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A multi-period distribution network design model under demand uncertainty

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Abstract

Supply chain management is taken into account as an inseparable component in satisfying customers' requirements. This paper deals with the distribution network design (DND) problem which is a critical issue in achieving supply chain accomplishments. A capable DND can guarantee the success of the entire network performance. However, there are many factors that can cause fluctuations in input data determining market treatment, with respect to short-term planning, on the one hand. On the other hand, network performance may be threatened by the changes that take place within practicing periods, with respect to long-term planning. Thus, in order to bring both kinds of changes under control, we considered a new multi-period, multi-commodity, multi-source DND problem in circumstances where the network encounters uncertain demands. The fuzzy logic is applied here as an efficient tool for controlling the potential customers' demand risk. The defuzzifying framework leads the practitioners and decision-makers to interact with the solution procedure continuously. The fuzzy model is then validated by a sensitivity analysis test, and a typical problem is solved in order to illustrate the implementation steps. Finally, the formulation is tested by some different-sized problems to show its total performance.

Keywords: Supply chain, Distribution network design, Uncertain demand, Fuzzy logic

Introduction

Moving towards competitive markets compels the manufacturers to increase their qualifications so that they can best fulfill the customers' needs. The supply chain management (SCM) concept arose as an effective managerial tool to enhance the customers' satisfaction within the last 20 years in particular (Azaron et al. 2008; Jayaraman and Ross 2003). Generally, a supply chain (SC) can be defined as a set of plants, distribution centers, and customers in which the products are completed once they are transmitted downstream the chain and delivered to the customers (Altiparmak et al. 2006; Blackhurst et al. 2004).

Distribution network design (DND) is one of the primitive principles in establishing a successful SC. It is associated with the category of production-distribution and facility location-allocation problems (Altiparmak et al. 2009). The researchers investigate the distribution networks under two distinctive categories including strategic and tactical levels. The first level concerns with whatever affects the entire network configuration (e.g., the numbers, capacities, and locations), and the second level deals with whatever affects the aggregate quantities (e.g., material handling, processing, and distribution) (Santoso et al. 2005). Consequently, considering the two planning levels simultaneously is very fruitful in enhancing the network performance.

A DND problem can also incorporate five aspects according to the necessity (please refer to Tang (2006) for more description):

- Network configuration (NC)
- Product assignment (PA)
- Customer assignment (CA)
- Production planning (PP)
- Transportation planning (TP)

Regarding the above, the proposed model covers the first four aspects at the same time.

Some other issues are expected in the model structure likewise (e.g., the number of consideration periods, sources, echelons, and commodities). For example, Amiri (2006) suggested a multi-capacity distribution

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network in deterministic environments so that both strategic and tactical levels were included. Gupta and Maranas (2003) concentrated on the tactical level in a multi-commodity, multi-echelon, multi-period problem. The authors considered demand as the uncertain parameter. Selim and Ozkarahan (2006) developed a fuzzy multi-echelon, multi-commodity, multi-capacity problem by considering both levels. They introduced the demand as the uncertain part likewise. Liu et al. (2006) introduced a multi-commodity DND problem and dealt with the potential uncertainties by the combination of gray and fuzzy factors. Azaron et al. (2008) proposed a multi-commodity, multi-source model and solved it by the goal attainment technique. However, they extended the uncertainty and vagueness to all three demand-, supply-, and process-side risks. Gumus et al. (2009) applied a neuro-fuzzy technique for demand uncertainty in a multi-echelon condition. Xu et al. (2009) developed a multi-echelon, multi-source problem where the demand, supply, and process follow a fuzzy nature. They applied a heuristic solution approach called spanning tree technique that originated from genetic algorithm. Peidro et al. (2010) proposed a multi-echelon, multi-period, multi-commodity, multi-source mathematical modeling under fuzzy programming so that all three mentioned sides were uncertain. Compared with the mentioned researches, our developed model lies in a three-echelon, multi-source, multi-period, multi-commodity, single-capacity category in which the two levels are planned. The environment under consideration faces a demand-side risk which is tackled with by the fuzzy mathematical modeling. A brief overview on DND literature is depicted in Table 1.

