Multivariate process capability indices on the presence of priority for quality characteristics

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Abstract

Multivariate Process Capability Indices (MPCI) show how well a manufacturing process can meet specification limits when quality characteristics enclose a relative correlation. Process capability is an important and commonly used metric for assessing and improving the quality of a production process. When quality characteristics of a product are correlated then an attractive comes close to MPCI methods, which are not usually an easy task to carry out. In this investigation after a full reviewing of the MPCI, a simple method to estimate product capability indices based on ridge regression models in the presence of priority for quality characteristics is presented. The technique is demonstrated for evaluation of product capability through the use of an example which shows performance of the proposed method.

Keywords: Multivariate process capability indices; Product capability; Ridge regression method; Quality characteristic priority; Multivariate statistical process control

1. Introduction

Process produces products according to a certain quality characteristic, for example weight, length, hardness, viscosity, etc. The degree a process is producing data within tolerance limits, can be measured using Process Capability Indices (PCIs). PCIs are generally used in industry to measure characteristics that are independence of each others. A standard practice in Statistical Process Control (SPC) programs is to ensure that the process is under statistical control prior to conducting a process capability analysis. Unfortunately, it is a fairly common practice to perform capability analysis using a sample of historical process data without any consideration of whether or not the process is in statistical control. As Montgomery [25] stated, if the process is not in control then its parameters are unstable and the value of these parameters in the future is uncertain. Hence, the predictive aspects of the process capability indices regarding the number of nonconforming items produced are lost.

The most frequently used univariate PCIs including C_P, C_{PK}, C_{PM}, and C_{PMK} have been proposed in the manufacturing industry to provide numerical measures on process capability and performance,

which are effective tools for quality assurance. These indices are defined as:

$$C_P = \frac{USL - LSL_i}{6\sigma}$$

$$C_{PK} = Min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\}$$

$$C_{PM} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} \qquad i = 1, 2, ..., p \qquad (1)$$

$$C_{PMK} = Min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}$$

where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, σ is the process standard deviation and T is target amount anywhere within the specification interval.

PCIs have recently received a considerable amount of attention in the literature of SPC. Numerous authors including Kane [20], Marcucci and

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Beazley [24], Chan and Cheng [3], Choi and Owen [8], Spring [37], Koons [21], Wheeler and Chamber [43], Pearn [32], Bissel [2], Wright [45], Pearn and Chen [29], Stoumbos [38], Pearn and Shu [33], Chen and Chen [6], Perakis and Xekalaki [34] and Chou *et al.* [10] have discussed theories and applications of univariate PCIs, when process normally distributed.

Extensive studies have also been conducted to determine the effects of non-normality on the various PCIs. Gunter [13,14,15,16] in a series of articles pointed out many flaws of the indices particularly C_{PK} when applied to non-normal data. Interested readers are referred to Munechika [26], Clemets [11], Wright [44], Somerville and Montgomery [36], Bai and Choi [1], Chen and Ding [7] and Chou *et al.* [9] for more discussions on the univariate process capability indices when normality assumption is violated.

During the past decade, there has been a growing concern about the normality and independence assumptions required to compute univariate capability indices. In practice, it is common to use two or more related quality characteristics of a product to evaluate the performance of a manufacturing process. Since the early work of Hotelling [17], it has become evident that such problems, due to the correlation that exists among quality characteristics, need to be addressed in multivariate context to ensure proper evaluation. Similar to the univariate case, the Multivariate Process Capability Indices (MPCIs) have also captured attentions of many researchers including Hubele *et al.* [19], Chan *et al.* [4], Taam *et al.* [39], Nickerson [27], Chen [14], Karl *et al.* [23], Niverthi and Dey [28], Shahriari *et al.* [35], Wang *et al.* [42], Wang and Du [40], Frey et al. [12], Wang and Hubele [41], Pearn *et al.* [31] and Pearn and Chien-Wei [30]. For a quick survey and interpretations on univariate and multivariate process capability indices see Kotz and Johnson [22].

