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# Correlation coefficient of intuitionistic fuzzy sets

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**Abstract**

Based on the point of view of geometrical representation of an intuitionistic fuzzy set, we take into account all three parameters describing intuitionistic fuzzy set, propose a kind of new method to calculate correlation and correlation coefficient of intuitionistic fuzzy sets which is similar to the cosine of the intersectional angle in finite sets and probability space, respectively. Further, we discuss some of their properties and give three numerical examples to illustrate our proposed method reasonable.

**Keywords:** Intuitionistic fuzzy set; Fuzzy set; Correlation; Correlation coefficient

## 1. Introduction

Fuzzy set theory pioneered by Zadeh [16] is a powerful tool to model imprecise and vague situations where exact analysis is either difficult or impossible. Considering that the concept called correlation plays an important role in statistics and engineering sciences, by correlation analysis, the joint relationship of two variables can be examined with the aid of a measure of interdependency of the two variables. Several authors have discussed and investigated the concept of correlation in fuzzy set theory. For example, Murthy and Pal [13] studied the correlation between two fuzzy membership functions, Chiang and Lin [5,6] studied the correlation and partial correlation of fuzzy sets, Chaudhuri and Bhattacharya [4] investigated the correlation between two fuzzy sets on the same universal support.

Yu [15] defined the correlation of fuzzy numbers $A, B$ in the collection $F([a,b])$ of all fuzzy numbers whose supports are included in a closed interval $[a,b]$ as follows:

$$C_Y(A,B)=\frac{1}{b-a}\int_a^b(\mu_A(x)\mu_B(x)+\nu_A(x)\nu_B(x))dx\,, \quad (1)$$

where $\mu_A(x)+\nu_A(x)=1$ and the correlation coefficient of fuzzy numbers $A, B$ was defined by:

$$\rho_Y(A,B)=\frac{C_Y(A,B)}{\sqrt{C_Y(A,A).C_Y(B,B)}}\,. \quad (2)$$

As a generalization of fuzzy set, intuitionistic fuzzy set was first introduced by Atanassov [1]. Some relevant basic notions can be found in [1,2].

Let $X$ be a fixed nonempty set. An intuitionistic fuzzy set $A$ in $X$ is an object having the form

$$A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle \mid x\in X\right\}, \quad (3)$$

where the functions $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$ define the degree of membership and degree of non-membership, respectively, of the element $x\in X$ to the set $A$, and for every element $x\in X, 0\le\mu_A(x)+\nu_A(x)\le 1$.

In 1991, Gerstenkorn and Manko [7] defined the correlation of intuitionistic fuzzy sets $A$ and $B$ in a finite set $X=\{x_1,x_2,...,x_n\}$ as follows:

$$C_{GM}(A,B)=\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i)+\nu_A(x_i)\nu_B(x_i))\,, \quad (4)$$

and the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$ was given by:

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$$\rho_{GM}(A,B) = \frac{C_{GM}(A,B)}{\sqrt{T(A)\cdot T(B)}}, \quad (5)$$

where,

$$T(A) = \sum_{i=1}^{n} (\mu_A^2(x_i) + \nu_A^2(x_i)). \quad (6)$$

In 1995, Hong and Hwang [9] defined the correlation of intuitionistic fuzzy sets $A$ and $B$ in a probability space $(X, B, P)$ as follows:

$$C_{HH}(A,B) = \int_X (\mu_A\mu_B + \nu_A\nu_B)dP, \quad (7)$$

and the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$ was given by:

$$\rho_{HH}(A,B) = \frac{C_{HH}(A,B)}{\sqrt{C_{HH}(A,A).C_{HH}(B,B)}}. \quad (8)$$

