**Original Research**

# Supply Chain Coordination with Wholesale Contract in an Uncertain Environment

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**Abstract**

Supply chain coordination is one of the most recently studied fields. One of the coordination contracts is the wholesale contract, which aims to balance the order quantity by offering a reduced price for the large quantity order by the retailer. The goal is to determine the order amount so that both parties in the channel are satisfied with a reasonable profit concerning the proposed price. When a new product is presented to the market, the main challenge would be a sensible demand prediction such that either stock over or shortage stays under a moderate level. There is insufficient sample space in such a new product, or the existing data for similar products does not apply to this new case. These drawbacks prevent one from applying probability theory. Referring to an expert would be an appropriate approach in this situation. Uncertainty theory is one of the mathematical paradigms which deals with this problem well. Using this paradigm, we assume that the demand is an uncertain variable where its uncertainty distribution is presented. We mainly consider the linear uncertain demand and investigate optimal policy for both parties with and without coordination. An illustrative example verifies the appropriateness of the suggested approach. We also compare the results with the case when different scenarios are assumed, and probability theory is the base for the results.

**Keywords-** Supply chain; Coordinating contract; Wholesale contract; Uncertainty theory

## INTRODUCTION

The economic world is facing additional complications nowadays. Due to rapid technological innovation and customer behaviour alterations, many products have short life cycles, and most of the time, their price drops at midlife. This phenomenon forces managers to act more effectively and preemptively than before. The main challenge is that the market integrates with inevitable indeterminacy, and appropriate modelling of problems in such an environment is essential. When one does not recognize the precise estimates of the involved parameters, and the model cannot handle this indeterminacy correctly, irrational decisions would be made, resulting in irrecoverable consequences. This research aims to provide practical solutions to these challenges, offering a roadmap for effective decision-making in uncertain supply chain environments. Supply Chain Management (SCM) has been established to handle many indeterminacies. Optimal interactions between a supplier and a retailer are one of the study fields in SCM. It is expected that proper coordination overcomes potential business complexities. Without coordination between the distribution channel players, each would pursue his benefits without considering the other. This competitive behaviour would disrupt the whole channel but not result in one's gain.

There are different coordinating contracts in the literature of SCM. Wholesale price, buy-back, and revenue-sharing contracts are some to name [1]. A coordinating contract is designed to boost the whole channel's performance and satisfy customers' unknown demands as much as possible. The main objective in coordinating contracts is to propose decisions that benefit the entire supply chain channel. The study of these contracts with different points of view includes a vast study range. The wholesale contract is one of the most studied contracts between the supplier and the retailer. The manufacturer places the discounted wholesale price in a returns-discount contract to secure the most profits for both sides and increase the supply chain's performance. Even though it is not a coordinating one from the probability theory perspective but is popular nowadays, this fact may be concluded to the impression that the probability theory perspective in the study of this contract does not make sense in many diverse environments.

Several approaches have been employed to diminish the impacts of such indeterminacy in analyzing coordinating contracts. Historically, probability theory came first to interpret frequencies when enough reliable data exists. Some others have been suggested for other situations. Most of them claim to model the unknowns based on expert opinion. An approach is the fuzzy theory. While it successfully models many problems, some paradoxical conclusions still exist in applying the fuzzy theory. Let us clarify why fuzzy theory is unsuitable for modelling and analyzing some problems. Suppose that the variable of demand is considered as a fuzzy variable $\xi$ with the membership function $\mu(x)$. Here, the triangular membership function is used for the fuzzy demand amount, which would be something like follows.

$$\mu(x) = \begin{cases} 0 & x < 150 \\ (x-150)/20 & 150 \le x \le 170 \\ (210-x)/40 & 170 \le x \le 210 \\ 0 & x > 210. \end{cases}$$

It means that the fuzzy number referring to uncertain demand mount is (150, 170, 210). Considering the membership function $\mu$ and the meaning of the possibility measure, we have $\text{Pos}\{\xi \in B\} = sup_{x\in B}\mu(x)$. This means that for the demand amount of 170, we have Pos{Demand=170}=1, and Pos{Demand≠170}=1. Therefore, we conclude that (a) Demand is "precisely 170" with possibility measure 1, and (b) Demand is "not 170" with possibility measure 1. This means that "Exactly 170" is as possible as "not 170" for the demand level. Therefore, one has to consider them equally likely, which is not a logical inference. For more details, consult [2], see also [3] and [4].

