

Joint pricing and production management: a geometric programming approach with consideration of cubic production cost function

Seyed Jafar Sadjadi · Aghil Hamidi Hesarsorkh · Mehdi Mohammadi · Ali Bonyadi Naeini

Received: 2 March 2014 / Accepted: 3 July 2014 / Published online: 2 August 2014
© The Author(s) 2014. This article is published with open access at Springerlink.com

Abstract Coordination and harmony between different departments of a company can be an important factor in achieving competitive advantage if the company corrects alignment between strategies of different departments. This paper presents an integrated decision model based on recent advances of geometric programming technique. The demand of a product considers as a power function of factors such as product's price, marketing expenditures, and consumer service expenditures. Furthermore, production cost considers as a cubic power function of outputs. The model will be solved by recent advances in convex optimization tools. Finally, the solution procedure is illustrated by numerical example.

Keywords Geometric programming · Production management · Cubic cost production function · Jointness

Introduction

In today's world, manufacturing organizations and competitive intensity have been largely increased. In such a situation, the manufacturer can be a formidable competitive weapon if, incorporates into different departments decisions. In particular, it has proposed that alliance between marketing and operation functions can makes the company more flexible in facing environmental changes (Shapiro 1977; Weir et al. 2000; Kahn and Mentzer 1998). This notwithstanding, much of the body of literature about this issue shows, marketing leads tend to focus outwardly on consumer and advertising issue and they may shy away from technical issue. On the other hand, manufacturing leads tend to be inwardly focused, concentrating on efficiencies, flexibility and capacity issues (Swink and Song 2007; Maltz and Kohli 2000). Mismatch between these two functions leads to company failure to achieve the overall goals such as competitive advantage achievement and sustainable profit (Crittenden and Crittenden 1995; Douglas and Strutton 2009; Ho and Tang 2004; Kahn and Mentzer 1998). To understand the represented reality, see Fig. 1. Since, a number of researchers and practitioners from diver's disciplines accepted and have championed that inter-functional coordination are crucial antecedents to business performance (Kahn and Mentzer 1998; O'Leary-Kelly and Flores 2002; Swink and Song 2007; Gupta et al. 1991; Konijnendijk 1994; Bahri and Tarokh 2012). Nevertheless, there is still lack of formal procedures outlining how functional decisions should be integrated. This paper deals with general problems of optimal decisions regarding manufacturing and marketing decisions integration. Therefore, the objective of this paper is to propose an integrated optimal model that wants to determine jointly and simultaneously optimal lot-sizing, pricing, and

S. J. Sadjadi
Department of Industrial Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran
e-mail: sjsadjadi@iust.ac.ir

A. Hamidi Hesarsorkh (✉) · A. Bonyadi Naeini
Department of Progress Engineering, Iran University of Science and Technology, Narmak, Tehran, Iran
e-mail: Hamidi1001@ymail.com

A. Bonyadi Naeini
e-mail: Bonyadi@iust.ac.ir

M. Mohammadi
Department of New Sciences and Technologies, University of Tehran, Amir Abad, Tehran, Iran
e-mail: memohammadi@ut.ac.ir

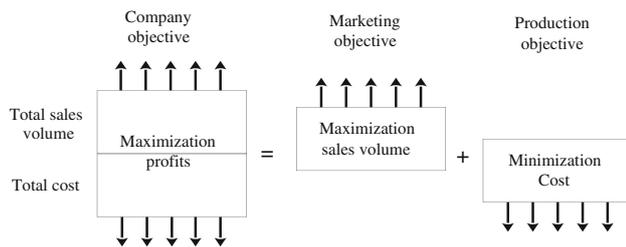


Fig. 1 Conflict between departments' objective

marketing decisions. In this context, due to the relationship between different parameters and variable decisions, nonlinear relationship can well represent reality.

Based on the literature in this area, the GP method is an excellent method to solve a typical algebraic nonlinear optimization (Duffin et al. 1967; Boyd et al. 2007). One of the notable properties of GP is that new solution methods can solve even large-scale GPs extremely efficiently and reliably (Boyd et al. 2007).

The theory of GP was first developed by Duffin et al. (1967). Later GP has undergone rapid development (Bazaraa and Shetty 1976; Zener 1971; Beightler and Phillips 1976). Subsequently, GP has been studied by a number of researchers and practitioners in various fields (Nazemi and Sharifi 2012). Some of the studies are in production planning, marketing and inventory fields. Cheng (1991) investigated an inventory decision problem with sell-dependent unit production cost and imperfect production processes. Lee and Kim (1993) investigated the effects of collaboration between production and marketing departments for a short-range planning horizon in a profit maximizing firm. Lee (1993) applied GP technique with both no quantity discounts and continuous quantity discounts cases to determine optimal price and order quantity for a retailer. Hariri et al. (1995) used GP technique to solve an inventory model with multiple items that having price discount. Lee et al. (1996) developed a profit-maximization model that considered process reliability improvement is an important managerial concern. Kim and Lee (1998) investigated the behavior of the optimal pricing, lot-sizing and economic order quantity for the joint pricing and lot-sizing model with both fixed and variable labor capacity. Chen (2000) used GP technique to determine the quality level, the selling quantity and the purchasing price of a product for the intermediate firms. Jung and Klein (2001) investigated two economic order quantity base in inventory model via GP technique. The very notable thing in this study is deep sensitivity analysis which has been conducted on the models. Abuo-El-Ata et al. (2003) described a probabilistic multi-item with varying order cost under two restrictions and solved by GP technique. Sadjadi et al.

(2005) proposed an integrated model with the consideration of production, marketing, setup and inventory cost items. Jung and Klein (2005) compared the cost minimization model to the profit-maximization model and investigated the difference between the optimal order quantities via GP technique. Liu (2006) used GP technique to determine long-run and short-run maximum profit, the highlighted point of this paper is demand and variable costs per unit of production as a range are considered. Mandal and Roy (2006) investigated a deteriorating multi-item inventory model with limited storage space. Jung and Klein (2006) compared three inventory models under various power functions of cost. Leung (2007) proposed an economic production quantity model with consideration of flexible and imperfect production process. Islam (2008) formulated a multi-objective marketing planning inventory model under the limitations of space capacity and the total allowable shortage cost constraints. Fathian et al. (2009) investigated a pricing model with consideration of the service quality for electronic goods via GP technique. Ghazi Nezami et al. (2009) presented a comprehensive model of joint marketing and production decisions. They considered some cost functions such as market share loss. Sadjadi et al. (2012) investigated an integrated pricing, lot-sizing and marketing planning model in which optimal levels for product's quality along with flexibility and reliability of the production process also need to be determined. Kotb and Fergany (2011) considered a multi-item inventory model with decreasing holding cost Ghosh and Roy (2013) used GP technique to reformulate a goal programming problem. A review of article on production planning and marketing fields that applied GP is summarized in Table 1.

Most of the GP applications above are posynomial GP with zero or a few degrees of difficulty. The degree of difficulty is the difference between the number of dual variables and the number of independent linear equations; and the greater the degrees of difficulty, the more difficult would be the solution (Creese 2010). GP requires that the expressions used are posynomials (Creese 2010; Boyd et al. 2007). In this study, an integrated model with the consideration of demand, cost, marketing and services given to customer is proposed, where total variable cost is a cubic power function of output. In fact, the current analysis goes beyond the existing literature, so that first, it introduces a significant innovation to the field by considering a new form of cost function in the context of production management. Also, we have developed a model in this paper to fill this gap in the literature by using a novel methodology in the environment of GP to get optimal solutions for the signomial models based on a transformation of these models to standard posynomial GP by using the concepts behind the relations between geometric

Table 1 Literature review summary on application of geometric programming

Author (s)	DD	DV	Power functional relations					Reliability
			Production (purchase) cost		Inventory holding cost		Demand	
			Function	Fixe	Function	Fixe	Function	
Cheng (1991)	0	D, Q, r	$C = BD^{-\lambda}r^{\kappa}$		*		*	*
Lee (1993)	1	p, Q	$C = BQ^{-\sigma}$			$h = i_h C$	$D = kp^{-\alpha}$	
Lee and Kim (1993)	0	p, M, Q	$C = BD^{-\lambda}$			$h = i_h C$	$D = kp^{-\alpha}M^{\beta}$	*
Hariri et al. (1995)	2N-1	Q		*		*		*
Lee et al. (1996)	0	p, Q, r	$C = BD^{-\lambda}$			*	$D = kp^{-\alpha}$	*
	3							
Kim and Lee (1998)	2	$p, Q, K_{r(x)}$		*		$h = i_h C$	$D = kp^{-\alpha}$	
	4							
Chen (2000)	1	p, Q, r	$P = Br^{-\theta}p^{\alpha}$			*		*
Jung and Klein (2001)	0	Q	$C = BD^{-\lambda}$			$h = i_h C$		*
	1	Q, D						*
Abuo-El-Ata et al. (2003)	4N-1	N_i		*		*	random	
Sadjadi et al. (2005)	1	p, M, Q	$C = BQ^{-\sigma}$			$h = i_h C$	$D = kp^{-\alpha}M^{\beta}$	
Jung and Klein (2005)	0	Q	$C = BD^{-\lambda}$			$h = i_h C$		*
	1	Q	$C = BQ^{-\sigma}$			$h = i_h C$		*
	1	Q	$C = BQ^{-\sigma}D^{-\lambda}$			$h = i_h C$		*
Liu (2006)	1	p, Q	$C = BQ^{-\sigma}$			*	$D = kp^{-\alpha}$	
Mandal and Roy (2006)	3N-1	T_i		Fuzzy		Hybrid		*
	4N-1	T_i		Fuzzy		Hybrid		*
Jung and Klein (2006)	1	p, Q	$C = BD^{-\lambda}$			*		*
	1	p, Q				*		*
	1	p, Q	$C = BQ^{-\sigma}D^{-\lambda}$			*		*
Leung (2007)	0	C_1, Q, r		*		*		*
Islam (2008)	1	C_2, M, Q	$C = BD^{-\lambda}$			$h = i_h C$	$D = kM^{\beta}$	
Fathian et al. (2009)	0	p, M, S	$C = BD^{-\lambda}$			*	$D = kp^{-\alpha}M^{\beta}S^{\tau}$	
Ghazi Nezami (2009)	4	p, M, Q	$C = BQ^{-\sigma}$			$h = i_h C$	$D = kp^{-\alpha}M^{\beta}$	
Sadjadi et al. (2012)	6	p, Q, q, M, C_1, r	$C = BQ^{-\sigma}q^{\theta}r^{\beta}$			$h = i_h C$	$D = kp^{-\alpha} \prod_{i=1}^I M^{\beta} q^{\theta}$	*
Kotb and Fergany (2011)	3N-1	D, L, Q	$C = BD^{-\lambda}$					*
Ghosh and Roy (2013)	0	P, M, Q, C_1	$C = BQ^{-\sigma}$			$h = i_h C$	$D = kp^{-\alpha}M^{\beta}$	

