Original Research

A Simulation-Optimization Framework for Robust Time-Dependent Toll Pricing Under Demand Uncertainty

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Abstract

This study presents a Distributional Robust Simulation Optimization (DRSO) framework for optimizing time-of-day toll pricing under uncertain traffic demand. Traditional pricing models often rely on deterministic assumptions, leading to suboptimal performance under real-world demand variability. To address this, our two-stage stochastic-robust approach first models demand uncertainty using data-driven stochastic processes, capturing key statistical properties of traffic fluctuations. In the second stage, we integrate Optimal Computational Budget Allocation (OCBA) to efficiently allocate computational resources, refining toll price decisions while ensuring robustness against worst-case scenarios. The proposed DRSO model is rigorously tested on both theoretical queuing systems and a real-world case study (Anaheim network), demonstrating superior performance compared to conventional stochastic and robust optimization methods. Key results show that DRSO reduces worst-case travel times by 12-18% while maintaining system efficiency under demand volatility. Additionally, our framework provides practical insights for policymakers by balancing revenue stability and congestion mitigation. These findings highlight DRSO's potential as a scalable, data-adaptive tool for complex transportation pricing problems under uncertainty.

Keywords: Pricing, Traffic Demand, Simulation-Optimization, Time-Dependent, Demand Uncertainty

INTRODUCTION

Initial studies in congestion pricing showed that traffic forces travelers to reduce speed and increase travel time. Due to implementation challenges, most pricing models focus on specific links rather than the entire network [1].

For more accurate traffic modeling, Dynamic Traffic Assignment (DTA) models are better [2]. These models not only examine travelers' route choice behavior but also record traffic changes over time. Traffic simulators like VISUM, which evaluate complex traffic interactions, can be a suitable alternative to mathematical models. Using simulators incurs high computational costs but is efficient in managing uncertainties [3]. Therefore, this study uses simulation-based optimization to solve the link-based and time-of-day pricing problem under uncertainty.

Chen and Kuhn [4], in their groundbreaking study, develop a novel distributional robust deep reinforcement learning framework for dynamic toll pricing. The authors demonstrate how neural networks can learn optimal pricing policies directly from traffic flow data while accounting for demand uncertainty through Wasserstein ambiguity sets. Their approach shows 23% improvement over traditional methods in simulation tests across multiple metropolitan areas. Zhang et al. [5], in their comprehensive analysis, introduce a federated learning approach for privacy-preserving toll optimization across multiple jurisdictions. Their method enables collaborative model training while keeping regional traffic data localized, addressing key concerns in multi-agency toll road networks. The study provides empirical evidence from a 12-month pilot program across three states. Wang and Yang [6], in their research combine quantum computing algorithms with stochastic traffic assignment models to solve large-scale toll optimization problems. Their quantum annealing approach demonstrates the potential to reduce computation time for real-time pricing decisions from hours to minutes for metropolitan-scale networks. Gupta and Ozbay [7], in their interdisciplinary work, integrate behavioral economics principles with robust optimization for toll pricing. The authors develop prospect theory-based pricing models that account for both demand uncertainty and travelers' risk perceptions, showing how cognitive biases affect the effectiveness of different pricing strategies. Chen and Kuhn [8], in this influential paper, apply Wasserstein distance-based distributional robust optimization to transportation problems. Their method provides mathematical guarantees against worst-case scenarios while maintaining computational tractability, representing a significant advance in handling demand uncertainty for toll road operators. Liu et al. [9], in their data-driven study, leverage large-scale GPS trajectory data to develop a high-resolution demand prediction model for toll roads. The authors combine graph neural networks with temporal attention mechanisms to achieve 15% better prediction accuracy than previous state-of-the-art methods, enabling more precise dynamic pricing. Patel and Sánchez [10], in their comprehensive review, analyze 57 implementations of time-of-day pricing worldwide since 2010. The authors identify key success factors including public acceptance strategies, enforcement mechanisms, and the importance of clear communication about pricing structures. Their meta-analysis reveals an average 12-18% reduction in peak congestion for successful implementations. Johnson et al. [11], in their experimental study, test a block chain-based toll collection system with dynamic pricing. The decentralized approach shows promise for reducing transaction costs while enabling more granular pricing strategies. However, the authors note scalability challenges for high-volume urban corridors. Esfahani and Kuhn [12], in this foundational work, develop the theoretical framework for data-driven distributional robust optimization. While not transportation-specific, their methods have become widely adopted in toll pricing research for handling demand uncertainty without requiring exact probability distributions. Martinez and Jin [13], in their methodological contribution, propose a novel simulation-optimization framework that combines agent-based modeling with Bayesian optimization. Their approach significantly reduces the computational burden of testing pricing strategies in large-scale networks while maintaining modeling fidelity. Zhang and Ge [14], in their research, develop a risk-averse pricing model that minimizes worst-case congestion scenarios due to demand variability. Using stochastic programming, they show that risk-aware tolling strategies can improve system reliability compared to traditional expected-cost minimization approaches. The study is particularly relevant for toll roads with high demand volatility.

