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# Stochastic extension of cellular manufacturing systems: a queuing-based analysis

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## Abstract

Clustering parts and machines into part families and machine cells is a major decision in the design of cellular manufacturing systems which is defined as cell formation. This paper presents a non-linear mixed integer programming model to design cellular manufacturing systems which assumes that the arrival rate of parts into cells and machine service rate are stochastic parameters and described by exponential distribution. Uncertain situations may create a queue behind each machine; therefore, we will consider the average waiting time of parts behind each machine in order to have an efficient system. The objective function will minimize summation of idleness cost of machines, sub-contracting cost for exceptional parts, non-utilizing machine cost, and holding cost of parts in the cells. Finally, the linearized model will be solved by the Cplex solver of GAMS, and sensitivity analysis will be performed to illustrate the effectiveness of the parameters.

**Keywords:** Cellular manufacturing system; Stochastic arrival rate and service rate; Average waiting time; Queuing theory

## Introduction

Cellular manufacturing system (CMS) is an application of the group technology concept, which classifies parts with closest features and processes into the part families and assigns machines into the cells, with the goal of increasing production efficiency while decreasing the unit cost. Some advantages of CMS such as simplification of material flow, reduction of transportation and queuing times, reduction of material handling cost and setup times, and the increase of machine utilization and throughput rates are declared in literatures (Muruganandam et al. 2005; Olorunniwo and Udo 2002; Wemmerlov and Hyer 1989). The four major decisions in the implementation of cellular manufacturing systems are the following:

1. Cell formation: grouping parts with the similar processes and features into part families and allocating machines to the cells (Mahdavi et al. 2007; Muruganandam et al. 2005; Yasuda et al. 2005)

2. Group layout: laying out cells and machines within each cell (Mahdavi and Mahadevan 2008; Tavakkoli-Moghaddam et al. 2007; Wu et al. 2007a)
3. Group scheduling: operating and managing the cell operation (Mak and Wang 2002; Tavakkoli-Moghaddam et al. 2010)
4. Resource allocation: assigning resources, such as tools, materials, and human resources, to the cells (Cesani and Steudel (2005; Mahdavi et al. 2010)

Wu et al. (2007b) considered cell formation, group layout, and group scheduling decisions simultaneously in their model, which minimize the makespan. They presented a hierarchical genetic algorithm to solve it. Logendran (1993) developed a mathematical programming model to minimize part inter-cell and intra-cell movements and proposed a heuristic algorithm to solve it. Chen (1998) proposed an integer programming model to minimize material handling and machine cost and reconfiguration cost to design a sustainable cellular manufacturing system in a dynamic environment. Most of the developed models in cellular manufacturing systems are cost-based, but there are some models in which machine reliability is considered simultaneously with different cost types. A multi-objective mixed

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integer programming model was presented by Das et al. (2006) that minimizes variable cost of machine and penalty cost of non-utilizing machine as well as inter-cell material handling cost, and maximizes system reliability with minimizing failure rate. Machine breakdown cost (Jabal-Ameli and Arkat 2007) and inverse of the reliability of the system (Das et al. 2007) are two more objective components in order to maximize machine reliability, which will develop cell performance.

Cellular manufacturing problems can be under static or dynamic conditions. In static conditions, cell formation is done for a single-period planning horizon, where product mix and demand rate are constant. However, in dynamic conditions, the planning horizon is considered as a multi-period planning horizon where the product mix and demand rate differ from one period to another. In order to reach the best efficiency, there will be different cell formations for each period. Some recent