An existing serious problem in multivariate quality control is in complexity of methodology for assessing MPCIs. The purpose of this paper is to provide a relatively simple method for estimating the most well-known relevant PCIs (priory showed through set of Eq. (1) in multivariate environment which define by MC_P, MC_{PK}, MC_{PM}, and MC_{PMK} respectively. A brief discussion to some MPCIs is presented in Section 2 and the proposed methodology to estimate MPCIs are offered in Section 3. The fourth part discusses a numerical example. Conclusions are provided in the final section.

2. Some techniques in MPCIs evaluations

Various authors have proposed alternative approaches to assess process capability in multivariate environment. Taam *et al.* [39] recommend using a multivariate capability index that is defined as a ratio of two volumes:

$$MC_{PM} = \frac{Vol.(R_1)}{Vol.(R_2)} \tag{2}$$

where R_1 is a modified tolerance region and R_2 is a scaled 99.73 percent process region. In particular, if the process data are multivariate normal, then R_2 is an elliptical region. A process region and modified tolerance region is shown in Figure 1 at appendix. The modified tolerance region is defined as the largest ellipsoid that is centered at the target completely located inside the original tolerance region. The estimate for MC_{PM} is given by:

$$M\hat{C}_{PM} = \frac{\hat{C}_P}{\hat{D}} \tag{3}$$

where,

$$\hat{C}_P = \frac{Vol.(tolerance\ region)}{Vol.(estimated\ 99.73\%\ of\ process\ region)}$$
$$= \frac{Vol.(tolerance\ region)}{|S|^{\frac{1}{2}}(\pi K)^{\frac{v}{2}}\left[\Gamma(\frac{v}{2}+1)\right]^{-1}} \tag{4}$$

and,

$$\hat{D} = \left[1 + \frac{n}{n-1}\left(\overline{\mathbf{X}} - \boldsymbol{\mu_0}\right)'\mathbf{S}^{-1}\left(\overline{\mathbf{X}} - \boldsymbol{\mu_0}\right)\right]^{\frac{1}{2}} \tag{5}$$

where K is the 99.73 percent quantile of a χ^2 distribution and $|\mathbf{S}|$ denotes the determinant of sample variance-covariance matrix.

Shahriari *et al.* [35] proposed a vector consisting of three components. The first two components use the assumption that the process data is from a multivariate normal distribution with elliptical contours defining probability regions and the third component is based on a geometric understanding of process relative to the engineering specifications. The first component is defined as:

$$MC_{PM} = \left[\frac{\text{vol. of engineering tolerance region}}{\text{vol. of modified process region}} \right]^{\frac{1}{v}}$$

$$= \left[\frac{\prod_{i=1}^{v}(USL_i - LSL_i)}{\prod_{i=1}^{v}(UPL_i - LPL_i)} \right]^{\frac{1}{v}} \quad i = 1,2,...,v \quad (6)$$

Figure 2 at appendix illustrates their method for a product in which the engineering specifications define a rectangular tolerance region but bi-variate normal process variables define an elliptical probability contour referred to as process region. Their proposed method forms a modified process region by drawing the smallest rectangle around the elliptical process region. The edges of the modified process region are defined as the lower and upper process limits (LPL_i and UPL_i, respectively, where $i=1,2,...v$) and are given by:

$$UPL_i = \mu_i + \sqrt{\frac{\chi^2_{v,\alpha}|\Sigma_i^{-1}|}{|\Sigma^{-1}|}} \tag{7}$$

$$LPL_i = \mu_i - \sqrt{\frac{\chi^2_{v,\alpha}|\Sigma_i^{-1}|}{|\Sigma^{-1}|}} \quad ; \ i = 1,2,...,v \tag{8}$$

where $\chi^2_{v,\alpha}$ denotes the upper $100(\alpha)$ % of a χ^2 distribution with v degrees of freedom and $|\Sigma_i^{-1}|$ is the determinant of the variance-covariance matrix with its ith row and column deleted.

The second component of the proposed vector is based on the assumption that the center of the specification limits denotes the process mean. This component is defined as the significance level of a Hotelling's T^2 statistic, which is computed as follows:

$$PV = P\left(T^2 \geq \frac{v(n-1)}{n-v} F_{(v,n-v)} \right) \tag{9}$$

where,

$$T^2 = n(\bar{X} - \mu_0)' S^{-1}(\bar{X} - \mu_0) \tag{10}$$

$F_{(v,n-v)}$ denotes the *value of F* distribution with v and n-v degrees of freedom. It should be pointed out that large values of *PV* indicate the closeness of the center of the process to the pre-specified target value.