Hung and Wu [10,11] introduced the concepts of positively and negatively correlated and used the concept of centroid to define the correlation coefficient of intuitionistic fuzzy sets which lies in the interval [-1,1], and the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$ was given by:

$$\rho_{HW}(A,B) = \frac{C_{HW}(A,B)}{\sqrt{C_{HW}(A,A).C_{HW}(B,B)}}, \quad (9)$$

where,

$$C_{HW} = m(\mu_A)m(\mu_B) + m(\nu_A)m(\nu_B), \quad (10)$$

$$m(\mu_A) = \frac{\int x\mu_A(x)dx}{\int \mu_A(x)dx}, \quad (11)$$

$$m(\nu_A) = \frac{\int x\nu_A(x)dx}{\int \nu_A(x)dx}, \quad (12)$$

$$m(\mu_B) = \frac{\int x\mu_B(x)dx}{\int \mu_B(x)dx}, \quad (13)$$

$$m(\nu_B) = \frac{\int x\nu_B(x)dx}{\int \nu_B(x)dx}. \quad (14)$$

Recently, Mitchell [12] adopted a statistical viewpoint to interpret intuitionistic fuzzy set as an ensemble of ordinary fuzzy set, and defined the correlation coefficient of intuitionistic fuzzy sets by using the correlation coefficient of two ordinary fuzzy sets and a mean aggregation function, thus, the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$ was given by:

$$\rho_M(A,B) = F(\rho_{h,k} \mid h,k \in \{1,2,...,N\}), \quad (15)$$

where $F$ is a mean aggregation function, and

$$\phi_A^{(h)}(u) = \mu_A(u) + \pi_A(u) \times p_h(u) \qquad u \in X, \quad (16)$$

$$\phi_B^{(k)}(u) = \mu_B(u) + \pi_B(u) \times p_k(u) \qquad u \in X. \quad (17)$$

$p_h(u)$ and $p_k(u)$ are two uniform random numbers chose from the interval $[0,1]$, and:

$$\rho_{h,k} = \frac{\int(\phi_A^{(h)}(u) - \overline{\phi}_A^{(h)})(\phi_B^{(k)}(u) - \overline{\phi}_B^{(k)})du}{\sqrt{\int(\phi_A^{(h)}(u) - \overline{\phi}_A^{(h)})^2 du} \cdot \sqrt{\int(\phi_B^{(k)}(u) - \overline{\phi}_B^{(k)})^2 du}}. \quad (18)$$

Here, $\overline{\phi}_A^{(h)}$ and $\overline{\phi}_B^{(k)}$ are average value of $\phi_A^{(h)}$ and $\phi_B^{(k)}$, respectively.

Furthermore, Bustince and Burillo [3] investigated the correlation of interval-valued intuitionistic fuzzy sets in finite universal set, Hong [8] studied correlation of interval-valued intuitionistic fuzzy sets in probability spaces.

In 2000, Szmidt and Kacprzyk [14] gave a geometrical representation of the intuitionistic fuzzy set and introduced three parameters to describe the distance between intuitionistic fuzzy sets. Based on the kind of geometrical background, in this paper we take into account all three parameters describing intuitionistic fuzzy set and propose a kind of new method to calculate correlation and correlation coefficient of intuitionistic fuzzy sets which is similar to the cosine of the intersectional angle in finite set and probability space, respectively.

The rest of this work is organized as follows. In Section 2, a geometrical interpretation of intuitionistic fuzzy set is reviewed. In Section 3, the correlation and correlation coefficient of intuitionistic fuzzy sets in finite set are discussed and some properties are given. In Section 4, the correlation and correlation coefficient of intuitionistic fuzzy sets in probability space are discussed and some properties are given. In Section 5, three numerical examples are compared based on several different correlation coefficients of intuitionistic fuzzy sets. The conclusion is in Section 6.