DD degree of difficulty, DV decision of variables

and arithmetic means, which allows us to solve problem with high degree of difficulty (Ghazi Nezami et al. 2009; Duffin et al. 1967). And also unlike the conventional approach, the standard GP model is solved by recent advances in optimization tools (Boyd and Michael 2009). The reminder of this paper is organized as follows. In the next section, the necessary notations, assumptions and the model formulation are presented. The model formulation is discussed in “Mathematical formulation”. In “Solution approach”, the signomial GP model transformed to a standard posynomial GP model. In “A numerical example”, a numerical example is solved in order to show the

implementation of the algorithm and analyze our model’s parameters behavior. Finally, a conclusion is drawn in “Conclusion”.

Model formulation

To illustrate the idea proposed in this paper, consider a company who produces a single product using a given production process. Due to growing company’s size and escalating market competition, the company directors are trying to increase their share of the market demand. To this

end, they have several options to choose generally. On the one hand, they can propel the investments outward their company, so they promote their marketing and customer service capability. In other words, in order to meet better requirements of the new fierce business environment, they can make investments to increase flexibility and reliability of its production process. They can also choose a combination of the overall strategy. In this condition, the manufacturer must make several decisions simultaneously, so that it leads to increasing company profit.

Notations and assumptions

The following notation and assumptions are considered to develop the model:

D	Annual demand
C_2	Production cost per unit
h	Inventory holding cost rate (%/per unit time) ($0 < h < 1$)
f	Storage area requirement for each item
F	Available floor/storage area
B	Total budget available to the marketing methods and customer service level
$A(r)$	Maintenance costs per production cycle
$E(C_1, r)$	Total cost of interest and depreciation for a production process per production cycle
ψ	Percent of total market demand that should be covered by manufacturer
P_M	Total market demand
R	Total resources available
b	Resource requirements for each item
C_y	The goal associated to number of production cycles
H	The upper limit on the reliability of the production process
L	The lower limit on the reliability of the production process
Decision variables	
p	Selling price per unit
M_j	Volume of investments in marketing method $j = 1, \dots, J$, per unit time
S_l	Volume of investments in customer service strategy $l = 1, \dots, L$, per unit time
Q	Economic production quantity
r	Production process reliability, in other word percent of non-defective items in a batch
C_1	Setup cost (representing process flexibility)
Assumptions	
I	Replenishment is instantaneous
II	No excess stock is carried, and no shortage and lost sales are allowed
III	The production quantity is produced in batches (lots)
IV	All batches are subject to a 100 % inspection policy and all defective items are discarded

In addition, the following power function relations are defined for the model:

1. In current model, the demand per unit time is described as a decreasing power function of price per unit, and marketing expenditures in various approaches, and customer service expenditures in different scenario act as increasing power functions according to the following equation:

$$D(p, M_j, S_l) = kp^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l}, \quad \text{where } k \quad (1)$$

and $\alpha > 0$ and $0 < \beta_j, \tau_l < 1$

In actuality, Eq. 1 indicates that when selling price increases, the demand per unit time decreases. On the other hand, when marketing and customer service expenditures increase, the demand per unit time increases. Also, requiring $k > 0$ is an obvious condition since D must be nonnegative.

2. The total variable cost (C_2Q) considered as a cubic power function of output. Generally, a cost function a function that shows the relation between the magnitude of cost and of output. The existence of such a function is postulated upon the following assumptions: (1) there is a fixed body of plant and equipment; (2) the prices of input factors such as wage rates and raw material prices remain constant; (3) no changes occur in the skill of the workers, managerial efficiency, or in the technical methods of production.

Money expenses of production depend upon the prices and quantities of the factors of production used. Since prices are assumed to remain unchanged, the shape of the cost function will be determined by the physical quantities of the factors used up at different levels of operation.

And since these quantities are functionally related to output, their relation to cost can be represented by a cost-output function. Thus, the underlying determinant of cost behavior is the pattern of change in the factor ingredients as output varies.

This pattern of change will be determined by the technical conditions of production. In general, when there exist fixed productive facilities to which variable resources are applied, the law of diminishing returns Queryis assumed to govern the behavior of returns (Dean 1941). Under this law, the relationship between total cost and output resembles the elongated S curve; from Fig. 2, it can be seen that how the total cost curve first increases gradually and then rapidly. Hence, by considering Eq. 2 as the equation of the total variable cost curve form explained above, $e_1, e_3 > 0$ and $e_2 < 0$ (Gujarati 2003).

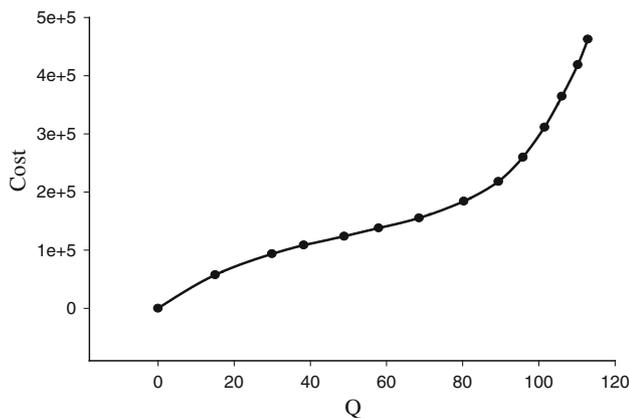


Fig. 2 Total variable cost of production as function of Q

$$C_T = e_1Q^3 + e_2Q^2 + e_3Q, \text{ where } e_1, e_3 > 0 \text{ and } e_2 < 0 \tag{2}$$

3. Total cost of interest and depreciation per production cycle is inversely related to a setup cost and directly related to production process reliability according to the following equation:

$$E(C_1, r) = dC_1^{-\delta}r^\theta, \text{ where } d, \delta, \theta \geq 0 \tag{3}$$

In the equation, b is a scaling constant, δ is setup elasticity to interest and depreciation cost, and θ is production process reliability elasticity to interest and depreciation cost.

In this relationship, the cost of maintenance is a function of production process reliability where it has an inverse relation with production process reliability as follows:

$$A(r) = ar^{-\gamma}, \text{ where } \alpha > 0 \text{ and } 0 < \gamma < 1 \tag{4}$$

This is generally true, since by increasing the reliability of the production process the less failure-prone the machinery will become and also reduce machine downtime duration which, in turn, results in decreasing of maintenance costs. Applied structures for the power functions in current article were used by many researchers with a little difference (Sadjadi et al. 2005, 2010, 2012; Lee and Kim 1993; Jung and Klein 2001; Leung 2007; Panda and Maiti 2009).

Mathematical formulation

Profit is defined as revenue minus cost. Therefore, manufacturer’s profit per cycle is determined as follows:

$$\begin{aligned} \text{profit} = & \underbrace{prQ}_{\text{SalesRevenue}} - \underbrace{C_1}_{\text{Set-upCost}} - \underbrace{C_2Q}_{\text{ProductionCost}} - \underbrace{hC_2 \frac{1}{2}rQT}_{\text{Inventory HoldingCost}} \\ & - \underbrace{E(C_1, r)}_{\text{Interest/DepreciationCost}} - \underbrace{A(r)}_{\text{MaintenanceCost}} \\ & - T \times \underbrace{\left(\sum_j^J M_j\right)}_{\text{Marketing Cost}} - T \times \underbrace{\left(\sum_l^L S_l\right)}_{\text{customer service cost}} \end{aligned} \tag{5}$$

In Eq. (5) inventory holding cost in per cycle

$$\begin{aligned} &= hC_2 \int_0^T q(t)dt = hC_2 \int_0^T (rQ - Dt)dt = hC_2 \left(rQT - \frac{DT^2}{2} \right) \\ &= hC_2 \frac{1}{2}rQT \end{aligned}$$

In this model $q(t)$ is inventory level in time $\left(q(t) = \begin{cases} rQ & \text{at } t = 0 \\ 0 & \text{at } t = T \end{cases} \right)$ and $T = \frac{rQ}{D}$, is the cycle length. This model is similar to those proposed by Panda and Maiti (2009).