The rest of the paper is organized as follows: model formulation is presented in the next section. Solution methodology is described in the third section, which also demonstrates how the fuzzy steps can be implemented. The fourth section pertains to the computational study, and finally, the conclusions and future research interests are discussed in the 'Conclusions' section.

Model formulation

In this section, the mixed-integer mathematical model is defined and the fuzzy approach is described. Before we construct the mathematical formulation, the assumptions, indices, parameters, and decision variables are introduced as follows:

1. Assumptions
   - Demand parameter follows a fuzzy nature.
   - A given plant or distributor can be open or close in each horizon.
   - The number of practicing facilities cannot exceed the predefined upper bound.
   - The processing capacity of each plant and distributor is known with respect to the commodities.
   - The facility capacity is fixed during the total horizon periods.
   - Each tier can supply its needs from more than one member of the upper tier.
2. Indices
   - C: set of commodities ($c \in C$)
   - P: set of plants ($p \in P$)
   - J: set of distributors ($j \in J$)
   - I: set of customers ($i \in I$)
   - H: set of planning horizons ($h \in H$)
3. Parameters
   - FC_{ph}: annual fixed cost of plant p in period h
   - FC'_{jh}: annual fixed cost of distributor j in period h
   - LC_{pchj}: logistics cost for supplying commodity c in period h from plant p to distributor j
   - LC'_{jchi}: logistics cost for supplying commodity c in period h from distributor j to customer i
   - d_{ich}: demand of customer i for commodity c in period h
   - α_{pc}: capacity of plant p for commodity c
   - α'_{jc}: capacity of distributor j for commodity c
   - P_{Upper}: the upper bound for the number of open plants
   - J_{Upper}: the upper bound for the number of open distributors
4. Decision variables
   - x_{pchj}: the proportion of commodity c in period h transported from plant p to distributor j
   - y_{jchi}: the proportion of commodity c in period h transported from distributor j to customer i
   - Z_{ph}: the binary variable that takes 1 if plant p is open in period h and 0, otherwise
   - Z'_{jh}: the binary variable that takes 1 if distributor j is open in period h and 0, otherwise
   - LQ_{chj}: the left quantity of commodity c in period h for distributor j

Now, the model can be developed by Equations 1 to 8, as follows:

$$\mathrm{Min} = \begin{array}{c} \sum_{p=1}^{P}\sum_{c=1}^{C}\sum_{h=1}^{H}\sum_{j=1}^{J}\mathrm{LC}_{pchj}x_{pchj}a'_{jc}+ \\ \sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{h=1}^{H}\sum_{i=1}^{I}\mathrm{LC}'_{jchi}y_{jchi}d_{ich}+ \\ \sum_{p=1}^{P}\sum_{h=1}^{H}\mathrm{FC}_{ph}Z_{ph}+\sum_{j=1}^{J}\sum_{h=1}^{H}\mathrm{FC}'_{jh}Z'_{jh} \end{array} \qquad (1)$$