## 2. Intuitionistic fuzzy sets: A geometrical interpretation

Throughout this paper, we write $X$ to denote the discourse set, IFSs stands for the set of all intuitionistic fuzzy subsets in $X$, $F(X)$ stands for the set of fuzzy subsets in $X$. $A$ expresses an intuitionistic fuzzy set, $A'$ expresses a fuzzy set. For a given intuitionistic fuzzy set $A$:

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle \,|\, x \in X\}. \tag{19}$$

Then $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is hesitancy degree of $x$ to $A$ [2]. Here, we call it ignorance degree of $x$ to $A$. It is obvious that $0 \le \pi_A(x) \le 1$ for every $x \in X$. Specially, for each fuzzy set $A'$ in $X$, we have $\pi_A(x) = 1 - \mu_A(x) - (1 - \mu_A(x)) = 0$ for every $x \in X$.

Therefore, for an ordinary intuitionistic fuzzy set, it is easy to see that the third parameter $\pi_A(x)$ cannot be casually omitted, thus, we can give a convenient representation of intuitionistic fuzzy set as:

$$A = \{(\mu_A(x), \nu_A(x), \pi_A(x)) \,|\, x \in X\}. \tag{20}$$

One of the traditional geometrical interpretation of the intuitionistic fuzzy sets [2] is shown in Figure1. Atanassov [2] considers a universe $E$ and subset $F$ in the Euclidean plane with the Cartesian coordinates.

For a fixed intuitionistic fuzzy set $A$, a function $f_A$ from $E$ to $F$ can be considered, such that if $x \in E$, then $p = f_A(x) \in F$ and the point $p \in F$ has the coordinates $\langle a', b' \rangle$ for which $0 \le a', b' \le 1$, where $a' = \mu_A(x), b' = \nu_A(x)$.

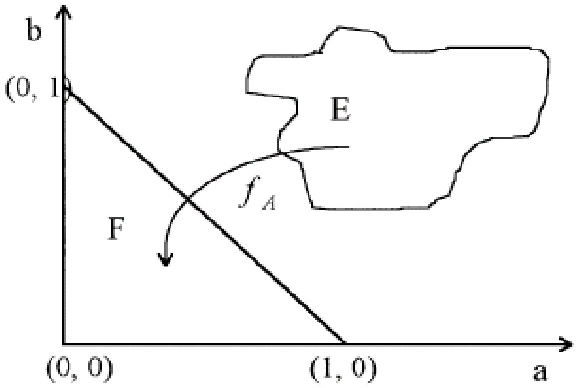


**Figure 1.** A geometrical interpretation of an intuitionistic fuzzy set.

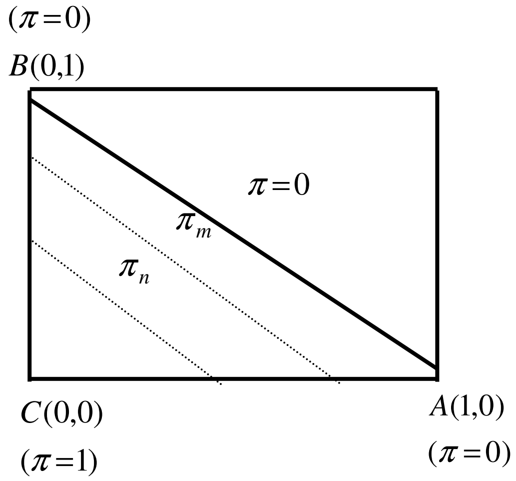


**Figure 2.** An orthogonal projection of the real (three-dimension) representation (triangle ABD in Fig. .3) of an intuitionistic fuzzy set.