**Table 1 Summarized cellular manufacturing system review**

| Publication                           | Objective   | Problem definition | Solution method | Output                        |
|---------------------------------------|---|--------------------|-----------------|-------------------------------|
| Sarker and Li (1997)                  | Min INTER + OC  | SO, MIP            | B&B             | CF, PR                        |
| Wicks and Reasor (1999)               | Min INTER + RCC + CI  | SO, DP, MPP, MIP   | GA              | CF, MD                        |
| Chen (1998)                           | Min INTER + OC + MHC + RCC                                      | SO, DP, MPP, IP    | DBH             | CF, RCP                       |
| Chen (2001)                           | Min INTER + OC + IC + SC  | SO, MINLP          | SMH             | CF, IL, PL                    |
| Baykasoglu et al. (2001)              | Min DS + CLV + ECR  | MO, NLP            | SA              | CF, PRP                       |
| Mak and Wang (2002)                   | Min TDT   | SO, MINLP          | GA              | CF, PS                        |
| Muruganandam et al. (2005)            | Min CLV + MM  | CO, NLP            | MA              | CF                            |
| Tavakkoli-Moghaddam et al. (2005)     | Min RCC + MAC + OC + INTER                                      | SO, DP, NLP        | GA, SA, TS      | CF, RCP                       |
| Das et al. (2006)                     | Min 1.FRM<br>2.VCM + INTER + NUC                                | MO, MIP            | SA              | CF, PR                        |
| Chan et al. (2006)                    | Min VO + EE, TDT  | SO, NLP, QAP, 2SP  | GA              | CF, MSE                       |
| Saidi-Mehrabad and Safaei (2007)      | Min MAC + INTER + OC + RCC                                      | SO, DP, MPP, NLP   | NA              | CF, CS, PR, RCP, MD           |
| Jabal-Ameli and Arkat (2007)          | Min INTER + MBC   | SO, IP             | B&B             | CF, PR                        |
| Wu et al. (2007a)                     | Min 1.MM, 2.EE  | MO, IP             | HGA             | CF, GL                        |
| Wu et al. (2007b)                     | Min CM  | SO, NLP            | HGA             | CF, GL, PS                    |
| Das et al. (2007)                     | Min 1.ISR<br>2.VCM + INTER + NUC                                | MO, NLP            | HIA             | CF, OPMI                      |
| Mahdavi et al. (2007)                 | Min EE + VO   | SO, NLP            | B&B             | CF                            |
| Schaller (2007)                       | Min PC + MAC + RCC  | SO, DP, IP         | CBP, TS         | CF, RCP                       |
| Safaei et al. (2008)                  | Min INTER + INTRA + CCM + VCM + RCC                             | SO, DP, MPP, MIP   | MFA-SA          | CF, RCP                       |
| Bajestani et al. (2009)               | Min 1.CLV<br>2.INTER + RCC + PUC                                | MO, DP, MINLP      | MOSS            | CF, RCP                       |
| Kioon et al. (2009)                   | Min RCC + PM + OC + SCC + HC + PC + INTER + INTRA               | SO, DP, MINLP      | B&C             | CF, PP, PR, RCP, IL           |
| Safaei and Tavakkoli-Moghaddam (2009) | Min CCM + VCM + IT + INTER + INTRA + RCC + SCC                  | SO, DP, MINLP      | B&B             | CF, RCP, BL, IL               |
| Saidi-Mehrabad and Ghezavati (2009)   | Min ICM + SCC + NUC   | SO, STP, MINLP     | B&B             | CF                            |
| Mahdavi et al. (2010)                 | Min VO + EE   | SO, NLP            | GA              | CF                            |
| (Tavakkoli-Moghaddam et al. 2010)     | Min INTRAT + TT + CSC + CM                                      | MC, MINLP          | SS              | CF, PS                        |
| Mahdavi et al. (2010)                 | Min HC + BC + INTER<br>MC + RCC + SAC + HIC + FIC               | SO, DP, MPP, NLP   | B&B             | CF, RCP, WA, IL<br>PL, BL, MD |
| Rafiee et al. (2011)                  | Min INTER + INTRA<br>OC + PUC + RCC + PRS + SCC + HC + CRC + PM | SO, DP, MIP        | PSO             | CF, RCP, PR, PP, IL, SBQ      |
| Ariaifar et al. (2011)                | Min INTER + INTRA   | SO, STP, MINLP     | B&B             | GL                            |

researches are under dynamic conditions (Kioon et al. 2009; Mahdavi et al. 2010; Schaller 2007).

Inputs of classical cellular manufacturing system problems are certain, but in real problems, some input parameters such as costs, demands, processing times, and setup times are uncertain so that this uncertainty can affect on the results. Some approaches such as stochastic programming, queuing theory, and robust optimization can be applied for uncertain models (Saidi-Mehrabad and Ghezavati 2009). Ariaifar et al. (2011) proposed a mathematical model for layout problem in cellular manufacturing systems that the demand of each product was described by uniform distribution. The objective function of their model minimizes total cost of inter-cell and intra-cell material handling, concurrently. It was solved by the Lingo software. Saidi-Mehrabad and Ghezavati (2009) presented a stochastic CMS problem applying queuing theory approach. They assumed parts as customers and machines as servers, in which the arrival rate of parts and service rate of machines were identified by the exponential distribution. Their objective function components are the following: non-utilization cost, machine idleness cost, and sub-contracting cost. They computed the utilization factor for each machine which shows the probability that each machine is busy.

