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ORIGINAL RESEARCH

Selection of energy source and evolutionary stable strategies for power plants under financial intervention of government

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Abstract Currently, many socially responsible governments adopt economic incentives and deterrents to manage environmental impacts of electricity suppliers. Considering the Stackelberg leadership of the government, the government's role in the competition of power plants in an electricity market is investigated. A one-population evolutionary game model of power plants is developed to study how their production strategy depends on tariffs levied by the government. We establish that a unique evolutionary stable strategy (ESS) for the population exists. Numerical examples demonstrate that revenue maximization and environment protection policies of the government significantly affect the production ESS of competitive power plants. The results reveal that the government can introduce a green energy source as an ESS of the competitive power plants by imposing appropriate tariffs.

Keywords Evolutionary game theory · Green electricity · Power plant · Government intervention · Energy source selection

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Introduction

The evolutionary game theory (Smith and Price 1973) naturally applies to biology; however, it can be adopted to explain and predict many phenomena in economics, business, and other issues in social and political areas. In interactions among players, evolutionary game suggests that the better strategies would finally evolve and dominate among the players (Barron 2013). For the first time, this paper proposes evolutionary game theory for evaluation of green and non-green production strategies of power plant's population. The evolutionary stable strategy (ESS) for power plants is a good strategy that results in a stable situation for the population. Moreover, we will study how the ESS may be affected from the government financial intervention.

The power plants as the largest polluting industry have encouraged a lot of scientific researches. To promote the green electricity, the government should take effective actions to compensate for extra production costs of the renewable (green) energy and impose penalties for the nonrenewable energies. For example, the pollution tax levied on carbon dioxide emission is a powerful policy mechanism that can address market failures in energy industry (Wu et al. 2006). The role of the government's green policies in the polluting industries is considered in several studies. By constructing a theoretical game model with incomplete information, Cerqueti and Coppier (2014) discussed the effects of interaction between polluting firms, tax inspectors, and government politicians on environmental policy. Dong et al. (2010) presented a framework for analyzing the conflicts between a local government and a potentially pollution producer using the game theory. They investigated the effects of environmental subsidy and penalty policies on implementation of a clean production.

Liu et al. (2007) studied quantity and price competitions of two power plants. Hafezalkotob (2015) modeled the competition of two green and regular supply chains under different government policies. They considered three strategies for government including environmental protection, revenue seeking, weighted sum model of environmental protection and revenue-seeking policies. The equilibrium prices of each supply chain under government intervention have been obtained. Huang et al. (2016) applied game theory to study the impacts of product line design, supplier selection, transportation mode selection and pricing strategies on profits and greenhouse gases emissions in a green supply chain with multiple suppliers, a single manufacturer and multiple retailers. Guo et al. (2016) analyzed a supply chain system that consists of supplier, manufacturer, and government, and then investigates the effects of government subsidies on social welfare and the profits of supply chain members. Their results showed that a government's green tariffs depends on the sensitivity of consumers to prices. Under government financial intervention, Hafezalkotob (2017) developed price-energy-saving competition and cooperation models for two green supply chains. Their results showed that the government can lead the green supply chains to achieve the sustainability objectives by an appropriate tariff mechanism. Considering the government's role in the competition of two power plants, Mahmoudi et al. (2014) proposed a Nash bargaining game model to help the government to determine the taxes and subsidies. Their proposed approach demonstrated how the government can intervene in a competitive market of electricity to achieve the environmental objectives.

In the game theory framework, several oligopoly models have also been proposed to evaluate the strategic behavior in the electricity markets, including Bertrand, Cournot, and Supply Function Equilibrium (SFE). For instance, Cournot equilibria in oligopolistic electricity markets have been studied by Vespucci et al. (2009). By assuming a linear demand curve, they presented a model that describes the strategic interactions of firms based on this assumption that the generation firms are Cournot oligopolists. Li et al. (2004) used the SFE model to evaluate the power supplier's bidding behavior. They modeled the market power of an independent system operator (ISO) as a bi-level multi-objective problem. Hinz (2006) obtained the equilibrium strategies in random-demand procurement auctions in the electricity market and presented a method for explicit calculation of the bid strategies.

A review on the previous studies indicates that the proposed approach of this research covers two new features in comparison with the other existing models. First, the government is regarded as the leading player who intervenes in the competitive electricity market. Although the governmental economic incentives such as promoting and preventing policies for the environmental protection purposes have been investigated in some particular industries (Dobbs 1991; Dinan 1993; Ulph 1996; Fullerton and Wu 1998; Walls and Palmer 2001), the incentives have been rarely studied in the electricity industry. Second, to the best of the author's knowledge, no research was found in the context of green electricity market that uses the evolutionary game theory to model the energy source choosing strategy of the power plants.

This paper especially investigates the government's role as the Stackelberg leader in the strategies of the power plants as the Stackelberg followers. A bi-level programming model is proposed for the hierarchical decision-making framework. The main objective of this study is to evaluate the evolutionary production strategies of the power plants regarding the governmental financial interventions to fulfill the environmental protection purposes.

This paper particularly uses the mathematical game theory model to address the following research questions:

1. Using financial instruments such as tax and subsidy, how can a government intervene in competition of the power plants such that green purposes can be achieved?
2. Under the governmental interventions, what are the evolutionary responses of the competitive power plants and which strategy is used by the majority of the plants?

The rest of the paper is organized as follows. In Sect. 2, the proposed model and the elements of evolutionary game theory are presented. The ESS and derived equilibrium solutions are also introduced in this section. In Sect. 3, a numerical example is considered. Eventually, the concluding remarks and suggestions for the future research are given in Sect. 4.

