

The optimal number of yard cranes in container terminal

Gholam Reza Amin*

Assistant Professor, Department of Computer Engineering, Postgraduate Engineering Center, Islamic Azad University, South Tehran Branch, Tehran, Iran

Hamed Golchubian

Research Scholar, Department of Transportation Engineering, Islamic Azad University, Science and Research Branch, Tehran, Iran

Abstract

The aim of this study is to propose a new mixed integer linear programming (MILP) model to find the minimum required number of yard cranes, *Rubber tyred gantry cranes* (RTGCs), for completion the total amount of works at the end of planning horizon. In other word, we find the optimal number of yard cranes in a container terminal which completes the total amount of works. The paper supports the proposed method using a numerical example.

Keywords: Container terminal; Yard crane; Mixed integer linear programming

1. Introduction

A container terminal in a port is the place where container vessels dock on berths and unload inbound containers and load outbound containers. Three different types of containers are handled in a container terminal: *inbound*, *outbound*, and *transshipment* containers [4, 7, 8]. Inbound containers which are usually import containers from abroad are discharged from a vessel in a short time when the vessel arrives at a berth, and kept in temporary storage at a container yard for a few days until they are transferred to outside trucks. Outbound containers which are usually export containers bound for other countries arrive at the terminal from several days before the arrival of the corresponding vessel and are loaded onto the vessel in a short amount of time when she arrives at a berth. Transshipment containers are unloaded from a vessel, stay in the yard for several days, and loaded onto another vessel [4, 7, 8]. The storage yard in a terminal is usually divided into blocks. A typical block has seven rows of spaces, six of which are used for storing containers and the seventh reserved for truck passing. The placing of a container in a yard is carried out by huge cranes called yard cranes. The most commonly used yard cranes are Rubber Tyred Gantry Cranes (RTGCs) [10]. The daily operations of

a container terminal are complex and involve a variety of decisions to be made under conditions varying with time. To perform a large number of container handling jobs a series of planning problems need to be solved and several expensive terminal resources should be allocated as well [8, 10]. Therefore managing a container terminal is a challenging task. Researchers have paid attention to the operational viewpoints of container terminals.

Kim and Kim [2] suggested a method of determining the optimal amount of storage space and the number of transfer cranes for handling import containers as well. Zhang et al. [10] assumed the given of workload of each block in each period of a day and formulated a dynamic crane deployment problem. Their mathematical model [10] minimized the total delayed workload in the yard and also proposed the optimal routes of crane movements among blocks. Also Kim and Park [4] discussed a dynamic space-allocation method for outbound containers. Recently, Wang [11] introduced container information tracing system. A comprehensive literature review on the application of operations research for container terminal is presented by Steenken et al. [9]. Recently, Kim and Kim [3] have offered a method of determining the optimal price schedule for storing inbound containers in a container yard.

* Corresponding author. E-mail: g_amin@azad.ac.ir

Also Lee et al. [5] introduced a new simulated annealing based algorithm for scheduling of port container terminals. More recently, Canonaco et al. [1] introduced a network model for managing the berth crane operations. Further by introducing the non-interference constraints in port container terminals a quay crane scheduling scheme is proposed [6].

This paper borrows the mixed integer linear programming (MILP) model initiated by Zhang *et al.* [10] for dynamic crane deployment in a container storage yards and extends a new model for finding the minimum required number of yard cranes for completion the total amount of works at the end of planning horizon. Therefore the contribution of the current paper is that it provides the optimal number of yard cranes in a container terminal for completion the total amount of works. The rest of this paper is organized as follows:

Section 2 shows a quick explanation of dynamic crane deployment proposed by Zhang *et al.* [10]. The minimum required number of yard cranes need to be considered for completion the total amount of works is given in Section 3. Section 4 illustrates the proposed model by a numerical example. Section 5 ends the paper with conclusion.

2. Dynamic crane deployment

In a container terminal for optimal deployment of yard cranes or minimizing the total works delayed at the end of a planning horizon, Zhang *et al.* [10] proposed an integer linear programming model as well as a Lagrangean relaxation heuristic algorithm for finding the near-optimal solution. Their model [10] has some important practical assumptions given in Zhang *et al.* [10], pages 542. As they assumed in the container terminal a yard is divided into some blocks. Also there is a specified number of yard cranes and a planning horizon containing some periods. The following notations (including the model parameters and decision variables) are also introduced in their paper [10].