Table 1 illustrates the summarization of more reviewed literatures consisting objectives, model definitions, solution methods, and outputs. This helps to compare our work with previous researches. Table 2 describes the abbreviations used in Table 1.

In this paper, we will develop a CMS problem as a queue system considering the average waiting time of parts behind each machine and holding cost of parts in cells. The goal of this model is to minimize summation of four cost types: (1) the idleness costs for machines, (2) total cost of sub-contracting, (3) non-utilization cost of machines in cells, and (4) holding cost of parts in cells. The rest of the paper is organized as follows: The proposed model is described in the 'Mathematical modeling' section. In the 'Experimental results' section, the experimental results and sensitivity analysis are presented. The last section is the 'Conclusions and future directions.'

## Mathematical modeling

### Problem description

In this section, a stochastic cellular manufacturing system problem will be formulated as a queue system, which considers parts and machines as customers and servers with the arrival and service rate of  $\lambda$  and  $\mu$ , respectively. Also, we consider that at each time, only one part can be processed by a machine; thus, when a machine is processing a part, the others should wait, and a queue will be created behind the machine. The

population of this queue as a new part arrives (birth) can be increased, or it can be decreased (death) by service completion.

In a steady system, to avoid infinite growth of queue, the service rate must be greater than the arrival rate, so the utilization factor (probability of being busy) of each machine ( $\rho = \lambda / \mu$ ) will be less than 1. Also, as each machine processes different parts with different arrival rates, according to this property, the minimum of some independent exponential random variables with the arrival rate of  $\lambda_1, \lambda_2, \dots, \lambda_m$  is also exponential with the arrival rate of  $\lambda = \sum_{i=1}^m \lambda_i$ . Hence, the utilization factor is  $\rho = \sum \lambda_i / \mu$ .

Our queue system is formulated as  $M / M / 1$ , where the arrival and processing time of parts are uncertain and described by exponential distribution, and as it was mentioned earlier, each machine can process at most one part at a time. In this problem, the decision maker needs to allocate parts and machines to cells in order to minimize objective function value. In the previous work of Saidi-Mehrabad and Ghezavati (2009), only the impact of utilization factor in designing CMS is considered, but our study shows the impact of utilization factor and maximum waiting time of parts simultaneously. Due to the uncertainty of arrival and service time of parts, the time that each part spends in the cell is uncertain, and as the time passes, the parts will be broken. Thus, in order to avoid long waiting time, a chance constraint is considered to show that the probability that the average waiting time of a part behind each machine exceeds the critical time is less than the service level ( $\alpha$ ). By knowing that, this probability affects on the utilization factor.

## Notation

### Indexes

The following are the indexes:

- $i$  Part index,  $i = 1, \dots, p$
- $j$  Machine index,  $j = 1, \dots, m$
- $k$  Cell index,  $k = 1, \dots, c$

- $$a_{ij} = \begin{cases} 1 & \text{if machine } j \text{ processes part } i \\ 0 & \text{otherwise} \end{cases}$$

- $C_i$  Sub-contracting cost per unit for part  $i$
- $u_j$  Idleness cost of machine  $j$
- $M_m$  Maximum number of machines permitted in a cell
- $\alpha$  Maximum allowed probability that the waiting time behind each machine can be more than the critical time
- $t_{qj}$  Average time parts spend behind machine  $j$
- $t$  Critical waiting time
- $np_i$  Total number of part  $i$