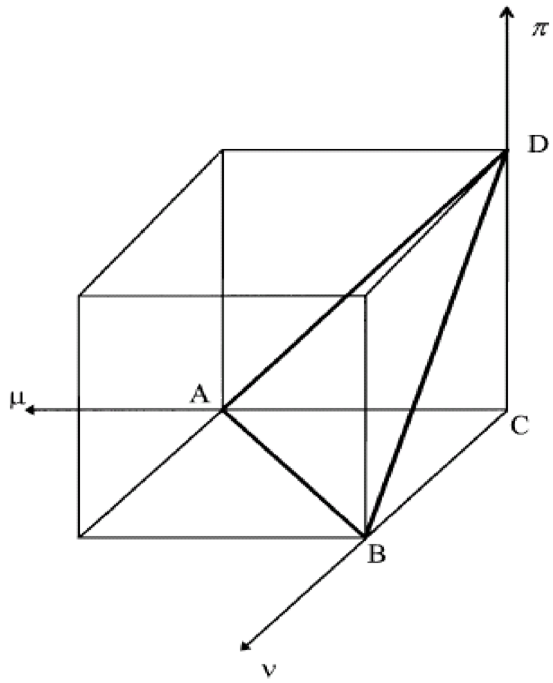


**Figure 3.** A geometrical interpretation of an intuitionistic fuzzy set.

Figure 3 can be used as an example when considering a situation at the beginning of negotiations - cf. Figure 2 (applications of intuitionistic fuzzy sets for group decision making, negotiations and other real situations are presented in [2]).

Each expert $i$ is represented as a point having coordinates $(\mu_i, \nu_i, \pi_i)$. Expert A: (1,0,0) - fully accepts a discussed idea. Expert B: (0,1,0) - fully rejects it. The experts placed on the segment AB fixed their points of view( their hesitation margins equal zero for segment AB, so each expert is convinced to the extent $\mu_i$, is against the extent $\nu_i$, and $\mu_i + \nu_i = 1$; segment AB represents a fuzzy set). Expert C: (0,0,1) is absolutely hesitant i.e. undecided – he/she is the most open to the influence of the arguments presented.

A line parallel to AB describes a set of experts with the same level of hesitancy. For example, in Figure 2, two sets are presented with ignorance degree equal to $\pi_m$, and $\pi_n$, where $\pi_n > \pi_m$.

In other words, Figure 2 (the triangle ABC) is an orthogonal projection of the real situation (the triangle ABD) presented in Figure 3.

An element of an intuitionistic fuzzy set has three coordinates $(\mu_i, \nu_i, \pi_i)$, hence the most natural representation of an intuitionistic fuzzy set is to draw a cube (with edge length equal to 1), and the triangle ABD (Figure 3) represents an intuitionistic fuzzy set. As before (Figure 2), the triangle ABC is the orthogonal projection of ABD.

This representation of an intuitionistic fuzzy set (Figure 3) will be a point of departure for considering the correlation coefficient of intuitionistic fuzzy sets.

For a finite set $X$, if $\mu_A(x_i) = 1$ or $\nu_A(x_i) = 1$ or $\pi_A(x_i) = 1$ ; or for a probability space, if $P\{\mu_A(x) = 1 \text{ or } \nu_A(x) = 1 \text{ or } \pi_A(x) = 1\} = 1$, then an intuitionistic fuzzy set $A$ is called a non-fuzzy set.

## 3. Correlation and correlation coefficient in finite set

At first, let us see the following example:

**Example 1.** For a given universal set $X = \{x_1, x_2, x_3\}$, *A, B, C* and *D* are four intuitionistic fuzzy sets in *X*, their membership functions are represented, respectively:

$$A = \{\langle x_1, 0, 0\rangle, \langle x_2, 0, 0\rangle, \langle x_3, 0, 0\rangle\},$$

$$B = \{\langle x_1, 0, 0\rangle, \langle x_2, 0, 0\rangle, \langle x_3, 0, 0\rangle\},$$

$$C = \{\langle x_1, 0, 1\rangle, \langle x_2, 0, 1\rangle, \langle x_3, 0, 1\rangle\},$$

$$D = \{\langle x_1, 0, 0\rangle, \langle x_2, 0, 0\rangle, \langle x_3, 0, 0\rangle\}. \quad (21)$$

Aimed at above mentioned conventional correlation coefficient for intuitionistic fuzzy sets, we cannot get a reasonable result for the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$ and intuitionistic fuzzy sets $C$ and $D$. However, known intuitively, it is easy to see that correlation coefficient for $A$ and $B$ should be 1 and correlation coefficient for $C$ and $D$ should be 0. Based on this view of point, we think that it is necessary to modify the definition of correlation and correlation coefficient for intuitionistic fuzzy sets. In this paper, we take into account all three parameters describing intuitionistic fuzzy set introduced by Szmidt and Kacprzyk [14] and propose a kind of new method to calculate correlation coefficient of intuitionistic fuzzy sets.

