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ORIGINAL RESEARCH

Analysis of two production inventory systems with buffer, retrials and different production rates

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Abstract This paper considers the comparison of two (s, S) production inventory systems with retrials of unsatisfied customers. The time for producing and adding each item to the inventory is exponentially distributed with rate β. However, a production rate $\alpha\beta$ higher than β is used at the beginning of the production. The higher production rate will reduce customers' loss when inventory level approaches zero. The demand from customers is according to a Poisson process. Service times are exponentially distributed. Upon arrival, the customers enter into a buffer of finite capacity. An arriving customer, who finds the buffer full, moves to an orbit. They can retry from there and inter-retrial times are exponentially distributed. The two models differ in the capacity of the buffer. The aim is to find the minimum value of total cost by varying different parameters and compare the efficiency of the models. The optimum value of α corresponding to minimum total cost is an important evaluation. Matrix analytic method is used to find an algorithmic solution to the problem. We also provide several numerical or graphical illustrations.

Keywords Production inventory · Buffer · Retrial · Matrix analytic method · Cost analysis

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Introduction

A queue is formed when either there is positive service time or there are no sufficient servers for the arriving customers. Queuing systems in which an arriving customer finds the server busy and waiting positions (if any) occupied leaves the service area but repeats his demand after some random time are called retrial queues. Between trials, customer is said to be in an orbit. Retrial queues play an important role in communication and computer networks. Other applications include stacked aircraft waiting to land, ticket reservation for trains and flights and queues of retail shoppers who may leave a long waiting line hoping to return later when the line may be shorter. For detailed discussion on retrial queues, one may refer to the monograph by Falin and Templeton (1997) and the bibliography by Artalejo (2010).

The analysis of inventory systems with retrials has received little attention of researchers in recent decades. Inventory is the raw materials, goods in different stages of production and finished goods, owned by a company that are ready or will be ready for sale. When customers arrive into a system and if the demanded item is available the same is provided with negligible or positive service time. If the item is out of stock, such customers need not be backlogged or lost; otherwise they move to an orbit and may retry from there.

However, retrial in production inventory has received little attention of the researchers in stochastic analysis. So we considered a mathematical model in which the main contributions of this paper are summarized as follows.

- Two production inventory systems with buffer are developed.
- Matrix analytic method is used to solve the systems.

- Some important performance measures of the systems are derived and a cost function is defined.
- The optimum value of α corresponding to the minimum expected total cost is found.
- The minimum value of expected total cost is found by varying different parameters of the model.
- The models are compared numerically and suggested best model for practical purposes.

These models can be applied to manufacturing systems with stochastic environment.

The rest of the paper is organized as follows. In Sect. 2, a brief review of literature is presented. In Sect. 3, we formulate the problem. In Sect. 4, we describe model I and its stability. We provide performance measures of model I in Sect. 5. We describe model II and its stability in Sect. 6 and the performance measures of model II in Sect. 7. Cost analysis is described in Sect. 8. Numerical results and graphical illustrations are presented in Sects. 9 and 10. In Sect. 11, we incorporate concluding remarks and future research.

Literature review

Artalejo et al. (2006) introduced retrial of unsatisfied customers in inventory systems with positive lead time. They compared numerically the efficiency of the generalized truncated model with a model based on finite truncation. There after some important works Krishnamoorthy and Jose (2007), Yadavalli et al. (2012), Jeganathan et al. (2013), etc., were reported in this direction. Recently, Padmavathi et al. (2015) analyzed a continuous review stochastic (s, S) inventory system. They considered two models which differ in the way that the server goes for vacation. Here the joint probability distribution of the inventory level, the number of demands in the orbit and the server status is obtained in the steady state case. Vijaya Laxmi and Soujanya (2015) described an (s, S) inventory system with service interruptions and retrial of negative customers. The arrival and service interruptions were according to a Poisson process. The lead time and inter-retrial times were exponentially distributed and solution was obtained in the steady state. Krishnamoorthy et al. (2015a) analyzed a queuing-inventory system with common life time and retrial of unsatisfied customers. The arrival of customers followed a Poisson process and all the underlying distributions were assumed to be exponential. In this, reservation and cancellation of inventory is permitted. Expected number of revisits to the maximum inventory level and sojourn times in the maximum inventory level as well as zero inventory are also computed.

The main area of literature related to this paper is that of inventory systems with production. Krishnamoorthy and Jose (2008) compared three production inventory systems with positive service time and retrial of customers by assuming all the underlying distributions to be exponential. They obtained that the model with buffer size equal to the inventoried items is the best profitable model for practical purposes. Benjaafar et al. (2010) analyzed a production inventory system as a Markov decision process and compared the performance of the optimal policy against several other policies and obtained that performance is poor for those models that ignore impatience of the customer. Chang and Lu (2011) studied a serial production inventory system by providing a phase-type approximation and obtained good estimates for performance measures such as fill rate and mean queue-length distributions of each station. An efficient production and service scheduling rule to a flexible production service system was proposed by Wang et al. (2013). They extended the optimal service scheduling policy in the classical service system. Yu and Dong (2014) considered a production lot size problem as a renewal process and used a numerical approach to find out the optimal solution to the problem. Karimi-Nasab and Sabri-Laghaie (2014) constructed three randomised approximation algorithms to optimize an imperfect production problem that creates defectives randomly. The algorithms can find the global optimum in polynomial time under certain conditions. Anoop and Jacob (2015) studied a multiserver Markovian queuing system by considering the servers as a standard (s, S) production inventory and they obtained the condition for checking ergodicity and the steady state solutions. Krishnamoorthy et al. (2015b) analyzed an (s, S) production inventory system where interruptions to both service process and production process may occur and obtained an explicit expression for the stability of the system. They studied numerically the dependence of system performance measures on the system parameters. Rashid et al. (2015) considered a single item inventory system and extended it to multi item by proposing a new heuristic algorithm. They considered demand and production times as stochastic parameters to calculate long run inventory costs. Baek and Moon (2016) analyzed an (s, S) production inventory system and found out an explicit stationary joint probability in product form and proposed probabilistic interpretations for the inventory model.