The third component of the vector that is referred to as location index (LI) compares the location of the modified process region to the tolerance region. This index has a value of one if the entire modified process region is contained within the tolerance region indicating that all the manufactured products conform to the specification limits, otherwise it will take a value of zero.

Chen [5] proposed a multivariate process capability index based on a multiple bilateral tolerance zone defined by:

$$V = \left\{ X \in R^V : |X_i - \mu_0| \leq r_i, i = 1,2,...,v \right\} (11)$$

where μ_{0_i} is the specification limit and r_i is a constant. The multivariate process capability index is given by $MC_P = 1/r$, where *r* is defined such that:

$$P\left(Max\left(\frac{|X_i - \mu_{0_i}|}{r_i}, i = 1,2,...,v \right) \leq r \right) = 1-\alpha \ (12)$$

Let F be the cumulative distribution function of,

$$h(X - \mu_0) = Max\left\{ \frac{|X_i - \mu_{0_i}|}{r_i} \leq r_i, i = 1,2,...,v \right\} \tag{13}$$

Then $r = F^{-1}(1-\alpha)$. If the value of MC_P is greater than or equal to 1, the process is capable with a certain confidence level.

Frey [12] proposed a matrix of dimensionless parameters (**C**), which represents a linear mapping of noise variables (**n_j** ; *j=1,2,...,n*) to quality characteristics (**q_i** ; *i=1,2,...,m*), for evaluating process capability in multivariate environment. The elements of this matrix, **C_{ij}**, are defined as:

$$\mathbf{C}_{ij} = \frac{6S_j \left(\frac{\partial q}{\partial n}\Big|_{n=t} \right)_{ij}}{USL_i - LSL_i} \tag{14}$$
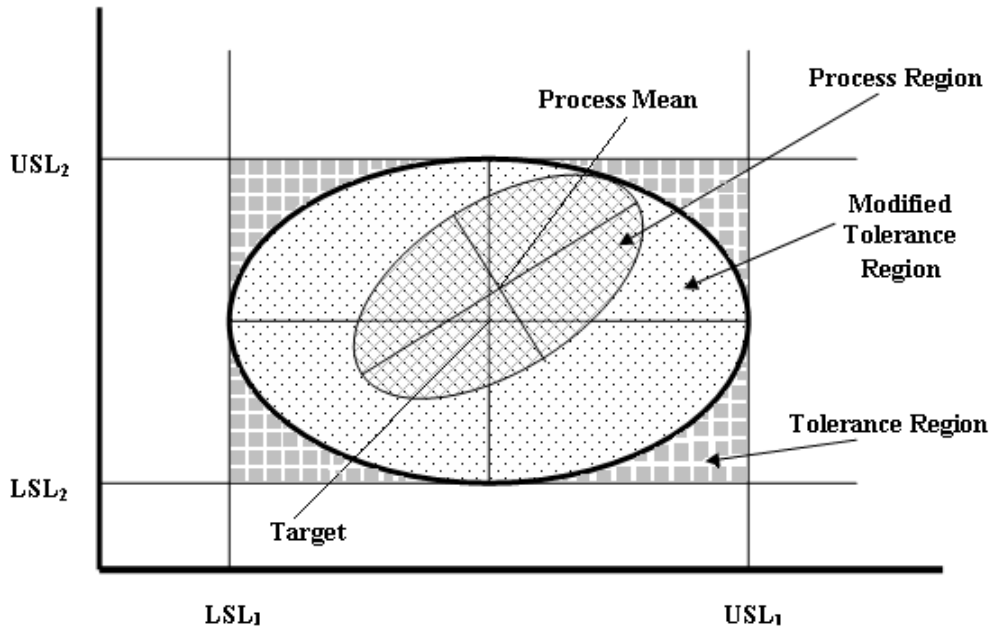
Figure 1. Typical modified tolerance region (R_1) versus estimated 99.73 % process region (R_2) in a bivariate case.
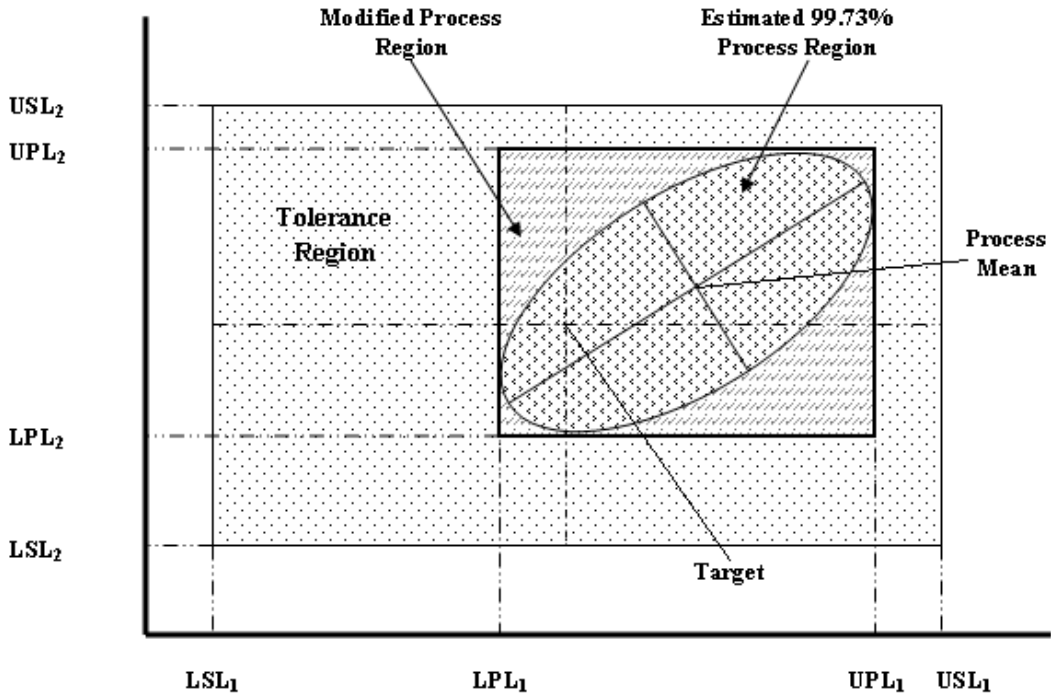