Let $X$ be a finite set such that $X = \{x_1, x_2, \ldots, x_n\}$ and $n < \infty$. For each $A \in$ IFSs, then:

$$A = \{(\mu_A(x_i), \nu_A(x_i), \pi_A(x_i)) \mid x_i \in X\}. \quad (22)$$

**Definition 1.** If $A, B \in$ IFSs, we define:

$$C_1(A,B) = \frac{1}{n}\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)), \quad (23)$$

and we call the correlation of intuitionistic fuzzy sets $A$ and $B$. Furthermore, we call

$$\rho_1(A,B) = \frac{C_1(A,B)}{\sqrt{C_1(A,A).C_1(B,B)}}, \quad (24)$$

the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$. This correlation coefficient is similar to the cosine of the intersectional angle of two vectors.

The following proposition is immediate from the definition.

**Proposition 1.** For $A, B \in$ IFSs, then we have:

$0 \le C_1(A,B) \le 1$,

$C_1(A,B) = C_1(B,A), \rho_1(A,B) = \rho_1(B,A)$,

If $A = B$, then $\rho_1(A,B) = 1$. (25)

**Theorem 1.** For $A, B \in$ IFSs, then we have:

$0 \le \rho_1(A,B) \le 1$.

**Proof.** Since $C_1(A,B) \ge 0$, we will prove that $\rho_1(A,B) \le 1$. For an arbitrary real number $k$, we have:

$$0 \le \sum_{i=1}^{n}((\mu_A(x_i) - k\mu_B(x_i))^2 + (\nu_A(x_i) - k\nu_B(x_i))^2 + (\pi_A(x_i) - k\pi_B(x_i))^2)$$
$$= \sum_{i=1}^{n}(\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)) - 2k\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)) + k^2\sum_{i=1}^{n}(\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)). \quad (26)$$

Thus, we can get:

$$(\sum_{i=1}^{n}(\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i)))^2 \le \sum_{i=1}^{n}(\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i))\sum_{i=1}^{n}(\mu_B^2(x_i) + \nu_B^2(x_i) + \pi_B^2(x_i)). \quad (27)$$

Therefore, we have $\rho_1(A,B) \le 1$.

**Theorem 2.** $\rho_1(A,B) = 1$ if and only if $A = B$.

**Proof.** If we consider the inequality in the proof of Theorem 1, then the equality holds if and only if we have $\mu_A(x_i) = k\mu_B(x_i)$, $\nu_A(x_i) = k\nu_B(x_i)$ and $\pi_A(x_i) = k\pi_B(x_i)$ for some positive real number $k$. As $\mu_A(x_i) + \nu_A(x_i) + \pi_A(x_i) = \mu_B(x_i) + \nu_B(x_i) + \pi_B(x_i) = 1$, it means $k = 1$. Namely, $A = B$.

**Theorem 3.** $C_1(A,B) = 0$ if and only if $A$ and $B$ are non-fuzzy sets and satisfy the condition $\mu_A(x_i) + \mu_B(x_i) = 1$ or $\nu_A(x_i) + \nu_B(x_i) = 1$ or $\pi_A(x_i) + \pi_B(x_i) = 1$, $\forall x_i \in X$.