Table 1 An overview on DND literature

Articles	Aspects					Features					Nature	Uncertain parameter(s)	Solution approach
	NC	PA	CA	PP	TP	Product	Stage	Period	Source	Capacity			
Jayaraman and Ross (2003)	***	***	***			Multiple	Three	Single	Single	Single	Deterministic	-	Simulated annealing
Santoso et al. (2005)	***	***	***	***		Multiple	Two	Single	Multiple	Single	Stochastic	Demand, supply, and process	Sample average approximation scheme and accelerated Benders' decomposition
Altiparmak et al. (2006)	***		***	***		Single	Multiple	Single	Single	Single	Deterministic	-	Genetic algorithm
Selim and Ozkarahan (2006)	***	***		***		Multiple	Three	Single	Single	Multiple	Fuzzy	Demand	Fuzzy multi-objective programming
Goh et al. (2007)	***		***			Single	Two	Single	Multiple	Single	Stochastic	Demand and supply	Heuristic algorithm
Chen et al. (2007)		***	***	***		Multiple	Three	Multiple	Multiple	Single	Fuzzy	Demand	Fuzzy programming
You and Grossmann (2008)	***	***		***		Multiple	Multiple	Multiple	Single	Single	Probabilistic	Demand	ε-Constraint
Cakir (2009)	***	***		***	***	Multiple	Two	Single	Multiple	Single	Deterministic	-	Benders' decomposition
Altiparmak et al. (2009)	***	***	***	***		Multiple	Multiple	Single	Single	Single	Deterministic	-	Genetic algorithm
Georgiadis et al. (2011)	***	***	***	***		Multiple	Three	Single	Multiple	Single	Stochastic	Demand	Branch-and-bound
Cintron et al. (2010)	***		***			Single	Three	Single	Single	Multiple	Stochastic	Demand	Goal programming
Hajiaghaei-Keshteli (2011)	***		***			Single	Two	Single	Multiple	Single	Deterministic	-	Genetic algorithm and artificial immune algorithm
Cardona-Valdes et al. (2011)	***		***	***	***	Single	Three	Single	Multiple	Single	Stochastic	Demand	L-shaped algorithm
Rezapour and Farahani (2010)	***		***			Single	Three	Single	Multiple	Multiple	Deterministic	-	A modified projection method
Park et al. (2010)	***		***			Single	Three	Single	Single	Single	Deterministic	-	A two-phase heuristic algorithm
Our proposed model	***	***	***	***		Multiple	Three	Multiple	Multiple	Single	Fuzzy	Demand, supply, and process	Fuzzy programming

S.t.:

$$\sum_{j-1}^{J} y_{jchi} \leq 1 \quad ; \forall i, c, h \tag{2}$$

$$\sum_{j-1}^{J} x_{pchj} a'_{jc} \leq Z_{ph} a_{pc} \quad ; \forall p, h, c \tag{3}$$

$$\mathrm{LQ}_{chj} = \sum_{p=1}^{P} x_{pchj} a'_{jc} - \sum_{i=1}^{I} y_{jchi} d_{ich} + \mathrm{LQ}_{c(h-1)j} \quad ; \forall c, h, j \tag{4}$$

$$\sum_{i=1}^{I} y_{jchi} d_{ich} \leq \mathrm{LQ}_{chj} \leq Z'_{jh} a'_{jc} \quad ; \forall j, c, h \tag{5}$$

$$\sum_{p=1}^{P} Z_{ph} = P_{\mathrm{Upper}} \quad ; \forall h \tag{6}$$

$$\sum_{j=1}^{J} Z'_{jh} = J_{\mathrm{Upper}} \quad ; \forall h \tag{7}$$

$$Z_{ph}, Z'_{jh} \in \{0.1\}, \quad 0 \leq x_{pchj}, y_{jchi} \leq 1. \tag{8}$$

Considering the presented formulation, Equation 1 is the objective function (OF) which consisted of logistic and annual fixed costs. In other words, the first two parts deal with the total sum of logistic costs (i.e., variable costs including processing, transportation, etc.). It is calculated for both the practicing plants and distributors separately. This type of cost is obtained from the commodity volume multiplied by the associated capacity. For example, the logistic cost of plants is determined by the corresponding unit logistic cost, to supply a given commodity from a plant to a distributor in a specific period, multiplied by the satisfied quantities. The values of responded needs are obtained from the multiplication of the given capacity by the satisfied proportion. The same calculations can be presented for distributors likewise. Furthermore, the fixed cost is realized for each of the plants and distributors with respect to the extant cost once the facility is practicing in the corresponding period. Equation 2 shows the fraction of satisfied demands for each of the commodities in each planning horizon with respect to the existing customers. However, a potential customer can acquire its demanded commodities from a set of distributors instead of focusing on a single one. Equation 3 guarantees that the transportation quantity from plants to distributors cannot surpass the given plant capacity, if it is open in the mentioned period. Equation 4 denotes that the left quantity for each commodity and distributor in each period can be calculated by the sum of transported quantities to customers subtracted from the sum of transported quantities to a distributor added to the left quantity in the previous period. Equation 5 is the same as Equation 3 but concerns the distributors and affirms that the output of each distribution center does not exceed its extant capacity, restricted by the remaining commodities. Moreover, the volume of the left commodities cannot exceed the distributor capacity. Equations 6 and 7 state that the number of open plants and distribution centers in each of the planning horizons should not exceed their upper bounds (i.e., the maximum allowed numbers), respectively. Furthermore, Equation 8 deals with the nature of existing variables.