Figure 2. Rectangular Tolerance Region versus Modified Process Region.

where $\mathbf{S_j}$ is the jth noise variable standard deviation and $\mathbf{t}$ denotes the vector of target values. The quantity $d\mathbf{q}=\mathbf{C}.d\mathbf{n}+\mathbf{K}$ describes the sensitivity of each quality characteristic with respect to the independent noise variables.

They describe the elements of the bias vector $\mathbf{K}$ by:

$$K_i = \frac{2}{USL_i - LSL_i}$$

$$\times\left(\left(\sum_{j=1}^{n}\left[m_j.\left(\frac{d\mathbf{q}}{d\mathbf{n}}\Big|_{\mathbf{n}=\mathbf{t}}\right)_{ij}\right]\right)-\frac{USL_i - LSL_i}{2}\right) \quad (15)$$

If the rows of the matrix $\mathbf{C}$ are orthogonal then the quality characteristics will be statistically independent of each other and the process yield can be expressed as:

$$Y_{RT}=\prod_{i=1}^{m}\frac{1}{2}\left[erf\left(\frac{3\sqrt{2}(1-\mathbf{K}_i)}{2\sqrt{\sum_{j=1}^{n}\mathbf{C}_{ij}^2}}\right)+erf\left(\frac{3\sqrt{2}(1+\mathbf{K}_i)}{2\sqrt{\sum_{j=1}^{2}\mathbf{C}_{ij}^2}}\right)\right] \quad (16)$$

If quality characteristics of interests are correlated, then the correlation coefficient K_{ij} between the ith and j^{th} quality characteristics is defined by:

$$K_{ij}=\frac{\mathbf{C}_i.\mathbf{C}_j}{\left|\mathbf{C}_i.\mathbf{C}_j\right|} \qquad i,j=1,2,...,m \qquad (17)$$

where $\mathbf{C}_i$ and $\mathbf{C}_j$ denote the ith and j^{th} rows of $\mathbf{C}$. If the diagonal elements of $\mathbf{K}$ are non-zero then quality characteristics will be correlated and process yield may be estimated through Monte Carlo simulation using the following expression:

$$Y_{RT}\cong\frac{1}{Trial}\sum_{k=1}^{Trial}\left\|\mathbf{C}_{ij}.rand(\tfrac{1}{3})+\mathbf{K}_i\right\|_{\infty}<1 \quad (18)$$

where $rand$ (1/3) is a function that generates a vector of random numbers from a population having the functional form of the nth noise variable with a mean of zero and standard deviation 1/3.