But it is quite obvious that manufacture to increase their profits is faced with a series of restrictions including:

Production capacity limit according to available resource (Eq. 6). Limited budget in order to marketing by different channels and types of service that will be given to customers (Eq. 7). Limited storage space (Eq. 8). The upper limit on the reliability of the production process due to lack of affordability (Eq. 9). The lower limit on the reliability of the production process due to avoid lengthy stop production process (Eq. 10). Necessary to achieve a minimum market share is planned (Eq. 11). Limitation of production cycles numbers (Eq. 12). Finally, all of the decisions variables should be positive (Eq. 13).

$$bQ \leq R, \tag{6}$$

$$\sum_j^J M_j + \sum_l^L S_l \leq B, \tag{7}$$

$$frQ \leq F, \tag{8}$$

$$r \leq H, \tag{9}$$

$$r \geq L, \tag{10}$$

$$\sum_i^I D \geq \psi P_M, \tag{11}$$

$$\frac{D}{rQ} \leq t, \tag{12}$$

$$p, M_j, S_l, Q, C_1, r, > 0, \forall j, l \tag{13}$$

With regard to the constraints, assumptions and power functions relations explained in the previous section, the signomial GP form of total profit maximization is defined as follows:

$$\begin{aligned} & \text{Max } \pi(p, M_j, S_l, Q, r, C_1) \\ & = \frac{1}{T} \left[prQ - C_1 - C_2Q - hC_2 \frac{1}{2} rQT - E(C_1, r) - A(r) - T \right. \\ & \quad \left. \times \left(\sum_j M_j + \sum_l S_l \right) \right] \\ & = \left[kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} - kC_1 p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} r^{-1} Q^{-1} - ke_1 r^{-1} p^{-\alpha} \right. \\ & \quad \times \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^2 + ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q - ke_3 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} \\ & \quad - 0.5he_1 Q^3 r + 0.5he_2 Q^2 r - 0.5he_3 Qr - kdC_1^{-\delta} r^{\theta-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} \\ & \quad \left. - kar^{-\gamma-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^{-1} - \left(\sum_j M_j + \sum_l S_l \right) \right] \tag{14} \end{aligned}$$

Subject to:

$$bQR^{-1} \leq 1, \tag{15}$$

$$\left(\sum_j M_j + \sum_l S_l \right) B^{-1} \leq 1, \tag{16}$$

$$frQF^{-1} \leq 1, \tag{17}$$

$$rH^{-1} \leq 1, \tag{18}$$

$$Lr^{-1} \leq 1, \tag{19}$$

$$\psi P_M k^{-1} p^\alpha \prod_{j=1}^J M^{-\beta_j} \prod_{l=1}^L S^{-\tau_l} \leq 1, \tag{20}$$

$$kp^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} r^{-1} Q^{-1} t^{-1} \leq 1, \tag{21}$$

$$p, C_1, Q, M_j, S_l, r, > 0, \forall j, l \tag{22}$$

Solution approach

In this section, the above-mentioned model converted to a posynomial GP form, using the concepts behind the relations between geometric and arithmetic means (Eq. 23).

$$\prod_{n=1}^N (v_n)^{k_n} \leq \sum_{n=1}^N \kappa_n \cdot v_n \tag{23}$$

In Eq. (23), v_n are positive numbers and k_n are nonnegative weights which their summation must equal to one,

$\left(\sum_{n=1}^N \kappa_n = 1 \right)$, in this equation if taking $\kappa_n \cdot v_n \equiv u_n$ in mind, result will be as follows:

$$\prod_{n=1}^N \left(\frac{u_n}{\kappa_n} \right)^{\kappa_n} \leq \sum_{n=1}^N u_n \tag{24}$$

In order to use this concept, the objective function of signomial GP problem which described in the previous section can be converted into the following conventional form with considering two functions U_1 and U_2 .

$$\begin{aligned} & \text{max } \pi(p, M_j, C_1, S_l, r, Q) = U_1 - U_2 \\ & \left(kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} + ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q + 0.5he_2 Q^2 r \right) = U_1 \\ & \left(kC_1 p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} r^{-1} Q^{-1} + ke_1 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^2 \right. \\ & \quad \left. + ke_3 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} S_l^{\tau_l} + 0.5he_1 Q^3 r + 0.5he_3 Qr + kdC_1^{-\delta} r^{\theta-1} \right. \\ & \quad \left. p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} + kar^{-\gamma-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^{-1} + \left(\sum_j M_j \right) + \left(\sum_l S_l \right) \right) \\ & = U_2 \tag{25} \end{aligned}$$

subject to the same constraints as in Eqs.(16–23). Model (25) is equivalent to the following model, after defining two extra constraints and auxiliary variable (Z).

$$\text{max } \pi(p, M, S, C_1, r, Q) = Z \tag{26}$$

$$\begin{aligned} U_1 - U_2 \geq Z & \rightarrow U_1 \geq U_2 + Z \rightarrow U_2 + Z \leq X \leq U_1 \\ & \rightarrow \begin{cases} U_1 \geq X \rightarrow U_1 X^{-1} \geq 1, \\ U_2 + Z \leq x \rightarrow U_2 X^{-1} + ZX^{-1} \leq 1, \end{cases} \tag{27} \end{aligned}$$

subject to the same constraints as in Eqs.(16–23).

As a result of Eqs. 23 and 24, the first extra constraint can be rewritten as follows:

$$U_1 = U_{11} + U_{12} + U_{13} \tag{28}$$

$$U_1 X^{-1} \geq 1 \rightarrow U_{11} X^{-1} + U_{12} X^{-1} + U_{13} X^{-1} \geq 1 \tag{29}$$

$$(U_{11} X^{-1} + U_{12} X^{-1} + U_{13} X^{-1})^{-1} \leq 1 \tag{30}$$

$$\begin{aligned} (U_{11} X^{-1} + U_{12} X^{-1} + U_{13} X^{-1})^{-1} & \leq \left(\frac{U_{11} X^{-1}}{Y_1} \right)^{-Y_1} \\ & \times \left(\frac{U_{12} X^{-1}}{Y_2} \right)^{-Y_2} \left(\frac{U_{13} X^{-1}}{Y_3} \right)^{-Y_3} \tag{31} \end{aligned}$$

$$Y_i = \frac{U_{1i}}{\sum_{i=1}^3 U_{1i}} \tag{32}$$

Equation (29) is true if and only if: $\sum_{i=1}^3 Y_i = 1$

Therefore, the weights of all positive terms of the objective function can be obtained as follows:

$$Y_1 = \frac{kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1}}{kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1} + ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1} + 0.5he_2 Q^2 r X^{-1}}, \tag{33}$$

$$Y_2 = \frac{ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1}}{kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1} + ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1} + 0.5he_2 Q^2 r X^{-1}}, \tag{34}$$

$$Y_3 = \frac{0.5fe_2 Q^2 r X^{-1}}{kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1} + ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1} + 0.5he_2 Q^2 r X^{-1}}, \tag{35}$$

So, it will be apparent determining the related weighs as the above, will made both sides of the relation (24) to be equivalent, then, As a result,

$$\left(\frac{U_{11}X^{-1}}{Y_1}\right)^{-Y_1} \left(\frac{U_{12}X^{-1}}{Y_2}\right)^{-Y_2} \left(\frac{U_{13}X^{-1}}{Y_3}\right)^{-Y_3} \leq 1, \tag{36}$$

Also, the second extra constraint according to negative terms of the objective function can be rewritten as follows:

$$U_2 X^{-1} + ZX^{-1} \leq 1 \rightarrow ZX^{-1} + \sum_{i=1}^I U_{2i} X^{-1} \leq 1, \tag{37}$$

After all transformations, the original model can be changed to following problem:

$$\min Z^{-1} \tag{38}$$

$$\left(\frac{U_{11}X^{-1}}{Y_1}\right)^{-Y_1} \left(\frac{U_{12}X^{-1}}{Y_2}\right)^{-Y_2} \left(\frac{U_{13}X^{-1}}{Y_3}\right)^{-Y_3} \leq 1, \tag{39}$$

$$U_2 X^{-1} + ZX^{-1} \leq 1 \rightarrow ZX^{-1} + \sum_{i=1}^I U_{2i} X^{-1} \leq 1, \tag{40}$$

subject to the another constraints as in Eqs. (16–23).

The above problem can be rewritten as follows:

$$\min Z^{-1} \tag{41}$$

$$\left(\frac{kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1}}{Y_1}\right)^{-Y_1} \left(\frac{ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1}}{Y_2}\right)^{-Y_2} \left(\frac{0.5he_2 Q^2 r X^{-1}}{Y_3}\right)^{-Y_3} \leq 1 \tag{42}$$

$$\begin{aligned} & ZX^{-1} + kC_1 p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} r^{-1} Q^{-1} X^{-1} \\ & + ke_1 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^2 X^{-1} \\ & + ke_3 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1} + 0.5he_1 Q^3 r X^{-1} \\ & + 0.5he_3 Q r X^{-1} + kdC_1^{-\delta} r^{\theta-1} p^{-\alpha} \\ & \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1} + kar^{-\gamma-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} Q^{-1} X^{-1} \\ & + \left(\sum_j M_j\right) X^{-1} + \left(\sum_l S_l\right) X^{-1} \leq 1, \end{aligned} \tag{43}$$

subject to the another constraints as in Eqs. (16–23).

