Journal of Industrial Engineering International
a SpringerOpen Journal

TECHNICAL ARTICLE Open Access

Optimizing the warranty period by cuckoo meta-heuristic algorithm in heterogeneous customers' population

Ali Roozitalab[1*] and Ezzatollah Asgharizadeh[2]

Abstract

Warranty is now an integral part of each product. Since its length is directly related to the cost of production, it should be set in such a way that it would maximize revenue generation and customers' satisfaction. Furthermore, based on the behavior of customers, it is assumed that increasing the warranty period to earn the trust of more customers leads to more sales until the market is saturated. We should bear in mind that different groups of consumers have different consumption behaviors and that performance of the product has a direct impact on the failure rate over the life of the product. Therefore, the optimum duration for every group is different. In fact, we cannot present different warranty periods for various customer groups. In conclusion, using cuckoo meta-heuristic optimization algorithm, we try to find a common period for the entire population. Results with high convergence offer a term length that will maximize the aforementioned goals simultaneously. The study was tested using real data from Appliance Company. The results indicate a significant increase in sales when the optimization approach was applied; it provides a longer warranty through increased revenue from selling, not only reducing profit margins but also increasing it.

Keywords: Free replacement warranty policy; Cuckoo optimization; Heterogeneous population; Warranty period

Introduction

In international competitive markets, the life and profitability of the manufacturers' market are to protect the rights of our clients, in addition to profitability, to earn their satisfaction and loyalty. Today, warranty is an integral part and a strategic competitive tool among producers and is one of the factors that influence the buying decisions of customers. Warranty is very important to initiate the company's commitment to support customers after the sale of the product. Since the cost of warranty obligations is very high and increases with increasing duration length, making profit needs scientific planning and appropriate policy. For example, US$23.6 billion was reported in 2010 to cover worldwide warranty claims of US-based companies (Warranty Week 2011).

Literature review

Based on the research of Murthy and Blischke (2005) 'warranty refers to the obligations of the manufacturer or seller for the occurrence of any possible problem resulting from poor quality materials or poor construction process under the specified conditions and period of time through the repair or replacement of damaged parts or restoring some of the best products'. Many studies have been undertaken in the warranty field. They often equate customers' behavior and ignore performance of different customers and the impact on the failure rate for the purpose of simplification (Murthy and Djamaludin 2002). The conditions that the producers intend to provide for after-sales services are called policies including free replacement warranty (FRW) and PRW.

In earlier researches, different types of warranty policies have been offered to cover specific types. More than 30 sample policies have been introduced by An (1992), and there are numerous others that may be offered depending on a particular product or service. Mathematical models to estimate the cost and analyze expense of the manufacturer's warranty have been studied for a variety of different policies (Blischke and Murthy 1996). This study optimizes the FRW policy in which the manufacturer has commitment to accept the

* Correspondence: Roozitalab@ut.ac.ir
[1]Kish International campus, University of Tehran, Kish Island 71778-5777, Iran
Full list of author information is available at the end of the article

component failure or misusage of the product if it occurs in warranty time. Warranty coverage will end after a time commitment. If an item cannot be repaired and must be replaced during the warranty, date of the swap does not start from the replacement date and the period is nonrenewable.

Sahin and Polatoglu (1996, 1998) offered a random distribution of products based on failure occurrence and variables related to the estimated costs. Murthy and Blischke (2005) studied the terms of warranty obligations of cost models and warranty models based on different platforms. St. John and Cassady (2010) considered a heterogeneous population of customers based on exponentially distributed failures to develop this study and form the basis of theoretical research.

St. John and Cassady (2010) categorized customers based on customer behavior and the effect of usage on the failure rate and considered a different warranty period for each group. In reality, we cannot offer different warranty periods to customers. In addition, his exponential distribution model is not generalized to resolve warranty issues. His model which was developed using Weibull distribution, has great flexibility in solving problems of reliability and warranty due to shape and scale parameters, and it can be extended in various issues.

In recent years, many studies were conducted on evolutionary algorithms inspired from nature and human societies such as genetic algorithms, swarm optimization, and ant colony. One of the most recent evolutionary algorithms that have been recently introduced is cuckoo optimization algorithm with good properties in convergence speed and high accuracy in solving optimization problems and accessing to the optimum overall.