3. Suggested approach

As it was shown in the previous section, MPCIs is not always an easy task to perform. As a matter of fact, in certain cases this task could become work intensive. In this section, a relatively simple method is proposed. It can help quality engineers to respectively determine an interval estimate for some most important process capability indices (C_P, C_{PK}, C_{PM} and C_{PMK}) in multivariate situation which will be showed by MC_P, MC_{PK}, MC_{PM} and MC_{PMK} later by using standard statistical packages without experiencing too much computational difficulties.

Suppose in multivariate circumstances, X defines a vector of p correlated quality characteristics of a product. The proposed procedure helps to evaluate the potential and performance process capability indices in multivariate environments. This course of action consists of the following three steps:

3.1. Step 1: Main parameters estimation when process is under control

Houshmand and Javaheri [18] proposed the Multivariate Ridge Residual Chart (MRRC) to ensure whether the multivariate process has been under statistical control. In SPC, this is referred to as phase I and construction of control charts during this phase is usually iterative. This control chart is applied to the standardized data which attain by subtracting data from their means and dividing them by their standard deviations. Houshmand and Javaheri [18] due to the presence of multicollinearity, use ridge regression to model each quality characteristic as a function of the remaining ones and construct Shewhart / EWMA (Exponentially Weighted Moving Average) charts of the residuals to monitor the stability of a process. The MRRC is a very effective tool to detect trends and shifts of any magnitude in the components of the mean vector of multivariate processes and also determines which variable caused out of control signal.

Following their method, a set of ridge regression equations is constructed for each quality characteristics as a function of the rest as follows:

$$X_1 = a_{12}X_2 + a_{13}X_3 + ... + a_{1p}X_p + \varepsilon_1$$

$$X_2 = a_{21}X_1 + a_{23}X_3 + ... + a_{2p}X_p + \varepsilon_2 \quad (19)$$

$$.....................................$$

$$X_p = a_{p1}X_1 + a_{p2}X_2 + ... + a_{p,p-1}X_{p-1} + \varepsilon_p$$

where ε_i ($i=1,...,p$) denotes random error term that follows a normal distribution with mean zero and a constant variance σ^2. An out-of-control signal in either the Shewhart or EWMA charts of the residuals are interpreted as an out-of-control signal in the corresponding dependent quality characteristic.

Once the process is announced under statistical control, the set of Equations in (19) along with the expected value are used to determine the mean estimates for the correlated quality characteristics as follows:

$$\hat{\mu}_{X_1} = a_{12}\hat{\mu}_{X_2} + ... + a_{1p}\hat{\mu}_{X_P} \tag{20}$$

$$\hat{\mu}_{X_{12}} = a_{21}\hat{\mu}_{X_1} + ... + a_{2p}\hat{\mu}_{X_P}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\hat{\mu}_{X_{1p}} = a_{p1}\hat{\mu}_{X_1} + ... + a_{p,p-1}\hat{\mu}_{X_{P-1}}$$

Also an estimate for the standard deviation of each quality characteristic may be computed by getting this operator on the previous set of equations as follows:

$$\hat{\sigma}_{X_1}^2 = a_{12}^2\hat{\sigma}_{X_2}^2 + a_{13}^2\hat{\sigma}_{X_3}^2 + ... + a_{1p}^2\hat{\sigma}_{X_P}^2 \tag{21}$$

$$+2a_{12}a_{13}\hat{\sigma}_{X_2X_3} + ... + 2a_{1,p-1}a_{1p}\hat{\sigma}_{X_{p-1}X_p}$$