Consequently, posynomial GP model can be formulated more clearly as follows:

$$\min Z^{-1} \tag{44}$$

$$\begin{aligned} & \left(kp^{1-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} X^{-1}\right)^{-Y_1} \\ & \left(ke_2 r^{-1} p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\tau_l} QX^{-1}\right)^{-Y_2} \\ & (0.5he_2 Q^2 r X^{-1})^{-Y_3} Y_1^{Y_1} Y_2^{Y_2} Y_3^{Y_3} \leq 1 \end{aligned} \tag{45}$$

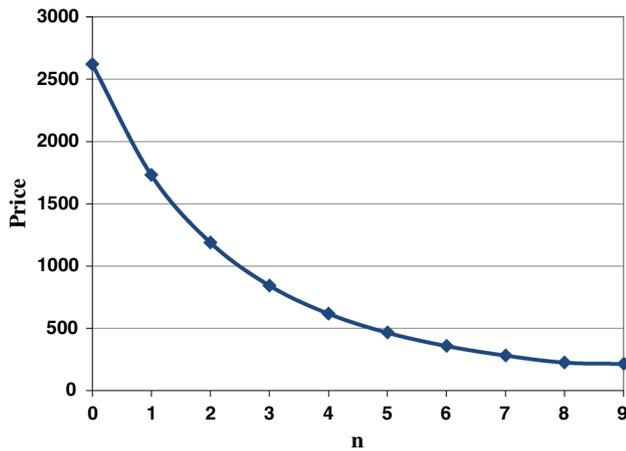


Fig. 3 Changes in price regarding changes in α

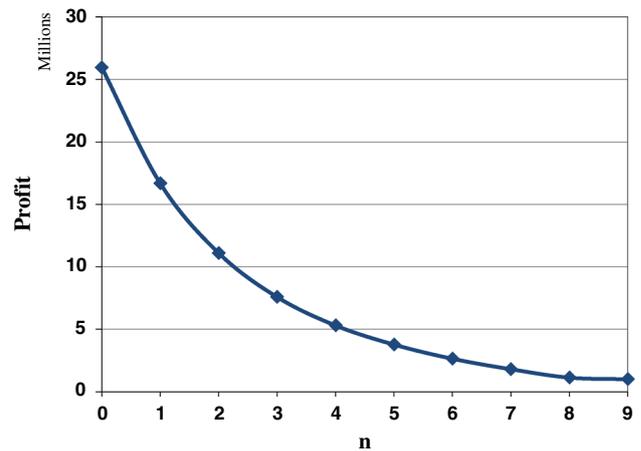


Fig. 4 Changes in profit regarding changes in α

$$\begin{aligned}
 & ZX^{-1} + kC_1p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\alpha_l} r^{-1} Q^{-1} X^{-1} \\
 & + ke_1r^{-1}p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\alpha_l} Q^2 X^{-1} \\
 & + ke_3r^{-1}p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\alpha_l} X^{-1} + 0.5he_1Q^3rX^{-1} \\
 & + 0.5he_3QrX^{-1} + kdC_1^{-\delta}r^{\theta-1}p^{-\alpha} \\
 & \times \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\alpha_l} X^{-1} + kar^{-\gamma-1}p^{-\alpha} \prod_{j=1}^J M_j^{\beta_j} \prod_{l=1}^L S_l^{\alpha_l} Q^{-1} X^{-1} \\
 & + \left(\sum_j M_j\right)X^{-1} + \left(\sum_l S_l\right)X^{-1} \leq 1,
 \end{aligned}
 \tag{46}$$

subject to the other constraints as in Eqs. (16–23).

A numerical example

To illustrate the validity of the proposed model and the usefulness of the proposed solution method, a simple numerical experiment is presented and the related results are reported in this section. To this end, consider a manufacturing company which wants to sell its product regarding two methods for advertising (TV, Newspaper) and providing two types services for consumers (Buying advice, Product Warranty). The required information to decision-making is set from marketing and production departments as follows:

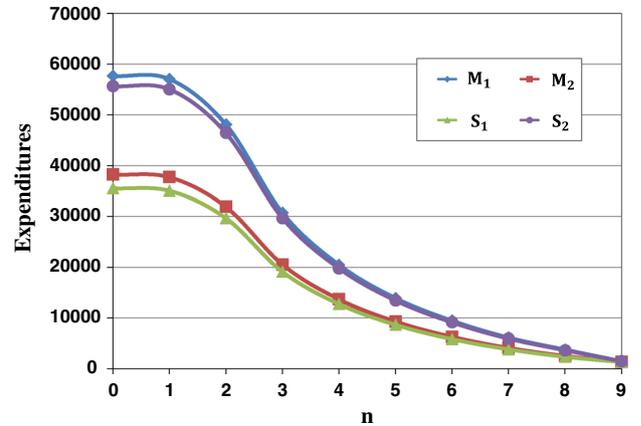


Fig. 5 Changes in customer service and marketing expenditures regarding changes in α

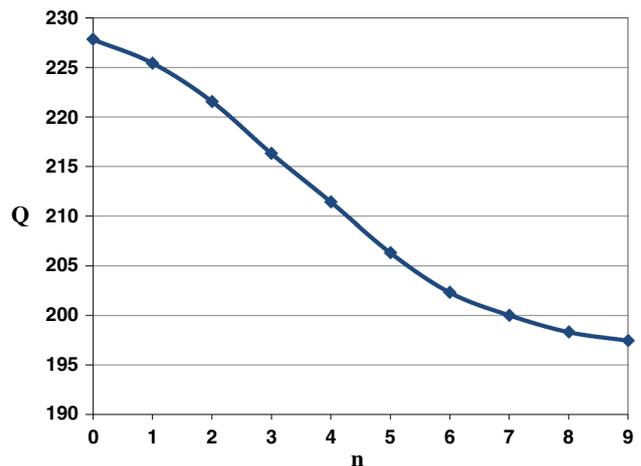


Fig. 6 Changes in lot-size regarding changes in α

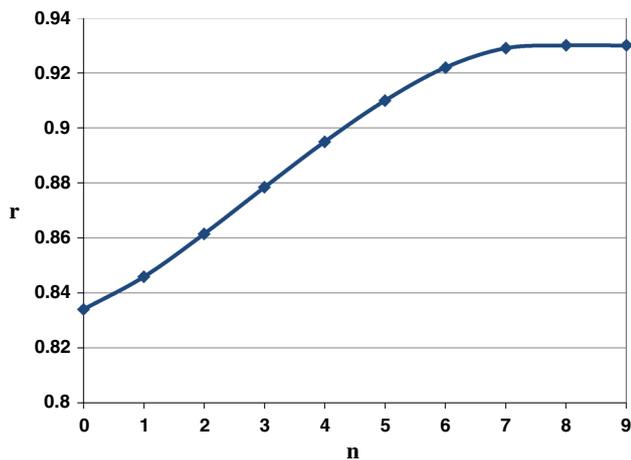


Fig. 7 Changes in reliability regarding changes in α

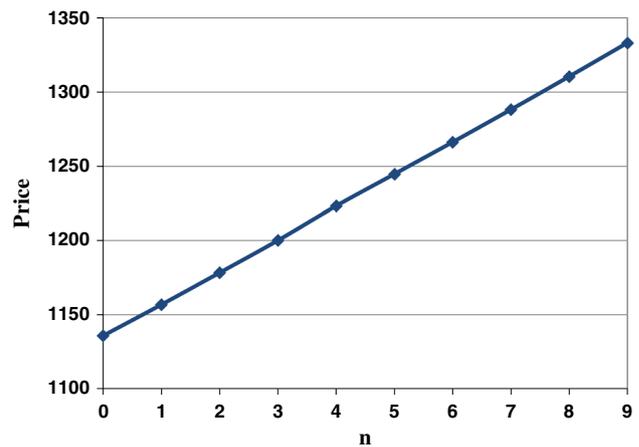


Fig. 9 Changes in price regarding changes in β_j and τ_l

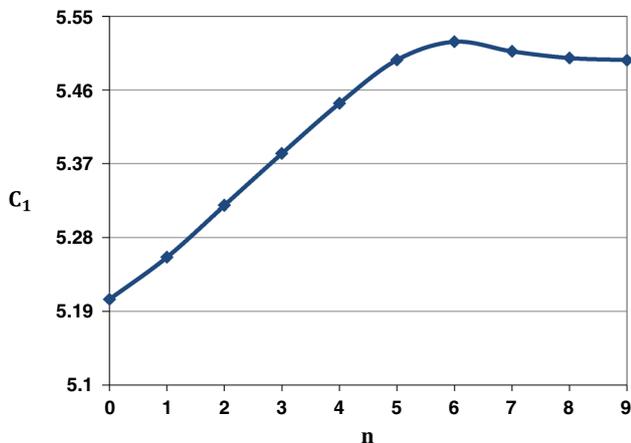


Fig. 8 Changes in setup cost regarding changes in α

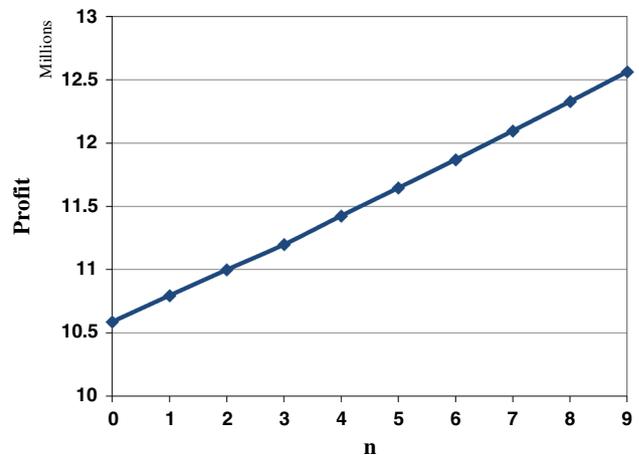


Fig. 10 Changes in profit regarding changes in β_j and τ_l

$k = 11 \times 10^9,$	$\alpha = 2,$	$\beta_1 = 0.0086,$	$\beta_2 = 0.0057,$	$\tau_1 = 0.0053,$
$\tau_2 = 0.0083,$	$e_1 = 0.03,$	$e_2 = 0.452,$	$e_3 = 2.703,$	$h = 0.15,$
$t = 55,$	$P_M = 5 \times 10^3,$	$\psi = 0.22,$	$B = 187000,$	$d = 50,$
$H = 0.93,$	$L = 0.71,$	$\delta = 1.5,$	$\theta = 1,$	$\gamma = 0.2,$
$a = 1000,$	$f = 10,$	$F = 1900,$	$b = 4,$	$R = 1200,$