Rajabioun (2011) and Yang and Deb (2009) offered cuckoo nonlinear meta-heuristic optimization algorithm based on cuckoo lives. This algorithm is important in specific issues with speed and precision of convergence to optimality. In this study, we use that to obtain the optimal revenue and optimal duration based on the research model. The use of meta-heuristic optimization techniques in the field is quite new in warranty.

The next section describes the acronyms used in this research. In the third part, we have the theoretical model with Weibull distribution and FRW policy. In the fourth section, we optimize the theoretical model in order to achieve an equal optimization period that maximizes the overall revenue of all groups through COA algorithm. The theoretical results obtained are discussed in the fifth section, and model of the real data obtained from an industrial appliance company are tested to optimize the procedure. Finally, conclusion and suggestions for future research are presented in the sixth section.

Theoretical framework

The theoretical model is presented in this section to form the basis of theoretical research.

Product failure model

Each product has a certain life span, and most products experience defects in their life depending on usage conditions, rate of usage, quality of construction, quality of designing and materials, and many other factors. Higher rate of usage will quicken the occurrence of defects that can be used as an index to budget the after-sales services.

If t is a random variable for the time of failure occurrence of the product t, the failure distribution usage is $F(t)$ and the density usage will be $f(t)$; the reliability usage will be as follows:

$$R(t) = 1 - f(t) \tag{1}$$

When a product is offered by FRW policy, all costs of repair or replacement of defective parts to cope with the after-sales service department is paid by the manufacturer or seller of the product. If you do not repair the defective product or part, it must be replaced with a new piece. We assume that the time of replacement parts and service speed is low and negligible; as a result, the failures in this case are expressed by $F(t)$ (Ross 1972).

Reliability

Reliability refers to the likelihood of appropriate operation of the product based on defined tasks for the product under certain conditions in a period of time, i.e., the probability that a defect or an interruption does not occur in a defined interval under certain circumstances. The reliability function is displayed by $R(t)$. Distribution function of nonfailure or reliability function is complementary to the distribution failure function:

$$R(t) + F(t) = 1. \tag{2}$$

Therefore, we can conclude

$$\begin{aligned} R(t \geq a) &= \text{١} - F(t \leq a) \\ &= \text{١} - \textstyle\int_0^t f(t)d(t) = \int_t^\infty f(t)d(t) \end{aligned} \tag{3}$$

You should consider the product reliability until failure time $t = \alpha$ means that the system will not be experiencing fault at least until a while after this time. Failure rate function λ(t) represents the instantaneous failure rate. The equation of this usage is as follows:

$$\lambda(t) = \frac{\mathrm{D}R(t)}{\mathrm{D}(t)} \times \frac{1}{R(t)} = \frac{f(t)}{R(t)} \tag{4}$$

If we solve the relationship of $R(t)$, the following equation is obtained:

$$R(t) = e^{-\int_0^t \lambda(t)dt'} \tag{5}$$

The warranty cost model under FRW policy

In this policy, if failure occurs, a new one replaces a defective piece. If μ (t, u) represents the number of expected failures during the commitment period (t) of warranty under usage rate of u, the period time is [0, t] where $0 < t < W$, and $U = u$; we have the following relation based on renewal theory (Hong-Zhong et al. 2008):

$$\mu(T,u) = F(T,U) + \int_0^{\tau} \mu(t,u)dF(t,U). \tag{6}$$

If it is not conditioned, the equation is as follows:

$$\mu(t) = \int_{u_{\min}}^{U_{\max}} \mu(t,u)dG(u). \tag{7}$$

Repairable products

These products can be fixed or repaired, and the fault will be resolved by replacing the defective part or the entire event. In this case, we assume that maintenance is minimal. The correct model here is based on Blischke and Murthy (1996) with the following equation. Accordingly, it is assumed that S (t, u) represents the expected number of failures in the interval [0, t]. If the condition $0 \le t \le W$ and $U = u$, we have

$$S(t,u) = \int_0^t r(t,u)dt. \tag{8}$$

If the condition is eliminated,

$$S(t) = \int_{u_{\min}}^{u_{\max}} dG(u) \int_0^{\tau} r(t,u)dt. \tag{9}$$

Class model of Weibull failure time

This model has lots of flexibility due to its shape as well as the shape and scale parameters that define it. Many processes of product failures are either modeled by the Weibull distribution. Due to the high flexibility of Weibull distribution parameters, we can have fixed, increased, or decreased failure rates. Weibull belongs to limited failure group in the period with the classification model of Musa and Okumoto (1983) that is of the binomial kind. Thus, the reliability of the distribution will be as follows:

$$r_j(t) = e^{-\left(\frac{\tau-\gamma}{\beta}\right)^{\alpha}}. \tag{10}$$

Weibull distribution of failure rate is calculated according to the following equation:

$$\lambda(T) = \frac{f(t)}{R(t)} \frac{\alpha}{\beta} \left(\frac{\tau-\gamma}{\beta}\right)^{\alpha-1}. \tag{11}$$