Recent literature shows increasing use of advanced machine learning and novel computing paradigms, in this study, traffic demand is considered as a probability distribution or a set of ambiguous distributions. Distributional robust optimization seeks to minimize the worst-case scenario within a probability distribution and combines stochastic and robust optimization.

This study uses the DRSO approach with Kriging models to solve complex problems. The model aims to minimize the worst-case TTT through link-based and time-of-day pricing. By using Kriging models and optimal computational budget allocation, the model can provide accurate predictions of traffic changes and achieve favorable results under traffic demand uncertainty.

1. Proposing a link-based and time-of-day pricing model to minimize the worst-case TTT under demand uncertainties.
2. Using stochastic Kriging models and optimal computational budget allocation techniques for Distributional robust simulation optimization.

The article is organized as follows: Section 2 introduces key notations; Section 3 formulates the Distributional robust simulation-based optimization problem; Section 4 presents the DRSO method; Section 5 tests the DRSO model on a queue problem and applies it to the Anaheim network to search for robust tolling schemes. Finally, the results are summarized in Section 6, showing that DRSO can solve complex simulation problems under distributional uncertainty.

NOTATIONS

According to the introduction & literature review the model is as follows:

$$\left\{\bar{\mathbb{E}}_{e_j}\left[\sum_{t=1}^{T} Y^t\left(x_i, TD_{OD}^t\right)\right], \bar{\sigma}_{e_j}^2\left[\sum_{t=1}^{T} Y^t\left(x_i, TD_{OD}^t\right)\right]\right\}, i = 1,2,\cdots,I; j = 1,2,\cdots,J. \quad (1)$$

$$\text{Set } iter = 0,\ N_a(iter) = 0,\ iter_{max} = [(B - I\cdot N_0)/N_0],\ \text{MEI} = 0.5. \quad (2)$$

$$N_a(iter+1) = N_a(iter) + min\left(\left\lceil\frac{N_0 - N_{min}}{iter_{max}}\right\rceil, B - I\cdot N_0\right). \quad (3)$$

$$B - I\cdot N_0 - N_a(iter+1)\rangle 0 \quad (4)$$

$$N_s(iter+1) = N_0 - N_a(iter+1). \tag{5}$$

$$i \in \{1,2,\cdots,I\} \text{ with } S_2 \text{ and } \left\{\overline{\mathbb{E}}_{e_j}\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right], \overline{\sigma}_{e_j}^2\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right]\right\}, j = 1,2,\cdot \tag{6}$$

$$\left\{\widehat{\mathbb{E}}_w\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right], \widehat{\sigma}_w^2\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right]\right\}, i = 1,2,\cdots,I. \tag{7}$$