The results of the running CVX on the proposed GP model are as follows:

$Z = \$11,103,436$	$Q^* = 220,$	$p^* = \$1189.3,$	$M_1^* = \$48068,$
$M_2^* = \$31874.75,$	$S_1^* = \$29640.53,$	$S_2^* = \$46392.96,$	$r^* = 0.86,$
$C_1^* = \$5.29,$			

Thus, the posynomial GP model according to the above parameters is a very difficult GP problem (degree difficulty = 14).

Intrinsically, it is complex to solve the model by solution procedures discussed in literature for a GP (Duffin et al. 1967). Hence, in order to find the optimal solution of the problem a MATLAB-based modeling system (CVX) (Boyd and Michael 2009) is used.

Sensitivity analysis

In this sub-section, due to high degrees of nonlinearities of the model, the behavior of the decision variables in situation optimal with regard to changes in some parameters of the model are analyzed. In other words, the study discusses some managerial insights by studying how the optimal solution would vary as the inputs values change.

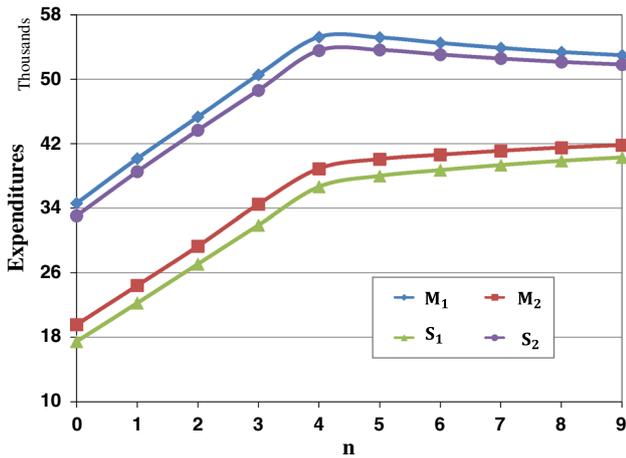


Fig. 11 Changes in customer service and marketing expenditures regarding changes in β_j and τ_l

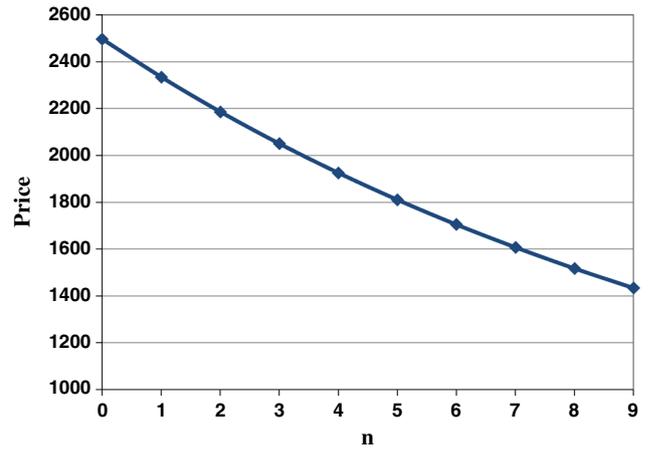


Fig. 12 Changes in price regarding changes in β_j , τ_l and α

Effects of changes in α on optimal solution

In this sub-section, the effect of changes in values price elasticity of demand on the optimal solution is investigated. In order to implement this investigation, the different value of the price elasticity of demand is considered as follows: $\alpha_k = \alpha_0 + (n \times 0.01)$ where $\alpha_0 = 1.8$, and $n = 0, \dots, 9$. According to, Figs 3, 4, 5, 6, respectively, reveal that as price elasticity of demand increases, the manufacturer has to decrease its product price with proper intensity in order to maintain the acceptable market share (Fig. 3). As a result of the reduced optimal selling price, manufacturer’s profit decreases as well (Fig. 4), so that, the manufacturer has to decrease marketing and customer service expenditures in order to escape from the loss limits (Fig. 5). Also, the manufacturer in order to minimize additional cost of production (i.e., inventory holding cost) and maintenance costs need to reduce lot-size (Fig. 6) and increases reliability of process (Fig. 7). Generally, when the lot-size decreases, setup cost will increase as well (Fig. 8).

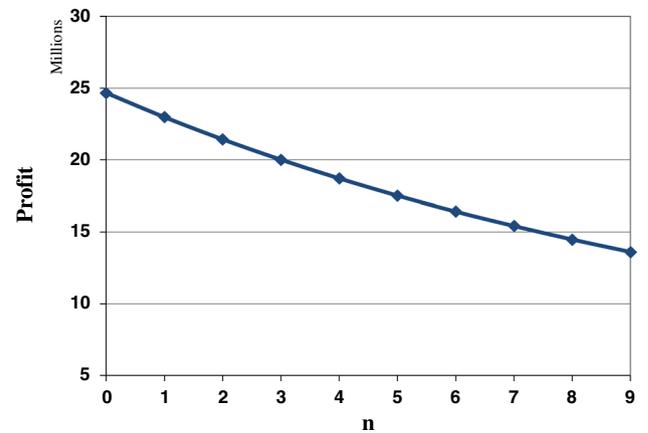


Fig. 13 Changes in profit regarding changes in β_j , τ_l and α

Effects of changes in β_j and τ_l on the optimal solution

This sub-section organized as follows. First, we investigate the effect of increasing in values of β_j ($j = 1, 2$) and τ_l ($l = 1, 2$) on the optimal solution. Second, we investigate the effect of increasing in values β_j ($j = 1, 2$) and τ_l ($l = 1, 2$), while simultaneously, α will also increases. Finally, we investigate a gradual increases in both β_2 and τ_1 while decreases β_1 and τ_2 simultaneously such that $\beta_j + \tau_l = \text{constant}$.

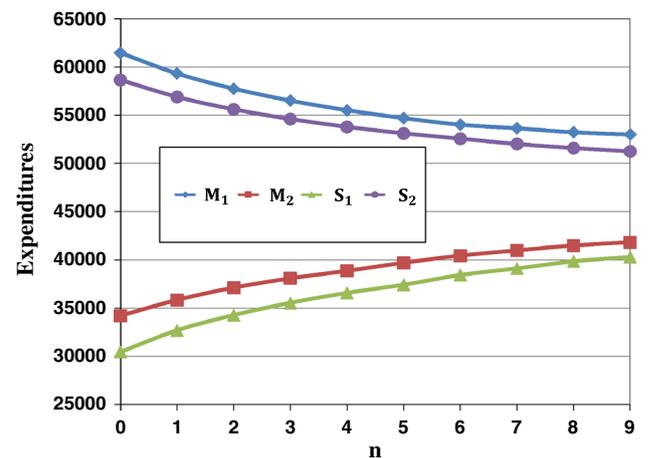


Fig. 14 Changes in customer service and marketing expenditures regarding changes in β_j , τ_l and α

Increasing both β_j ($j = 1, 2$) and τ_l ($l = 1, 2$) Different values of β_j ($j = 1, 2$) and τ_l ($l = 1, 2$) are examined as

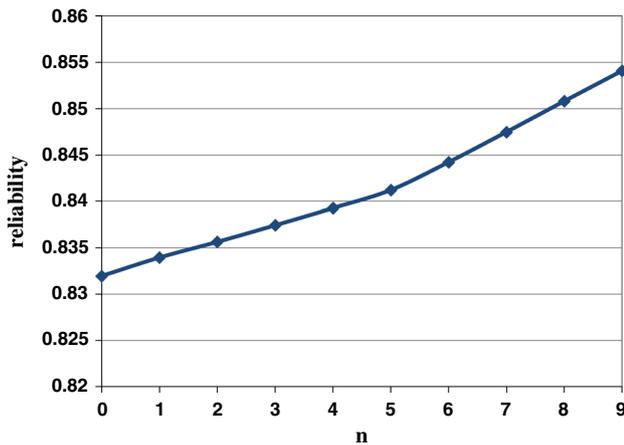


Fig. 15 Changes in reliability regarding changes in β_j , τ_l and α

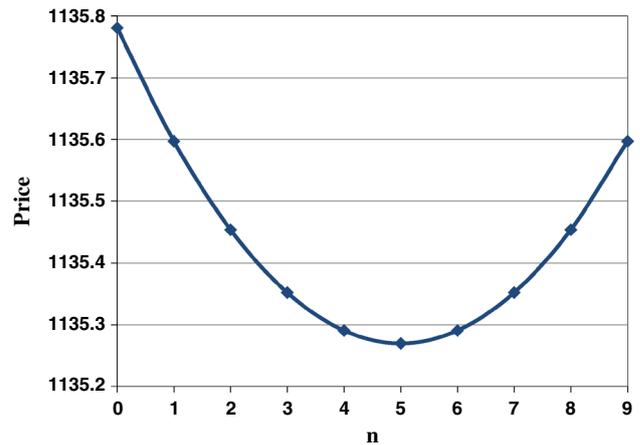


Fig. 17 Changes in price regarding changes in β_j and τ_l ($\beta_j + \tau_l = \text{constant}$)