Since we assume that customers use the products in different intensities, all customers are divided into subgroups based on their behavior. This will exert the influence of the intensity use of each subgroup on its failure rate. Therefore, Weibull is considered as a statistical distribution usage for the maximum flexibility due to its shape and scale parameters and can be extended to other distributions. The failure rate of each subgroup of customers is as follows:

$$\lambda(t) = \lambda_0 \times \delta_i. \tag{12}$$

In this equation, $\lambda_0 > 0$ is the basic failure rate, a coefficient that is based on the Weibull distribution, and application intensity δi is based on customer usage rates in subgroup i.

The warranty literature indicates that customer purchases based on the increased warranty period is going to saturate the market; after that, increasing the warranty period is not effective on elevating sales. Let d_i denote the proportion of customers in subgroup i who purchased the product. We use a shifted logistic function, as shown in Equation 13, to describe d_i as a function of the duration of the warranty period:

$$d_i = d_{\max}\left(\frac{1}{1+\exp\left(2-\frac{\tau_i}{\gamma}\right)} - \frac{1}{1+\exp(2)}\right). \tag{13}$$

In this function, $0 < d_{\max} < \frac{1+\exp}{\exp(1)}$ and γ is the parameter of the logistic shift function that is met based on the demand; $d_{\max}$ is the parameter that limits d_{I} between [1,0].

τ_i is the warranty period related to subgroup i. According to the statistical distribution of breakdown products, the reliability function will be

$$R_I(t) = e^{-\left(\frac{T-\gamma}{B}\right)^{A}}. \tag{14}$$

By knowing the cost per unit for producing and the number of products sold in each subgroup, we can calculate the total income:

$$n_j = d_j \times q_i. \tag{15}$$

n_I represents the number of products expected to be demanded by each subgroup i, and then the total revenue equals to

$$\rho_I = m \times r_i \times n_I. \tag{16}$$

ρ_I represents the income of each i subgroup. Improved earnings in accordance with the above equation are obtained for each subgroup corresponding to warranty period for the group.

Optimization by cuckoo algorithm

Since in reality the duration of different periods cannot be offered for different customers, by cuckoo optimization algorithm, we try to maximize time length and provide a common optimal time for all subgroup charges in order to issue the variables making up the array. These arrays are the same as chromosomes in genetic algorithms which are called *habitat* here. Prior researches (Rajabioun 2011; Yang and Deb 2009, 2010) constitute theoretical studies of this section. In order to determine the primary residence of the cuckoo, we define the problem of optimization variables related to earnings per subgroup, which represents the primary profits of those subgroups (Rajabioun 2011).

$$\text{habitat} = [\rho_1, \rho_2, ..., \rho_n].$$

Each location with a cuckoo has a special profit that is calculated using the O_P function:

$$\text{Profit} = O_P(\rho_1, \rho_2, ..., \rho_n).$$

To start this algorithm, inherently seeking maximal profits function, a matrix of the habitat with $N_P \times N_\rho$ dimension is developed, where N_P is the number of cuckoo and N_ρ is the variable income for each subgroup. In this nature, each cuckoo lays about 5 to 20 eggs; the numbers are the maximum and minimum numbers of eggs laid at a time. The cuckoo lays eggs in a particular domain and range (distance from home). This distance is equal to ELR:

$$\text{ELR} = W \times \frac{\text{Number of current cuckoo eggs}}{\text{Total number of eggs}} \times (\text{Var high} - \text{Var low}). \tag{17}$$

W is the ratio of the maximum ELR set up to maximize the convergence.

The cuckoo begins to lay eggs in its coverage range in other bird's nests, with each nest having certain benefits. Locations with less profit are excluded. The rest of the chicks grow in the nest by a huge profit, and merely one bird can grow in every nest.

When the birds grow, they go to the most profitable areas (most likely to survive) and lay eggs. To know each ρ belongs to which group, we use clustering technique of *k*-means. Clustering is one of the common techniques for data to categorize groups of similar objects, or more precisely, the partitioning of a data set or a subset of a cluster so that each cluster of data in a series has common features. Most of these similarities are defined based on the criteria. Due to the large number of data, sometimes, managing this widespread amount of data would be impossible. Therefore, this method can facilitate data analysis by categorizing into groups with common characteristics of the entities.