Model formulation

In this paper, many (a sufficiently large number of) independent (geographically dispersed) markets are considered. It is assumed that all the markets are identical and the individuals in the population randomly compete with each other in pairs to play a game. In other words, there are exactly two power plants in every market. All one/two-population models assume that two individuals in one-shot game and representative market are copied several times such that the structures for all one-shot game are the same. This is a common assumption in the one/two-population evolutionary game model (Bester and Güth 1998; Xiao and Chen 2009): each power plant has two options for the type of its energy source which are called green and non-green energy sources. Green energy is the renewable energy sources that can be solar power, wind power, small-scale hydroelectric power, tidal power, or biomass power. These sources mostly do not produce pollutants; hence, they are called environmentally friendly or green sources. Renewable energies are regarded as a key factor in tackling with global climate changes and energy shortage crisis (Guler 2009). To keep generality of the proposed evolutionary game model, the model is not limited to a specific energy source; hence, the terms “green” and “non-green sources” are employed throughout the paper.

Government levies different levels of tariff for the power plants with respect to their energy sources. The government is considered as a profit-seeking agent which monitors pollution of the power plant population as well. Two scenarios are considered for the government decision procedures. In the first one, the government has an environmental protection behavior, i.e., its decisions are based on the goal of minimizing the pollution by considering a minimum level for its revenue. In the second one, the government has a revenue-seeking behavior, that is, its decisions are based on the goal of maximizing the revenue by considering a maximum level for the pollution. On the other hand, each power plant determines the electricity production strategy to maximize its profit. The goal of this paper is to find the ESS of the power plant’s production decisions and to determine the optimal government’s tariff with regard to evolutionary responses of the power plants.

For lucidity and simplicity, the subscript “g” is used for the green source and “ng” for the non-green one. Moreover, the indexes i and j indicate the production strategies of the competitive power plants where $i,j \in \{g, ng\}$. The parameters and variables used in the model formulae are as follows:

Parameters

- C_i — the unit production cost of the power plants when using the energy source i, $C_i > 0$;
- F_i — the initial setup cost of the power plants when using the energy source i, $F_i > 0$;
- η_{in} — The emission amount of pollutant gas n from the power plants using the energy source i, $\eta_{in} > 0$;
- w_{in} — the relative importance of the pollutant gas n that is produced from the power plant using the energy source i, $w_{in} > 0$;
- α_{ij} — the constant market base for the power plant that employs the energy source i versus the one which uses the energy source j, $\alpha_{ij} \geq 0$.
- Lb_{GNR} — the lower bound of the Government Net Revenue (GNR);
- Ub_{EIs} — the upper bound of the Environmental Impacts (EIs) according to the national or international standards;
- R — the reservation payoff for the power plants;
- β_{ij} — the demand sensitivity of the power plant to its own price, $\beta_{ij} > 0$;
- γ_{ij} — the demand sensitivity of the power plant to its rival’s price, $\gamma_{ij} > 0$;

Variables

- p_{ij} — The electricity price of the power plant that uses the energy source i versus the power plant that employs the energy source j, ($p_{ij} > C_i$)
- t_i — The tariff imposed by the government on the power plants using the energy source i (t_i is free in sign). If $t_i < 0$, then the government has provided a subsidy for consumers of this power plants; however, $t_i > 0$ indicates that the government has imposed a tax on the electricity
- D_{ij} — the demand of the power plant which employs the energy source $i \in \{g, ng\}$, against the power plant which uses the energy source $j \in \{g, ng\}$

The proposed game theory model is established on the following assumptions:

Assumption 1 The power plants play a symmetric two-person benefit matrix (bi-matrix) game, i.e., $B = A^{\mathrm{T}}$. A and B are the payoff matrixes of the first and second power plants, respectively. Practically, it means that in a symmetric game it does not matter who is the player I and who is the player II and the players can switch their roles. This assumption differs from the two-population evolutionary models (see Weibull 1997 for more information).

Assumption 2 It is assumed that the competitive power plants follow the government’s financial legislations and have the capability to produce electricity using two different energy sources. They are able to set up facilities for generating electricity from the specific sources. When they install and start up the corresponding power generations equipment, the production capacity is ample for the market demand. That is, the production rate of the power plants is equal to the corresponding demand rate. Moreover, they have a negligible internal consumption and waste rate.

Assumption 3 The demand function for each power plant is assumed continuous which takes the following forms:

$$D_{ij} = \alpha_{ij} - \beta_{ij}(p_{ij} + t_i) + \gamma_{ij}(p_{ji} + t_j)\ i,j \in \{g, ng\}. \quad (1)$$

D_{ij} is the demand function for the power plants employing the energy source $i \in \{g, ng\}$, against the power plant which uses the energy source $j \in \{g, ng\}$.

This function is a general linear demand function used in most of the previous studies (Shy 2003). The parameters β_{ij} and γ_{ij} denote independent and positive values that indicate the demand sensitivity to the prices of a power plant and its rival, respectively. Equation (1) states that the market demand of each power plant is an increasing function of its rival price, though a decreasing function of its own price.

Assumption 4 Regarding the leader role of the government, the time order of this game is assumed as follows:

Step 1. The government determines tariffs for the electricity generated from different sources. The government's tariffs are unchanged for a long time.

Step 2. Considering the tariffs, each power plant in each period adopts pricing strategy for the selected source.

Backward induction technique is used to investigate this dynamic game. In this regard, optimal electricity prices and ESS of the power plants were analyzed given the government's tariffs, then the government's decisions will be studied.

Profit function of power plant

The profit function for each power plant is formulated as follows:

$$\Pi_{ij} = (p_{ij} - C_i)D_{ij} - F_i = (p_{ij} - C_i)(\alpha_{ij} - \beta_{ij}(p_{ij} + t_i) + \gamma_{ij}(p_{ji} + t_j)) - F_i, i,j \in \{g, ng\}. \tag{2}$$

This function shows how the profit of each power plant depends on the electricity prices as well as the government's tariffs.

Bertrand game

In each iteration of evolutionary game, the two matched power plants play a one-shot, non-zero sum game which represents the benchmark game of the population. These power plants adopt Bertland competition in each market. According to the Bertland game model (Vives 1985), a simultaneous-move game is considered where they independently choose the electricity prices. Let (p_{ij}, p_{ji})Proposition be the prices of the power plants, respectively. 1 presents the Nash equilibrium of prices for the two matched power plants.