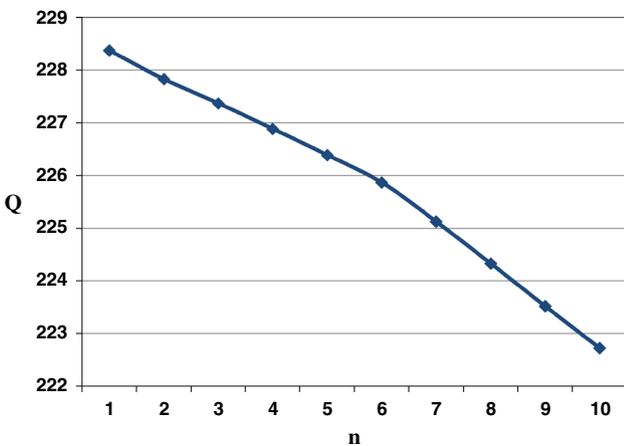


Fig. 16 Changes in lot-size regarding changes in β_j , τ_l and α

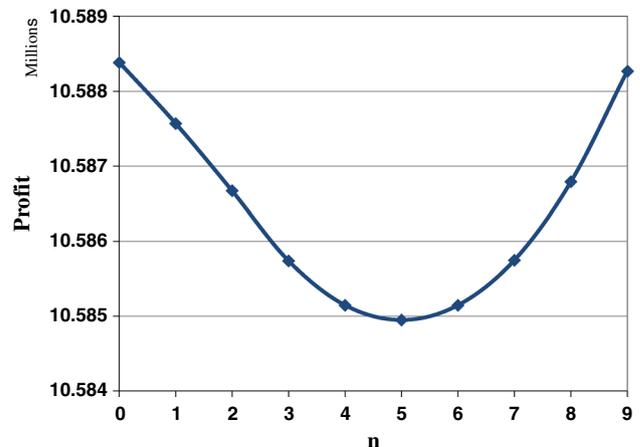


Fig. 18 Changes in profit regarding changes in β_j and τ_l ($\beta_j + \tau_l = \text{constant}$)

follows: $\beta_j(\tau_l) = \beta_{0j}(\tau_{0l}) + (0.0008) \times n$ where $\beta_{01} = 0.0066$, $\tau_{01} = 0.0033$, $\beta_{02} = 0.0037$, $\tau_{02} = 0.0063$, and $n = 0, 1, \dots, 9$. According to Figs. 9 and 10, it is seen that when β_j ($j = 1, 2$) and τ_l ($l = 1, 2$) increase simultaneously, it results in increasing price of product. Therefore, price increase results in increasing profit. This is because of the rise in marketing and customer service elasticity of demand, the marketing and customer service expenditures are increasing in response to customers' requests as well (Fig. 11). As a result, the company that gain more profit, has to increase its product price.

Increasing β_j ($j = 1, 2$), τ_l ($l = 1, 2$) and α The effect of changes in β_j ($j = 1, 2$), τ_l ($l = 1, 2$) and α on the optimal values of decision variables are discussed in this part. The values of β_j ($j = 1, 2$), τ_l ($l = 1, 2$) are increased according to the equation $\beta_j(\tau_l) = \beta_{0j}(\tau_{0l}) + (0.0008 \times n)$, while simultaneously, α is increased according to

$\alpha_k = \alpha_0 + (n \times 0.02)$, for $n = 0, \dots, 9$ such that $\beta_{01} = 0.0066$, $\tau_{01} = 0.0033$,

$\beta_{02} = 0.0037$, $\tau_{02} = 0.0063$ and $\alpha_0 = 1.8$. We can observe from Figs. 12 and 13, with increase in β_j ($j = 1, 2$), τ_l ($l = 1, 2$) and α , the optimal selling price and total profit decreases subsequently. This is because of company response to combination results of changes in β_j ($j = 1, 2$), τ_l ($l = 1, 2$) and α [i.e., where effects of changes α overcome on the effects of changes in β_j ($j = 1, 2$) and τ_l ($l = 1, 2$)]. Also, Fig. 14 shows that the company should not be oblivious to the effects of changes in values of marketing and customer service elasticity, and in order to minimize the total cost, the company has to decrease the marketing expenditures through marketing method 1 and also decrease the customer's service expenditures through customer's service type 2. On the other hand, in order to

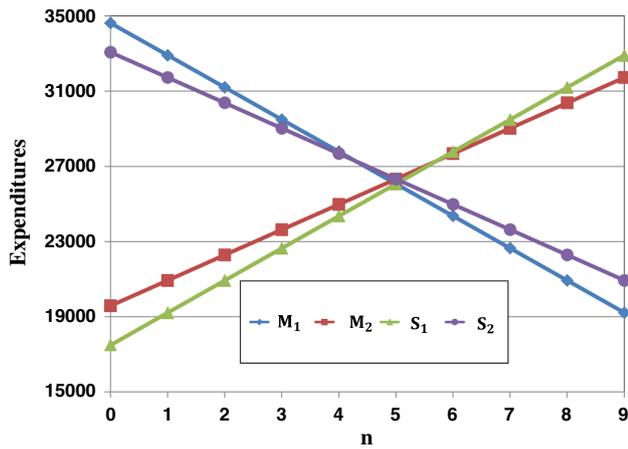


Fig. 19 Changes in customer service and marketing expenditures regarding changes in β_j and τ_l ($\beta_j + \tau_l = \text{constant}$)

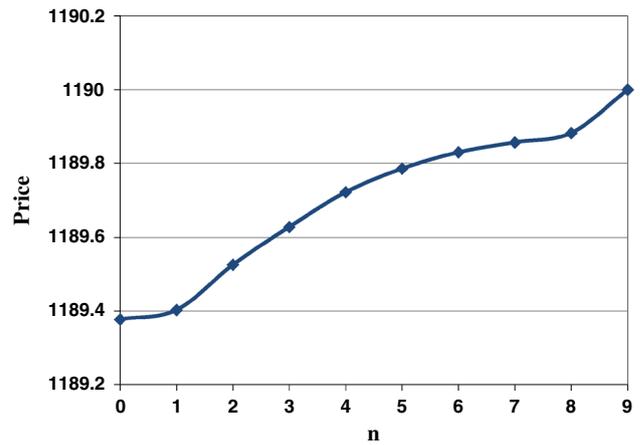


Fig. 21 Changes in price regarding changing γ

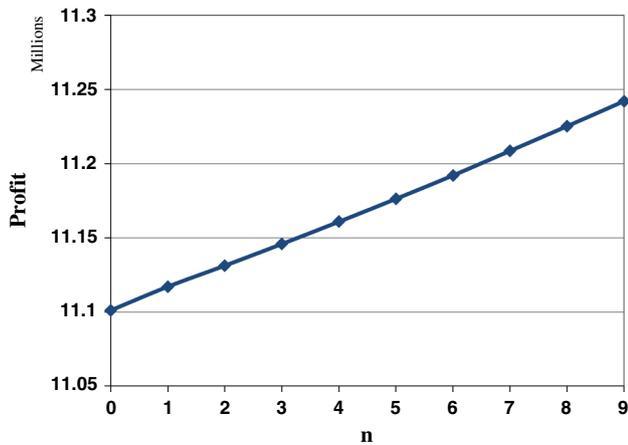


Fig. 20 Changes in profit regarding changes in γ

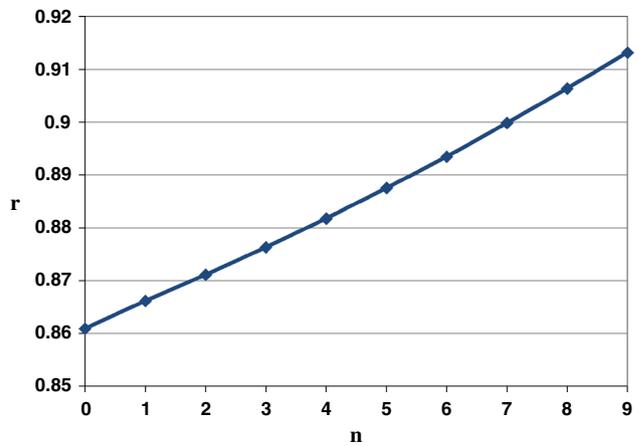


Fig. 22 Changes in reliability regarding changes in γ

respond to the needs of different customer segments, the company has to increase the marketing expenditures through marketing method 1, and also increase the customer's service expenditures through customer's service type 2.