**Proof.** Since $\forall x_i \in X$ we have $\mu_A(x_i)\mu_B(x_i) + \nu_A(x_i)\nu_B(x_i) + \pi_A(x_i)\pi_B(x_i) \ge 0$, then $C_1(A,B) = 0$ implies for every $x_i \in X$, we have $\mu_A(x_i)\mu_B(x_i) = 0$, $\nu_A(x_i)\nu_B(x_i) = 0$ and $\pi_A(x_i)\pi_B(x_i) = 0$. If $\mu_A(x_i) = 1$, then $\mu_B(x_i) = 0$ and $\nu_A(x_i) = \pi_A(x_i) = 0$. And if $\mu_B(x_i) = 1$, then $\mu_A(x_i) = 0$ and $\nu_B(x_i) = \pi_B(x_i) = 0$, hence, we have $\mu_A(x_i) + \mu_B(x_i) = 1$.

Conversely, when $A$ and $B$ are non-fuzzy sets and $\mu_A(x_i) + \mu_B(x_i) = 1$, if $\mu_A(x_i) = 1$, we have $\mu_B(x_i) = 0$ and $\nu_A(x_i) = \pi_A(x_i) = 0$; if $\mu_B(x_i) = 1$, we have $\mu_A(x_i) = 0$ and $\nu_B(x_i) = \pi_B(x_i) = 0$, which implies $C_1(A,B) = 0$.

Similarly we can give the proof when $\nu_A(x_i) + \nu_B(x_i) = 1$ or $\pi_A(x_i) + \pi_B(x_i) = 1$.

**Theorem 4.** $C_1(A,A) = 1$ if and only if $A$ is a non-fuzzzy set.

**Proof.** If $A$ is non-fuzzy set, $C_1(A,A) = 1$ is obvious. Conversely, we use proof by contradiction. Assume that $A$ is not a non-fuzzy set, then we have $0 \le \mu_A(x_i) < 1$, $0 \le \nu_A(x_i) < 1$ and $0 \le \pi_A(x_i) < 1$

for some $x_i$, hence, $\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i) < 1$, then:

$$C_1(A,A) = \frac{1}{n}\sum_{i=1}^{n}(\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A^2(x_i)) < 1, \quad (28)$$

which is contradiction. It shows that $A$ is a non-fuzzy set.

## 4. Correlation and correlation coefficient in probability space

Let $(X,B,P)$ be a probability space and $A$ be an intuitionistic fuzzy set in a probability space $X$, $A=\{(\mu_A(x),\nu_A(x),\pi_A(x)) \mid x\in X\}$, where $\mu_A,\nu_A : X \to [0,1]$. Supposed both functions $\mu_A(x)$ and $\nu_A(x)$ be Borel measurable functions, then $\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ is also Borel measurable function.

**Definition 2.** If $A,B\in$ IFSs, we define:

$$C_2(A,B)=\int_X (\mu_A\mu_B+\nu_A\nu_B+\pi_A\pi_B)dP, \tag{29}$$

and we call the correlation of intuitionistic fuzzy sets $A$ and $B$. Furthermore, we call:

$$\rho_2(A,B)=\frac{C_2(A,B)}{\sqrt{C_2(A,A).C_2(B,B)}}, \tag{30}$$

the correlation coefficient of intuitionistic fuzzy sets $A$ and $B$.

If intuitionistic fuzzy sets $A$ and $B$ satisfy $P=\{\mu_A=\mu_B,\nu_A=\nu_B\}=1$, then we denote by $A\overset{a.e}{=}B$.

**Remark 1.** In particular, if $X=\{x_1,x_2,...,x_n\}$ and probability $P$ is given by $P(A)=|A|/n$, where $|A|$ is the cardinality of $A$, then we have $C_2(A,B)=C_1(A,B)$ and $\rho_2(A,B)=\rho_1(A,B)$.

**Remark 2.** In particular, if $X=\{x_1,x_2,...,x_n\}$ and probability $P$ is given by $P(A)=|A|/n$, where $|A|$ is the cardinality of $A$, then we have $C_{HH}(A,B)=C_{GM}(A,B)$.