The schematic performance of our multi-commodity, multi-period, multi-source network is depicted in Figure 1 for a given planning horizon.

Solution methodology

Since we presumed that the developed distribution network practices in a vague environment, demand uncertainty can be represented by fuzzy numbers efficiently. In order to solve the model, we applied the fuzzy approach

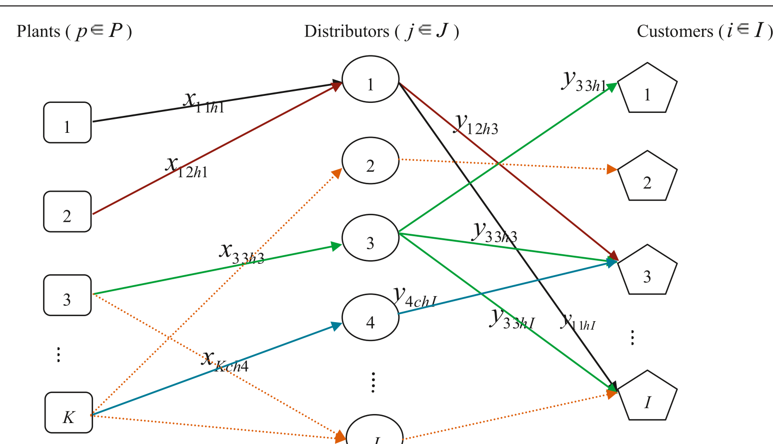

Figure 1 The proposed three-stage distribution network performance in period h.

suggested by Jimenez et al. (2007) which consists of a three-phase interaction algorithm.

Converting the fuzzy model into its crisp equivalent

In this section, we show the main framework of the implementation method of fuzzy approach proposed by Jimenez et al. (2007). Regarding the risk threatening estimated demands, all OF coefficients, technological multipliers, and right-hand side coefficients may take a vague quantity. For instance, consider the following problem in which $\tilde{C}' = (\tilde{c}_1', \tilde{c}_2', \ldots \tilde{c}_n')$, $\tilde{A}'' = \left[\tilde{a}_{ij}''\right]_{m\times n}$, and $\tilde{B}' = (\tilde{b}_1', \tilde{b}', \ldots \tilde{b}_m')^t$ are the fuzzy parameters representing the OF coefficients, technological multiplies, and right-hand side coefficients, respectively. Therefore, the fuzzy mathematical model can be represented by Equation 9:

$$\begin{aligned} &\text{Min} = \tilde{C}'^t x \\ &\text{S.t.}: \tilde{A}'' x \geq \tilde{B}_i' \qquad ; i = 1, 2, \ldots, m \\ &x \in R^n, \geq 0. \end{aligned} \tag{9}$$

The uncertainty in the nature of fuzzy problems makes the decision-makers (DMs) find a solution so that both feasibility and optimality conditions can be satisfied efficiently. Consequently, there has been a great effort on determining different methods which could fulfill the two above characteristics (e.g., refer to Rommelfanger and Slowinski (1998)). Some papers have also discussed the ranking methods besides their justifications. We applied the method introduced by Jimenez (1996) to our problem.

It is assumed that the applied fuzzy parameters follow a trapezoidal nature in order to provide a broader range of potential values. Figure 2 shows the trapezoidal fuzzy number n, in which the α-cut equals the feasibility degree for a specific decision.