According to Figs. 15 and 16, with increases in β_j ($j = 1, 2$), τ_l ($l = 1, 2$) and α , the lot-size decreases and the process reliability increases. Generally, when that the effect of change in α overcomes the effects of changes in β_j ($j = 1, 2$) and τ_l ($l = 1, 2$), it can be concluded that customers are highly sensitive to the price of product and company cannot just retain its customer through marketing and offering services. Therefore, as explained in the previous section, the company has to decrease its product price in order to keep its customer. On the other hand, by increasing process reliability and reducing lot-size, the

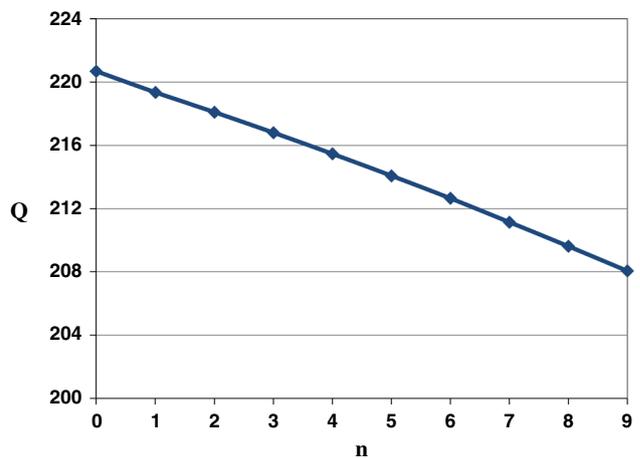


Fig. 23 Changes in lot-size regarding changes in γ

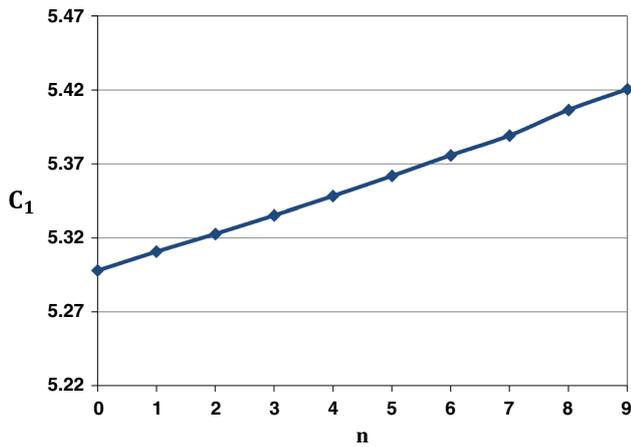


Fig. 24 Changes in setup cost regarding changes in γ

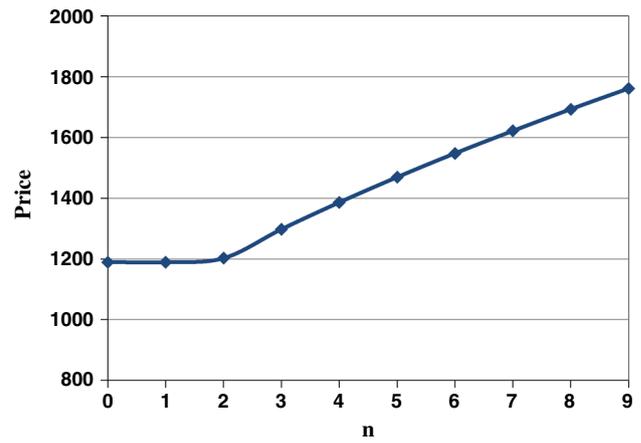


Fig. 26 Changes in price regarding changes in b

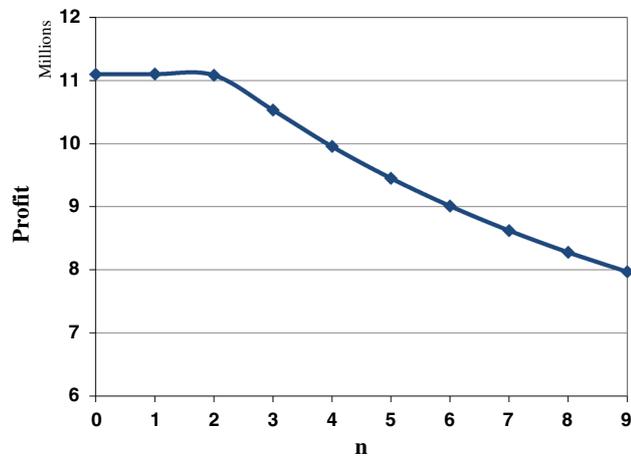


Fig. 25 Changes in profit regarding changes in b

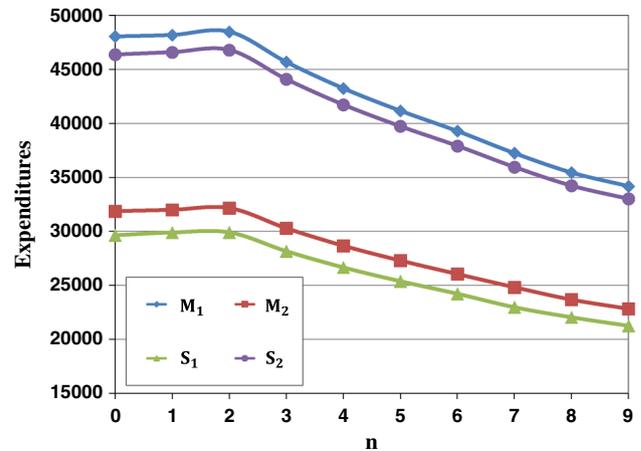


Fig. 27 Changes in customer service and marketing expenditures regarding changes in b

company has to reduce its total cost such as maintenance costs and inventory holding cost in order to escape from the limit of losses.

Increasing β_2 and τ_1 while decreasing β_1 and τ_2 The values of β_1 and τ_1 are calculated according to the equation $\beta_1 (\tau_1) = \beta_{01} (\tau_{01}) \pm (0.00033 \times n)$, while simultaneously, β_2 and τ_2 are calculated according to $\beta_2 (\tau_2) = \beta_{02} (\tau_{02}) \pm (0.00026 \times n)$, where $\beta_{01} = 0.0066$, $\tau_{01} = 0.0033$, $\beta_{02} = 0.0037$, $\tau_{02} = 0.0063$ and $n = 0, \dots, 9$. According to Figs. 17, 18, 19, as $\beta_j (j = 1, 2)$ and $\tau_l (l = 1, 2)$ get closer to each other (i.e., $n = 5$), the product price, and total annual profit decreases. This is mainly due to the fact that when different methods of marketing and various types of customer's service are equally important from the customer's perspective, increasing investment on all options are useless. From this point of view, it is advisable

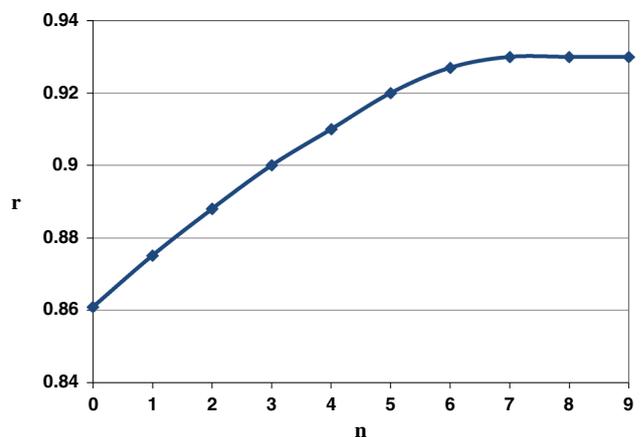


Fig. 28 Changes in reliability regarding changes in b

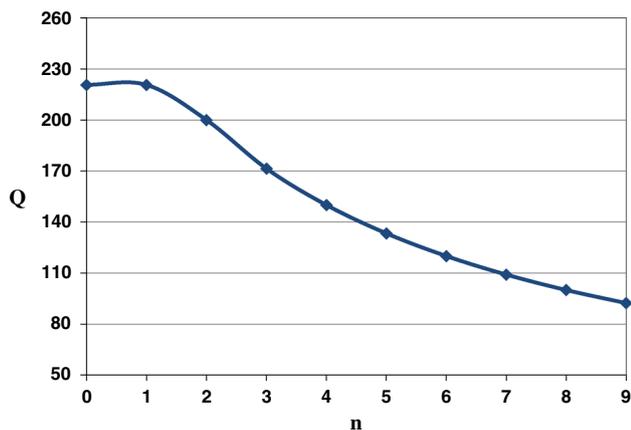


Fig. 29 Changes in lot-size regarding changes in b

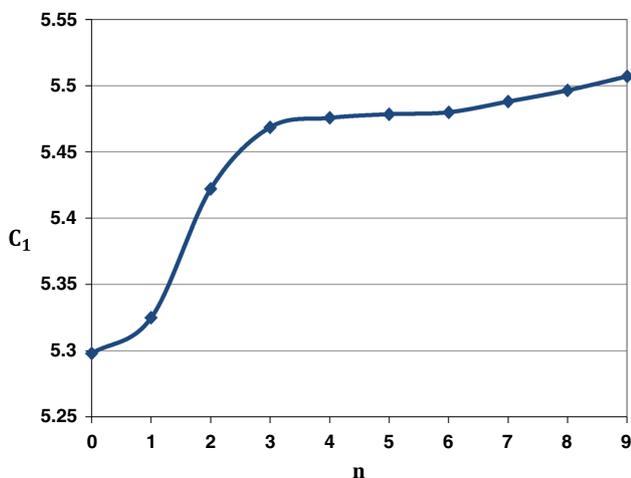


Fig. 30 Changes in setup cost regarding changes in b

to invest in a diverse range of marketing methods and customer service types in order to efficient use of limited budget.