Proposition 1 *The equilibrium price for the power plants under the given government tariffs* $(t_g,\ t_{ng})$ *is as follows:*

$$p_{ij} = M_{ij} + C_i, \tag{3}$$

where $M_{ij} = [2\beta_{ji}\alpha_{ij} + \gamma_{ij}\alpha_{ji} + \beta_{ji}\gamma_{ij}(t_j + C_j) + (\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij})(t_i + C_i)]/(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji})$.

Proofs of all Propositions are given in Appendix A.

Proposition 2 *Power plant's demand and profit at the equilibrium prices for the given government's tariffs* $(t_g,\ t_{ng})$ *are obtained as follows:*

$$D^*_{ij} = \beta_{ij}M^*_{ij} = \beta_{ij}(\theta_{ij} + \tau_{ij}t_i + \chi_{ij}t_j), \tag{4}$$

$$\Pi^*_{ij} = \beta_{ij}M^{*^2}_{ij} - F_i = \beta_{ij}(\theta_{ij} + \tau_{ij}t_i + \chi_{ij}t_j)^2 - F_i, \tag{5}$$

where $\theta_{ij} = [2\beta_{ji}\alpha_{ij} + \gamma_{ij}\alpha_{ji} + \beta_{ji}\gamma_{ij}C_j + (\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij})C_i]/(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji})$, $\tau_{ij} = (\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij})/(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji})$ and $\chi_{ij} = \beta_{ji}\gamma_{ij}/(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji})$.

ESS of production decisions of power plants

In comparison with the traditional games, the evolutionary game theory emphasizes on the dynamics of strategy change more than the properties of strategy equilibrium. A strategy is called evolutionarily stable strategies (ESS), if it outperforms any invading strategy (Riechmann 2001). Nowadays, the evolutionary game theory is applied to analyze various gaming behaviors such as behaviors of firms and industries, biological and dynamical systems, and economic growth. Especially in the electricity market, Menniti et al. (2008) suggested the evolutionary game model to obtain near Nash equilibrium when more than two producers exist. Whenever there are only two pure strategies used in the population, the ESS definition is as follows (Barron, 2013):

2.3.1 Definition S^* is an ESS against S if and only if either (6) or (7) holds:

$$U(S^*, S^*) > U(S, S^*),\ \forall 0 \leq S \leq 1,\ S \neq S^*, \tag{6}$$

$$U(S^*, S^*) = U(S, S^*) \Rightarrow U(S^*, S) > U(S, S),\ \forall S \neq S^*. \tag{7}$$

An important idea of ESS is that eventually, the strategies will be chosen by the players who produce a better-than-average payoff. Let s_j denote the fraction of power plants in the population who are using the strategy *j*. If the strategy *j*, is an ESS, then the small fraction of individuals adopting other strategies in the population cannot obtain higher profit than the one adopting the strategy *j*. In the one-population evolutionary game with two actions, Friedman (1991) and Weibull (1997) showed that a locally asymptotically stable fixed point of any weak compatible dynamics is an ESS. Behavior of the power plants can evolve to an ESS through an imitating successful behavior following any weak compatible dynamics.

2.3.2. Definition The expected payoff of a player playing the strategy $i = 1,\ 2,\ \ldots,$ n is as follows:

Table 1 Bi-matrix for two power plants by different energy sources

		Power plant II	
Power plant I	Production strategy	Green	Non-green
	Green	$(\Pi_{g,g}, \Pi_{g,g})$	$(\Pi_{g,ng}, \Pi_{ng,g})$
	Non-green	$(\Pi_{ng,g}, \Pi_{g,ng})$	$(\Pi_{ng,ng}, \Pi_{ng,ng})$

$$E(i,\pi) = \sum_{k=1}^{n} a_{i,k} s_k = {}_i A\pi. \tag{8}$$

$$\text{where } \pi \in \Pi = \left\{ \pi = (s_1, s_2, \ldots, s_n) \middle| s_j \geq 0, j = 1,2,\ldots,n, \sum_{j=1}^{n} s_j = 1 \right\}. \tag{9}$$

A one-population model (please refer to Weibull 1997; Xiao and Chen 2009; Barron 2013) is assumed, in which two matched power plants play a symmetric two-person bi-matrix game in random contest. Then, the payoff (utility) bi-matrix of the matched power plants is studied regarding their adopted strategies (Table 1).

From Eqs. (5), it is understood that the payoff matrix of the power plant I is given by

$$A = \begin{bmatrix} a_{11} a_{12} \\ a_{21} a_{22} \end{bmatrix} = \begin{bmatrix} \Pi_{g,g} \Pi_{g,ng} \\ \Pi_{ng,g} \Pi_{ng,ng} \end{bmatrix} = \begin{bmatrix} \beta_{g,g} M_{g,g}^2 - F_g; \beta_{g,ng} M_{g,ng}^2 - F_g \\ \beta_{ng,g} M_{ng,g}^2 - F_{ng} \beta_{ng,ng} M_{ng,ng}^2 - F_{ng} \end{bmatrix}. \tag{10}$$

Obviously, the bi-matrix of the power plant II is $B = A^T$(Barron 2013). From (8), it is found that

$$E(I,\pi) = {}_1A\pi = a_{11}s_1 + a_{12}s_2 = (a_{11} - a_{12})s_1 + a_{12}, \tag{11}$$

$$E(II,\pi) = {}_2A\pi = a_{21}s_1 + a_{22}s_2 = (a_{21} - a_{22})s_1 + a_{22}. \tag{12}$$