Considering the above definitions, the corresponding parts of the OF and constraints (i.e., the parts that include demand parameter) can be transformed to their relevant crisp values due to Equations 10 to 12, respectively:

$$\text{MinEI}\left(\tilde{C}'\right)x = \left[\frac{1}{2}\left(c_1' + c_2'\right) + \frac{1}{2}\left(c_3' + c_4'\right)\right]x \tag{10}$$

$$\left[(1-\alpha)\frac{a_1'' + a_2''}{2} + \alpha\frac{a_3'' + a_4''}{2}\right]x \leq \alpha\frac{b_1' + b_2'}{2} + (1-\alpha)\frac{b_3' + b_4'}{2}. \tag{11}$$

$$\left[\left(1-\frac{\alpha}{2}\right)\frac{a_1'' + a_2''}{2} + \frac{\alpha}{2}\frac{a_3'' + a_4''}{2}\right]x \leq \frac{\alpha}{2}\frac{b_1' + b_2'}{2} + \left(1-\frac{\alpha}{2}\right)\frac{b_3' + b_4'}{2} \tag{12}$$

$$\left[\left(1-\frac{\alpha}{2}\right)\frac{a_3'' + a_4''}{2} + \frac{\alpha}{2}\frac{a_1'' + a_2''}{2}\right]x \geq \frac{\alpha}{2}\frac{b_3' + b_4'}{2} + \left(1-\frac{\alpha}{2}\right)\frac{b_1' + b_2'}{2}$$

It should be noted that EI stands for the expected interval for the corresponding fuzzy number and can be computed by finding the half point of the given fuzzy number. We can therefore calculate the expected value of a triangular or any other shaped fuzzy numbers by considering the mentioned concept likewise. Equation 11 can be applied for less than or equal type constraints, and Equation 12 can be applied for the equality type constraints likewise. However, the crisp form of the proposed model can be rewritten as Equations 13 to 16 based on Equations 2, 3, 6, 7, and 8:

$$\begin{aligned} \text{Min} = &\sum_{p=1}^{P}\sum_{c=1}^{C}\sum_{h=1}^{H}\sum_{j=1}^{J}\text{LC}_{pchj}x_{pchj}a_{jc}' + \\ &\sum_{j=1}^{J}\sum_{c=1}^{C}\sum_{h=1}^{H}\sum_{i=1}^{I}\text{LC}_{jchi}'y_{jchi}\left(\frac{d_{ich}^1 + d_{ich}^2 + d_{ich}^3 + d_{ich}^4}{4}\right) \\ &+\sum_{p=1}^{P}\sum_{h=1}^{H}\text{FC}_{ph}Z_{ph} + \sum_{j=1}^{J}\sum_{h=1}^{H}\text{FC}_{jh}'Z_{jh}' \end{aligned} \tag{13}$$

S.t.:

$$\text{LQ}_{chj} \leq \sum_{p=1}^{P}x_{pchj}a_{jc}' - \sum_{i=1}^{I}y_{jchi}\left[\frac{\alpha}{2}\left(\frac{d_{ich}^1 + d_{ich}^2}{2}\right) + \left(1-\frac{\alpha}{2}\right)\left(\frac{d_{ich}^3 + d_{ich}^4}{2}\right)\right] + \text{LQ}_{c(h-1)j};\ \forall c, h, i \tag{14}$$

$$\text{LQ}_{chj} \geq \sum_{p=1}^{P}x_{pchj}a_{jc}' - \sum_{i=1}^{I}y_{jchi}\left[\frac{\alpha}{2}\left(\frac{d_{ich}^3 + d_{ich}^4}{2}\right) + \left(1-\frac{\alpha}{2}\right)\left(\frac{d_{ich}^1 + d_{ich}^2}{2}\right)\right] + \text{LQ}_{c(h-1)j};\ \forall c, h, i \tag{15}$$

$$\sum_{i=1}^{I}y_{jchi}\left[(1-\alpha)\left(\frac{d_{ich}^1 + d_{ich}^2}{2}\right) + \alpha\left(\frac{d_{ich}^3 + d_{ich}^4}{2}\right)\right] \leq \text{LQ}_{chj} \quad ; \forall j, c, h \tag{16}$$