In this study, we used *k*-means clustering method that is an unsupervised learning method known to solve clustering problems. This algorithm is a simple way to cluster a data set in a prespecified number *k* clusters. The main idea is to define the *k* center for each cluster. These centers should be selected very carefully because different centers create different results.

Therefore, the best choice is putting them (centers) at the farthest possible distance from one another. The next step is to assign each pattern to the closest center. When all the points were assigned to the center, the first phase has been completed and an early grouping is done. Here, we need to calculate a new center cluster level to calculate *k* for the clusters of the previous levels. After determining the new center *k*, then we assign the data to the

Table 1 Basic data related to population, average temperature, and failure rate of subgroups

Subgroups (provinces)	Total population	Ratio of total population to subgroup population	Average of annual temperature (°C)	Product failure rate (λ_1)
Tehran	8,791,378	0:240	17.3 ≃ 17	0.0629
Gilan	2,480,874	0:067	16 ≃ 15.5	0.0592
Esfahan	4,879,312	0:133	16	0.0592
Fars	4,596,658	0:125	18	0.0666
Khorasan	5,994,402	0:163	14	0.0518
Mazandaran	30,773,943	0:083	15	0.0555
Khoozestan	4,531,720	0:123	26 ≃ 25.5	0.0962
Yazd	1,074,428	0:029	19	0.0703
Ghazvin	1,201,565	0:032	13	0.0481
Total	36,624,280	100	-	-

Table 2 Income of each group and corresponding optimal warranty period

Subgroups (provinces)	Warranty period	Sales
Tehran	52.48	$4/5{,}025 \times 10^8$
Gilan	14.49	$1/2{,}898 \times 10^8$
Esfahan	14.49	$2/5{,}368 \times 10^8$
Fars	95.47	$2/3{,}194 \times 10^8$
Khorasan	50.46	$3/2{,}137 \times 10^8$
Mazandaran	77.44	$1/6{,}227 \times 10^8$
Khoozestan	44.24	$2/415 \times 10^8$
Yazd	47.41	$5/3{,}423 \times 10^7$
Ghazvin	51.19	$6/5{,}435 \times 10^7$
Total revenue	1.871498×10^9	

Table 3 Total revenue results according to different policies

Kind of population	Warranty period length	Total revenue
Homogeneous population	51	1.1707×10^9
Heterogeneous population with different periods of warranty for each subgroup	-	1.871498×10^9
Heterogeneous population with optimized warranty period length (optimized by cuckoo algorithm)	856/50 ≃ 51	1.9782×10^9
Homogeneous population with average usage intensity	836/49 ≃ 50	1.9360×10^9

appropriate centers again. This process is repeated so many times that *k* of the center does not move.

In every turn, average earnings are calculated for cuckoo groups (clusters) to obtain the optimization of the place, and the group that has the most benefits attracts other cuckoos. The maximum number of cuckoos that live in a moment to kill cuckoos living in inappropriate areas is 200 (Rajabioun 2011).

After moving several times and laying eggs with high convergence, cuckoos go to an optimal point, which has a higher profit than elsewhere, and this is the optimal solution (Rajabioun 2011; Yang and Deb 2009).

Numerical example

Numerical examples of fan failures for an industrial production company have been studied in nine provinces in Iran. The failure rate that was based on Equation 11: $\lambda_0 = 0.037$. Since the intensity of a fan's usage is depended on temperature of each zone, the warmer region has a higher intensity of usage. The mean of annual temperature was taken from the Iran weather organization, and average temperatures of the regions were considered as a usage of the effective failure rate. Each fan is sold at US$103. Table 1 shows the statistics on the population of each subgroup to the total population (Static Center of Iran 2011), the mean annual temperature, and the corresponding risk rate for each group.

Relationship between the demand of fans of the company and duration of warranty period is considered: $d_{max} = 0.75$ and $\gamma = 8$. This number is assigned regarding the problem and the target market; however, the maximum saturation of the market's share is taken at 75%. Table 2 represents the amount of each group's income and corresponding optimal warranty period.