Recently, Cheng et al. (2016) considered the problem of minimizing the total cost, which includes the production, delivery and inventory costs. They proposed a fast approximation algorithm with three different absolute and asymptotic worst case ratios for the jobs have identical sizes, identical processing times and both arbitrary sizes and arbitrary processing times. De et al. (2016) developed a

Table for literature of retrial and production inventory

References	Demand	Retrial inventory	Production inventory	Replenishment policy		
				Policy	Single rate	Different rates
Artalejo et al. (2006)	Stochastic	Yes		(s, S)	Yes	
Krishnamoorthy and Jose (2007)	Stochastic	Yes		(s, S)	Yes	
Krishnamoorthy and Jose (2008)	Stochastic	Yes	Yes	(s, S)	Yes	
Benjaafar et al. (2010)	Stochastic		Yes	(s, S)	Yes	
Chang and Yang-Shu (2011)	Stochastic		Yes	$(S-1, S)$	Yes	
Yadavalli et al. (2012)	Stochastic	Yes		(s, S)	Yes	
Wang et al. (2013)	Stochastic		Yes	(s, S)	Yes	
Jeganathan et al. (2013)	Stochastic	Yes		(s, S)	Yes	
Karimi-Nasab and Sabri-Laghaie (2014)	Deterministic		Yes	(R, Q)	Yes	
Yu and Dong (2014)	Stochastic		Yes	(R, Q)	Yes	
Krishnamoorthy et al. (2015a)	Stochastic	Yes		(s, S)	Yes	
Krishnamoorthy et al. (2015b)	Stochastic		Yes	(s, S)	Yes	
Anoop and Jacob (2015)	Stochastic		Yes	(s, S)	Yes	
Vijaya Laxmi and Soujanya (2015)	Stochastic	Yes		(s, S)	Yes	
Rashid et al. (2015)	Stochastic		Yes	(R, Q)	Yes	
Padmavathi et al. (2015)	Stochastic	Yes		(s, S)	Yes	
Baek and Moon (2016)	Stochastic		Yes	(s, S)	Yes	
De et al. (2016)	Stochastic				Yes	
Cheng et al. (2016)	Stochastic		Yes		Yes	
Our Model	Stochastic	Yes	Yes	(s, S)		Yes

mathematical model for ship routing problem for varying demand and supply scenario at different ports. They presented a Mixed Integer Non-Linear Programming (MINLP) model which includes the issues pertaining to multiple time horizons, sustainability aspects and varying demand and supply at various ports. They solved the model using Particle Swarm Optimization of Composite Particle (PSO-CP), basic Composite Particle (CP) and Genetic Algorithm (GA).

As presented in above table, there are some papers which dealt with retrial in inventory. Some others dealt with production inventory. Even among those researchers who worked on retrial and production inventory, assumption of different production rates is new.

Mathematical formulation

We consider two production inventory systems where items are produced one unit at a time according to (s, S) policy. That is, when the inventory level falls to s production starts and it stops when the inventory level reaches back to S. The time for producing each item to the inventory is exponentially distributed. The production rate is $\alpha\beta$ when production starts, where $\alpha \in [1, k]$ and k is a finite value greater than 1; but the rate is β, when level crosses above s, i.e., for the level from $s + 1$ to S. Items in inventory incur a holding cost c_2 per unit per unit time. The demand from customers is according to a Poisson process with rate λ. Upon arrival, the customers enter into a buffer of finite capacity. Orders are fulfilled if inventory is available. Service times are exponentially distributed with parameter μ. In model I, we provide a buffer having capacity equal to the maximum inventory level S and in model II a buffer having varying capacity equal to the current inventory level. The system incurs holding cost of customers c_4 in the buffer per unit per unit time. When a customer enters into the system and finds the buffer full, he moves to an orbit of infinite capacity with probability γ and is lost forever with probability $(1—\gamma)$. The system incurs holding cost of customers c_3 in the orbit per unit per unit time. If a customer retries from the orbit and finds the buffer full, he returns to the orbit with probability δ and is lost forever with probability $(1—\delta)$. Inter-retrial times follow an exponential distribution with linear rate $i\theta$ when there are i customers in the orbit.

Assumptions

(i) Inter-arrival times of demands are exponentially distributed with parameter λ.
(ii) Service times are exponentially distributed with rate μ.
(iii) Production time inventory is exponentially distributed as $\alpha\beta$, when the inventory level lies between 0 and s; otherwise it is β.
(iv) Inter-retrial times are exponential with linear rate $i\theta$, when there are i customers in the orbit.

Notations

$I(t)$: inventory level at time t.
$N(t)$: Number of customers in the orbit at time t.
$M(t)$: Number of customers in the buffer at time t.

$$J(t) = \begin{cases} 0, & \text{if the production is in OFF mode} \\ 1, & \text{if the production is in ON mode} \end{cases}$$

$e : (1, 1, \ldots 1)'$ a column vector of 1's of appropriate order.

Description of model I

In this model, we provide a buffer having capacity equal to the maximum inventory level S. Let $I(t)$ be the inventory level and $N(t)$ be the number of customers in the orbit at time t. Let $M(t)$ be the number of customers in the buffer at time t. Let $J(t)$ be the production status which is equal to 1 if the production is in ON mode and 0 if the production is in OFF mode. Now $\{X(t), t \geq 0\}$, where $X(t) = (N(t), J(t), I(t), M(t))$ is a level dependent quasi birth–death process on the state space $\{(i, 0, j, k);\ i \geq 0; j = s+1, \ldots S,\ k = 0, 1, \ldots, S\} U \{(i, 1, j, k); i \geq 0; j = 0, \ldots, S-1, k = 0, 1, \ldots, S\}$. The infinitesimal generator Q, of the process is a block tri-diagonal matrix and it has the following form:

$$Q = \begin{bmatrix} A_{1,0} & A_0 & & & & \\ A_{2,1} & A_{1,1} & A_0 & & & \\ & A_{2,2} & A_{1,2} & A_0 & & \\ & & A_{2,3} & A_{1,3} & A_0 & \\ & & & \ddots & \ddots & \ddots \end{bmatrix} \quad (1)$$

where the blocks $A_0, A_{1,i} (i \geq 0)$ and $A_{2,i} (i \geq 1)$ are square matrices, each of order $(S+1)(2S-s)$; they are given by

$$A_0 = \begin{array}{c} \underline{0, s+1} \\ \underline{0, s+2} \\ \vdots \\ \underline{0, S} \\ \underline{1, 0} \\ \underline{1, 1} \\ \vdots \\ \underline{1, S-1} \end{array} \begin{bmatrix} F & & & & & & & \\ & F & & & & & & \\ & & \ddots & & & & & \\ & & & F & & & & \\ & & & & F & & & \\ & & & & & F & & \\ & & & & & & \ddots & \\ & & & & & & & F \end{bmatrix}$$

$$A_{1,i} = \begin{array}{c} \underline{0, s+1} \\ \underline{0, s+2} \\ \vdots \\ \underline{0, S} \\ \underline{1, 0} \\ \underline{1, 1} \\ \vdots \\ \underline{1, s} \\ \underline{1, s+1} \\ \underline{1, s+2} \\ \vdots \\ \underline{1, S-1} \end{array}$$

$$\begin{bmatrix} K & & & & & & & & J & & & \\ J & K & & & & & & & & & & \\ & \ddots & \ddots & & & & & & & & & \\ & & J & K & & & & & & & & \\ & & & & L_0 & M_1 & & & & & & \\ & & & & J & L_1 & M_1 & & & & & \\ & & & & & \ddots & \ddots & \ddots & & & & \\ & & & & & & J & L_1 & M_1 & & & \\ & & & & & & & J & L & M & & \\ & & & & & & & & J & L & \ddots & \\ & & & & & & & & & \ddots & \ddots & M \\ & & & M & & & & & & & J & L \end{bmatrix}$$

$$A_{2,i} = \begin{array}{c} \underline{0, s+1} \\ \underline{0, s+2} \\ \vdots \\ \underline{0, S} \\ \underline{1, 0} \\ \underline{1, 1} \\ \vdots \\ \underline{1, S-1} \end{array} \begin{bmatrix} V & & & & & & & \\ & V & & & & & & \\ & & \ddots & & & & & \\ & & & V & & & & \\ & & & & V & & & \\ & & & & & V & & \\ & & & & & & \ddots & \\ & & & & & & & V \end{bmatrix}$$

