = X\backslash V$ and $\Phi^s(X\backslash V) \subseteq X\backslash V$. Then $x \notin \Phi^s(X\backslash V)$ and there exists $U \in SO(X, \tau)$ such that $(X\backslash V) \cap U \notin G$. Let $A = (X\backslash V) \cap U$, then $x \notin A$ and $A \notin G$. Thus $x \in U\backslash A = U\backslash[(X\backslash V) \cap U] = U\backslash(X\backslash V) \subseteq U$, where $U\backslash A \in \beta(\tau_G^s)$. Hence, we prove that $\beta(\tau_G^s)$ is semi-closed under finite intersection. Let $U_1\backslash A, U_2\backslash A \in \beta(\tau_G^s)$. that is $U_1, U_2 \in SO(X, \tau)$ and $A, B \notin G$. Then $U_1 \cap U_2 \in SO(X, \tau)$ and $A \cup B \notin G$. Now, $(U_1 - A) \cap (U_2\backslash B) = (U_1 \cap U_2)\backslash(A \cup B) \notin \beta(\tau_G^s)$, then the prove is completed. ■

Corollary 4.4 For any grill G defined on a space (X, τ), $\tau \subseteq \beta(\tau_G^s) \subseteq \tau_G^s$.

***Example* 4.5** Let $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$ and $G = \{\{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$. Then , we have $\tau_G = \{\{a\}, \{b\}, \{c\}, \phi, X, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$ and $\tau_G^s = \{\{a\}, \{b\}, \{c\}, \phi, X, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}\}$ that is $\tau \subseteq \tau_G \subseteq \tau_G^s$. Also, let $G_2 = \{\{b, c\}.\{a, b, c\}, \{b, c, d\}\}$ and $G_1 = \{\{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$, then $\tau_{G_2}^s \subseteq \tau_{G_1}^s$.

Note: This example shows that τ_G^s is a generalization for τ_G.

Theorem 4.6 Let G be a grill on a space (X, τ) and A be any subset of X such that $A \subseteq \Phi_G^s$. Then $sCl(A) = \tau_G^s - Cl(A) = sCl(\Phi_G^s(A)) = \Phi_G^s(A)$

Proof. Since τ_G^s is finer than τ, $\tau_G^s - Cl(A) \subseteq sCl(A)$, we need to prove that $sCl(A) \subseteq \tau_G^s - Cl(A)$. Let $x \notin \tau_G^s - Cl(A)$, then exists $V \in SO(X, \tau)$ and $B \notin G$ such that $x \in V\backslash B$ and $(V\backslash B) \cap A = \phi$ (by Theorem 4.3), so, $\Phi^s((V\backslash B) \cap A) = \phi$ implies to $\Phi^s((V \cap A)\backslash B) = \phi$. Then $\Phi^s(V \cap A) = \phi$ (by Corollary 3.6)this implies that $V \cap \Phi^s(A) = \phi$ (by Theorem 3.3), that is $V \cap A = \phi$ (as $A \subseteq \Phi^s(A)$), then $x \notin sCl(A)$. Thus $sCl(A) \subseteq \tau_G^s - Cl(A)$.
By Theorem 3.2 $\Phi^s(A) = sCl\Phi^s(A)$ and $\Phi^s(A) \subseteq sCl(A)$.implies to $sCl\Phi^s(A) \subseteq sCl(sCl(A)) = sCl(A)$. Also, $A \subseteq \Phi^s(A)$ implies to $sCl(A) \subseteq sCl\Phi^s(A)$. Thus $sCl(A) = sCl\Phi^s(A) = \Phi^s(A)$. ■

5. Conclusion

The study of grill topological spaces generalized most of near open sets and near continuous function. So we introduced a new operators Φ_G^s , Ψ_G^s and relationships between this operators and the old operators Φ_G, Ψ_G. Also, we introduce a generalization to a grill topological space (X, τ, G).

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