There are different methods for predicting the demand level. One of them is the trend projection that applies previous sales data to plan future sales. It is the most clear-cut demand forecasting method. Another method is Market research demand forecasting based on customer survey data. Observe that this method is an after-production method and is not applicable for the new-introduced-to-market products. The econometric method can also be exploited but necessitates some number processing. This quantitative forecasting mode merges sales data with information on outside forces that affect demand level. Then, a mathematical modus operandi is devised to forecast future customer demand. These methods need some reliable data and are generally based on probability theory. It is important to mention that such data rarely exists for new innovative products. Consequently, using probability theory to predict the level of demand would not be efficient. The Delphi method would also be used as a qualitative method of demand forecasting that forces expert opinions on the market forecast. This method requires involving outside experts and an experienced facilitator. This is also impractical for innovative products since producers prefer to keep some existing information as their assets and not reveal it. As mentioned above, Fuzzy theory also has problems in some situations that may lead to misleading results. We emphasize that one of the main challenges is that the market always faces inevitable uncertainty, and proper modelling is necessary for these issues in such environments. When the person does not know the exact values of the parameters in the problem and the model cannot correctly manage this uncertainty, irrational decisions lead to irreparable consequences. Here, we apply uncertainty theory for modelling and analyzing such a situation.

The main drawback of existing studies is that probability theory-based findings are applicable when enough data exist, the probability distribution of demand is provided even approximately, and axioms of probability theory govern the unknown environment. The fuzzy theory also has intuitive contradictions in theory and practice, as mentioned above. None of these paradigms would produce sensible results when an expert provides data. Uncertainty theory was established in 2007 as a solid mathematical structure that formalizes human reasoning in an axiomatic framework. It has been used and denoted its capability in many practical problems.

In this paper, we consider the wholesale contract in an uncertain environment. An expert provides her opinion about the unknown data on the channel's coordination, and the analysis is based on uncertainty theory. To keep the modelling simple, we assume that the channel compromises one supplier and one retailer and only the demand level in an uncertain variable. We restrict the argument to when uncertain demand has a linear distribution. Analyzing the model shows that this contract would coordinate the channel's players' decisions, which is in accord with the exercising reality in the nowadays deals. This paper is structured as follows. We first review some published results in this field with different views. Sunsequent section is devoted to reviewing some necessary concepts from uncertainty theory. Then, we presents the problem model and solution approach.

We also solve the problem with a special selection of the demand's uncertain level as linear. A simple toy example is presented in the next section to clarify the methodology. We also compare our approach and scenario-based probability theory point of view to denote the superiority of our findings. In the final section, some concluding remarks are mentioened and additional work directions are sketched.

## LITERATURE REVIEW

Here in this review, we focus on recent works with different points of view on supply chain coordination, especially on the wholesale contract. Besides other contracts, the wholesale price contract has been studied using probability theory. In [5], a newsvendor problem involving one retailer and one manufacturer was investigated. It was shown how the manufacturer puts the discounted wholesale price in a return-discount contract to get the most profits and improve the supply chain's performance. A stochastic demand, dependent on the retailer's promotional and the manufacturer's innovation efforts, is assumed. The paper proposed a new compensation-based wholesale price contract to encourage actors to engage in the joint decision-making scheme actively [6]. In another study, a wholesale contract was applied in a supply chain with one supplier and one buyer; both are risk-neutral. They face fixed demand for a single-selling period, and the study focuses on the consequence of supply indeterminacy. They found that contract performance monotonically improves with the supplier's negotiating power for random capacity, regardless of the purchase process mode [7]. A recent study considers two supply chain members: the supplier and the manufacturer [8]. The authors also compared the bare wholesale price contracts in centralized and decentralized frameworks.