2.1. Parameters

1) The number of cranes assigned for block i ($i = 1, \dots, N$) at the beginning of the planning horizon denoted by x_{ii0} . Therefore the total number of available cranes is $\sum_{i=1}^N x_{ii0}$.

2) The capacity of each crane within a planning period is shown by $C = 240$ minutes.

3) The total number of blocks under consideration is denoted by N .

4) The total number of planning periods in a planning horizon is $T = 6$ periods.

5) The workload in block i within planning period t is given by b_{it} , for each $i = 1, \dots, N$ and $t = 1, \dots, T$.

6) The traveling time of a crane from block i to block j is indicated by t_{ij} , $i, j = 1, \dots, N, i \neq j$.

2.2. Decision variables

1) The number of cranes moving from block i to block j during a planning period t is denoted by x_{ijt} . Note that when $i = j$, these cranes stay in the same block during period t , for each $i, j = 1, \dots, N$ and $t = 1, \dots, T$.

2) The workload fulfilled in block i by cranes that move from block i to block j during planning period t is indicated by z_{ijt} , $i, j = 1, \dots, N, t = 1, \dots, T$.

3) The workload fulfilled in block j by cranes that move from block i to block j during planning period t is denoted by y_{ijt} , $i, j = 1, \dots, N, t = 1, \dots, T$.

4) The workload left in block i at the end of planning period t denoted by w_{it} , $i = 1, \dots, N, t = 1, \dots, T$.

Now we give the constraints of the dynamic crane deployment model used in Zhang *et al.* [10].

Constraints (1) ensure the crane flow or movement conservation in each block when cranes are deployed from one period to the next. That is

$$\sum_{j=1}^N x_{ijt} = \sum_{j=1}^N x_{ji(t-1)} \quad i = 1, \dots, N, t = 1, \dots, T \quad (1)$$

Constraints (2) warrant that only two cranes can serve a block in a planning period. These constraints assure us the number of existing cranes in each block at the beginning of a planning period (and during a period) is at most two.

$$\sum_{j=1}^N x_{ijt} + \sum_{\substack{j=1 \\ j \neq i}}^N x_{jit} \leq 2 \quad i=1, \dots, N, t=1, \dots, T \quad (2)$$

Constraints (3) imply the balance between the workload that should be finished ($w_{i(t-1)} + b_{it}$) and the workload that can be finished ($\sum_{j=1}^N (z_{ijt} + y_{jit})$) in each block by using slack variables w_{it} . That is

$$w_{i(t-1)} + b_{it} - \left(\sum_{j=1}^N z_{ijt} + \sum_{j=1}^N y_{jit} \right) - w_{it} = 0$$

$$i=1, \dots, N, t=1, \dots, T \quad (3)$$

Constraints (4) assure that the total workload fulfilled by a crane in each planning period cannot exceed its total net crane capacity.

$$z_{ijt} + y_{jit} - (C - t_{ij})x_{ijt} \leq 0$$

$$i=1, \dots, N, j=1, \dots, N, t=1, \dots, T \quad (4)$$

Constraints (5) initialize workload at the beginning of a planning horizon; any delayed workload from the previous planning horizon is counted as the workload in the first planning period.

$$w_{i0} = 0 \quad i=1, \dots, N \quad (5)$$

Constraints (6), together with the parameters x_{ii0} , define the initial locations of the cranes.

$$x_{ij0} = 0 \quad i=1, \dots, N, j=1, \dots, N, i \neq j \quad (6)$$

Constraints (7) and (8) are non-negative and integer constraints. Constraints (8) also restrict that at most one RTGC (crane) can be moved from one block to another in a period.

$$w_{it} \geq 0, z_{ijt} \geq 0, y_{jit} \geq 0$$

$$i=1, \dots, N, j=1, \dots, N, t=1, \dots, T \quad (7)$$

$$x_{iit} \in \{0, 1, 2\}, x_{ijt} \in \{0, 1\}$$

$$i=1, \dots, N, j=1, \dots, N, i \neq j, t=1, \dots, T \quad (8)$$

Therefore Zhang et al. [10] introduced the following mathematical model for the optimal crane deployment in a container storage yard.

$$\min \sum_{t=1}^T \sum_{i=1}^N w_{it}$$

subject to

$$(9)$$

Constraints (1), (2), ..., (8)

The next section of this paper proposes a new MILP model to find the minimum required number of yard cranes for completion the total amount of works at the end of planning horizon.