**Remark 3.** If we define probability $P$ on $X=[a,b]$ by $P(A)=\frac{1}{b-a}\int_A 1dx, A\in F([a,b])$, then we have:

$$C_2(A,B)=C_Y(A,B), \tag{31}$$

$$\rho_2(A,B)=\rho_Y(A,B). \tag{32}$$

**Remark 4.** For an ordinary fuzzy sets, $A$ and $B$, $\mu_A(x)+\nu_A(x)=1$, then $\rho_1(A,B)=\rho_{GM}(A,B)$.

**Remark 5.** For an ordinary fuzzy sets, $A$ and $B$, $\mu_A(x)+\nu_A(x)=1$, then $\rho_2(A,B)=\rho_{HH}(A,B)$. Then the following proposition is immediate from the definition.

**Proposition 2.** For $A,B\in$ IFSs, then we have:

$0\le C_2(A,B)\le 1$,

$C_2(A,B)=C_2(B,A), \rho_2(A,B)=\rho_2(B,A)$,

If $A=B$, then $\rho_2(A,B)=1$. (33)

**Theorem 5.** For $A,B\in$ IFSs, then we have:

$0\le \rho_2(A,B)\le 1$.

**Proof.** Since $C_2(A,B)\ge 0$, we only need to prove $\rho_2(A,B)\le 1$.

For an arbitrary real number $k$, we have

$$\begin{aligned}0 &\le \int_X(\mu_A-k\mu_B)^2dP+\int_X(\nu_A-k\nu_B)^2dP\\ &+\int_X(\pi_A-k\pi_B)^2dP=\int_X(\mu_A^2+\nu_A^2+\pi_A^2)dP\\ &-2k\int_X(\mu_A\mu_B+\nu_A\nu_B+\pi_A\pi_B)dP\\ &+k^2\int_X(\mu_B^2+\nu_B^2+\pi_B^2)dP.\end{aligned} \tag{34}$$

Thus, we can get:

$$\begin{aligned}(\int_X(\mu_A\mu_B+\nu_A\nu_B+\pi_A\pi_B)dP)^2 &\le \int_X(\mu_A^2\\ +\nu_A^2+\pi_A^2)dP.\int_X(\mu_B^2+\nu_B^2+\pi_B^2)dP.\end{aligned} \tag{35}$$

Therefore, we have $\rho_2(A,B)\le 1$.

**Theorem 6.** $\rho_2(A,B)=1$ if and only if $A\overset{a.e}{=}B$.

**Proof.** Considering the inequality in the proof of Theorem 5, then the equality holds if and only if $P\{\mu_A=k\mu_B\}=P\{\nu_A=k\nu_B\}=P\{\pi_A=k\pi_B\}=1$ for some positive real number $k$. And

$\mu_A+\nu_A+\pi_A=\mu_B+\nu_B+\pi_B=1$, it means $k=1$. Namely, $A\overset{a.e}{=}B$.

**Theorem 7.** $C_2(A,B)=0$ if and only if $A$ and $B$ are non-fuzzy sets and they satisfy the condition: $\mu_A+\mu_B\overset{a.e}{=}1$ or $\nu_A+\nu_B\overset{a.e}{=}1$ or $\pi_A+\pi_B\overset{a.e}{=}1$.

**Proof.** Since $\mu_A\mu_B+\nu_A\nu_B+\pi_A\pi_B\geq 0$, then $C_2(A,B)=0$ implies $P\{\mu_A\mu_B+\nu_A\nu_B+\pi_A\pi_B=0\}=1$, which means $P\{\mu_A\mu_B=0\}=1$, $P\{\nu_A\nu_B=0\}=1$ and $P\{\pi_A\pi_B=0\}=1$. If $\mu_A(x)=1$, then we have $\mu_B(x)=0$ and $\nu_A(x)=\pi_A(x)=0$. At the same time, if $\mu_B(x)=1$, then we can get $\mu_A(x)=0$ and $\nu_B(x)=\pi_B(x)=0$, hence, we have $\mu_A(x)+\mu_B(x)=1$.