**Table 2 Abbreviations used in Table 1**

| Objective function                | Abbreviation | Problem definition               | Abbreviation | Solution and outputs                         | Abbreviation |
|-----------------------------------|--------------|----------------------------------|--------------|--|--------------|
| Inter-cell material handling cost | INTER        | Single objective                 | SO           | Branch and bound                             | B&B          |
| Intra-cell material handling cost | INTRA        | Multi-objective                  | MO           | Branch and cut                               | B&C          |
| Machine operating cost            | OC           | Combined objective               | CO           | Genetic algorithm                            | GA           |
| Material moves                    | MM           | Multi-criteria                   | MC           | Decomposition base heuristic                 | DBH          |
| Backorder cost                    | BC           | Multi-period planning            | MPP          | Silver meal heuristic                        | SMH          |
| Production cost                   | PC           | Stochastic problem               | STP          | Tabu search                                  | TS           |
| Inventory cost                    | IC           | Dynamic programming              | DP           | Hierarchical genetic algorithm               | AGA          |
| Reconfiguration cost              | RCC          | Integer program                  | IP           | Mean field annealing and simulated annealing | MFA-SA       |
| Capital investment                | CI           | Mixed integer program            | MIP          | Multi-objective scatter search               | MOSS         |
| Machine holding cost              | MHC          | Non-linear program               | NLP          | Hierarchical approach                        | HIA          |
| System setup cost                 | SC           | Mixed integer non-linear program | MINLP        | CB procedure                                 | CBP          |
| Dissimilarity between parts       | DS           | Quadratic assignment problem     | QAP          | Particle swarm optimization                  | PSO          |
| Cell load variation               | CLV          | Two-stage scheduling problem     | 2SP          | Scatter search                               | SS           |
| Extra capacity requirement        | ECR          |                                  |              | Neural approach                              | NA           |
| Total distance traveled           | TDT          |                                  |              | Cell formation                               | CF           |
| Cell setup cost                   | CSC          |                                  |              | Inventory level                              | IL           |
| Holding cost                      | HC           |                                  |              | Production level                             | PL           |
| Machine cost                      | MC           |                                  |              | Backorder level                              | BL           |
| Machine amortization cost         | MAC          |                                  |              | Reconfiguration plan                         | RCP          |
| Machine breakdown cost            | MBC          |                                  |              | Machine sequence                             | MSE          |
| Failure rate of machine           | FRM          |                                  |              | Cell size                                    | CS           |
| Machine variable cost             | VCM          |                                  |              | Group layout                                 | GL           |
| Machine constant cost             | CCM          |                                  |              | Process route                                | PR           |
| Machine non-utilization cost      | NUC          |                                  |              | Machine duplication                          | MD           |
| Total number of voids             | VO           |                                  |              | Production plan                              | PP           |
| Total exceptional elements        | EE           |                                  |              | Production scheduling                        | PS           |
| Completion time                   | CM           |                                  |              | Processing part requirement                  | PRP          |
| Inverse of system reliability     | ISR          |                                  |              | Optimal preventive maintenance interval      | OPMI         |
| Purchase cost of machine          | PUC          |                                  |              | Subcontracted quantity                       | SBQ          |
| Preventive maintenance cost       | PM           |                                  |              | Worker assignment                            | WA           |
| Sub-contracting cost              | SCC          |                                  |              |  |              |
| Inventory transportation          | IT           |                                  |              |  |              |
| Intra-cell move time              | INTRAT       |                                  |              |  |              |
| Tardiness time                    | TT           |                                  |              |  |              |
| Salary cost                       | SAC          |                                  |              |  |              |
| Firing cost                       | FIC          |                                  |              |  |              |

**Table 2 Abbreviations used in Table 1 (Continued)**

|                           |     |
|---------------------------|-----|
| Hiring cost               | HIC |
| Process routes setup cost | PRS |
| Corrective repair cost    | CRC |
| Machine idleness cost     | ICM |

- $C_u$  Maximum number of cells
- $\lambda_i$  Mean arrival rate for part  $i$
- $\mu_j$  Mean service rate for machine  $j$
- $U_{ij}$  Cost of part  $i$  not utilizing machine  $j$
- $h_i$  Holding cost per unit for part  $i$

**Decision variables**

The following are the decision variables:

- $X_{ik} = \begin{cases} 1 & \text{if part } i \text{ is assigned to cell } k \\ 0 & \text{otherwise} \end{cases}$
- $Y_{jk} = \begin{cases} 1 & \text{if machine } j \text{ is assigned to cell } k \\ 0 & \text{otherwise} \end{cases}$
- $\rho_j$  Utilization factor for machine  $j$  (the value of  $\rho$  indicates the probability in which machine  $j$  is busy)