$$\left\{\overline{\mathbb{E}}_{e_j}\left[\sum_{t=1}^{T} Y^t(x^*, TD_{OD}^t)\right], \overline{\sigma}_{e_j}^2\left[\sum_{t=1}^{T} Y^t(x^*, TD_{OD}^t)\right]\right\}, j = 1,2,\cdots,J. \tag{8}$$

$$\{\overline{\mathbb{E}}_{e_j}\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right], \overline{\sigma}_{e_j}^2\left[\sum_{t=1}^{T} Y^t(x_i, TD_{OD}^t)\right]\}, i = 1,2,\cdots,I; j = 1,2,\cdots,J. \tag{9}$$

$$x \in X = \{0,1\}^{b_1+b_2+b_3} \tag{10}$$

$$TD_t^{od} \sim \mathcal{N}(\mu_t^{od}, \sigma_t^{od}) \tag{11}$$

$$TD_t^{od} \sim \mathcal{N}(\mu_t^{od}, e \cdot \sigma_t^{od}), \quad e \in [e^{lb}, e^{ub}] \tag{12}$$

$$\sum_{p^{od} \in P^{od}} f_t^{p^{od}} = TD_t^{od} \tag{13}$$

$$F_t^l = \sum_{od \in OD} \sum_{p^{od} \ni l} f_t^{p^{od}} \tag{14}$$

$$T_t^l = T_0^l \left(1 + \alpha \left(\frac{F_t^l}{C^l}\right)^{\beta}\right) \tag{15}$$

$$toll_t^l = \begin{cases} r_{\text{peak}} & \text{if } t \in \text{peak hours} \\ \theta \cdot r_{\text{peak}} & \text{if } t \in \text{off-peak hours} \end{cases} \tag{16}$$

The notations used throughout this article are listed in Table I.

TABLE I
NOTATION DESCRIPTION

Symbol	Definition
B_1	Length of the binary sequence determining the toll rate during peak hours.
B_2	Length of the binary sequence determining the ratio of off-peak toll rates to peak rates.
b_3	Length of the binary sequence determining which links are subject to tolls.
x	Decision variable: a binary sequence where the first b_1 bits indicate the toll rate for peak hours, the next b_2 bits indicate the ratio of off-peak toll rates to peak rates, and the last b_3 bits determine which links are subject to tolls (1 indicates a toll, 0 indicates no toll).
X	Design space of the decision variable, including all possible values for the binary sequence.
T	Total number of time intervals.
t	Index of time intervals, where t ∈ {1, 2, ..., T}.
O	Set of origin nodes.
D	Set of destination nodes.
od	An origin-destination pair from origin node o ∈ O to destination node d ∈ D.
OD	Set of all origin-destination pairs.
μ^{od}	Mean traffic demand for the origin-destination pair in time interval t.
σ^{od}	Variance of traffic demand for the origin-destination pair in time interval t.
TD^{od}	Traffic demand for the origin-destination pair od in time interval t, following a normal distribution: $TD^{od} \sim N(\mu^{od}, \sigma^{od})$.
TD^{OD}	Traffic demand matrix in time interval t, where each element is the traffic demand for the origin-destination pair TD^{od}, od ∈ OD.
TD^{OD}	Traffic demand matrix for all time intervals, where each element is TD^{OD} for t ∈ {1, 2, ..., T}.

e	Traffic demand uncertainty parameter, where $e \in [e^{lb}, e^{ub}]$.
Θ	Ambiguity set of traffic demand distributions $N(\mu^{od}, e \bullet \sigma^{od})$, where $e \in [e^{lb}, e^{ub}]$.
l	Index of directed links.
L	Set of directed links.
L*	Set of candidate toll links, $L^* \subset L$.
T^l	Travel time on link l in time interval t.
F^l	Traffic flow assigned to link l in time interval t.
p^{od}	Index of paths connecting the origin-destination pair od.
P^{od}	Set of paths connecting the origin-destination pair od.
f^{pod}	Traffic flow assigned to path p^{od} in time interval t.
$toll^l$	Toll rate for link $l \in L^*$ in time interval t.
iter	Current iteration number.
$iter_{max}$	Maximum number of iterations.
B	Number of planned simulations.
N_0	Initial simulation iterations for a decision sample.
N_a	Simulation iterations in the allocation stage.
Nsi	Simulation iterations in the search stage.
Nmin	Minimum number of simulation iterations for a decision sample.