Owning to symmetry of the one-population evolutionary game, it is found that $E(I,\pi) = E(g)$ and $E(II,\pi) = E(ng)$. If the demand matrix is considered for the power plant I, D, as $\begin{bmatrix} D_{g,g} & D_{g,ng} \\ D_{ng,g} & D_{ng,ng} \end{bmatrix}$, it is known that the demand matrix of the power plant II is D^{T}. Similar to Eqs. (11) and (12), $E(D_g)$ and $E(D_{ng})$ can be computed as:

$$E(D_g) = {}_1D\pi = (D_{g,g} - D_{g,ng})s_1 + D_{g,ng}, \tag{13}$$

$$E(D_{ng}) = {}_2D\pi = (D_{ng,g} - D_{ng,ng})s_1 + D_{ng,ng}. \tag{14}$$

It is supposed that the frequencies $\pi = (s_1, s_2, \ldots, s_n) = \pi(t) \in \Pi$ can change over time. Changes in the frequencies over time are described by the following system of differential equations (Barron 2013):

$$\frac{ds_i(t)}{dt} = s_i(t)[E(i,\pi(t)) - E(\pi(t),\pi(t))], \quad i = 1,2,\ldots,n. \tag{15}$$

A solution that does not change over time will be a steady-state equilibrium, or stationary solution. When $s_i(t)[E(i,\pi(t)) - E(\pi(t),\pi(t))] = 0$, then $ds_i(t)/dt = 0$ and $s_i(t)$ is not changing over time. If there are only two strategies in the population, it can be simplified down to one equation $s(t)$ using the substitutions $s_1(t) = s(t)$, $s_2(t) = 1 - s(t)$. Then:

$$\frac{ds(t)}{dt} = s(t)(1 - s(t))(E(1,\pi) - E(2,\pi)), \; \pi = (s, 1 - s), \tag{16}$$

where $0 \leq s(t) \leq 1$.

Inserting (11) and (12) into Eq. (16) yields:

$$ds(t)/dt = s(t)(1 - s(t))((a_{11} + a_{22} - a_{12} - a_{21})s(t) + a_{12} - a_{22}). \tag{17}$$

For the stationary solution, $ds(t)/dt = 0$ is solved:

$$s(t) = 0, \; s(t) = 1, \; s(t) = (a_{22} - a_{12})/(a_{11} + a_{22} - a_{12} - a_{21}). \tag{18}$$

Inserting $s(t)$ in Eqs. (11) and (12) shows that in the mixed Nash solution, the expected payoff is the same for each power plant:

$$E(\Pi) = E(1,\pi) = E(2,\pi) = (a_{11}a_{22} - a_{12}a_{21})/(a_{11} + a_{22} - a_{12} - a_{21}). \tag{19}$$

Proposition 3 *The Eq. (17) can be solved implicitly using integration by parts to give the implicitly defined solution:*

$$\frac{((a_{11} - a_{21})s - (a_{22} - a_{12})(1 - s))^{1/(a_{11} - a_{21}) + 1/(a_{22} - a_{12})}}{|1 - s|^{1/(a_{11} - a_{21})} s^{1/(a_{22} - a_{12})}} = Ce^t. \tag{20}$$

The Proposition 3 provides a function of time and equilibrium tariffs which can illustrate the behavior of power plants during the time. In other words, the equilibrium in the behavior of the power plant and their evolutionary learning during the time, until they reach a stable state, can be viewed using this function.

Lemma 1 *The two-player symmetric game with the matrix* $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, $B = A^{\mathrm{T}}$ *, is equivalent to the symmetric game with the matrix* $A = \begin{pmatrix} a_{11} - a & a_{12} - b \\ a_{21} - a & a_{22} - b \end{pmatrix}$, $B = A^{\mathrm{T}}$, *for any a, b, in the sense that they have the same set of Nash equilibria.*

After calculating the Nash equilibrium point (s), the following Proposition can be employed to investigate the ESS condition of the obtained point(s).

Proposition 4 *At the two-player symmetric game with the matrices* $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$, *and* $B = A^{\mathrm{T}}$, *when* $(a_{11} - a_{21})(a_{22} - a_{12}) \neq 0$, *the ESS* s^* *of the evolutionary game between power plants is computed by*

$$s^* = \begin{cases} 1 \; if \; (a_{11} - a_{21}) > 0, \; (a_{22} - a_{12}) < 0 \\ \dfrac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{21} - a_{12}} \\ 0 \\ 0 \; and \; 1 \end{cases} \tag{21}$$

$$s^* = \begin{cases} 1 \\ \dfrac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{21} - a_{12}} \; if \; (a_{11} - a_{21}) < 0, \; (a_{22} - a_{12}) < 0 \\ 0 \\ 0 \; and \; 1 \end{cases} \tag{22}$$

$$s^* = \begin{cases} 1 \\ \dfrac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{21} - a_{12}} \; if \; (a_{11} - a_{21}) < 0, \; (a_{22} - a_{12}) > 0 \\ 0 \\ 0 \; and \; 1 \end{cases} \tag{23}$$

$$s^* = \begin{cases} 1 \\ \dfrac{a_{22} - a_{12}}{a_{11} + a_{22} - a_{21} - a_{12}} \; .if \; (a_{11} - a_{21}) > 0, \; (a_{22} - a_{12}) > 0. \\ 0 \\ 0 \; and \; 1 \end{cases} \tag{24}$$

If $(a_{11} - a_{21}) > 0$, $(a_{22} - a_{12}) > 0$, the mixed Nash is not an ESS, then there are two evolutionary stable strategies, namely $s_1^* = (1, 0)$ and $s_2^* = (0, 1)$. To determine which one will be eventually chosen by the community of power plants, the Proposition 3 has to be used. In this case, the stationary solution of the evolutionary game of the power plants depends on the initial condition of the power plants.