**Mathematical model**

In this section, details of the mathematical formulation which we are interested will be described. For this purpose, the following formulations are presented:

$$\begin{aligned} \text{Min } Z = & \sum_j (1-\rho_j) u_j + \sum_k \sum_j \sum_i C_i a_{ij} X_{ik} (1-Y_{jk}) \\ & + \sum_k \sum_j \sum_i U_{ij} X_{ik} Y_{jk} (1-a_{ij}) \\ & + \sum_k \sum_j \sum_i \frac{h_i a_{ij} X_{ik} Y_{jk}}{np_i} \end{aligned}$$

subject to

$$\sum_k X_{ik} = 1 \quad i = 1, 2, \dots, p, \tag{2}$$

$$\sum_k Y_{jk} = 1 \quad j = 1, 2, \dots, m, \tag{3}$$

$$\rho_j - \sum_k \frac{\sum_i \lambda_i a_{ij} X_{ik} Y_{jk}}{\mu_j} = 0 \quad j = 1, \dots, m, \tag{4}$$

$$\sum_j Y_{jk} \leq M_m \quad k = 1, 2, \dots, c, \tag{5}$$

$$P(t_{qj} > t) \leq \alpha, \tag{6}$$

$$\rho_j \leq 1 \quad j = 1, 2, \dots, m, \tag{7}$$

$$X_{ik}, Y_{jk} \in \{0, 1\}, \rho_j \geq 0. \tag{8}$$

Equation 1 indicates the objective function which can compute the total idleness cost for machines in cells,

sub-contracting cost, resource underutilization cost, and holding cost of parts in cells. Set constraint (2) restricts that each part is allocated to only one cell. Set constraint (3) ensures that each machine is allocated to only one cell. Set constraint (4) computes the utilization factor. Set constraint (5) guarantees that the number of machines in each cell will not exceed the maximum number. Equation (6) is a chance constraint and ensures that the probability that the average waiting time of parts behind each machine exceeds the critical time is less than the service level ( $\alpha$ ). Set constraint (7) says that the proportion of time that the machine is processing a part must be less than or equal to 1. Set constraint (8) specifies binary and non-negative variables.

**Linearization of the model**

As the objective function has non-linear terms, we can change it to a mixed integer linear programming. For this purpose, we replace a new binary variable  $Z_{ijk}$  instead of the multiple of  $X_{ik}$  and  $Y_{jk}$ . We reformulate the model, and the three auxiliary constraints (10), (11), and (12) are added to the model to guarantee the correctness of the replacement. Constraint (6) is equal to  $e^{-\mu_j(1-\rho_j)t} \leq \alpha$  and contains a non-linear equation; therefore, Equation 14 denotes the linear form of this constraint.

$$\begin{aligned} \text{Min } Z = & \sum_j (1-\rho_j) u_j + \sum_k \sum_j \sum_i C_i a_{ij} X_{ik} \\ & - \sum_k \sum_j \sum_i C_i a_{ij} Z_{ijk} \\ & + \sum_k \sum_j \sum_i U_{ij} (1-a_{ij}) Z_{ijk} \\ & + \sum_k \sum_j \sum_i \frac{a_{ij} Z_{ijk}}{np_i}. \end{aligned} \tag{9}$$

Subject to constraints (2), (3), (5), (7), and (8),

$$Z_{ijk} \leq X_{ik} \forall i, j, k, \tag{10}$$

$$Z_{ijk} \leq Y_{jk} \forall i, j, k, \tag{11}$$

$$X_{ik} + Y_{jk} - Z_{ijk} \leq 1 \forall i, j, k. \tag{12}$$

Constraints (4) and (6) are changed as follows:

$$\rho_j - \sum_k \frac{\sum_i \lambda_i a_{ij} Z_{ijk}}{\mu_j} = 0 \quad \forall j \tag{13}$$