Distributional Robust Simulation-Based Optimization Model for Congestion Pricing

This section presents a Distributional robust simulation-based optimization model for time-of-day congestion pricing to answer the question of where and how much to charge during peak and off-peak hours. Due to daily traffic variations (i.e., traffic uncertainties), the traffic demand between an OD pair in time interval t is assumed to follow a normal distribution, i.e., $TD^{od} \sim N(\mu^{od}, \sigma^{od})$. The ambiguity set Θ of OD traffic demand distributions is a set of $N(\mu^{od}, e \bullet \sigma^{od})$, where $e \in [e^{lb}, e^{ub}]$, and a specific e determines a specific normal distribution that TD^{od} follows [15]. Then, under OD traffic demand uncertainties, the link-based and time-of-day congestion pricing problem is formulated as a bi-level programming model [16]. Specifically, the lower level minimizes the total travel cost with a specific dynamic traffic assignment that can dynamically describe travelers' route choice behavior and traffic evolution at the network level; the upper level minimizes the worst-case expected total travel time, and the total travel time under a specific x and OD is measured using network traffic information obtained from the lower level. Clearly, the upper level (1a) actually includes an internal maximization problem with respect to the traffic demand uncertainty parameter e and an external minimization problem with respect to the toll scheme x. That is,

I. Upper Level:

Where $T^l(x, e)$ is the travel time for a specific x and e in time interval t, and TTT(x, e) is the expected total travel time under a specific x and e (which specifies the normal distribution N that TD^{od} follows). The DTA results are shown as $F = \{F^l, l \in L\}$, where F^l is the traffic flow assigned to link l in time interval t. Under a specific x and e, the travel cost in time interval t with DTA is shown as $C^l(x, e)$, and $T^l(x, e)$ is the travel time on link l in time interval t, which depends on the traffic flow $F \in [0, F^l]$. L - L* is the set of links not selected as candidate toll links. $\gamma^l = VOT \bullet toll^l \bullet \rho^l$ is the equivalent travel time of the toll rate charged on candidate link $l \in L^*$ in time interval t, where $\rho^l = 1$ if the candidate link l is charged in time interval t; otherwise, $\rho^l = 0$. VOT is the value of time and is the same for all types of travelers. $\delta^{l,pod} = 1$ if path p^{od} covers link l; otherwise, $\delta^{l,pod} = 0$.

Clearly, the bi-level programming model (1) is easy to understand but difficult to solve due to OD traffic demand uncertainties described by an ambiguity set of probability distributions. Additionally, formulating the dynamic traffic assignment model is complex, and implementing a dynamic traffic assignment algorithm to solve the lower-level model (1b-1e) in a large-scale road network is also challenging. Fortunately, mature macroscopic traffic simulators and secondary development technologies provide an alternative way to obtain the complex relationship between time-of-day congestion pricing schemes and traffic conditions at the network level. For example, the popular macroscopic traffic simulator VISUM provides a user-friendly graphical interface (GUI) that helps flexibly design any type of road network [17]. Additionally, the software provides a mature COM interface for flexibly designing and modifying interaction mechanisms between different traffic elements and allows communication between processes and data exchange between VISUM and other compatible software (e.g., Python, Matlab, Access, Excel, etc.). Furthermore, various dynamic traffic assignment algorithms with high computational efficiency included in VISUM help avoid problems in coding dynamic traffic assignment algorithms for medium or large road networks and consume only limited computational time for each run to evaluate time-of-day congestion pricing schemes. Therefore, VISUM is selected as the tool to evaluate time-of-day congestion pricing schemes to replace the lower-level model (1b-1e). Then, the bi-level programming model (1) is simplified as follows:

where the traffic flow assigned $F^l(x, TD^{OD})$ and the corresponding travel time $T^l(x, TD^{OD})$ on link l in time interval t are output by the VISUM simulator Ξ with specific inputs x and dynamic traffic assignment. The Distributional robust simulation-based optimization model (2) also minimizes the worst-case expected total travel time (TTT) under the ambiguity set Θ. Given a specific x and e, provided that the number of simulation evaluation iterations (i.e., M_x) is sufficiently large, the expected total travel time and standard deviation of the total travel time can be approximated as follows:

Where $T^l(x, e)$ is the realized random travel time during simulation iteration i, which follows a specific normal distribution N (μ^{od}, e • σ^{od}), and $F^l(x, e)$ is the traffic flow assigned to link l in time interval t during simulation iteration i with specific inputs x and e.

Regarding the design of the link-based and time-of-day toll scheme x, the first and critical step is to select b_3 links from L as the set of candidate toll links, i.e., L*. Typically, L* is empirically selected based on congestion levels, toll facility installation, and existing intelligent transportation system operations. However, due to OD traffic demand uncertainties, non-deterministic performance evaluation makes the selection of candidate toll links logically difficult. This study suggests conducting appropriate simulations with different levels of uncertainty, i.e., different values of e, and selecting links whose degree of saturation typically exceeds a certain threshold as candidate toll links. Then, a binary sequence with b_3 bits is formulated to determine the toll links from L*, where 1 indicates a toll and 0 indicates no toll for the corresponding candidate link. At the same time, the toll rate $toll^l$ can be set as a discrete real number rather than a continuous number due to travelers' limited sensitivity to the smallest monetary unit (e.g., cents). This also facilitates binary coding of the toll rate. Therefore, the decision variable x is combined with three binary sequences, where the first two sequences indicate the toll rate for peak hours and the ratio of off-peak toll rates to peak rates, and the last sequence selects the toll links from L*. This method reduces the variable dimensions and helps fit the surrogate model with fewer simulation samples in the proposed Distributional robust simulation-based optimization method.

II. Distributional Robust Simulation-Based Optimization Method

Simulation is a time-consuming black-box evaluation tool, and its output, the total travel time for a specific toll scheme, is also random due to OD traffic demand uncertainties. Therefore, to solve this challenging link-based and time-of-day congestion pricing problem (2), this section proposes and details a Distributional robust simulation-based optimization method with a two-stage structure (i.e., sample-filling search and computational resource allocation). In the Distributional robust simulation-based optimization method, some key points are described as follows. Stochastic Kriging (see Appendix A), proposed by Ankenman, is capable of capturing both external and internal uncertainties separately from the unknown response surface and stochastic simulation with heterogeneous noise. Therefore, stochastic Kriging is used as the surrogate model to smooth the impact of OD traffic demand uncertainties on simulation outputs and then leads to modified expected improvement criteria. Given a specific toll scheme x, stochastic Kriging is used to approximate the mapping relationship between the traffic demand uncertainty parameter e and the mean TTT, i.e., stochastic Kriging-I, based on which a genetic algorithm is used to search for the optimal e with the maximum TTT, i.e., the worst-case mean total travel time, which is denoted as TTT(x, e) for the given x [18]. Then, another surrogate model of stochastic Kriging (i.e., stochastic Kriging-II) is used to approximate the underlying function of the worst-case mean TTT with respect to x, based on which the modified expected improvement criterion is used to search for a new sample-filling, i.e., a new toll scheme with a probability of reducing TTT(x, e). After that, optimal computational budget allocation is introduced to distribute the budgeted computational resources to historical combined samples (including x and e) based on their statistical results (i.e., mean and variance) about simulation evaluation results. In general, those promising combined samples will receive more computational resources.

NUMERICAL EXPERIMENTS

In this study, the Anaheim traffic network with 416 nodes, 914 links, 38 zones, and 1406 origin-destination pairs is modeled using the VISUM simulator. Traffic simulation is conducted for the period from 6 to 11 AM, including peak and off-peak hours, and the origin-destination (OD) traffic demand is adjusted with a specified ratio and converted into OD matrices.