Model of government

A government normally aims to take a measure which optimizes the pollution level and its net revenue. Two different scenarios are assumed for these objective functions. First, the government minimizes the Environmental Impacts (EIs) subject to specific conditions on its net revenue and power plant's profit. According to the Kyoto protocol in 1992, governments should take actions to reduce pollution by raising the percentage of green electricity supply (Yoo and Kwak 2009). The total amount of pollution generated by the power plants is an important factor for the policy makers of any government. In the second scenario of the developed model, it is assumed that the government considers a value, Ub_{EIs}, for the maximum permissible level of total pollution generated by the power plants. Thereby, the government maximizes its net revenue owing to the upper bound of EIs and the lower bound on utility of the power plant. The proposed model in the first scenario can be expressed as:

$$\begin{aligned} &\min \mathrm{EIs} = N \sum_{m=1}^{M} w_{g,m} \eta_{g,m} E(D_g) + N \sum_{m=1}^{M} w_{ng,m} \eta_{ng,m} E(D_{ng}), \\ &s.t \\ &N t_g E(D_g) + N t_{ng} E(D_{ng}) \geq Lb_{\mathrm{GNR}} \\ &E(\Pi) \geq R, \\ &t_g, \; t_{ng} \text{ free in sign.} \end{aligned} \tag{25}$$

It is noteworthy that there are N power plants in the population and M types of the pollutants are considered with different importance weights. In this nonlinear programming problem, the objective function represents the EIs for pollution of the power plants. According to the green policy, the government would minimize the total weighted pollutant. The first constraint assures that the government net revenue (GNR) from the power plants does not become smaller than Lb_{GNR}. The second constraint is individual rational constraint (IR) for the expected payoff of the power plants. Under this condition, the power plants would like to accept government's tariffs; otherwise, they will reject the tariffs and withdraw from the electricity market. In the other words, IR constraint guarantees that the power plants would like to have a long-term relationship with the government. The suggested model for the second scenario can be expressed as:

$$\begin{aligned} &Max \, \mathrm{GNR} = N t_g E(D_g) + N t_{ng} E(D_{ng}), \\ &s.t \\ &N \sum_{m=1}^{M} w_{g,m} \eta_{g,m} E(D_g) + N \sum_{m=1}^{M} w_{ng,m} \eta_{ng,m} E(D_{ng}) \leq Ub_{\mathrm{EIs}} \\ &E(\Pi) \geq R, \\ &t_g, \; t_{ng} \text{ free in sign.} \end{aligned} \tag{26}$$

In this optimization problem, the objective function represents the GNR; hence, the government would maximize its net revenue from both the green and non-green power plants. The first constraint assures that the environmental impacts of the power generation activities do not exceed the upper bound Ub_{EIs}. The second constraint is IR condition of the power plants. To obtain optimal policy of the government, its models at the equilibrium prices should be solved.

Table 2 Power plants data

Parameters	Energy source		Parameters	Energy source	
	Green	Non-green		Green	Non-green
C	10	13	η_2	15	20
F	700	350	w_1	0.5	0.6
η_1	20	25	w_2	0.6	0.7

Table 3 Data of demand function $(\alpha_{ij}, \beta_{ij}, \gamma_{ij})$

		Power plant j	
Power plant i	Production strategy	Green	Non-green
	Green	(1400, 16, 17)	(1300, 18, 15)
	Non-green	(1700, 14, 18)	(1600, 15, 16)

In the models (25) and (26), all the object functions and constraints are nonlinear functions in $t_{g,g}$, $t_{g,ng}$, $t_{ng,g}$ and $t_{ng,ng}$. Therefore, the problems (25)–(26) are nonlinear programming problems which can be simply solved by a nonlinear programming solver. We perform all the numerical calculations by optimization toolbox of MATLAB 14.

Numerical example

In this section, a numerical example is provided to demonstrate how the theoretical results, in this paper, can be applied in practice. It is supposed that there are a population of 100 power plants in a competitive market. All these power plants have the same market and structural characteristics. For power generation, each power plant has two options for the green or the non-green energy sources. To analyze the sensitivity of the model to characteristics of being green, the data of numerical examples were presented in a way that the advantage of green energy source over non-green energy source was the environmental features. Moreover, the market characteristics for the non-green energy source were evaluated better than those of the green energy source. Parameters in this numerical example are listed in Tables 2 and 3.

It is assumed that $Lb_{\mathrm{GNR}} = 10,000$, $Ub_{\mathrm{EIs}} = 15,000$, $R = 1000$. First, the government model will be solved at the equilibrium price of the power plants to get tariffs of the power plant. Then, the game will be analyzed using the evolutionary game theory.

When $-10^5 \leq t_g,\ t_{ng} \leq 10^5$, Fig. 1 illustrates the surface of objective function (EIs) in the first scenario. From Fig. 1, it can be understood that when the maximum level of tax to the non-green energy source and the minimum level of subsidies for the green energy source are applied, EIs is minimal. On contrary, the EIs is maximal, when the maximum level of tax to the green energy source and the minimum level of subsidies for the non-green energy source are applied.

Figure 2 shows the surface of objective function in the second scenario. In comparison with $t_g > t_{ng}$, from Fig. 2, it is obvious that the government has the most revenue when $t_{ng} > t_g$. From Fig. 1, it is visible that in the first scenario, subsidy will be applied for the green energy source and the tax will be applied for the non-green energy source. However, Fig. 2 illustrates that the government imposes a rather high tax for the non-green energy source to maximize the GNR in the second scenario. The calculated values for this example are summarized in Table 4. The results of optimal, tariffs, electricity prices, profit

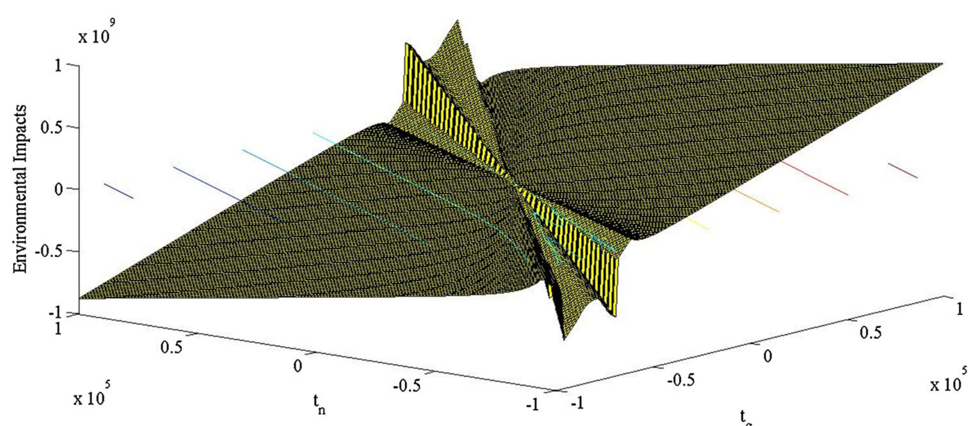

Fig. 1 Surface of objective function in first scenario (EIs)