$$\hat{\sigma}_{X_2}^2 = a_{21}^2\hat{\sigma}_{X_1}^2 + a_{23}^2\hat{\sigma}_{X_3}^2 + ... + a_{2p}^2\hat{\sigma}_{X_P}^2$$

$$+2a_{21}a_{23}\hat{\sigma}_{X_{12}X_3} + ... + 2a_{2,p-1}a_{2p}\hat{\sigma}_{X_{p-1}X_p}$$

$$\cdots\cdots\cdots\cdots$$

$$\hat{\sigma}_{X_p}^2 = a_{p1}^2\hat{\sigma}_{X_1}^2 + a_{p2}^2\hat{\sigma}_{X_2}^2 + ... + a_{p,p-1}^2\hat{\sigma}_{X_{p-1}}^2$$

$$+2a_{p1}a_{p2}\hat{\sigma}_{X_1X_2} + ... + 2a_{p,p-2}a_{1p}\hat{\sigma}_{X_{p-1}X_p}$$

3.2. Step 2: PCIs evaluation for all quality characteristics

Consider TT_{X_i} denotes technical tolerance for the ith quality characteristics. In general form every quality characteristic may have LSL, USL and a target amount (if it exists) as follows:

$$TT_{X_i} = \left[LSL_{X_i}, T_{X_i}, USL_{X_i} \right] \quad i = 1, 2, ..., p \tag{22}$$

For each quality characteristic a univariate PCIs is computed. The most common indices such as $C_P(X_i)$, $C_{PK}(X_i)$, $C_{PM}(X_i)$ and $C_{PMK}(X_i)$ can independently be estimated for all the quality characteristics as follows:

$$\hat{C}_P(X_i) = \frac{USL_{X_i} - LSL_{X_i}}{6\hat{\sigma}_{X_i}} \tag{23}$$

$$\hat{C}_{PK}(X_i) = Min\left\{ \frac{USL_{X_i} - \hat{\mu}_{X_i}}{3\hat{\sigma}_{X_i}}, \frac{\hat{\mu}_{X_i} - LSL_{X_i}}{3\hat{\sigma}_{X_i}} \right\}$$

$$\hat{C}_{PM}(X_i) = \frac{USL_{X_i} - LSL_{X_i}}{6\sqrt{\hat{\sigma}_{X_i}^2 + (\hat{\mu}_{X_i} - T_{X_i})^2}}$$

$$\hat{C}_{PMKi}(X_i) = Min\left\{ \frac{USL_{X_i} - \hat{\mu}_{X_i}}{3\sqrt{\hat{\sigma}_{X_i}^2 + (\hat{\mu}_{X_i} - T_{X_i})^2}}, \right.$$

$$\left. \frac{\hat{\mu}_{X_i} - LSL_{X_i}}{3\sqrt{\hat{\sigma}_{X_i}^2 + (\hat{\mu}_{X_i} - T_{X_i})^2}} \right\} \quad i = 1, 2, ..., p$$

3.3. Step 3: MPCIs estimate via applying weighting average method

In order to estimate the most known MPCIs based on the p estimates of PCIs, one may use weighting average method. Based on this routine, the product capability indices are defined by the following set of equations as:

$$MC_P = \sum_{i=1}^{p} W_i C_P(X_i)$$

$$MC_{PK} = \sum_{i=1}^{p} W_i C_{PK}(X_i)$$

$$MC_{PM} = \sum_{i=1}^{p} W_i C_{PM}(X_i) \tag{24}$$

$$MC_{PMK} = \sum_{i=1}^{p} W_i C_{PMK}(X_i)$$

where MC_P, MC_{PK}, MC_{PM} and MC_{PMK} respectively act as equivalents for C_P, C_{PKi}, C_{PMi} and C_{PMK}, in multivariate circumstances and W_i shows the normalized importance weight of the ith quality characteristic derived from the customer's points of view.

Note that: $$\sum_i^P W_i = 1 \qquad (25)$$

As will be shown in the next section, applying this approach to a set of multivariate data is much easier than going through the complicated equations.