Effect of changes in γ on optimal solution

The effects of changes in reliability elasticity of maintenance cost on the optimal solution are investigated by considering different values for γ as follow: $\gamma = 0.2 + (0.05 \times n)$, for $n = 0, \dots, 9$.

According to Figs. 20 and 21, the selling price and total profit are slightly sensitive to the changes in values of parameter γ that is negligible. Also, from Fig. 22, since maintenance cost has a reverse relationship with the process reliability, the process reliability should be increased to maximize the total profit. In order words, in order to maximize the profit, revenue has to be increased and other costs have to be decreased. This means that less investment on process flexibility must be implemented. And on the

other hand, by reducing the lot-size, the total production and inventory holding costs must be reduced, which in turn means higher setup cost and lower lot-size, as seen in Figs. 23 and 24.

Effects of changes in b on optimal solution

The effects of changes in b on the optimal solution are investigated by considering different values for b as follows: $b = 4 + (1 \times n)$, for $n = 0, \dots, 9$.

Figures 25, 26, 27, 28, 29, respectively, reveal that any increase in b leads to a higher optimal selling price, lower total profit, lower optimal marketing and customer service expenditures, higher process reliability, smaller optimal production lot-size, lower optimal setup costs per production cycle. The implication of these behaviors is that the company has to decrease additional cost in order to prevent of decreasing profit (Fig. 30).

Conclusions

To cope with the issue of integrating the marketing and production strategies for a company that produces a single product, an integrated model with using the GP technique is proposed in this paper. In this paper, the production cost is considered as a cubic power function of outputs. The model is restricted with available limited storage space constraint for acceptable products, available limited budget for marketing and service quality, and necessary to achieve a minimum market share is planned. Also, in this paper the defective items are considered, so that only % r of products can be good to be used. According to the descriptions, the resulting GP model is very hard, with the 14 degrees of difficulty, so it is very complex to solve the model intrinsically.

To the best of our knowledge, this research is one of the primary works using the concepts behind the relations between geometric–arithmetic means and CVX modeling system for solving type of models with a high degree of difficulty, hence the literature considering this approach in production planning is still scarce. Therefore, many possible future research avenues can be defined in this context, and will be solved using the toolbox and concepts.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

References

Abuo-El-Ata M, Fergany HA, El-Wakeel MF (2003) Probabilistic multi-item inventory model with varying order cost under two



- restrictions: a geometric programming approach. *Int J Prod Econ* 83(3):223–231
- Bahri M, Tarokh MJ (2012) A seller-buyer supply chain model with exponential distribution lead time. *J Ind Eng Int* 8(1):1–7
- Bazaraa MS, Shetty CM (1976) *Foundations of optimization*. Springer, Berlin
- Beightler CS, Phillips DT (1976) *Applied geometric programming*, vol 150. Wiley, New York
- Boyd S, Michael G (2009) MATLAB software for disciplined convex programming. <http://cvxr.com/cvx/>
- Boyd S, Kim S-J, Vandenberghe L, Hassibi A (2007) A tutorial on geometric programming. *Optim Eng* 8(1):67–127
- Chen C-K (2000) Optimal determination of quality level, selling quantity and purchasing price for intermediate firms. *Prod Plan Control* 11(7):706–712
- Cheng T (1991) An economic order quantity model with demand-dependent unit production cost and imperfect production processes. *IIE Trans* 23(1):23–28
- Creese RC (2010) Geometric programming for design and cost optimization (with illustrative case study problems and solutions). *Synth Lect Eng* 5(1):1–140
- Crittenden VL, Crittenden WF (1995) Examining the impact of manufacturing and marketing capacity decisions on firm profitability. *Int J Prod Econ* 40(1):57–72
- Dean J (1941) The relation of cost to output for a leather belt shop, with a memorandum on empirical cost studies by C. Reinold Noyes. National Bureau of Economic Research, Technical Paper, New York
- Douglas MA, Strutton D (2009) Going “purple”: Can military jointness principles provide a key to more successful integration at the marketing-manufacturing interface? *Bus Horiz* 52(3):251–263
- Duffin RJ, Peterson EL, Zener C (1967) *Geometric programming: theory and application*. Wiley, New York
- Fathian M, Sadjadi SJ, Sajadi S (2009) Optimal pricing model for electronic products. *Comput Ind Eng* 56(1):255–259
- Ghazi Nezami F, Sadjadi SJ, Aryanezhad Mir B (2009) A geometric programming approach for a nonlinear joint production—marketing problem. *International association of computer science and information technology—spring conference*
- Ghosh P, Roy TK (2013) A goal geometric programming problem (G 2 P 2) with logarithmic deviational variables and its applications on two industrial problems. *J Ind Eng Int* 9(1):1–9
- Gujarati DN (2003) *Basic econometrics*, 4th edn. McGraw-Hill, New York
- Gupta YP, Lonial SC, Mangold WG (1991) An examination of the relationship between manufacturing strategy and marketing objectives. *Int J Oper Prod Manag* 11(10):33–43
- Hariri A, Abou-El-Ata M, Goyal S (1995) The resource constrained multi-item inventory problem with price discount: a geometric programming approach. *Prod Plan Control* 6(4):374–377
- Ho TH, Tang CS (2004) Introduction to the special issue on marketing and operations management interfaces and coordination. *Manag Sci* 50(4):429–430
- Islam S (2008) Multi-objective marketing planning inventory model: a geometric programming approach. *Appl Math Comput* 205(1):238–246
- Jung H, Klein CM (2001) Optimal inventory policies under decreasing cost functions via geometric programming. *Eur J Oper Res* 132(3):628–642
- Jung H, Klein CM (2005) Optimal inventory policies for an economic order quantity model with decreasing cost functions. *Eur J Oper Res* 165(1):108–126
- Jung H, Klein CM (2006) Optimal inventory policies for profit maximizing EOQ models under various cost functions. *Eur J Oper Res* 174(2):689–705
- Kahn KB, Mentzer JT (1998) Marketing’s integration with other departments. *J Bus Res* 42(1):53–62
- Kim D, Lee WJ (1998) Optimal joint pricing and lot sizing with fixed and variable capacity. *Eur J Oper Res* 109(1):212–227
- Konijnendijk PA (1994) Coordinating marketing and manufacturing in ETO companies. *Int J Prod Econ* 37(1):19–26
- Kotb KA, Fergany HA (2011) Multi-item EOQ model with both demand-dependent unit cost and varying leading time via geometric programming. *Appl Math* 2(5):551–555
- Lee WJ (1993) Determining order quantity and selling price by geometric programming: optimal solution, bounds, and sensitivity. *Decis Sci* 24(1):76–87
- Lee WJ, Kim D (1993) Optimal and heuristic decision strategies for integrated production and marketing planning. *Decis Sci* 24(6):1203–1214
- Lee WJ, Kim D, Cabot AV (1996) Optimal demand rate, lot sizing, and process reliability improvement decisions. *IIE Trans* 28(11):941–952
- Leung K-NF (2007) A generalized geometric-programming solution to “An economic production quantity model with flexibility and reliability considerations”. *Eur J Oper Res* 176(1):240–251
- Liu S-T (2006) Computational method for the profit bounds of inventory model with interval demand and unit cost. *Appl Math Comput* 183(1):499–507
- Maltz E, Kohli AK (2000) Reducing marketing’s conflict with other functions: the differential effects of integrating mechanisms. *J Acad Mark Sci* 28(4):479–492
- Mandal NK, Roy TK (2006) Multi-item imperfect production lot size model with hybrid number cost parameters. *Appl Math Comput* 182(2):1219–1230
- Nazemi A, Sharifi E (2012) Solving a class of geometric programming problems by an efficient dynamic model. *Commun Nonlinear Sci Numer Simul* 18:692–709
- O’Leary-Kelly SW, Flores BE (2002) The integration of manufacturing and marketing/sales decisions: impact on organizational performance. *J Oper Manag* 20(3):221–240
- Panda D, Maiti M (2009) Multi-item inventory models with price dependent demand under flexibility and reliability consideration and imprecise space constraint: A geometric programming approach. *Math Comput Modell* 49(9):1733–1749
- Sadjadi SJ, Oroujee M, Aryanezhad MB (2005) Optimal production and marketing planning. *Comput Optim Appl* 30(2):195–203
- Sadjadi SJ, Aryanezhad M-B, Jabbarzadeh A (2010) Optimal marketing and production planning with reliability consideration. *Afr J Bus Manag* 4(17):3632–3640
- Sadjadi SJ, Yazdian SA, Shahanaghi K (2012) Optimal pricing, lot-sizing and marketing planning in a capacitated and imperfect production system. *Comput Ind Eng* 62(1):349–358
- Shapiro BP (1977) Can marketing and manufacturing coexist. *Harvard Bus Rev* 55(5):104–114
- Swink M, Song M (2007) Effects of marketing-manufacturing integration on new product development time and competitive advantage. *J Oper Manag* 25(1):203–217
- Weir K, Kochhar A, LeBeau S, Edgeley D (2000) An empirical study of the alignment between manufacturing and marketing strategies. *Long Range Plan* 33(6):831–848
- Zener C (1971) *Engineering design by geometric programming*. Wiley, New York