Table 4 Details of calculated values and example results

Variable	First scenario	Second scenario	Variable	Second scenario	First scenario
t_g	−22.283	391.64	$D_{ng,g}$	1795.4	1365
t_{ng}	24.024	500	$D_{ng,ng}$	2263.9	1753.9
$p_{g,g}$	102.51	130.1	$\Pi_{g,g}$	230,080	136,240
$p_{g,ng}$	108.31	112.55	$\Pi_{g,ng}$	188,580	173,250
$p_{ng,g}$	110.5	141.24	$\Pi_{ng,g}$	229,900	132,740
$p_{ng,ng}$	129.93	163.93	$\Pi_{ng,ng}$	341,340	204,730
$D_{g,g}$	1480.2	1921.6	GNR	1.6508e + 08	*
$D_{g,ng}$	1769.5	1845.8	Els	*	6,938,800
			S	1	0.8998

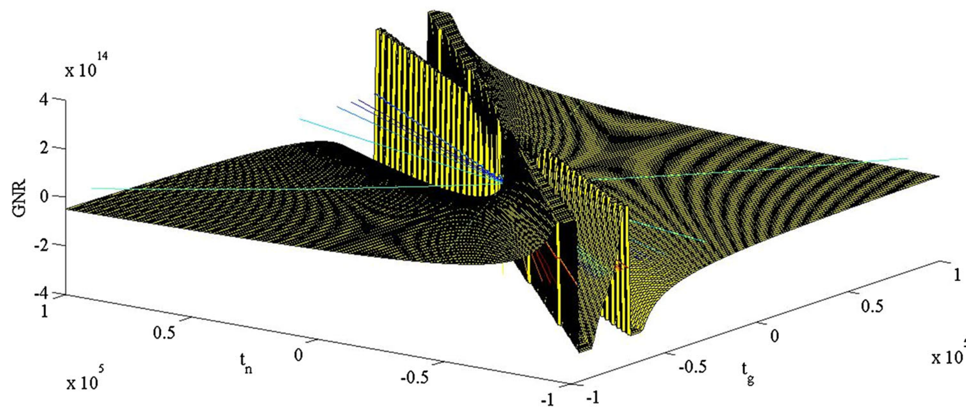

Fig. 2 Surface of objective function in second scenario (GNR)

value of power plants, and objective function, in each scenario, are given in the rows of this table.

From Proposition 4, it can be inferred that the Nash equilibria that are evolutionary stable are found as $X_1^* = (1,0), X_2^* = (0,1)$. Figure 3 shows how the strategies of the power plants converge to ESS (1, 0) or (0, 1). The trajectory of ds/dt for $x_3^* = (0.8998, 0.1002)$ and five different initial conditions have been shown in Fig. 4.

The example results shown in the second scenario, $x_1^* = (1,0)$, $x_2^* = (0,1)$ are symmetric Nash equilibria. Both of these Nash equilibria are evolutionary stable, because $\Pi_{g,g} - \Pi_{ng,g} = 180 > 0$ and $\Pi_{ng,ng} - \Pi_{g,ng} = 152760 > 0$. Figure 5 indicates how the strategies of the power plants converge to ESS (1, 0) or (0,1). The trajectory of ds/dt for $x_1^* = (1,0)$ and four different initial conditions have been shown in Fig. 6.

From Figs. 5 and 6, it is implied that under the government tariffs, $X_1^* = (1,0)$ is the ESS point of the game and all the power plants will be driven to adopt the green energy source in the long-term evolution. From the numerical example, it is found that the tariffs imposed by the government have important short-term and long-term effects on the source selection decisions of the power plants. Sensitivity analysis on the tariffs can determine the short-term strategies of the power plants. Furthermore, they can show how the strategies of the power plants evolve in long term. Therefore, the results of the sensitivity analyses can reveal appropriate decisions of the government with respect to the budget limitations and environmental standard considerations.

Conclusions

This study is a contribution to the growing research on the development of rigorous mathematical and game theory frameworks for the environmental-energy modeling. In a competitive electricity market, the proposed computational framework helps the governmental policy makers to determine appropriate tariffs for each of the individual electric power plants considering the energy source used by them. A numerical example was presented to analyze the performance of the model in different two scenarios of the model. This numerical example also demonstrates how the policy makers could determine the appropriate tariffs to achieve the desired short-term and long-term environmental objectives.

There are several directions and suggestions for future research. First of all, the proposed model can be extended to the case where more than two energy sources exist with different environmental effects. Secondly, in the present study, the demand function for each power plant was assumed in the linear form. However, other types of the