(p, q)th element of the matrices contained in A_0, $A_{1,i}$ and $A_{2,i}$ are given by $[F]_{pq} = \begin{cases} \lambda\gamma, & p = q = S+1 \\ 0, & \text{otherwise} \end{cases}$,

$$[K]_{pq}=\begin{cases}-(\lambda+i\theta), & p=q=1\\ -(\lambda+\mu+i\theta), & 2\le p\le S, q=p\\ -(\lambda\gamma+\mu+i\theta(1-\delta)), & p=q=S+1\\ \lambda, & 1\le p\le S, q=p+1\\ 0, & \text{otherwise}\end{cases}$$

$$[L_0]_{pq}=\begin{cases}-(\lambda+\alpha\beta+i\theta), & 1\le p\le S, q=p\\ -(\lambda\gamma+\alpha\beta+i\theta(1-\delta)), & p=q=S+1\\ \lambda, & 1\le p\le S, q=p+1\\ 0, & \text{otherwise}\end{cases}$$

$$[L_1]_{pq}=\begin{cases}-(\lambda+\alpha\beta+i\theta), & p=q=1\\ -(\lambda+\alpha\beta+\mu+i\theta), & 2\le p\le S, q=p\\ -(\lambda\gamma+\alpha\beta+\mu+i\theta(1-\delta)), & p=q=S+1\\ \lambda, & 1\le p\le S, q=p+1\\ 0, & \text{otherwise}\end{cases}$$

$$[L]_{pq}=\begin{cases}-(\lambda+\beta+i\theta), & p=q=1\\ -(\lambda+\beta+\mu+i\theta), & 2\le p\le S, q=p\\ -(\lambda\gamma+\beta+\mu+i\theta(1-\delta)), & p=q=S+1\\ \lambda, & 1\le p\le S, q=p+1\\ 0, & \text{otherwise}\end{cases}$$

$$[J]_{pq}=\begin{cases}\mu, & 2\le p\le S+1, q=p-1\\ 0, & \text{otherwise}\end{cases}$$

$$[M_1]_{pq}=\begin{cases}\alpha\beta, & 1\le p\le S+1, q=p\\ 0, & \text{otherwise}\end{cases}$$

$$[M]_{pq}=\begin{cases}\beta, & 1\le p\le S+1, q=p\\ 0, & \text{otherwise}\end{cases}$$

$$[V]_{pq}=\begin{cases}i\theta, & 1\le p\le S, q=p+1\\ i\theta(1-\delta), & p=q=S+1\\ 0, & \text{otherwise}\end{cases}$$

We use Neuts and Rao (1990) truncation method to modify the infinitesimal generator Q to the following form where $A_{1,i}=A_1$ and $A_{2,i}=A_2$ for $i\ge N$.

$$Q=\begin{bmatrix} A_{1,0} & A_0 & & & & & \\ A_{2,1} & A_{1,1} & A_0 & & & & \\ & A_{2,2} & A_{1,2} & A_0 & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & A_{2,N-1} & A_{1,N-1} & A_0 & \\ & & & & A_2 & A_1 & A_0 \\ & & & & & A_2 & A_1 & A_0 \end{bmatrix}$$

System stability

Using Lyapunov test function (Falin and Templeton 1997) we define $\varphi(s)=i$, if s is a state in the level i.

The mean drift y_s for any s belonging to the level $i\ge 1$ is given by

$$\begin{aligned} y_s &= \sum_{p\neq s} q_{sp}(\varphi(p)-\varphi(s))\\ &= \sum_u q_{su}(\varphi(p)-\varphi(s))+\sum_v q_{sv}(\varphi(v)-\varphi(s))\\ &\quad+\sum_w q_{sw}(\varphi(w)-\varphi(s)) \end{aligned}$$

where u, v, w vary over the states belonging to the levels $(i-1), i$ and $(i+1)$ respectively. Then by the definition of φ,$\varphi(u)=i-1, \varphi(v)=i$ and $\varphi(w)=i+1$ so that

$$y_s=-\sum_u q_{su}+\sum_w q_{sw}$$

$$=\begin{cases}-i\theta(1-\delta)+\lambda\gamma, & \text{if the buffer is full}\\ -i\theta, & \text{otherwise}\end{cases}$$

Since $(1-\delta)>0$, for any $\varepsilon>0$, we can find N' large enough that $y_s<-\varepsilon$ for any s belonging to the level $i\ge N'$. Then the system under consideration is stable by Tweedi's (1975) result.

Steady state probability vector

Let $x=(x_0,x_1,\ldots,x_{N-1},x_N,\ldots)$ be the steady state probability vector of Q. Under the stability condition, x_i's $(i\ge N)$ are given by

$$x_{N+r-1}=x_{N-1}R^r(r\ge 1)$$

where R is the unique non negative solution of the equation

$$R^2A_2+RA_1+A_0=0$$

for which the spectral radius is less than one and the vectors $x_0,x_1,\ldots,x_{N-1}$ are obtained by solving

$$\left.\begin{aligned} x_0A_{1,0}+x_1A_{2,1}&=0\\ x_{i-1}A_0+x_iA_{1,i}+x_{i+1}A_{2,i+1}&=0(1\le i\le N-2)\\ x_{N-2}A_0+x_{N-1}\left(A_{1,N-1}+RA_2\right)&=0 \end{aligned}\right\}\quad(2)$$

subject to the normalizing condition

$$\left[\sum_{i=0}^{N-2}x_i+x_{N-1}(I-R)^{-1}\right]\boldsymbol{e}=1\quad(3)$$

Algorithmic analysis

To find R, we use iterative method. Denote the sequence of R by $\{R_n(N)\}$ and is defined by $R_0(N)=0$ and $R_{n+1}(N)=\left(-R_n^2(N)A_2(N)-A_0(N)\right)A_1^{-1}(N))$. The value of N must be chosen such that $|\eta(N)-\eta(N+1)|<\varepsilon$, where ε is an arbitrarily small value and $\eta(R)$, spectral radius of $R(N)$. For detailed discussion of selection of the value of N, one can refer to Neuts (1981).