$$-\mu_j (1-\rho_j) t \leq \ln \alpha \forall j. \tag{14}$$

**Table 3 Effectiveness of queuing approach in a CMS problem**

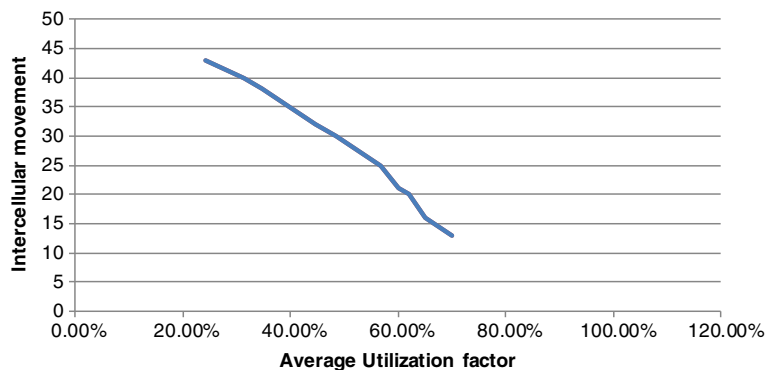
| Problem information |  |  |                    |                                |                                   |
|---------------------|--|--|--------------------|--------------------------------|-----------------------------------|
| Problem number      | Number of parts × number of machines × number of cells | Maximum number of machine allowed in each cell | Idleness rate cost | Average utilization factor (%) | Number of inter-cellular movement |
| P1                  | 30 × 10 × 3  | 4  | 300                | 24.27                          | 43                                |
| P2                  |  |  | 460                | 31.27                          | 40                                |
| P3                  |  |  | 654                | 34.66                          | 38                                |
| P4                  |  |  | 850                | 44.52                          | 32                                |
| P5                  |  |  | 905                | 48.36                          | 30                                |
| P6                  |  |  | 1,025              | 56.83                          | 25                                |
| P7                  |  |  | 1,350              | 59.90                          | 21                                |
| P8                  |  |  | 1,600              | 62.05                          | 20                                |
| P9                  |  |  | 2,105              | 65.12                          | 16                                |
| P10                 |  |  | 2,678              | 69.97                          | 13                                |
| P11                 | 38 × 10 × 3  | 4  | 300                | 46.91                          | 52                                |
| P12                 |  |  | 460                | 47.56                          | 51                                |
| P13                 |  |  | 654                | 48.55                          | 50                                |
| P14                 |  |  | 850                | 48.64                          | 50                                |
| P15                 |  |  | 905                | 48.71                          | 49                                |
| P16                 |  |  | 1,025              | 48.93                          | 49                                |
| P17                 |  |  | 1,350              | 49.63                          | 49                                |
| P18                 |  |  | 1,600              | 49.80                          | 48                                |
| P19                 |  |  | 2,105              | 49.96                          | 48                                |
| P20                 |  |  | 2,678              | 50.25                          | 48                                |

**Experimental results**

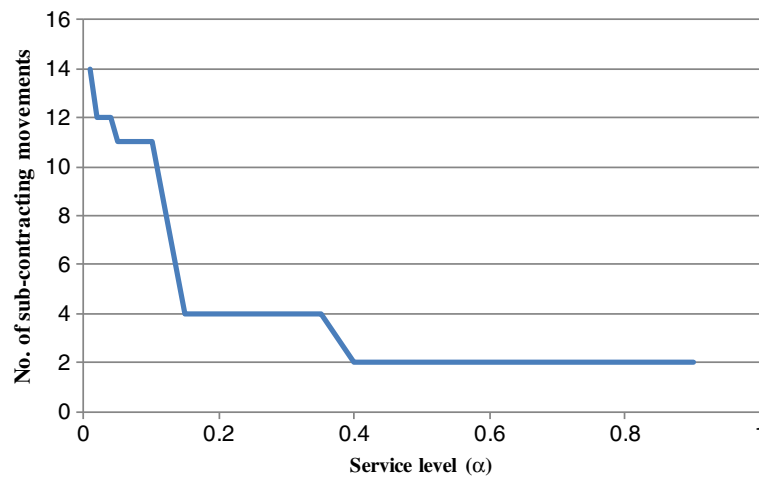
Consider a manufacturing system consisting of ten machines to process parts, wherein the decision maker should allocate machines and parts to three cells. Also, the maximum number of machines permitted to be located is four. In this section, we present some numerical examples which have been generated randomly, to illustrate the effect of changing the main parameters ( $\alpha$ ,  $t$ ,  $u_j$ ), on the number of sub-contracting movements and utilization factors. The proposed mixed integer model

was performed by GAMS and Cplex solver and was run on a processor Intel Core 2 Duo CPU running at 2 GHz with 2-GB RAM.