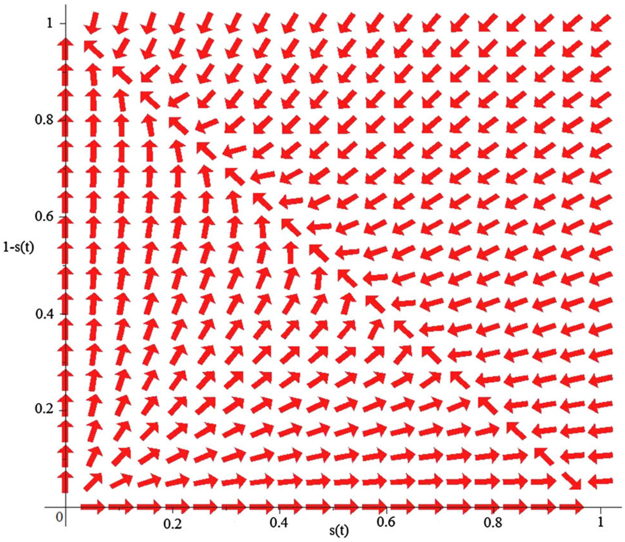

Fig. 3 Convergence to ESS (1, 0) or (0, 1) in first scenario

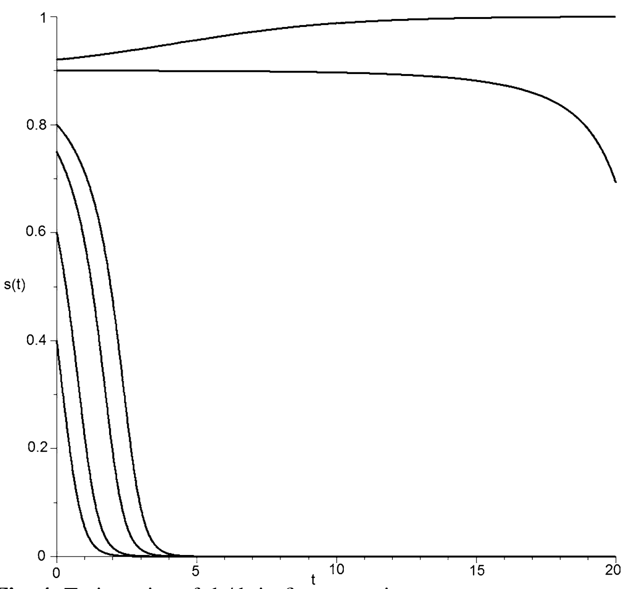

Fig. 4 Trajectories of ds/dt in first scenario

demand function can be assumed for future research. Moreover, it would be very interesting but challenging to consider the uncertainty on other model parameters such as electricity production costs or demand function of the power plants. Eventually, application of the proposed framework can be extended to some other markets.

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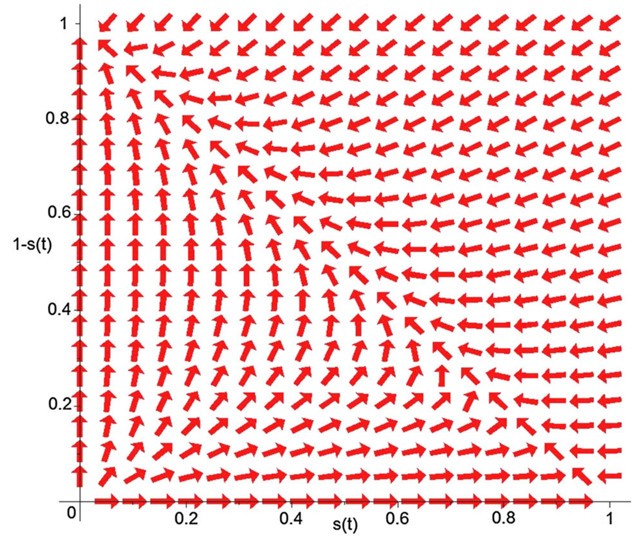

Fig. 5 Convergence to ESS (1, 0) or (0, 1) in second scenario

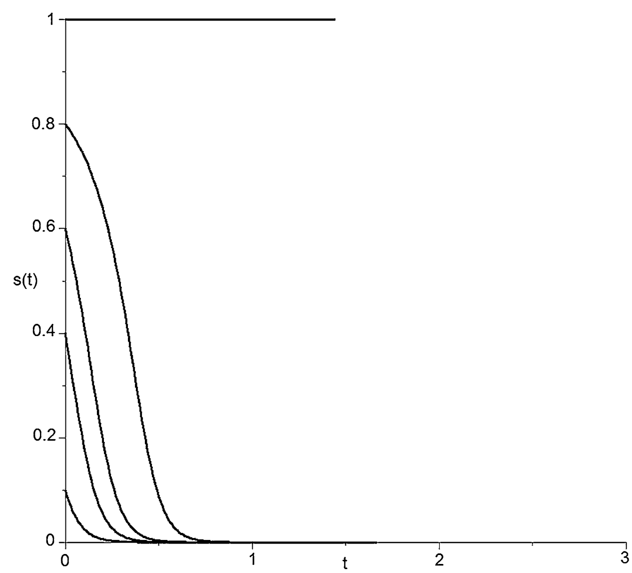

Fig. 6 Trajectories of d*s*/d*t* in second scenario

Appendix A

Proof of Proposition 1 The first-order conditions of profit of power plants are

$$\partial \Pi_{ij}/\partial p_{ij} = \alpha_{ij} - 2\beta_{ij}(p_{ij} - C_i) + \gamma_{ij}(p_{ji} - C_j) + \gamma_{ij}(t_j + C_j) - \beta_{ij}(t_i + C_i) = 0, \tag{27}$$

Now, let us define the following variables:

$$M_{ij} = p_{ij} - C_i, \tag{28}$$

Using M_{ij} and M_{ji}, we rewrite the first-order conditions as:

$$2\beta_{ij}M_{ij} - \gamma_{ij}M_{ji} = \alpha_{ij} + \gamma_{ij}(t_j + C_j) - \beta_{ij}(t_i + C_i), \tag{29}$$

$$2\beta_{ji}M_{ji} - \gamma_{ji}M_{ij} = \alpha_{ji} + \gamma_{ji}(t_i + C_i) - \beta_{ji}(t_j + C_j), \tag{30}$$

By solving (Eq. 31) and (Eq. 32) simultaneously, we have

$$M_{ij} = \frac{2\beta_{ji}\alpha_{ij} + \gamma_{ij}\alpha_{ji} + \beta_{ji}\gamma_{ij}(t_j + C_j) + (\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij})(t_i + C_i)}{(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji})}. \tag{31}$$

The p^*_{ij} and p^*_{ji} obtained from Eq. (3) are the optimum prices if the profit functions are concave on p_{ij} and p_{ji}. The second derivative of the function is as follows:

$$\partial^2 \Pi_{ij}/(\partial p_{ij})^2 = -\beta_{ij} - \beta_{ij} = -2\beta_{ij} \leq 0, \tag{32}$$