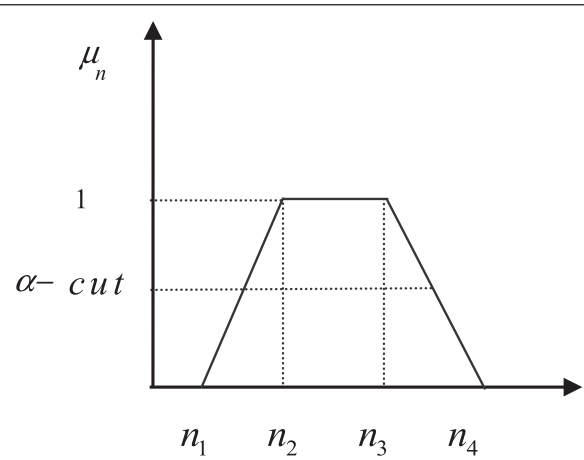

Figure 2 The membership function of trapezoidal fuzzy number *n*.

Interaction resolution method

In this section, we explain the three levels of the resolution method in which the DM is involved interactively.

Firstly, the linear fuzzy model transformed into its crisp mode should be solved regarding each different value of α-cuts independently. In order to make the comparison more clearly, the solutions are compared against three quantitative measures as follows:

- Fixed cost: it can be calculated by the sum of annual fixed costs of practicing facilities in planning horizons.
- Logistics cost: it can be calculated by the sum of cost of supplying commodities from plants to customers in the given periods.
- Average customers' satisfaction: it can be determined by Equation 17 so that the average satisfied demands are determined as follows:

$$\sum_{h=1}^{H} \frac{\left(\sum_{j=1}^{J} y_{jchi} d_{ich} / d_{ich}\right)}{H} \quad ; \forall c, i. \tag{17}$$

Secondly, a decision vector should be obtained so that the DM is satisfied with respect to two conflicting factors (i.e., *feasibility* degree and *global satisfaction*). In this phase, the DM is asked to provide an aspiration level z added to the relevant tolerance threshold t for the values obtained by the three measures introduced above. The DM's satisfaction level can be assessed by means of a fuzzy set $\tilde{z}$ whose membership function is depicted by Equation 16 or 17 for a 'less is better' or 'much is better' case, respectively. In other words, the first two measures exemplify the 'less is better' state and the last measure typifies the 'much is better' one. However, λ is projected in a descending and ascending manner for Equations 18 and 19, respectively:

$$\mu_{\tilde{z}}(\text{OF}) = \begin{matrix} 1, \text{if} & \text{OF} \le z \\ \lambda \in \{0,1\} \text{if} & z \le \text{OF} \le z + t \\ 0, \text{if} & \text{OF} \ge z + t \end{matrix} \tag{18}$$

$$\mu_{\tilde{z}}(\text{OF}) = \begin{matrix} 1, \text{if} & \text{OF} \ge z \\ \lambda \in \{0,1\} \text{if} & z - t \le \text{OF} \le z \\ 0, \text{if} & \text{OF} \le z - t\,. \end{matrix} \tag{19}$$

Since we have defined three measures $\lambda_k (k = 1, 2, 3)$ for our DND problem, we are required to specify the associated weights ω_k, with regard to the DM's opinion by one of the multi-criteria decision-making tools (e.g., AHP, ANP, or TOPSIS). At the end of the second level, the global satisfaction degree Ω can be determined by Equation 20:

$$\Omega = \lambda_1 \omega_1 + \lambda_2 \omega_2 + \lambda_3 \omega_3. \tag{20}$$

Finally, we should reach a favorite balanced solution considering the feasibility degree and the global satisfaction degree at the third level. This is carried out by using Equation 21 that consists of two fuzzy sets whose membership functions represent the magnitude the DM accepts for each feasibility degree ρ_α and satisfaction degree ρ_Ω importance:

$$P = \theta \rho_\alpha + (1 - \theta) \rho_\Omega \qquad \text{if } \rho_\alpha, \rho_\Omega \ne 0 \text{ and } 0, \text{otherwise.} \tag{21}$$

Note that P is the joint acceptation index obtained by the linear combination of the two mentioned degrees and $\theta \in [0,1]$ points to the relative importance the DM assigns to feasibility in comparison with global satisfaction.