To estimate the parameters of the Weibull distribution, we have different ways like the method of least squares or graphics in this study. Weibull distribution parameters are estimated using the *R* software and values of α = 1/274,262 and β = 44/199,218 were obtained, respectively. Since the shape parameter is greater than one, it can be concluded that the damages are more likely due to frazzle and long length of product usage, which are rare in new products.

The results of the testing data without optimization and with optimization by cuckoo algorithm are shown in Table 3. By optimizing the models obtained by cuckoo optimization algorithm, optimal time was considered at 50.856 months,

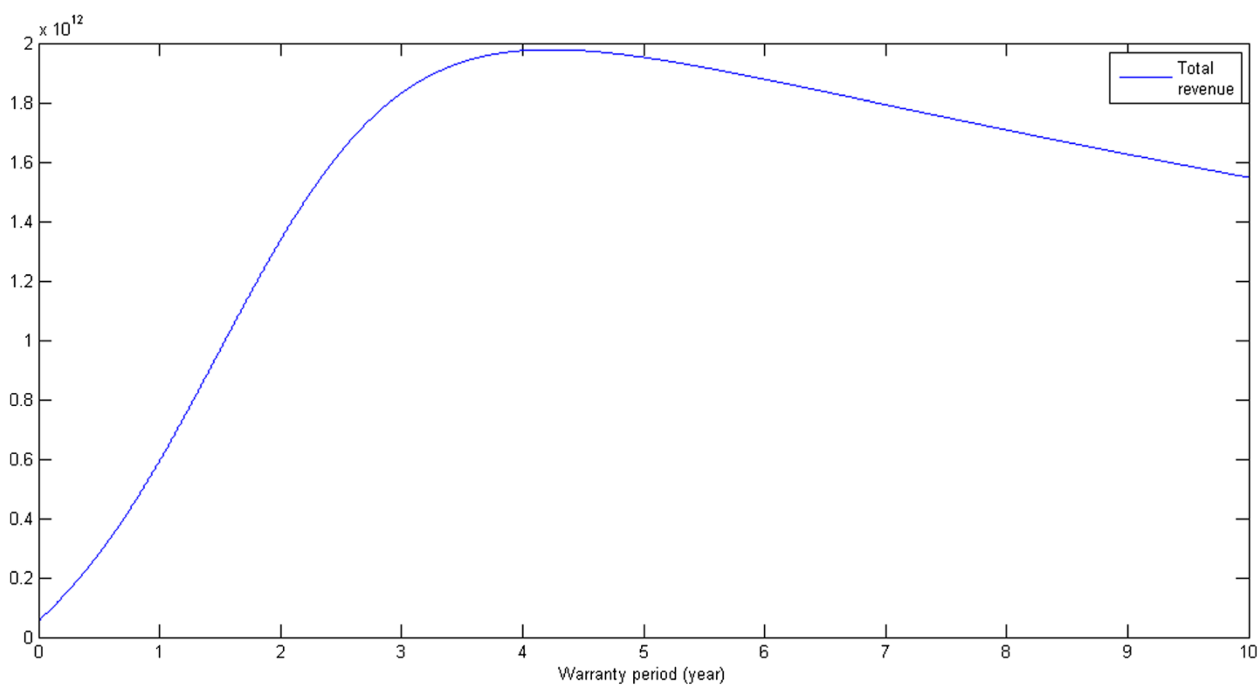

Figure 1 The total revenue corresponding to the optimal warranty period.

the longest period that the revenue is maximized as shown in Figure 1.

Total revenue results according to different policies are presented in Table 3. In the case of a homogeneous population (without the effect of usage intensity), the total income for the period of 51 months has been calculated as equal to FRW policy optimization. Heterogeneous population with different periods is used for each group in the model of St. John and Cassady (2010). Heterogeneous population with the same optimization period of the developed model and optimization point are presented in this study. In the last case, instead of considering the different usage intensity for each group, an average failure rate for all groups was selected to better compare the optimum performance with and without the effect of application intensity.

Conclusions and recommendations for future research

One of the key points in providing an appropriate policy of warranty is to estimate the optimized warranty coverage. This feature must be able to get more customers to buy these products and gain more trust from them. The profit of the company should not be threatened by the cost of repairing or replacing the defective products.

In this paper, FRW policy based on heterogeneous population of customers was developed based on different application intensities with statistical distribution of Weibull. To determine the optimal duration of warranty, optimal time was estimated using the cuckoo optimization algorithm. By comparing the amount of income earned during the cuckoo optimized period, heterogeneous population without optimization, homogeneous population without optimization, and the length of warranty period in numerical data, a significant increase was observed in the amount of the manufacturer's revenue. Furthermore, with the extended warranty period, we expect that costs imposed upon customers decrease and their satisfaction increases which will also bring about social and economic benefits for the producer in the long run.