System performance measures

We partition the steady state probability vector $\mathbf{x} = (x_0, x_1, \ldots, x_{N-1}, x_N, \ldots)$ such that its $((i+1)$th component is given by

$$x_i = (y_{i,0,s+1,0}, \ldots, y_{i,0,s+1,S}, y_{i,0,s+2,0}, \ldots, y_{i,0,s+2,S}, \ldots, y_{i,0,S,0}, \ldots, y_{i,0,S,S}, y_{i,1,0,0}, \ldots, y_{i,1,0,S}, y_{i,1,1,0}, \ldots, y_{i,1,1,S}, \ldots, y_{i,1,S-1,0}, \ldots, y_{i,1,S-1,S})$$

(i) Expected inventory level, EI, in the system is given by

$$\mathrm{EI} = \sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=0}^{S} j y_{i,0,j,k} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1}\sum_{k=0}^{S} j y_{i,1,j,k}$$

(ii) Expected number of customers, EC, in the orbit is given by

$$\mathrm{EC} = \left(\sum_{i=1}^{\infty} i x_i\right)\boldsymbol{e} = \left(\left(\sum_{i=1}^{N-1} i x_i\right) + x_N\left(N(I-R)^{-1} + R(I-R)^{-2}\right)\right)\boldsymbol{e}$$

(iii) Expected number of customers, EB, in the buffer is given by

$$\mathrm{EB} = \sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=0}^{S} k y_{i,0,j,k} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1}\sum_{k=0}^{S} k y_{i,1,j,k}$$

(iv) Expected switching rate, ESR, is given by

$$\mathrm{ESR} = \mu \sum_{i=0}^{\infty}\sum_{k=1}^{S} y_{i,0,s+1,k}$$

(v) Expected number of departures, EDS, after completing service is

$$\mathrm{EDS} = \mu\left[\sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=1}^{S} y_{i,0,j,k} + \sum_{i=0}^{\infty}\sum_{j=1}^{S-1}\sum_{k=1}^{S} y_{i,1,j,k}\right]$$

(vi) Expected number of external customers lost, EL_1, before entering the orbit per unit time is

$$\mathrm{EL}_1 = (1-\gamma)\lambda\left[\sum_{i=0}^{\infty}\sum_{j=s+1}^{S} y_{i,0,j,S} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1} y_{i,1,j,S}\right]$$

(vii) Expected number of customers lost, EL_2, due to retrials per unit time

$$\mathrm{EL}_2 = \theta(1-\delta)\left[\sum_{i=1}^{\infty}\sum_{j=s+1}^{S} i y_{i,0,j,S} + \sum_{i=1}^{\infty}\sum_{j=0}^{S-1} i y_{i,1,j,S}\right]$$

(viii) Overall rate of retrials,

$$\mathrm{ORR} = \theta\left[\sum_{i=1}^{\infty} i x_i\right]\boldsymbol{e}$$

(ix) Successful rate of retrials,

$$\mathrm{SRR} = \theta\sum_{i=1}^{\infty} i\left[\sum_{j=s+1}^{S}\sum_{k=0}^{S-1} y_{i,0,j,k} + \sum_{j=0}^{S-1}\sum_{k=0}^{S-1} y_{i,1,j,k}\right]$$

Description of model II

In this model, we assume that there is a buffer of varying (finite) capacity, equal to the current inventory level. Now $\{X(t), t \geq 0\}$, where $X(t) = (N(t), J(t), I(t), M(t))$ is a level dependent quasi birth–death process on the state space $\{(i,0,j,k);\ i \geq 0;\ j = s+1, \ldots S,\ k = 0,1,\ldots,j\}$ $U\{(i,1,j,k);\ i \geq 0;\ j = 0,\ldots,S-1, k = 0,1,\ldots,j\}$. Then the infinitesimal generator Q has the form (1) where the blocks $A_{0,}$ $A_{1,i}(i \geq 0)$ and $A_{2,i}(i \geq 1)$ are square matrices of the same order $\frac{1}{2}[(S-s)(S+s+3) + S(S+1)]$ and they are given by

$$A_{1,i} = \begin{array}{c} \underline{0,s+1} \\ \underline{0,s+2} \\ \vdots \\ \underline{0,S} \\ \underline{1,0} \\ \underline{1,1} \\ \vdots \\ \underline{1,s} \\ \underline{1,s+1} \\ \vdots \\ \underline{1,S-2} \\ \underline{1,S-1} \end{array} \begin{bmatrix} G_{s+1} & & & & & & & H_{s+1} & & & & \\ H_{s+2} & G_{s+2} & & & & & & & & & & \\ & \ddots & \ddots & & & & & & & & & \\ & & H_S & G_S & & & & & & & & \\ & & & & E_0 & T_0 & & & & & & \\ & & & & H_1 & E_1 & T_1 & & & & & \\ & & & & & \ddots & \ddots & \ddots & & & & \\ & & & & & & H_s & E_s & T_s & & & \\ & & & & & & & H_{s+1} & D_{s+1} & U_{s+1} & & \\ & & & & & & & & \ddots & \ddots & \ddots & \\ & & & & & & & & & H_{S-2} & D_{S-2} & U_{S-2} \\ & & & U_{S-1} & & & & & & & H_{S-1} & D_{S-1} \end{bmatrix}$$

$$A_0 = \begin{array}{c} \underline{0,s+1} \\ \underline{0,s+2} \\ \vdots \\ \underline{0,S} \\ \underline{1,0} \\ \underline{1,1} \\ \vdots \\ \underline{1,S-1} \end{array} \begin{bmatrix} B_{s+1} & & & & & & & \\ & B_{s+2} & & & & & & \\ & & \ddots & & & & & \\ & & & B_S & & & & \\ & & & & B_0 & & & \\ & & & & & B_1 & & \\ & & & & & & \ddots & \\ & & & & & & & B_{S-1} \end{bmatrix}$$

$$A_{2,i} = \begin{array}{c} \underline{0,s+1} \\ \underline{0,s+2} \\ \vdots \\ \underline{0,S} \\ \underline{1,0} \\ \underline{1,1} \\ \vdots \\ \underline{1,S-1} \end{array} \begin{bmatrix} C_{s+1} & & & & & & & \\ & C_{s+2} & & & & & & \\ & & \ddots & & & & & \\ & & & C_S & & & & \\ & & & & C_0 & & & \\ & & & & & C_1 & & \\ & & & & & & \ddots & \\ & & & & & & & C_{S-1} \end{bmatrix}$$

(p,q)th element of the matrices contained in A_0, $A_{1,i}$ and $A_{2,i}$ are given by

$$[B_0]_{pq} = \begin{cases} \lambda\gamma, & p=q=1 \\ 0, & \text{otherwise} \end{cases}$$

$$[B_n]_{pq} = \left.\begin{cases} \lambda\gamma, & p=q=n+1 \\ 0, & \text{otherwise} \end{cases}\right\} 1 \le n \le S$$

$$[C_0]_{pq} = \begin{cases} i\theta(1-\delta), & p=q=1 \\ 0, & \text{otherwise} \end{cases}$$

$$[C_n]_{pq} = \left.\begin{cases} i\theta, & 1 \le p \le n, q=p+1 \\ i\theta(1-\delta), & p=q=n+1 \\ 0, & \text{otherwise} \end{cases}\right\} 1 \le n \le S$$

$$[E_0]_{pq} = \begin{cases} -(\lambda\gamma+\alpha\beta+i\theta(1-\delta)), & p=q=1 \\ 0, & \text{otherwise} \end{cases}$$