Computational study

In this section, the developed fuzzy model is tested by the sensitivity analysis, and then a numerical example is solved to show its applicability and efficiency in real-world problems. Finally, the total performance of the model is investigated by different-sized problems. The calculations are run by Lingo 8 solver on a PC with characteristics of Intel®, Pentium® 4, central processing unit (CPU) 3.20 GHz, and 2,046 MB of RAM.

Sensitivity analysis

Applying the sensitivity analysis helps us understand how accurate the problem performance is. This is demonstrated by changes in model outputs according to manipulations in parameters. In other words, it is assumed that the input data may vary independently. Hence, we have to investigate the potential changes in values of model variables. To do so, a typical example whose data have been created randomly is considered here. The results are depicted in Table 2.

Implementation of interactive phases

This section is presented in order to show how the model can be implemented with respect to the three interactive solution steps. To do so, the DM is required to specify the aspiration level of each three introduced measures $\mu\tilde{z}(\text{OF})$ as follows. It should be noted that the measures' quantity is calculated according to the lower and upper bound results, obtained from different cuts:

- Fixed cost: z = US$17,650,000 and $z + t$ = US $18,800,000.

Table 2 The results of fuzzy sensitivity analysis

Variation source	Fixed costs (US$)	Logistic costs (US$)	OF (US$)	Average customers' satisfaction (%)
Fixed costs	18,095,016	947,500	19,042,516	89.2
Logistic costs	17,650,000	765,014	18,415,014	83.1
Demand	17,923,000	1,100,540	19,023,540	85.7
Capacities	17,025,000	945,000	17,970,000	86.2

Table 3 The comparison results between feasibility sets and deterministic state

α-cut	OF (US$)	CPU time (s)	Iteration	Fixed cost (US$)	Logistic cost (US$)	Average customers' satisfaction (%)
0	18,595,000	14	52,741	17,650,000	945,000	78.6
0.1	18,602,400	13	47,856	17,665,500	952,400	81.4
0.2	18,784,700	15	58,996	17,823,000	961,700	86.2
0.3	18,937,300	14	44,258	17,965,000	972,300	85.7
0.4	19,204,020	21	68,439	18,220,000	984,020	88.9
0.5	19,558,500	26	89,477	18,575,000	983,500	94.5
0.6	19,510,750	27	71,644	18,522,000	988,750	92.8
0.7	19,810,600	34	83,355	18,800,000	1,010,600	92.1
0.8	19,844,330	31	67,195	18,800,000	1,044,330	87.7
0.9	19,844,050	50	85,008	18,800,000	1,044,050	85.5
1	19,860,010	63	107,414	18,800,000	1,053,070	77.3
Deterministic	19,789,840	19	104,220	18,800,000	1,060,010	81.2

- Logistic cost: z = US$945,000 and $z + t$ = US $1,060,010.
- Average customers' satisfaction: $z - t$ = 77.3% and z = 94.5%.

The global satisfaction degree can be determined by weighting the measures which is done here by considering equal importance (i.e., $\omega_1 = \omega_2 = \omega_3$).

Afterwards, it is time to specify the joint acceptance indicator, shown by Equation 19. The results of our typical example are demonstrated in Tables 3 and 4, respectively, in which the former shows the output for each feasibility cut and the latter shows the final quantity regarding the feasibility and global satisfaction simultaneously. Note that θ is assumed to be 0.5 to give equal weights to both feasibility and global conditions.

Referring to Table 3, it can be observed that the fuzzy formulation led to lower costs for all cuts compared with the deterministic mode. However, cuts greater than 0.6 resulted in an equal value for the annual fixed cost. In order to compare the results with respect to the elapsed solution time (i.e., the CPU time), it is apparent that the deterministic mode needs a rather small time although the lower cuts do the same. However, it indicates an approximately ascending order, so the last cuts required much more time that the first ones. On the other hand, the average customers' satisfaction measure showed the third lowest value for the deterministic mode. However, the measure increases as the cut magnitude goes up until it reaches the maximum value and then starts to decrease.