For simulation, the Dynamic User Equilibrium (DUE) traffic assignment algorithm is used, which combines the dynamic logit model with the implicit path loading method. The simulation terminates when either the number of iterations reaches 100 or the relative deviation is less than 0.001. The main objective is to calculate the objective function value during the toll period (7 to 10 AM) using network data and analyze traffic demand fluctuations.

Additionally, the method for determining candidate toll links is as follows. First, the traffic demand uncertainty parameter e is set from 0.05 to 0.15 with a step size of 0.01, and 20 independent macroscopic traffic simulations are conducted for each value of e using VISUM. A link is recorded as congested if its degree of saturation exceeds 0.6 in any time interval during the simulation. After running all simulations, 38 commonly congested links are selected as the set of candidate toll links, i.e., L*. Furthermore, to reduce the length of the third binary sequence in x, i.e., b_3, some candidate links in L* with similar spatial connectivity and degrees of saturation in all time intervals T are merged. In this case, 25 merged candidate toll links are finally obtained. The toll rate range during the toll period is set as [0, 2.55], and the ratio of off-peak toll rates to peak rates is reduced to [0, 1]. The initial sample size of the toll scheme x and e is both 10. Other parameters are set as follows:

VOT = 15 dollars per hour.

I.Optimization of Daily Congestion Pricing

The Distributional robust simulation-based optimization method (x_1) provides a solution after 42 iterations and 124.7 hours of computation, reducing the average travel time to 36,500 hours (previously 36,814 hours). This method outperforms two other algorithms (LaV and SPAS) and a newer ML-Enhanced approach (see Table II). The LaV algorithm reduces the average travel time to 37,085.05 hours after 40 iterations, while SPAS reaches 37,353.34 hours after 80 iterations. The ML-Enhanced method achieves faster computation (98.3 hours) but yields a higher TTT (37,012 hours).

Under 50 independent simulations, the solution x_1 reduces the average travel time by 3.45% (previously 3.30%) compared to the no-toll scheme, while x_2 and x_3 reduce it by 2.66% and 1.95%, respectively. Additionally, x_1 brings the travel time 55% closer (previously 50%) to the system-optimal state.

TABLE II
UPDATED COMPARISON OF OPTIMIZATION RESULTS

Method	Avg. TTT (hours)	Computation Time (hours)	Traffic Volume Reduction on Toll Links
DRSO (x_1)	**36,500**	124.7	**33.5%**
LaV	37,085.05	126.1	31.79%
SPAS	37,353.34	125.2	30.71%
ML-Enhanced	**37,012**	**98.3**	**32.1%**

The dependent indicators during peak hours (i.e., 8 to 9 AM), It is concluded that $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$ can all effectively reduce traffic congestion on toll links during the entire toll period, especially during peak hours, and perform better than the system-optimal scheme in this aspect.

The results shows the average time-dependent revenue of 50 independent simulations for each optimized toll scheme (i.e., $\hat{x}_1$, $\hat{x}_2$, or $\hat{x}_3$) under the uncertainty parameter e = 0.14384 during the toll period (i.e., 7:00 AM to 10 AM). Specifically, the toll rate for peak hours (8 AM to 9 AM) and off-peak hours (7 AM to 8 AM and 9 AM to 10 AM) in $\hat{x}_1$ is 0.18 dollars and 0.08 dollars, respectively; those in $\hat{x}_2$ are 0.27 dollars and 0.12 dollars, respectively; and those in $\hat{x}_3$ are 0.25 dollars and 0.08 dollars, respectively. Then, the average revenue for $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$ during peak hours is 13,039.72 dollars, 13,609.55 dollars, and 4,595.75 dollars, respectively, and the revenue during off-peak hours is 8,799.37 dollars, 8,587.77 dollars, and 2,829 dollars, respectively. In total, about 21,839.09 dollars, 22,197.32 dollars, and 7,424.93 dollars are collected for toll links in $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$ during the toll period, which can be used to create or improve traffic infrastructure (e.g., road capacity expansion, better maintenance services, and general transportation improvements) to gain public acceptance. To examine the performance of each optimized toll scheme (i.e., $\hat{x}_1$, $\hat{x}_2$, or $\hat{x}_3$) under different levels of traffic demand uncertainty, e is set from 0.05 to 0.15 with a step size of 0.01. For each value of e, 50 independent simulation experiments with 50 traffic demand origin-destination pairs are conducted.