$$[E_n]_{pq} = \left.\begin{cases} -(\lambda+\alpha\beta+i\theta), & p=q=1 \\ -(\lambda+\alpha\beta+\mu+i\theta), & 2 \le p \le n, q=p \\ -(\lambda\gamma+\alpha\beta+\mu+i\theta(1-\delta)), & p=q=n+1 \\ \lambda, & 1 \le p \le n, q=p+1 \\ 0, & \text{otherwise} \end{cases}\right\} 1 \le n \le s$$

$$[D_n]_{pq} = \left.\begin{cases} -(\lambda+\beta+i\theta), & p=q=1 \\ -(\lambda+\beta+\mu+i\theta), & 2 \le p \le n, q=p \\ -(\lambda\gamma+\beta+\mu+i\theta(1-\delta)), & p=q=n+1 \\ \lambda, & 1 \le p \le n, q=p+1 \\ 0, & \text{otherwise} \end{cases}\right\}$$
$$s+1 \le n \le S-1$$

$$[G_n]_{pq} = \left.\begin{cases} -(\lambda+i\theta), & p=q=1 \\ -(\lambda+\mu+i\theta), & 2 \le p \le n, q=p \\ -(\lambda\gamma+\mu+i\theta(1-\delta)), & p=q=n+1 \\ \lambda, & 1 \le p \le n, q=p+1 \\ 0, & \text{otherwise} \end{cases}\right\}$$
$$s+1 \le n \le S$$

$$[H_n]_{pq} = \left.\begin{cases} \mu, & 2 \le p \le n+1, q=p-1 \\ 0, & \text{otherwise} \end{cases}\right\} 1 \le n \le S$$

$$[T_n]_{pq} = \left.\begin{cases} \alpha\beta, & 1 \le p \le n+1, q=p \\ 0, & \text{otherwise} \end{cases}\right\} 0 \le n \le s$$

$$[U_n]_{pq} = \left.\begin{cases} \beta, & 1 \le p \le n+1, q=p \\ 0, & \text{otherwise} \end{cases}\right\} s+1 \le n \le S-1$$

System stability

The mean drift y_s for any s belonging to the level $i \geq 1$ is given by

$$y_s = -\sum_u q_{su} + \sum_w q_{sw}$$
$$= \begin{cases} -i\theta(1-\delta) + \lambda\gamma, & \text{if the buffer is full} \\ -i\theta, & \text{otherwise} \end{cases}$$

Since $(1-\delta) > 0$, for any $\varepsilon > 0$, we can find N' large enough that $y_s < -\varepsilon$ for any s belonging to the level $i \geq N'$. Then the system under consideration is stable.

System performance measures

We partition the steady state probability vector $x = (x_0, x_1, \ldots, x_{N-1}, x_N, \ldots)$ such that its $(i+1)$th component is given by

$$x_i = \big(y_{i,0,s+1,0}, \ldots, y_{i,0,s+1,s+1}, y_{i,0,s+2,0}, \ldots, y_{i,0,s+2,s+2}, \ldots, y_{i,0,S,0}, \ldots, y_{i,0,S,S}, y_{i,1,0,0}, y_{i,1,1,0}, y_{i,1,1,1}, \ldots, y_{i,1,S-1,0}, \ldots, y_{i,1,S-1,S-1}\big)$$

Then,

(i) Expected inventory level, EI, in the system is given by

$$\mathrm{EI} = \sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=0}^{j} j y_{i,0,j,k} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1}\sum_{k=0}^{j} j y_{i,1,j,k}$$

(ii) Expected number of customers, EC, in the orbit is given by

$$\mathrm{EC} = \left(\sum_{i=1}^{\infty} i x_i\right)\boldsymbol{e} = \left(\left(\sum_{i=1}^{N-1} i x_i\right) + x_N\left(N(I-R)^{-1} + R(I-R)^{-2}\right)\right)\boldsymbol{e}$$

(iii) Expected number of customers, EB, in the buffer is given by

$$\mathrm{EB} = \sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=0}^{j} k y_{i,0,j,k} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1}\sum_{k=0}^{j} k y_{i,1,j,k}$$

(iv) Expected switching rate, ESR, is given by

$$\mathrm{ESR} = \mu \sum_{i=0}^{\infty}\sum_{k=1}^{s+1} y_{i,0,s+1,k}$$

(v) Expected number of departures, EDS, after completing service is

$$\mathrm{EDS} = \mu \sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{k=1}^{j} y_{i,0,j,k} + \mu \sum_{i=0}^{\infty}\sum_{j=1}^{S-1}\sum_{k=1}^{j} y_{i,1,j,k}$$

(vi) Expected number of external customers lost, EL_1, before entering the orbit per unit time is

$$\mathrm{EL}_1 = (1-\gamma)\lambda\left[\sum_{i=0}^{\infty}\sum_{j=s+1}^{S} y_{i,0,j,j} + \sum_{i=0}^{\infty}\sum_{j=0}^{S-1} y_{i,1,j,j}\right]$$

(vii) Expected number of customers lost, EL_2, due to retrials per unit time

$$\mathrm{EL}_2 = \theta(1-\delta)\left[\sum_{i=1}^{\infty}\sum_{j=s+1}^{S} i y_{i,0,j,j} + \sum_{i=1}^{\infty}\sum_{j=0}^{S-1} i y_{i,1,j,j}\right]$$

(viii) Overall rate of retrials,

$$\mathrm{ORR} = \theta\left[\sum_{i=1}^{\infty} i x_i\right] e$$

(ix) Successful rate of retrials,

$$\mathrm{SRR} = \theta \sum_{i=1}^{\infty} i\left[\sum_{j=s+1}^{S}\sum_{k=0}^{j-1} y_{i,0,j,k} + \sum_{j=1}^{S-1}\sum_{k=0}^{j-1} y_{i,1,j,k}\right]$$

Table 1 Variations in α

α	Model 1		Model 2	
	ORR	SRR	ORR	SRR
1.1	2.2655	0.9752	2.5856	0.9476
1.2	2.1966	1.0197	2.5293	0.9794
1.3	2.1363	1.0600	2.4839	1.0051
1.4	2.1026	1.0825	2.4479	1.0255
1.5	2.0893	1.0910	2.4194	1.0413
1.6	2.0840	1.0940	2.3971	1.0535
1.7	2.0813	1.0953	2.3796	1.0628
1.8	2.0797	1.0959	2.3658	1.0698
1.9	2.0786	1.0962	2.3548	1.0752

. $S = 20$; $s = 5$; $\lambda = 1.5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\delta = 0.7$; $\mu = 3$; $C = 20$; $c_1 = 1$; $c_2 = 1$; $c_3 = 1$; $c_4 = 1$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

Table 2 Variations in μ

μ	Model 1		Model 2	
	ORR	SRR	ORR	SRR
2.1	2.3901	0.9415	2.8332	0.8324
2.2	2.3457	0.9631	2.7637	0.8685
2.3	2.3044	0.9833	2.7012	0.9008
2.4	2.2661	1.0022	2.6454	0.9295
2.5	2.2305	1.0199	2.5958	0.9548
2.6	2.1975	1.0364	2.5518	0.9771
2.7	2.1670	1.0518	2.5126	0.9966
2.8	2.1388	1.0660	2.4779	1.0136
2.9	2.1129	1.0791	2.4470	1.0284