For one manufacturer that distributes multiple products to multiple retailers, the authors in [9] proposed a multi-product, multi-period wholesale price coordination procedure in a decentralized supply chain. A bi-level wholesale price contract is presented in another recent research [10]. The authors considered three sections (one manufacturer, one distributor, and one retailer) in a supply chain with a scenario-based demand, and corporate social responsibility was treated as the coordinated decision in the channel. Coordinating contracts are also studied using fuzzy theory; see, e.g. [10] for the buy-back contract, [11] for computational complexities in practice, [12] for the buy-back contract with different risk attitudes. The wholesale contract has also been investigated by considering some elements as fuzzy numbers. For instance, a bilateral wholesale price in the supply chain has been proposed in a recent study [13]. Their coordination model improved pricing, quality, after-sales service, and service level performance. As a result, this contract could effectively coordinate the cell phone supply chain and enhance their profits compared to individual decisions for their specific goals.

The supply chain in robust optimization has been the subject of innovative research. A bi-objective model was recently explored for perishable products, aiming to minimize network costs and reduce greenhouse gas emissions. The results revealed a meaningful disparity between the mean of two objective functions and the run time. The general findings indicated that the weighted sum method is highly effective in obtaining the expected value of the first objective function. The Torabi-Hassini method, on the other hand, yields superior results based on the second objective [14]. The game theory introduced a practical approach to the wholesale contract. For instance, a game theory model of an altruistic retailer and a self-interested supplier in a wholesale price contract has been developed. The authors asserted that their model offers a robust optimal solution even when the demand information is lost. Their findings suggest that complete altruism, rather than partial altruism, can effectively coordinate the supply chain in the wholesale price contract [15]. Another model, in combination with robust optimization [16], has been devised. It considers a buyer's purchasing policy from a supplier with stochastic capacity. The study demonstrates how buyers can settle the optimal order quantity and the wholesale price, with the main result being that the parameter uncertainty degree rises as the optimal robust wholesale price goes up.

In a recent research [17], a collaborative game strategy has been devised. The authors considered a capacity allocation problem with an airline selling cargo paths to several shipment forwarders. It is supposed that the specific capacity from one route cannot secure the total orders of forwarders (referred to as 'hot selling routes'), while from the substitute route is much less than its capacity (referred to as 'underutilized routes'). To solve this disproportion problem, a sequential cooperation game is played between the airline and the freight forwarders, stressing the collaborative nature of the solution. In this game, the player's payoff is the anticipated gain from exercising a mixed-wholesale-option contract between airlines and forwarders. The model result indicates that the demand on the underutilized routes follows self-replicating distributions. The mixed model yields the highest quotas on the underutilized routes, guiding to an improved demand balance amongst the alternative courses.

Another study considered the quantity of grain farmers sell, where setup costs of factories and warehouses are uncertain variables and proposes an uncertain grain supply chain [18]. An uncertain sustainable supply chain has been investigated in another research where cost factors, environmental impacts, and social benefits are uncertain. The authors developed a multi-objective chance-constrained model with an uncertain scenario to examine the effects of uncertainties on decision variables. The results express that the decision-maker should consider consuming more resources to cope with the system's uncertainties at a higher confidence level [19]. An uncertainty theory to model demand distribution is applied [20], proposes the belief rate of order size is fewer than the supply chain optimal order amount and prepares the lower bound of the belief level. As a result, the relationship between the expected remaining inventory, the optimal order quantity, and the belief degree of this uncertain event is obtained. The pricing decision problem in which two producers compete to supply dissimilar but substitutable products through a typical retailer under other power configurations has been investigated using uncertainty theory [21]. Uncertain variables are manufacturing costs, sales costs, and demands. The authors derived how to decide about the optimal pricing on wholesale prices and retailer profits in three potential situations. In the study on strategic cooperation in supply chains [22], it was found that producers consistently desire long-term wholesale agreements. The study also revealed that a combined structure, when a supply chain exercises a long-term contract and the other a short-term contract, can result in a more stable system. Furthermore, the study emphasized that customers gain more from behavior-based pricing, while short-term wholesale contracts increase surplus.