3. The optimal number of yard cranes

As it can be seen Zhang et al. [10] proposed a novel MILP model and a heuristic based solution procedure as well to solve the dynamic crane deployment problem in a container terminal. One important issue related to model (9) is assigning the initial values of yard cranes for each block $i=1, \dots, N$. As they have assigned $x_{ii0} \in \{0, 1, 2\}$ crane(s) to the i^{th} block subjectively ($i=1, \dots, N$). For example those authors [10] assigned the available yard cranes as follows:

Assuming $N=20$ blocks if the proportion of the available number of yard cranes and the number of blocks is equal then one crane assigns to each block. When the proportion is 1.5, that is the number of yard cranes is 30, then the order of assignment of cranes to the blocks is as 2, 1, ..., 2, 1, respectively. That is they assigned 2 cranes to the first block, one crane for the second block, and so on.

In order to find the optimal number of cranes or the minimum number of required yard cranes for completion the total amount of delayed works at the end of planning horizon, this paper considers the initial position of yard cranes as the new decision variables. Therefore Constraints (1) of model (9) is converted to the following constraints:

$$\sum_{j=1}^N x_{ijt} - \sum_{j=1}^N x_{ji(t-1)} = 0 \quad i=1, \dots, N, t=1, \dots, T \quad (1-1)$$

$$\sum_{i=1}^N x_{ii0} = k \quad (1-2)$$

$$x_{ii0} \in \{0, 1, 2\} \quad i=1, \dots, N \quad (1-3)$$

where, k is the number of initial yard cranes that we consider it as unknown. Here Constraints (2) of model (9) remain unchanged. Regarding the objective

of this paper Constraints (3) of model (9) is also converted to the following constraints:

$$w_{i(t-1)} + b_{it} - \left(\sum_{j=1}^N z_{ijt} + \sum_{j=1}^N y_{jit} \right) - w_{it} = 0$$

$$i = 1, \dots, N, t = 1, \dots, T-1 \quad (3-1)$$

$$w_{i(T-1)} + b_{iT} - \left(\sum_{j=1}^N z_{ijt} + \sum_{j=1}^N y_{jit} \right) = 0$$

$$i = 1, \dots, N \quad (3-2)$$

Because to find the minimum number of required cranes for completion the total amount of works at the end of planning horizon we have $w_{iT} = 0$ for each block $i = 1, \dots, N$. In fact Constraints (3) disjoined into two constraints. The first type for $t = 1, \dots, T-1$ is denoted as (3-1) remain unchanged and the second type for $t = T$ is shown as (3-2). Note that for $t = T$ the left hand side of (3-2) is equal to w_{iT} and therefore should be vanished. To find the optimal number of required yard cranes the remaining constraints of model (9) remain unchanged except for constraints (5) and (8) which are modified as follows:

$$w_{i0} = w_{iT} = 0 \quad i = 1, \dots, N \quad (5-1)$$

$$x_{iit} \in \{0, 1, 2\}, x_{ijt} \in \{0, 1\}$$

$$i = 1, \dots, N, j = 1, \dots, N, i \neq j, t = 0, 1, \dots, T \quad (8-1)$$

Because it is needed to finish the amount of works at the end of planning horizon for each of the blocks, in constraints (5), $w_{iT} = 0$ and also x_{i0} is considered as a new decision variable, in constraints (8). Therefore we propose the following model (10):

$$k^* = \min k$$

Subject to

$$\sum_{j=1}^N x_{ijt} - \sum_{j=1}^N x_{ji(t-1)} = 0 \quad i = 1, \dots, N, t = 1, \dots, T$$

$$\sum_{i=1}^N x_{i0} = k$$

$$x_{i0} \in \{0, 1, 2\} \quad i = 1, \dots, N$$

$$\sum_{j=1}^N x_{ijt} + \sum_{\substack{j=1 \\ j \neq i}}^N x_{jit} \leq 2 \quad i = 1, \dots, N, t = 1, \dots, T$$

$$w_{i(t-1)} + b_{it} - \left(\sum_{j=1}^N z_{ijt} + \sum_{j=1}^N y_{jit} \right) - w_{it} = 0$$

$$i = 1, \dots, N, t = 1, \dots, T-1$$

$$w_{i(T-1)} + b_{iT} - \left(\sum_{j=1}^N z_{ijt} + \sum_{j=1}^N y_{jit} \right) = 0 \quad i = 1, \dots, N$$