Table 4 The best cut with respect to the DM's opinion

α	ρ_α	Ω	ρ_Ω	P
0	0	0.02	0	0
0.1	0.05	0.09	0.1	0.075
0.2	0.15	0.26	0.33	0.24
0.3	0.25	0.32	0.4	0.325
0.4	0.4	0.47	0.55	0.475
0.5	0.55	0.71	0.78	0.765
0.6	0.7	0.65	0.7	0.7
0.7	0.85	0.81	0.93	0.89
0.8	0.9	0.82	0.95	0.925
0.9	0.95	0.77	0.86	0.905
1	1	0.66	0.72	0.86

Considering Table 4, it is obvious that α = 0.8 yielded the most acceptable result for the final solution. The DM's opinions can be seen for each of the different feasibility and satisfaction degrees likewise.

Total performance of the model

In order to understand the total performance of the mathematical formulation, some different-sized problems are solved and compared with respect to the elapsed solution time. The results are shown in Table 5.

Table 5 The results when considering different-sized problems

Problem size $p \times c \times j \times i \times h$	CPU time (s)	OF (US$)	Average customers' satisfaction (%)	Variables	Constraints
2 × 2 × 4 × 6 × 2	68	16,855,025	81	418	420
3 × 2 × 5 × 8 × 2	217	17,120,250	75	622	754
3 × 2 × 5 × 9 × 3	873	23,864,700	86	1,216	1,436
3 × 3 × 6 × 9 × 3	1,307	25,004,610	79	1,484	1,668
4 × 3 × 6 × 10 × 2	4,876	93,324,080	80	1,560	1,610
5 × 3 × 10 × 12 × 3	9,654	26,080,950	76	3,044	3,152
5 × 4 × 10 × 15 × 4	14,662	35,775,940	78	5,696	5,842

It is clear that because of intrinsic intricacy, it takes a long time for the model to respond as the dimension enlarges. These problems belong to the NP-hard category, and the solution time increases exponentially as the problem size rises, in other words. For example, in Table 5, there is a noticeable difference in CPU time while even one of the problem elements changes to a higher value. This can be easily observed as the required solution time gets multi-fold once the considered problems become larger. For instance, the second problem is larger than the first one just as one unit for the number of plants and two units for the number of distribution centers, but its elapsed time is 3.19 times as much as that of the first one. Like the previous comparison, the third problem needed a more remarkable time than the second one while the number of considered customers and planning horizons has been added by one unit. That same analysis can be carried out for the rest of the problems as well. Consequently, it is suggested that intelligent computation approaches be applied for large-sized cases in particular. This makes the mathematical model be applied efficiently for any real market conditions.

Conclusions

The noticeable increase in competition amongst the market holders makes the SCM a necessary tool in fulfilling the customers' needs. DND is of great importance as it establishes the first steps in handling the network. Consequently, a multi-period, multi-commodity, multi-source DND problem was considered in this paper. We extended our formulation to an uncertain environment, due to receiving demands, by fuzzy mathematical programming. The fuzzy model consisted of a three-interaction resolution method, so it included the DMs throughout the solution procedure. Afterwards, it was validated by a sensitivity analysis, and then a numerical example was solved in order to give a picture of how the method steps can be implemented. The consideration of different-sized problems was also carried out and indicated to the intrinsic complexity.

The presented model focused on increasing the network design efficiency so that it can help more to the drawn decisions. Considering that demand uncertainty causes the disruptions, it threatens the network (e.g., the Bullwhip effect) to decrease to a great extent as the practitioners are well equipped by demand fluctuations *a priori*. In fact, this issue helps in the necessity of the least changes in executive plans.

For future studies, it is suggested that the presented model be solved by more effective tolls like meta-heuristic or exact methods and even compared with each other. The model can also be enriched by added concepts like the inventory control management. Furthermore, the problem can be considered by other backgrounds like economic viewpoints at the same time.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

BHT carried out the problem formulation and its solution, and JR participated in its design and coordination. Both authors read and approved the final manuscript.

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