The theoretical model using actual data of a manufacturing plant in the field of home appliances was tested. Results show a dramatic increase in revenue due to the longer period of optimization; we have reached customers' satisfaction, trust, and indirect benefits of the proposed policy. The cuckoo meta-heuristic algorithm was used for the first time in the field to optimize the warranty that was quite new. We suggest a more frequent and wider use of such algorithms in warranty policies. Using these algorithms, we are able to take into account the dynamic price and inflation that will lend special attraction to the research studies in competitive markets.

This model was discussed in a manufacturing company in the field of home appliances; nonetheless, services can be covered under the same warranty policy, too. In this study, FRW policy is selected as the base policy to optimize. Optimizing other models of policies from previous studies with meta-heuristic algorithms can help industry managers and decision-makers in warranty policies to select and apply the most appropriate method and policy. Some of these problems are currently under investigation by the authors.

Competing interests

The authors declare they have no competing interest

Authors' contributions

EA provide the warranty literature and participated in data calculating. AR creates the mathematic model and has share in collecting raw data, also Case study analyses and the Conclusions and recommendations for future research part do by him. All authors read and approved the final manuscript.

Author details

[1]Kish International campus, University of Tehran, Kish Island 71778-5777, Iran.
[2]Tehran University, Gisha bridge, Jalale ale ahmad blvd, Tehran 6311-14155, Iran.

Received: 3 March 2013 Accepted: 28 August 2013
Published: 04 Dec 2013

References

An DL (1992) Asset management: remedy for addressing the fiscal challenges facing highway infrastructure. Int J Transp Manag 1:41–54
Blischke WR, Murthy DNP (1996) Warranties: concepts and classification. In: Blischke WR, Murthy DNP (eds) Product warranty handbook. Marcel Dekker, New York
Hong-Zhong H, Zhi-Jie L, Yanfeng L, Yu L, Liping H (2008) A warranty cost model with intermittent and heterogeneous usage. Eksploatacja I Niezawodnosc-Maintenance and Reliability 40:9–15
Murthy DNP, Blischke WR (2005) Warranty management and product manufacture. Springer Series in reliability engineering, USA. ISBN-10: 1852339330, chapter 1, 5, 6.
Murthy DNP, Djamaludin I (2002) New product warranty: a literature review. Int J Prod Econ 79(3):231–260
Musa JD, Okumoto K (1983) Software reliability models: concepts, classification, comparisons, and practice. In: Skwirzynski JK (ed) Electronic systems effectiveness and lifecycle costing, NATO ASI Series, F3. Springer-Verlag, Heidelberg, pp 395–424
Rajabioun R (2011) Cuckoo optimization algorithm. Appl Soft Comput J 11:5508–5518
Ross SM (1972) On optimal assembly of systems. Nav Res Logist 19:569–574
Sahin I, Polatoglu H (1996) Manufacturing quality, reliability and preventive maintenance. Prod Oper Manag 5:132–147
Sahin I, Polatoglu H (1998) Probability distributions of cost, revenue and profit over a warranty cycle. Eur J Oper Res 108:170–183
St. John D, Cassady RC (2010) Customized warranty policies for heterogeneous populations. In: 2010 Proceedings - Reliability and Maintainability Symposium (RAMS). San Jose, 25–28 Jan 2010
Static Center of Iran (2011) Population estimates until September of 2011. http://www.amar.org.ir/. Accessed 31 Sep 2012.
Warranty Week (2011) Eighth Annual Warranty Report., Totals and Averages http://www.warrantyweek.com/archive/ww20110401.html. Accessed 01 May 2012.
Yang X-S, Deb S (2009) Cuckoo search via Lévy flights. In: World Congress on Nature & Biologically Inspired Computing, Coimbatore, 9–11 Dec 2009. IEEE, Piscataway, pp p. 210–214
Yang X-S, Deb S (2010) Engineering optimization by cuckoo search. Int J Mathematical Modeling and Numerical Optimization 1(4):330–343

10.1186/2251-712X-9-27
Cite this article as: Roozitalab and Asghari zadeh: Optimizing the warranty period by cuckoo meta-heuristic algorithm in heterogeneous customers' population. *Journal of Industrial Engineering International* 2013, 9:27