$$z_{ijt} + y_{jit} - (C - t_{ij})x_{ijt} \leq 0$$

$$i = 1, \dots, N, j = 1, \dots, N, t = 1, \dots, T \quad (10)$$

$$w_{i0} = w_{iT} = 0 \quad i = 1, \dots, N$$

$$x_{ij0} = 0 \quad i = 1, \dots, N, j = 1, \dots, N, i \neq j$$

$$w_{it} \geq 0, z_{ijt} \geq 0, y_{jit} \geq 0$$

$$i = 1, \dots, N, j = 1, \dots, N, t = 1, \dots, T$$

$$x_{iit} \in \{0, 1, 2\}, x_{ijt} \in \{0, 1\}$$

$$i = 1, \dots, N, j = 1, \dots, N, i \neq j, t = 0, 1, \dots, T$$

Clearly the objective of the above model is finding the minimum required number of yard cranes in such a way that at the end of planning horizon the amount of workloads in each of the blocks is zero. According to the constraints of the above model, Constraints (2) and (8-1), at most two yard cranes can work in each of the blocks at every time period, so if the amount of workloads is very large, model (10) may be infeasible. In such case changing the parameter C rectifies this problem. The next section gives a numerical illustration for the new model (10).

Table 1. The optimal initial positions of yard cranes.

x_{110}^*	x_{220}^*	x_{330}^*	x_{440}^*	x_{550}^*	x_{660}^*	x_{770}^*	x_{880}^*	x_{990}^*	$x_{10,10,0}^*$
1	2	2	2	1	1	2	1	0	0
$x_{11,11,0}^*$	$x_{12,12,0}^*$	$x_{13,13,0}^*$	$x_{14,14,0}^*$	$x_{15,15,0}^*$	$x_{16,16,0}^*$	$x_{17,17,0}^*$	$x_{18,18,0}^*$	$x_{19,19,0}^*$	$x_{20,20,0}^*$
0	0	1	0	0	2	1	0	0	0

4. A numerical illustration

As a numerical demonstration for the proposed model, we use the numerical example regarding the container terminal consisting 20 blocks given in Zhang et al. [10]. The problem contains two tables, table 1 as appeared in Zhang et al. [10] gives the crane traveling time between blocks, t_{ij} , for $i, j = 1, \dots, N = 20$ and Table 1 shows the workload in each block i within every planning period t , b_{it} , for $i = 1, \dots, N = 20, t = 1, \dots, T = 6$, where $T = 6$ indicates the number of planning periods. Also Zhang et al. [10] assigned an initial position for the available 30 cranes into 20 blocks. Furthermore the capacity of one crane within a planning period is denoted as C . Two importance issues corresponding to comparison of the proposed model in this paper, model (10), and the model introduced by Zhang et al. [10], model (9), are, the proposed model (10) takes the initial position of yard cranes as decision variables, $x_{ii0}, i = 1, \dots, N = 20$, and the number of yard cranes is also considered as unknown, k , in our new model. Without loss of generality we consider $C=600$. Also to obtain the exact optimal solution of the proposed model we used LINGO software. Solving model (10) for the stated numerical data gives the optimal number of yard cranes $k^* = 16$. Table 1 shows the optimal initial position of yard cranes, $x_{ii0}^*, i = 1, \dots, 20$, when $C=600$ minutes and the optimal number of cranes is also identified as $k^* = 16$. As the above table shows assigning the initial position of yard cranes based on a subjective method, as Zhang et al. [10] assigned, may not produce an optimum position for yard cranes. For example if one uses the scheme used in Zhang et al. [10] it assigned the initial position of yard cranes for the given example as $x_{ii0} = 1$ for $i = 1, \dots, 16$ and $x_{ii0} = 0, i = 17, 18, 19, 20$. Furthermore, the optimal solution shows $w_{i6}^* = 0$ for

every block $i = 1, \dots, 20$. This indicates the completion amount of existing workloads at the end of planning horizon for each block is achieved.

5. Conclusion

This paper has proposed a modified mathematical model to obtain the optimal number of yard cranes for completion the amount of workloads at the end of planning horizon in a container terminal. We have formulated the problem as a mixed integer programming model and using a numerical example the results of the proposed model have been illustrated. Finding the optimal capacity of every crane within a planning period and relationship between C and k needs further investigation.

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