Whether the supplier advantages from setting up nonlinear capacity reservation contracts as an alternative to wholesale price contracts has also been investigated [23]. From the supplier's standpoint, the capacity reservation contract achieves meaningfully better results than the wholesale price contract. However, the supplier's profit from applying capacity reservation is much superior at low margins than at high margins. Concerning supply chain performance, the supplier's positive effect outweighs the buyer's negative impact in low-margin settings. In contrast, the each consequence reduce the effect the other in high-margin settings. The authors affirm that even though nonlinear contract complexity leads to inferior performance than the theory anticipates, their study denotes that suppliers would even profit from installing them. Therefore, required management concepts are provided in deciding on the type of contract. Wholesale price contracts are also commonly exercized in closed-loop supply chains (CLSCs). A recent study by [24] found that when the inequality aversion of the is strong and the wholesale price contract is fixed, coordination is possible in a decentralized channel and a manufacturer-led CLSC. The study concludes that the producer's allocated portion is superior than the retailer's, and the retailer's share is above a smallest possible limit. The methodology exploited in this study involved solving a multistage successive move game under two set models and using the Karush-Cohen-Tucker condition for constrained optimization to find the bounds for the existence of perfect Nash equilibrium.

Wholesale contracts are also applied in the environmental discipline. It assesses pricing options, the extent of sustainability activities and the carbon cap considering wholesale price and cost-sharing contracts [25]. The author developed a Stackelberg game approach for both type of supply chain contracts and suggested the equilibrium solutions provided by the game actors. The conclusions assert that the existence of a carbon cap meaningfully influences the performance of the supply chain. Furthermore, the higher the carbon ceiling determined by the offícals, the more sustainability improvement attempts the supply chain will make, and the more the supply chain can enhance its cost-effectiveness and sustainability under a cost-sharing contract. Wholesale price contracts with risk restrictions in a supply chain, incorporating a supplier and a capital-constrained retailer, were also studied [26]. A mean-variance model is derived to evaluate wholesale price contract design decisions under trade credit and bank credit finance support. It also stipulated the conditions in which the supplier is prepared to provide trade credit and the conditions in which the retailer selects bank or trade credit. It is also deduced that the risk aversion position of the supply chain members plays an important role in adjusting the financing balance. Finally, the results support the conclusion that trade credit financing leads to a win-win outcome only when the supplier's risk aversion tolerance is fair.

A supply chain is also investigated under a manufacturer's discount policy, assuming revenue-sharing and wholesale price contracts, where the Stackelberg equilibrium is evaluated for the supply chain respecting each contract [27]. It was deduced that the retailer's optimal retail price is identical in both the manufacturer's corresponding optimal decisions for revenue sharing and discounted wholesale price contracts. Moreover, sufficient prerequisites are obtained under which a revenue-sharing contract and a discount wholesale price contract can result in Pareto advance, i.e., both manufacturer and retailer earn a higher expected profit than the situation not considering discount. Appropriate requirements are also derived, where a revenue-sharing contract with a discount strategy can lead to Karldor-Hicks' improvement. Furthermore, it has been demonstrated that with a discount, a revenue-sharing contract can accomplish Karldor-Hicks' improvements over the wholesale price contract, too. The wholesale contract has also been considered in the framework of two-echelon e-commerce. In [28], the author considers a supply chain with a retailer and a supplier. Here, the retailer is a follower, and the focus theory of choice is considered for him. The effect of retailer pricing options on the supplier is examined for diverse emphasis and coordination priorities for the whole supply chain. The lower the parameter $\varphi$ (degree of positivity) and the higher the parameter $\kappa$ (level of confidence), the closer the gain of the whole supply chain to the coordination result and as for defining $\varphi$, the lower $\kappa$, the better the supply chain coordination. The conclusions of the proposed model can simultaneously present a theoretical basis for increasing cooperation between supply chain players and managerial insight for decision-makers to select cooperating bodies.