$S = 20$; $s = 5$; $\lambda = 1.5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\delta = 0.7$; $\alpha = 1.5$; $C = 20$; $c_1 = 1$; $c_2 = 1$; $c_3 = 1$; $c_4 = 1$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

Table 4 Variations in δ

δ	Model 1		Model 2	
	ORR	SRR	ORR	SRR
0.1	1.8089	0.9971	1.8984	0.8717
0.2	1.8340	1.0063	1.9400	0.8846
0.3	1.8640	1.0171	1.9914	0.9010
0.4	1.9006	1.0300	2.0565	0.9222
0.5	1.9465	1.0458	2.1412	0.9503
0.6	2.0065	1.0654	2.2558	0.9884
0.7	2.0893	1.0910	2.4194	1.0413
0.8	2.2150	1.1262	2.6806	1.1164
0.9	2.4416	1.1795	3.2220	1.2297

$S = 20$; $s = 5$; $\lambda = 1.5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\mu = 3$; $\alpha = 1.5$; $C = 20$;$c_1 = 1$; $c_2 = 1$; $c_3 = 1$; $c_4 = 1$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

Table 3 Variations in γ

γ	Model 1		Model 2	
	ORR	SRR	ORR	SRR
0.1	1.5532	0.8967	1.5642	0.8284
0.2	1.6109	0.9205	1.6399	0.8549
0.3	1.6742	0.9459	1.7305	0.8831
0.4	1.7437	0.9726	1.8367	0.9129
0.5	1.8199	1.0007	1.9588	0.9439
0.6	1.9028	1.0299	2.0965	0.9758
0.7	1.9926	1.0600	2.2501	1.0083
0.8	2.0893	1.0910	2.4194	1.0413
0.9	2.1931	1.1227	2.6046	1.0745

$S = 20$; $s = 5$; $\lambda = 1.5$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\delta = 0.7$; $\mu = 3$; $\alpha = 1.5$; $C = 20$; $c_1 = 1$; $c_2 = 1$; $c_3 = 1$; $c_4 = 1$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

Table 5 Variations in λ

λ	Model 1		Model 2	
	ORR	SRR	ORR	SRR
1.1	1.8575	1.0591	2.0378	1.0083
1.2	1.9089	1.0668	2.1230	1.0160
1.3	1.9644	1.0747	2.2149	1.0240
1.4	2.0245	1.0828	2.3137	1.0325
1.5	2.0893	1.0910	2.4194	1.0413
1.6	2.1594	1.0993	2.5322	1.0505
1.7	2.2348	1.1077	2.6519	1.0600
1.8	2.3161	1.1163	2.7789	1.0699
1.9	2.4033	1.1250	2.9133	1.0800

$S = 20$; $s = 5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\delta = 0.7$; $\mu = 3$; $\alpha = 1.5$; $C = 20$; $c_1 = 1$; $c_2 = 1$; $c_3 = 1$; $c_4 = 1$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

Cost analysis

Here we consider the following costs

$C =$ fixed cost

$c_1 =$ procurement cost/unit

$c_2 =$ holding cost of inventory/unit/unit time

$c_3 =$ holding cost of customers in the orbit/unit/unit time

$c_4 =$ holding cost of customers in the buffer/unit/unit time

$c_5 =$ cost due to loss of primary customers/unit/unit time

$c_6 =$ cost due to loss of retrial customers/unit/unit time

$c_7 =$ cost due to service/unit/unit time

$c_8 =$ revenue from service/unit/unit time

In terms of these costs we define the expected total cost function as

$$\begin{aligned}\mathrm{ETC} = {} & (C + (S - s)c_1)\mathrm{ESR} + c_2\mathrm{EI} + c_3\mathrm{EC} + c_4\mathrm{EB} \\ & + c_5\mathrm{EL}_1 + c_6\mathrm{EL}_2 + (c_7 - c_8)\mathrm{EDS}\end{aligned}$$

.

Numerical results and interpretation

Here we compare the overall rate of retrials and successful rate of retrials of model I and II by varying different parameters.

Tables 1 and 2 contain changes of overall rate of retrials (ORR) and successful rate of retrials (SRR) with respect to variations of α and μ. When the replenishment rate or service rate increases, the number of customers in the orbit decreases. Hence, overall rate of retrials decreases and the

Table 6 Variations in θ

θ	Model 1		Model 2	
	ORR	SRR	ORR	SRR
1.1	1.5776	0.9232	1.8664	0.9281
1.2	1.7081	0.9694	2.0089	0.9609
1.3	1.8367	1.0125	2.1485	0.9903
1.4	1.9638	1.0529	2.2852	1.0170
1.5	2.0893	1.0910	2.4194	1.0413
1.6	2.2136	1.1270	2.5513	1.0635
1.7	2.3367	1.1611	2.6810	1.0840
1.8	2.4586	1.1934	2.8088	1.1029
1.9	2.5796	1.2243	2.9347	1.1206

$S = 20;\ s = 5;\ \lambda = 1.5;\ \gamma = 0.8;\ N = 25;\ \beta = 2;\ \delta = 0.7;\ \mu = 3;\ \alpha = 1.5;\ C = 20;\ c_1 = 1;\ c_2 = 1;\ c_3 = 1;\ c_4 = 1\ c_5 = 1\ c_6 = 1;\ c_7 = 2;\ c_8 = 1$

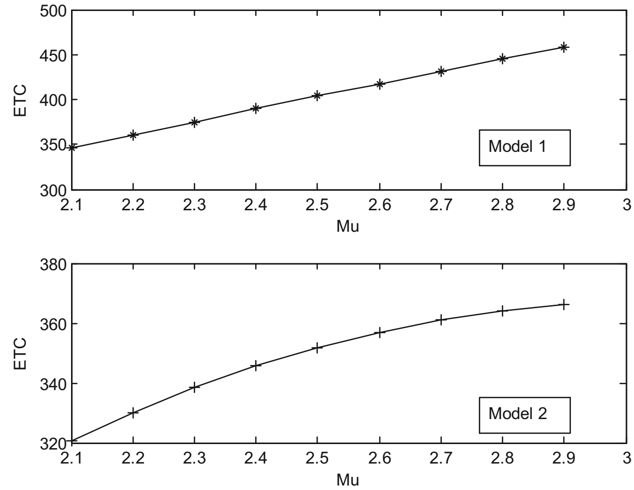

Fig. 2 ETC versus μ. $S = 20;\ s = 5;\ \lambda = 1.5;\ \gamma = 0.8;\ N = 25;\ \theta = 1.5;\ \beta = 2;\ \delta = 0.7;\ \alpha = 1.5;\ C = 20;\ c_1 = 1;\ c_2 = 1;\ c_3 = 1;\ c_4 = 1;\ c_5 = 1;\ c_6 = 1;\ c_7 = 150;\ c_8 = 1$