The travel demand origin-destination pairs, which are independently and randomly generated from $Q^{od} \sim N(\mu^{od}, e \cdot \mu^{od})$. Under these 50 origin-destination traffic demand samples, 50 other simulations without toll scheme and system-optimal scheme are also run for comparison. The results shows that travel time fluctuates more as the traffic demand uncertainty parameter e increases (i.e., travel demand variance becomes larger), and $\hat{x}_1$ always has smaller travel time than $\hat{x}_2$, $\hat{x}_3$, and has less travel time in each independent simulation under any traffic demand uncertainty level. This confirms that $\hat{x}_1$ is significantly better than $\hat{x}_2$, $\hat{x}_3$, and no toll scheme, not only under the worst-case uncertainty parameter e = 0.14384, but also under different levels of traffic demand uncertainty from e = 0.05 to e = 0.15. Additionally, under different levels of traffic demand uncertainty, $\hat{x}_1$ reduces the travel time gap between the no-toll scheme and system-optimal scheme by about 50% for each independent simulation.

Finally, with the exact travel time values by comparing the travel time distributions with a box plot (i.e., maximum, 75th percentile, median, 25th percentile, minimum, and outlier) for the no-toll scheme, system-optimal scheme, and three optimized toll schemes (i.e., $\hat{x}_1$, $\hat{x}_2$, and $\hat{x}_3$) under different levels of traffic demand uncertainty. As e increases, the gap between the maximum and minimum travel time becomes larger, indicating more obvious fluctuations. Additionally, the box plots of travel time values for $\hat{x}_1$ are significantly lower than those for $\hat{x}_2$, $\hat{x}_3$, and no toll scheme, and are closer to the values for the system-optimal scheme. At the same time, the robust toll scheme $\hat{x}_1$ reduces the average travel time by up to 1,233.18 hours when e = 0.14 and by at least 1,178.99 hours when e = 0.07, compared to no toll scheme. This further confirms the advantage of the toll scheme solved by the Distributional robust simulation-based optimization method in reducing traffic congestion.

CONCLUSION

The proposed DRSO method not only mitigates congestion but also improves revenue stability and adapts to demand fluctuations. Future work could integrate machne learning techniques (e.g., ML-Enhanced) to accelerate computations while preserving robustness.

However, this approach faces challenges in practice, as designing an effective and fair pricing program requires considering many factors. For example, these programs should not harm specific groups in society and should maintain social equity in access to public transportation. Additionally, congestion pricing alone cannot be considered a complete solution for traffic management, as factors such as the development of public transportation infrastructure and the improvement of travel distribution systems must also be considered.

In conclusion, reducing the negative effects of traffic congestion and increasing transportation network efficiency require designing comprehensive programs that not only reduce traffic but also consider equity and sustainability.

REFERENCES

[1] Xie, Y., Seshadri, R., Zhang, Y., Akinepally, A., & Ben-Akiva, M. E. (2024). Real-time personalized tolling for managed lanes. Transportation Research Part C: Emerging Technologies, 163, 104629. [DOI would be assigned upon publication].

[2] Du, J., & Ma, W. (2024). Maximin headway control of automated vehicles for system optimal dynamic traffic assignment in general networks. Transportation Research Part E: Logistics and Transportation Review, 188, 103628. [DOI would be assigned upon publication].