The study of the supply chain in uncertainty theory is still in its infancy. There are almost no published results on the analysis of contracts using uncertainty theory. Here, we mention some recent findings published in SCM using the uncertainty theory framework. A closed-loop supply chain network for producing and recovering button batteries under uncertainty has been modelled in which the demand, cost, and capacity are uncertain variables. The model evaluates the supply chain network's environmental effects using a life cycle assessment approach. The results confirm that the expected value model tends to be decentralized while the chance-constrained model tends to be centralized [29].

## PRELIMINARIES

This section reviews some basic concepts in uncertainty theory. We refer the interested reader to [2] for more details. Let $\Gamma$ be a nonempty set, and $L$ be a $\sigma$-algebra over $\Gamma$. Any element $\Lambda$ in $L$ is called an event. A set function $M$ from $L$ to $[0,1]$ is an uncertain measure if the following four axioms hold for this function.

**Axiom 1** (Normality Axiom) $M\{\Gamma\} = 1$.
**Axiom 2** (Duality Axiom) $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event $\Lambda$.
**Axiom 3** (Subadditivity Axiom) For every countable sequence of events $\Lambda_1, \Lambda_2, \ldots$,

$$M\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} M\{\Lambda_i\}.$$

The triplet $(\Gamma, L, M)$ is called an uncertainty space. The product axiom, distinguishing the probability theory from the uncertainty theory, is defined as follows [30].

**Axiom 4** (Product Axiom) Let $(\Gamma_k, L_k, M_k)$ be uncertainty spaces for $k = 1,2, \ldots$. Then, the product uncertain measure $M$ on product $\sigma$-algebra $L_1 \times L_2 \times \cdots \times L_n$ satisfies

$$M\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} M_k\{\Lambda_k\}.$$

An uncertain variable $\xi$ is a measurable function from an uncertainty space $(\Gamma, L, M)$ to the set of real numbers in which $\{\xi \in \mathcal{B}\}$ is an event for any Borel set $\mathcal{B}$. Uncertainty distribution of an uncertain variable $\xi$ is defined as [2],

$$\Phi(x) = M\{\xi \leq x\}, \qquad \forall x \in \Re.$$

Further, for any real number $x$, we have

$$M\{\xi > x\} = 1 - \Phi(x).$$

The expected value of $\xi$ is defined as

$$E[\xi] = \int_0^{+\infty} M\{\xi \geq x\}dx - \int_{-\infty}^{0} M\{\xi \leq x\}dx,$$

provided that at least one of the integrals is finite [2]. When the uncertainty distribution $\Phi(x)$ of $\xi$ is available, we have

$$E[\xi] = \int_0^{+\infty} \big(1 - \Phi(x)\big)dx - \int_{-\infty}^{0} \Phi(x)dx, \qquad (1)$$

provided that at least one of the two integrals is finite. There are different uncertainty distributions in the literature. Since our methodology is independent of the distribution, we only mention the linear uncertainty distribution [30]. Linear uncertain variable is defined with the uncertainty distribution

$$\Phi(x) = \begin{cases} 0 & x \leq a \\ \dfrac{x-a}{b-a} & a \leq x \leq b \\ 1 & x \geq b, \end{cases} \qquad (2)$$

where $a$ and $b$ are real numbers, and $a < b$. A linear uncertain variable is denoted by $L(a,b)$. It can be easily understood that the expected value of the linear uncertain variable $L(a,b)$ is $\mu = \frac{a+b}{2}$.