4. Numerical example

In the following, we will consider a numerical example to demonstrate how the MPCIs can be applied in processes with multiple characteristics. The example involves a process of turret lathe for manufacturing certain steel sleeves. The performance of this production process is evaluated by measuring three identifiable diameters of cylindrical sleeves, henceforth referred to as "A", "B" and "C" , respectively, reported in 0.0001 inches above nominal. Table 1 contains 28 sleeve measurements from an under control process for the three quality characteristics with the same weights of importance both for A and C and twice for B.

The lower and upper specification limits (LSL, USL) for the three quality characteristics are shown in Table 2.

Table 1: Diameters of the three steel sleeves in 0.0001 inch above nominal.

Sample	A	B	C	Sample	A	B	C
1	125	75	100	15	112.5	67.5	115
2	135	87.5	110	16	125	67.5	120
3	142.5	95	120	17	127.5	57.5	125
4	130	102.5	110	18	112.5	67.5	110
5	125	72.5	100	19	95	55	100
6	137.5	55	115	20	122.5	50	110
7	120	72.5	110	21	102.5	45	100
8	115	87.5	105	22	120	82.5	100
9	125	65	110	23	115	57.5	95
10	157.5	82.5	130	24	100	65	90
11	100	17.5	95	25	100	47.5	100
12	105	40	100	26	100	57.5	105
13	107.5	45	105	27	107.5	72.5	110
14	110	65	110	28	120	82.5	115

The sample mean vector $\overline{\mathbf{X}}$ and variance-covariance matrix $\mathbf{S}^2$ for the data in Table 2 are given by:

$$\overline{\mathbf{X}} = \begin{bmatrix} 117.68 \\ 65.63 \\ 107.68 \end{bmatrix}, \quad \mathbf{S}^2 = \begin{bmatrix} 218.0 & 162.4 & 99.0 \\ 162.4 & 336.9 & 69.6 \\ 99.0 & 69.6 & 87.9 \end{bmatrix}$$

The correlation coefficients presented in Table 3 is indicative of some meaningful correlation among the three quality characteristics.

Pass 2000® statistical software was applied to the data presented in Table 1 and the following ridge regression equations with a biasing factor of $K=0.005$ which helped the regression coefficients to remain constant was obtained:

$$\hat{A} = 2.689 + 0.297B + 0.887C$$

$$\hat{B} = -16.789 + 0.781A - 0.088C$$

$$\hat{C} = 54.107 + 0.465A - 0.018B$$

Figure 3 presents the normal probability plots with constant small variance for the residuals of the derived models. This figure confirms the normality and uncorrelated assumption and suffices validity of the models.

Ridge regression equations can be used to obtain estimates for the mean and variance of each quality characteristic as follows:

$$\hat{\mu}_A = 2.689 + 0.297(65.63) + 0.887(107.68) = 117.69$$

$$\hat{\mu}_B = -16.789 + 0.781(117.68) - 0.088(107.68) = 65.64$$

$$\hat{\mu}_C = 54.107 + 0.465(117.68) - 0.018(65.63) = 107.65$$

and,

$$\hat{\sigma}_A^2 = (0.297)^2(336.9) + (0.887)^2(87.9) + \\ 2(0.297)(0.887)(69.6) = 135.6 \Rightarrow \hat{\sigma}_A = 11.64$$

$$\hat{\sigma}_B^2 = (0.781)^2(218) + (-0.088)^2(87.9) + \\ 2(0.781)(-0.088)(99) = 120.06 \Rightarrow \hat{\sigma}_B = 10.96$$

$$\hat{\sigma}_C^2 = (0.465)^2(218) + (-0.018)^2(336.9) + \\ 2(0.465)(-0.018)(162.4) = 44.5 \Rightarrow \hat{\sigma}_C = 6.67$$