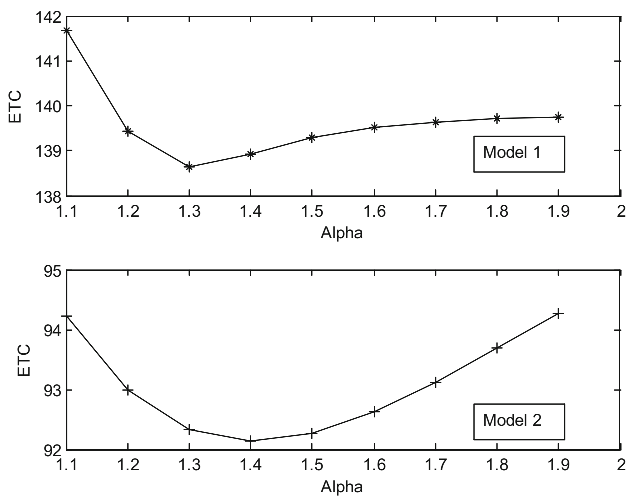

Fig. 1 ETC versus α. $S = 20;\ s = 5;\ \lambda = 1.5;\ \gamma = 0.8;\ N = 25;\ \theta = 1.5;\ \beta = 2;\ \delta = 0.7;\ \mu = 3\ C = 20;\ c_1 = 1;\ c_2 = 1;\ c_3 = 28;\ c_4 = 3.6;\ c_5 = 50;\ c_6 = 50;\ c_7 = 1.01;\ c_8 = 1$

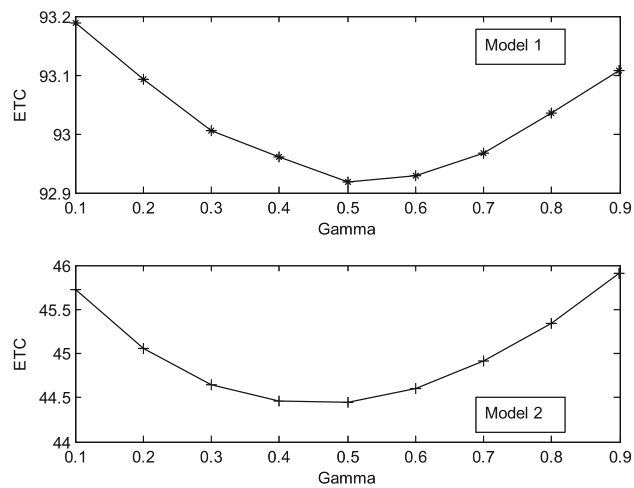

Fig. 3 ETC versus γ. $S = 20;\ s = 5;\ \lambda = 1.5;\ N = 25;\ \theta = 1.5;\ \beta = 2;\ \delta = 0.7;\ \mu = 3;\ \alpha = 1.5;\ C = 20;\ c_1 = 14;\ c_2 = 1;\ c_3 = 3;\ c_4 = 3;\ c_5 = 3.3;\ c_6 = 6;\ c_7 = 4;\ c_8 = 1$

successful rate of retrials increases. Tables 3, 4 and 5 show the changes of ORR and SRR with respect to variations of γ, δ and λ respectively. In all the cases as the variations of γ, δ and λ increases the number of customers in the orbit increases and hence the overall and successful rates of retrials increase. Table 6 shows that as the retrial rate θ of customers in the orbit increases, the overall and successful rate of retrials increase.

Graphical illustrations and interpretations

Here we compare the two models by calculating the expected total cost (ETC) per unit time by varying the parameters one at a time keeping others fixed. In Fig. 1, we compare the values of the cost function by varying the value of α. For given parameter values, the cost function has minimum values 138.6508 at $\alpha = 1.3$ for model I and 92.1336 at $\alpha = 1.4$ for model II. From Fig. 2, as μ varies ETC has minimum values 346.2704 at $\mu = 2.1$ for model I and 320.5516 at $\mu = 2.1$ for model II. From Fig. 3, as γ varies ETC has minimum values 92.9188 at $\gamma = 0.5$ for model I and 44.4500 at $\gamma = 0.5$ for model II. The minimum values of ETC are at $\delta = 0.6$ for model I and at $\delta = 0.8$ for model II as seen in Fig. 4. The optimum values are 73.1769 and 24.0576, respectively. In Figs. 5, 6 we can see that ETC is minimum for model II. Hence, model II is more efficient for practical purposes in the given range of parameter values.

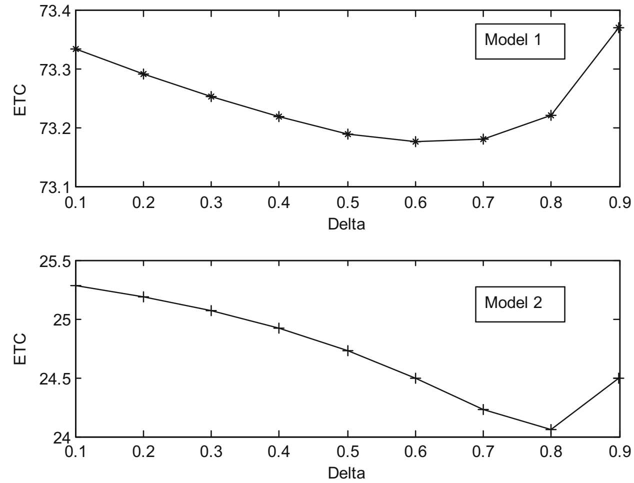

Fig. 4 ETC versus δ. $S = 20$; $s = 5$; $\lambda = 1.5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\mu = 3$; $\alpha = 1.5$; $C = 20$; $c_1 = 1$; $c_2 = 1$; $c_3 = 2.9$; $c_4 = 2.9$; $c_5 = 1$; $c_6 = 1$; $c_7 = 2$; $c_8 = 1$

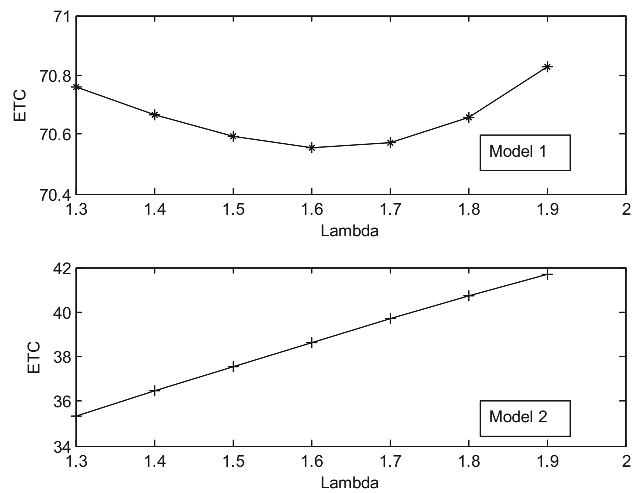

Fig. 5 ETC versus λ. $S = 20$; $s = 5$; $\gamma = 0.8$; $N = 25$; $\theta = 1.5$; $\beta = 2$; $\delta = 0.7$; $\mu = 3$; $\alpha = 1.5$; $C = 20$; $c_1 = 3$; $c_2 = 1$; $c_3 = 8$; $c_4 = 8$; $c_5 = 5$; $c_6 = 5$; $c_7 = 2$; $c_8 = 1$