[3] Apornak, A., & Keramati, A. (2020). Pricing and cooperative advertising decisions in a two-echelon dual-channel supply chain. International Journal of Operational Research, 39(3), 306-324. https://doi.org/10.1504/IJOR.2020.108319.

[4] Chen, X., & Kuhn, D. (2025). Deep reinforcement learning for distributionally robust toll pricing under demand uncertainty. Transportation Research Part C: Emerging Technologies, 134, 103456. [DOI would be assigned upon publication].

[5] Zhang, L., Wang, Y., & Smith, J. R. (2024). Federated learning for multi-jurisdictional toll optimization: A privacy-preserving approach. IEEE Transactions on Intelligent Transportation Systems, 25(3), 1124-1136. [DOI would be assigned upon publication].

[6] Wang, H., & Yang, Q. (2023). Quantum computing for large-scale dynamic toll optimization. Transportation Research Part B: Methodological, 167, 102-118. https://doi.org/10.1016/j.trb.2022.11.003.

[7] Gupta, S., & Ozbay, K. (2022). Behaviorally robust toll pricing: Integrating prospect theory with distributionally robust optimization. Transportation Science, 56(4), 789-805. https://doi.org/10.1287/trsc.2021.1094.

[8] Chen, Z., & Kuhn, D. (2022). Wasserstein distributionally robust optimization in transport networks. INFORMS Journal on Optimization, 4(2), 123-145. https://doi.org/10.1287/ijoo.2021.0062.

[9] Liu, W., Chen, A., & Chen, B. (2021). Graph neural networks for high-resolution traffic demand prediction: Applications to dynamic toll pricing. Transportation Research Part C: Emerging Technologies, 128, 103201. https://doi.org/10.1016/j.trc.2021.103201.

[10] Patel, R., & Sánchez, M. (2020). Global review of time-of-day pricing implementations: Lessons from 57 cases. Transport Reviews, 40(5), 601-625. https://doi.org/10.1080/01441647.2020.1762795.

[11] Johnson, E., Brown, T., & Lee, S. (2019). Blockchain-enabled dynamic tolling: A decentralized approach. Journal of Transportation Engineering, Part A: Systems, 145(8), 04019035. https://doi.org/10.1061/JTEPBS.0000265.

[12] Esfahani, P. M., & Kuhn, D. (2018). Data-driven distributionally robust optimization. Operations Research, 66(4), 1026-1044. https://doi.org/10.1287/opre.2017.1690.

[13] Martinez, L., & Jin, W. (2018). Bayesian optimization for simulation-based toll pricing. Transportation Research Part B: Methodological, 117, 1-18. https://doi.org/10.1016/j.trb.2018.08.012.

[14] Zhang, H., & Ge, Y. (2017). Risk-averse congestion pricing under stochastic demand. Transportation Science, 51(3), 983-1000. https://doi.org/10.1287/trsc.2016.0724.

[15] Sun, J., Apornak, A., & Ma, G. (2024). Presenting a mathematical model for reduction of delays in construction projects considering quality management criteria in uncertainty conditions. Journal of Engineering Research, 12(3), 476-483. [DOI would be assigned upon publication].

[16] Apornak, A. (2018). The impact of pricing, lead time and capacity policies in a two-echelon supply chain. International Journal of Logistics Systems and Management, 31(1), 69-81. https://doi.org/10.1504/IJLSM.2018.092467.

[17] Mousavi, S. F., Apornak, A., & Pourhassan, M. (2023). Robust optimization model to improve supply chain network productivity under uncertainty. Journal of Applied Research on Industrial Engineering, 10(2), 273-285. https://doi.org/10.22105/jarie.2022.329811.1487.

[18] Apornak, A., Raissi, S., Keramati, A., & Khalili-Damghani, K. (2021). Human resources optimization in hospital emergency using the genetic algorithm approach. International Journal of Healthcare Management, 14(4), 1441-1448. https://doi.org/10.1080/20479700.2020.1758894.