In Table 3, the results associated to solve two sets of examples for ten times for each is shown, where only idleness cost is not fix, and the impact of its changes on the average utilization factor and the number of sub-contracting movements are illustrated. As the utilization factor of the machines is directly related to the idleness cost, it can be found that the more the average of



**Figure 1 Relationship curve between average utilization factor and number of sub-contracting movements of set problem 1.**



**Figure 2** Relationship curve between service level and number of sub-contracting movements.

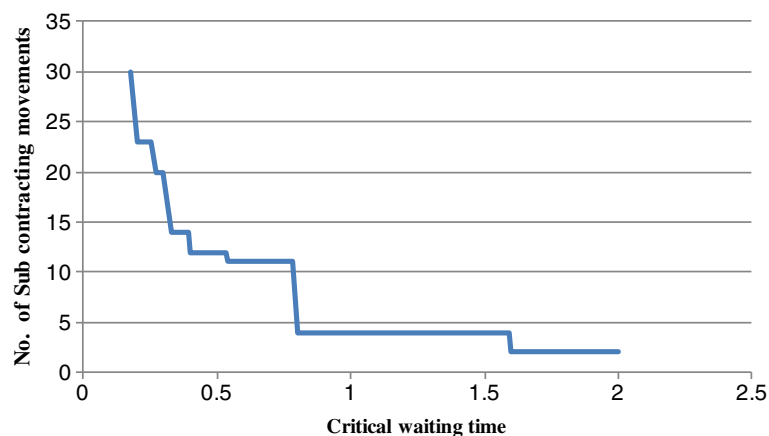
idleness cost leads to the better design of cellular manufacturing system. It means more costs lead to a higher utilization factor. Also, more utilization factor may lead to less number of sub-contracting movements. Results of both sets show the same changes.

Figure 1 indicates the relation between the average utilization factor and number of sub-contracting movements of set problem 1. It can be found that by increasing idleness cost, the average utilization factor will be increased, too. As the term  $\sum(1 - \rho_j)u_j$  indicates, the direct relation between idleness cost and utilization factor is established. Therefore, in order to minimize this term, by increasing idleness cost, idleness rate of machine must be decreased. This means that the probability that each machine is busy increases. Therefore, the total number of sub-contracting movements must be decreased in order to decrease idleness of each machine. The less total number of sub-contracting movements

makes the queue system be more populated. Therefore, the total objective function value will be minimized.

The effect of service level's changes (the maximum allowed probability that the waiting time behind each machine can be more than the critical time) on the sub-contracting movements is shown in Figure 2. This figure illustrates that for a fixed critical waiting time, if the service level ( $\alpha$ ) increases, the upper bound of utilization factor ( $\rho_j$ ) will increase, where this growth may cause decreasing in the total number of sub-contracting movements.

Figure 3 demonstrates the relation between changes of critical waiting time and number of sub-contracting movements. If we assume all the parameters to be fixed except the service level ( $\alpha$ ), increasing the critical waiting time may lead to the reduction of the number of sub-contracting movements, which is due to the increase of the upper bound of the utilization factor.



**Figure 3** Relationship between critical waiting time and number of sub-contracting movements.



### Conclusions and future directions

We have developed a stochastic CMS model that considers the arrival rate of parts into cells and machine service rate as uncertain parameters. The proposed non-linear mixed integer programming model was linearized using auxiliary variables. Then, the linearized model was solved using the Cplex solver of GAMS. As the CMS problem is NP-hard, by increasing the size of the problem, GAMS stops solving it; due to the increase of computational time, the branch-and-cut algorithm is unable to give good solutions. Therefore, it is necessary to present a heuristic or meta-heuristic approach to solve this model for large-scale problems. Also, the following directions can be applied for further considerations:

1. Developing the proposed model under new stochastic parameters such as capacities, lead times, and machine failures.
2. Analyzing the defined problem under scenario-based planning approach and robust optimization theory.
3. Incorporating our objectives with productivity and production planning aspects in uncertain situations.

### Competing interests

The authors declare that they have no competing interests.

### Authors' contributions

VG carried out the modeling, drafted the manuscript, involved in revising the manuscript, and gave final approval of the version to be published. FF was involved in performing the model and analyzing the results, participated in the sequence alignment, and helped draft the manuscript. AZ carried out the literature study and participated in the design of the study and the sequence alignment. All authors read and approved the final manuscript.

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