It is known that β_{ij} is the positive value. Therefore, it is obvious that the second derivative of the profit function in the equilibrium is negative; hence, the function is concave at this point. □

Proof of Proposition 2 By substituting M^*_{ij} obtained from Proposition 1 into Eqs. (1) and (2), after some mathematical manipulations, the demand and profit function can be simplified into

$$D^*_{ij} = \beta_{ij}M^*_{ij} = \beta_{ij}(\theta_{ij} + \tau_{ij}t^*_{ij} + \chi_{ij}t^*_j) \tag{33}$$

$$\Pi^*_{ij} = \beta_{ij}M^{*^2}_{ij} - F_i = \beta_i(\theta_{ij} + \tau_{ij}t^*_{ij} + \chi_{ij}t^*_j)^2 - F_i. \tag{34}$$

That θ_{ij}, τ_{ij} and χ_{ij} are defined as Proposition 2. □

Proof of Proposition 3 The Eq. (17) is equal to.

$$ds(t)/s(t)(1 - s(t))((a_{11} - a_{21}).s(t) - (a_{22} - a_{12}) .(1 - s(t))) = dt, \tag{35}$$

From (Barron 2013), it is known that the integration of $ds/s(1 - s)(as - b(1 - s)) = dt$ is

$$(as - b(1 - s))^{\frac{1}{a}+\frac{1}{b}}/(1 - s)^{\frac{1}{a}}s^{\frac{1}{b}} = Ce^t, \tag{36}$$

Therefore, from (35) and (36) we can get (20). □

Proof of Lemma 1 The set of Nash equilibria in the game with matrix $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is

$$\frac{ds(t)}{dt} = s(t)(1 - s(t))((a_{11} + a_{22} - a_{12} - a_{21}).s(t) + a_{12} - a_{22}). \tag{37}$$

And the set of Nash equilibria in the game with matrix $A = \begin{pmatrix} a_{11} - a & a_{12} - b \\ a_{21} - a & a_{22} - b \end{pmatrix}$ is

$$\begin{aligned}\frac{ds(t)}{dt} &= s(t)(1 - s(t))((a_{11} - a + a_{22} - b - a_{12} + b - a_{21} + a) \\ &\quad .s(t) + a_{12} - b - a_{22} + b) \\ &= s(t)(1 - s(t))((a_{11} + a_{22} - a_{12} - a_{21}).s(t) + a_{12} - a_{22}).\end{aligned} \tag{38}$$

(37) and (38) show that these games have the same set of Nash equilibria. □

Proof of Proposition 4 From the Lemma 1, for given any two-player symmetric game with matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ and $B = A^T$ then the game is equivalent to the symmetric game with matrix $A = \begin{bmatrix} a_{11} - a & a_{12} - b \\ a_{21} - a & a_{22} - b \end{bmatrix}$ and $B = A^T$. The ESS conditions given in Definition 1 are not influenced by this transformation (Webb 2007). In the power plant's payoff matrix (10), let $a = a_{21}$ and $b = a_{12}$, then we have the equivalent matrix $\mathbf{A} = \begin{bmatrix} a_{11} - a_{21} & 0 \\ 0 & a_{22} - a_{12} \end{bmatrix}$.

Now, we evaluate the cases of Proposition 2.

Cases (21) and **(23)** In each case, there is a unique strict symmetric Nash equilibrium. Therefore, the ESSs are either $(s, 1 - s) = (1, 0)$ (in Case 21) or $(s, 1 - s) = (0, 1)$ (in Case 23).

Case (24) In this case, $X_1 = (s, 1 - s) = (1, 0)$, $X_2 = (s, 1 - s) = (0, 1)$, and $X_3 = (s^*, 1 - s^*)$ are three symmetric Nash equilibriums, where s^* is defined by Eq. (18). The two pure Nash equilibriums X_1, X_2 are strict and, therefore, are ESSs. In view of Definition 2.3.1, the mixed Nash X_3 is not an ESS, because condition

$$\begin{aligned}(s^*, 1 - s^*)\mathbf{A}(s^*, 1 - s^*)^T &= (s, 1 - s)\mathbf{A}(s^*, 1 - s^*)^T \\ &= [a^*_{11} - a^*_{21}]s^*,\end{aligned} \tag{39}$$

is hold for each $s \in [0, 1]$ and

$$\begin{aligned}(s^*, 1 - s^*)\mathbf{A}(1, 0)^T &= [a^*_{11} - a^*_{21}]s^* < (1, 0)\mathbf{A}(1, 0)^T \\ &= a^*_{11} - a^*_{21}.\end{aligned} \tag{40}$$

Therefore, X_3 does not satisfy condition (7) of Definition 2.3.1.

Case (22) The symmetric Nash equilibrium $X_3 = (s^*, 1 - s^*)$ where s^* is defined by Eq. (18) is an ESS because

$$\begin{aligned}(s^*, 1 - s^*)\mathbf{A}(s^*, 1 - s^*)^T &= (s, 1 - s)\mathbf{A}(s^*, 1 - s^*)^T \\ &= [a^*_{11} - a^*_{21}]s^*.\end{aligned} \tag{41}$$

is hold for each $s \in [0, 1]$ and regarding $a^*_{11} < a^*_{21}$ and $a^*_{22} < a^*_{12}$ we know

$$\begin{aligned}(s^*, 1 - s^*)\mathbf{A}(s, 1 - s)^T &= [a^*_{11} - a^*_{21}]s^* > (s, 1 - s)\mathbf{A}(s, 1 - s)^T \\ &= [a^*_{11} - a^*_{21}]s^2 + [a^*_{22} - a^*_{12}](1 - s)^2,\end{aligned} \tag{42}$$

is hold for each $s \in [0, 1]$. Thus, X satisfies the conditions of Definition 2.3.1 and it is an ESS. □

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