Now, an estimate for PCIs corresponding to each quality characteristic can be determined. The results of the computations are provided in Table 4.
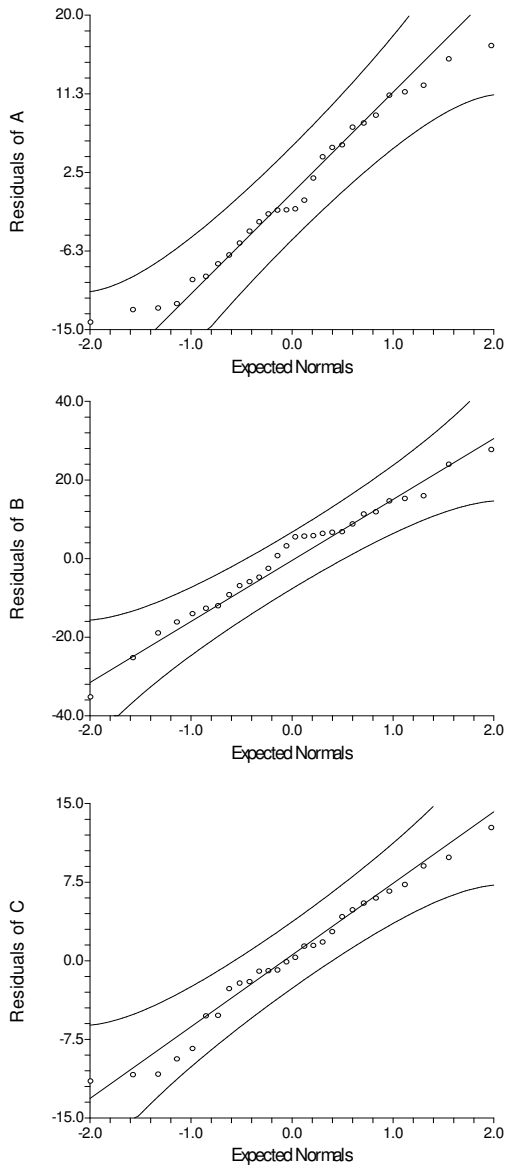
Figure 3. Normal probability plot of residuals with 95% confidence interval.

Table 2. The main aspects of customer requirements for the three steel sleeves.

Variables	A	B	C
USL	171	132	147
LSL	64	0	70
T	117	65.6	107
W	0.25	0.5	0.25

Table 3. Correlation coefficient matrix for the three quality characteristics.

	A	B	C
A	1.00	0. 60	0.72
B	0. 60	1.0	0.41
C	0.72	0.41	1.00

Table 4. Process capability indices estimation.

X_k	**A**	**B**	**C**
Mean	117.69	65.64	107.65
Standard Deviation	11.64	10.96	6.67
C_P	1.43	1.83	1.50
C_{Pk}	1.37	1.69	1.12
C_{PM}	1.40	1.70	0.99
C_{PMk}	1.34	1.57	0.73

Via weighting average method based on the normalized weights of 0.25, 0.5 and 0.25 for A, B and C respectively, the MPCIs could be carried out as:

$$MC_P = \sum_{i=1}^{3} w_i C_P(X_i) = 0.25(1.43) + 0.5(1.83)$$

$$+ 0.25(1.50) = 1.65$$

$$MC_{PK} = \sum_{i=1}^{3} w_i C_{PK}(X_i) = 0.25(1.37)$$

$$+ 0.5(1.69) + 0.25(1.12) = 1.47$$

$$MC_{PM} = \sum_{i=1}^{3} w_i C_{PM}(X_i) = 0.25(1.40)$$

$$+ 0.5(1.70) + 0.25(0.99) = 1.45$$

$$MC_{PMK} = \sum_{i=1}^{p} w_i C_{PMK}(X_i) = 0.25(1.34)$$

$$+ 0.5(1.57) + 0.25(0.73) = 1.30$$

5. Conclusions

PCIs determine the relation between the actual process performance and the technical tolorances, which enumerate process potential and process performance, are vital to any successful quality improvement activities. Capability indices measure for processes with single characteristic has broadly been looked, but is relatively deserted for product characteristics with different preferences on quality specifications. However, the lack of efficient indices in multivariate domain is levelheaded. The proposed MPCIs can be applied after validating under

control condition through use of multivariate ridge regression chart. This proposed approach has the following advantages:

1) It constructed based on recognized multiple regression method which can be easily planned using statistical packages.

2) It can be applied to estimate many familiar process capability indices.

3) The important workstations can be identified using the regression coefficients when ridge regression method is applied to the standardized data set.

4) The projected method could be applied when each quality specification has different weight of importance.

The numerical example was used to model the relationship among quality specifications to obtain overall product capability indices. This example was used to show the effectiveness of the proposed method in terms of the amount of computation involved.

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