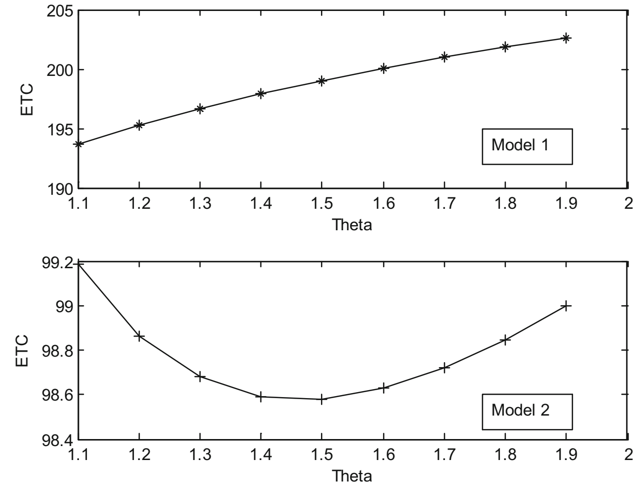

Fig. 6 ETC versus θ. $S = 20$; $s = 5$; $\lambda = 1.5$; $\gamma = 0.8$; $N = 25$; $\beta = 2$; $\delta = 0.7$; $\mu = 3$; $\alpha = 1.5$; $C = 20$; $c_1 = 1$; $c_2 = 15$; $c_3 = 13.5$; $c_4 = 1.5$; $c_5 = 0.5$; $c_6 = 0.5$; $c_7 = 2$; $c_8 = 1$

Concluding remarks and future research

The main aspect of the paper is to compare two production inventory systems with different production rates and retrials. We derived formulae for some important performance measures of the system and constructed a suitable cost function. The most striking feature of the paper is the evaluation of the optimum value of α, the coefficient of replenishment rate, corresponding to the minimum expected total cost. The minimum value of total cost is obtained by varying different parameters of the system. These evaluations are carried out in previous section of the paper. We found that the model with buffer of varying capacity is efficient for practical purposes in the given range of parameter values.

The models discussed in this paper have remarkably good applications in industries such as automobiles, textiles and drugs. For instance, when we consider an automobile tyre producing company, each tyre can be considered as an inventory. When the storage of tyres in the company is reduced to a particular lower level, we increase the production rate. The increased production rate will reduce the loss of customers from the company in the absence of tyres. The higher production rate also increases the rate of satisfaction of the customers and goodwill of the company. This situation leads to more profitable circumstances of the company.

For future research, one can extend this model in several ways. For instance, it could be of interest to extend the exponential service distribution to phase-type distribution or a general distribution. Another interesting extension may be the case with the change of both arrival and service distributions to some other suitable probability distributions.

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References

Andrew Junfang Y, Dong Y (2014) A numerical solution for a two-stage production and inventory system with random demand arrivals. Comput Oper Res 44:13–21

Artalejo JR (2010) Accessible bibliography on retrial queues: progress in 2000–2009. Math Comput Model 51:1071–1081

Artalejo JR, Krishnamoorthy A, Lopez Herrero MJ (2006) Numerical analysis of (s, S) inventory systems with repeated attempts. Ann Oper Res 141:67–83

Baek JW, Moon SK (2016) A production–inventory system with a Markovian service queue and lost sales. J Korean Stat Soc 45(1):14–24

Benjaafar S, Gayon J-P, Tepe S (2010) Optimal control of a production-inventory system with customer impatience. Oper Res Lett 38(4):267–272

Chang K-H, Lu Y-S (2011) Inventory management in a base-stock controlled serial production system with finite storage space. Math Comput Model 54(11–12):2750–2759

Cheng BY, Leung JYT, Li K (2016) Integrated scheduling on a batch machine to minimize production, inventory and distribution costs. Eur J Oper Res. doi:10.1016/j.ejor.2016.09.009

De A, Mamanduru VKR, Gunasekaran A, Subramanian N, Tiwari MK (2016) Composite particle algorithm for sustainable integrated dynamic ship routing and scheduling optimization. Comput Ind Eng 96:201–215

Falin GI, Templeton JGC (1997) Retrial queues. Chapman and Hall, London

Jeganathan K, Anbazhagan N, Kathiresan J (2013) A retrial inventory system with non-preemptive priority service. Int J Inf Manage Sci 24(1):57–77

Kangzhou W, Jiang Z, Wenming X (2013) Optimal scheduling policies for a production service system with two items. In: IFAC Proceedings volumes-7th IFAC conference on manufacturing modelling, management, and control 46(9):712–717

Karimi-Nasab M, Sabri-Laghaie K (2014) Developing approximate algorithms for EPQ problem with process compressibility and random error in production/inspection. Int J Prod Res 52(8):2388–2421

Krishnamoorthy A., Jose KP (2007) Comparison of inventory systems with service, positive lead time, loss and retrial of customers. J Appl Math Stoch Anal Article ID 37848: 23 p

Krishnamoorthy A, Jose KP (2008) Three production inventory systems with service, loss and retrial of customers. Int J Inf Manag Sci 19(3):367–389

Krishnamoorthy A, Shajin Dhanya, Lakshmy B (2015a) On a queueing-inventory with reservation, cancellation, common life time and retrial. Ann Oper Res. doi:10.1007/s10479-015-1849-x

Krishnamoorthy A, Nair SS, Narayanan VC (2015b) Production inventory with service time and interruptions. Int J Syst Sci 46(10):1800–1816

Nair AN, Jacob MJ (2015) An (s,S) production inventory controlled self-service queuing system. J Probab Stat Article ID 505082, 8 p

Neuts MF (1981) Matrix–geometric solutions in stochastic models: an algorithmic approach. The Johns Hopkins University Press, Baltimore

Neuts MF, Rao BM (1990) Numerical investigation of a multi server retrial model. Queueing Syst 42:377–400

Padmavathi I, Sivakumar B, Arivarignan G (2015) A retrial inventory system with single and modified multiple vacation for server. Ann Oper Res 233(1):335–364

Rashid R, Hoseini SF, Gholamian MR, Feizabadi M (2015) Application of queuing theory in production-inventory optimization. J Ind Eng Int 11(4):485–494

Tweedie RL (1975) Sufficient conditions for regularity, recurrence and ergodicity of Markov processes. Proc. Camb. Phil. Soc. 1975; 78 Part I

Vijaya Laxmi P, Soujanya ML (2015) Retrial inventory system with negative customers and service interruptions. Opsearch 52(2):212–224

Yadavalli VSS, Sivakumar B, Arivarignan G, Adetunji Olufemi (2012) A finite source multi-server inventory system with service facility. Comput Ind Eng 63(4):739–753