Conversely, if $A$ and $B$ are non-fuzzy sets and $\mu_A(x)+\mu_B(x)=1$, if $\mu_A(x)=1$, then we have $\mu_B(x)=0$ and $\nu_A(x)=\pi_A(x)=0$. On the other hand, if $\mu_B(x)=1$, then we have $\mu_A(x)=0$ and $\nu_B(x)=\pi_B(x)=0$, which implies $C_2(A,B)=0$.

Similarly we can give the proof when $\nu_A+\nu_B\overset{a.e}{=}1$ or $\pi_A+\pi_B\overset{a.e}{=}1$.

The theorem generalizes Theorem 3 of Hong and Hwang [15].

**Theorem 8.** If $A$ is a non-fuzzzy set, then $C_2(A,A)=1$. The proof is obvious.

## 5. Comparative example

In this section, we will give three examples to compare with several correlation coefficient of intuitionistic fuzzy sets.

**Example 2.** For a finite universal set $X=\{x_1,x_2,x_3\}$, four intuitionistic fuzzy sets are represented, respectively.

$$A=\{(0,0),(0,0),(0,0)\},$$

$$B=\{(0,0),(0,0),(0,0)\},$$

$$C=\{(0,1),(0,1),(0,1)\},$$

$$D=\{(0,0),(0,0),(0,0)\}.$$

Therefore, we have $\rho_1(A,B)=1$ and $\rho_1(C,D)=0$. This result coincides with our intuition.

**Example 3.** For a finite universal set $X=\{x_1,x_2,x_3\}$, if two intuitionistic fuzzy sets are written, respectively. $A=\{(1,0),(0.8,0),(0.7,0.1)\}$ and $B=\{(0.5,0.3),(0.6,0.2),(0.8,0.1)\}$, then we have:

$$\rho_1(A,B)=0.8882,$$

$$\rho_{GM}(A,B)=0.8987.$$

It shows that intuitionistic fuzzy sets $A$ and $B$ have a good positively correlated.

**Example 4.** For a continuous universal set $X=[1,5]$, if two intuitionistic fuzzy sets are represented in the following, respectively:

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in[1,5]\},$$

$$B=\{\langle x,\mu_B(x),\nu_B(x)\rangle \mid x\in[1,5]\},$$

where

$$\mu_A(x)=\begin{cases}0.5(x-1), & 1\leq x<2\\ (5-x)/6, & 2\leq x\leq 5\end{cases}$$

$$\nu_A(x)=\begin{cases}1.9-0.9x, & 1\leq x<2\\ 0.3x-0.5, & 2\leq x\leq 5\end{cases}$$

$$\mu_B(x)=\begin{cases}0.3(x-1), & 1\leq x<3\\ 0.3(5-x), & 3\leq x\leq 5\end{cases}$$

$$\nu_B(x)=\begin{cases}1.4-0.4x, & 1\leq x<3\\ 0.4x-1, & 3\leq x\leq 5\end{cases}$$

Thus, we have:

$$\rho_2(A,B)=0.9237,$$

$$\rho_{HH}(A,B) = 0.9389,$$

$$\rho_{HW}(A,B) = 0.9949.$$

It shows that intuitionistic fuzzy sets $A$ and $B$ have a high positively correlated.

## 6. Conclusion

In this paper, we take into account all three parameters describing intuitionistic fuzzy set, propose a kind of new method to calculate correlation and correlation coefficient of intuitionistic fuzzy sets which is similar to the cosine of the intersectional angle in finite set and probability space. Further, we discuss some of their properties and give three numerical examples to illustrate our proposed method reasonable. Thus, we generalize some conclusions in literature.

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