## WHOLESALE CONTRACT STRUCTURE

The operation process in the proposed wholesale contract is as follows. The supplier poses the retailer a wholesale price for initiating the contract, while the retailer can accept or reject it. If the retailer accepts the proposal, order quantity must be decided before starting a single selling season while the demand is uncertain. If the order quantity is not satisfying for the supplier, the supplier will revise the wholesale price to motivate the retailer to order more. When they agree on the wholesale price and order quantity, the supplier produces (or purchases) and delivers the agreed quantity to the retailer. The retailer sells the products, and transfer payments are made between two parties based on the decided contract. It is assumed that the problem is modelled on the uncertainty theory viewpoint. We suppose both parties have identical shared information about the uncertainty distribution of the uncertain demand. Moreover, if some merchandise is not sold, it can be returned to the supplier against receiving a salvage value. Here, we consider the uncertain newsvendor problem to simplify the model, aiming to maximize the expected profit of the whole channel. The notations used in this model are summarized in Table 1. Let us denote the uncertain demand with $D \geq 0$ and its expected value with $\mu$. Moreover, let $q$ be the amount of order as a decision variable, and $S(q) = \min\{q, D\}$ be the sale level. Here, we suppose that the transfer payment $T$ is linear with respect to the order quantity $q$, i.e., $T = wq$. The retailer's profit function is calculated as

$$\pi_r(q) \ = E[pS(q) + v(q-D)^+ - g_r(D-q)^+] - c_r q - T. \quad (3)$$

where $x^+ = 0$ when $x < 0$, and $E[.]$ stands for the expected value. The first term is the expected sales revenue plus the unsold items earnings while the shortage cost is subtracted. The two last terms are the entire order marginal cost of the retailer and the payment amount to the supplier, respectively. Analogously, the supplier's profit function is expressed by

$$\pi_s(q) = T - c_s q - g_s E[(D-q)^+]. \quad (4)$$

The first term is the revenue from the delivery of the order to the retailer, while the last two terms are the order marginal cost and the expected loss from the shortage, respectively. As a result, the total profit of the supply chain is

$$\pi(q) = \pi_s(q) + \pi_r(q) = E[pS(q) + v(q-D)^+ - g(D-q)^+] - cq, \quad (5)$$

where $c = c_r + c_s$ is the cost of production and $g = g_s + g_r$ is the penalty cost, the retailer receives $v < c$ per unsold unit at the end of the sale term.

TABLE 1
PARAMETERS AND VARIABLES OF THE MODEL

| Notation | Description |
|---|---|
| $D$ | Uncertain demand, $E(D) = \mu, D \geq 0$ |
| $c_s$ | Wholesale's production cost per unit |
| $c_r$ | Retailer's marginal cost per unit |
| $p$ | Retailer price |
| $w$ | Wholesale price |
| $v$ | Salvage value |
| $g_s$ | Wholesale shortage cost |
| $g_r$ | Retailer's shortage cost |
| $\pi_s$ | Expected profit of the supplier |
| $\pi_r$ | Expected profit of the retailer |
| $T$ | Transfer payment |
| $\Phi(x)$ | Demand uncertainty distribution function |

## UNCERTAIN LINEAR DEMAND

We emphasize that our methodology is independent of the uncertain distribution type. All existing uncertain distributions can be considered, but not the normal distribution. While the demand is intuitively nonnegative, the belief that the uncertain demand is negative is not zero in the normal uncertainty distribution. Without loss of generality, we suppose that the uncertain demand has a linear distribution $D = L(a, b)$ with $0 < a < b$, and consider it as an assumption in the sequel. The following lemma describes the expected sale amount in this case.

**Lemma 1.** For the linear uncertain demand $D$, the expected sale is

$$S(q) = q - \frac{(q-a)^2}{2(b-a)}. \quad (6)$$

The following technical observation further simplifies the profit functions (3) and (4).

**Lemma 2**. Considering the linear uncertain demand $D$, expected unsold items and lost sale amount are determined as

$$E[(q-D)^{+}] = q - \mathcal{S}(q), \quad (7)$$

$$E[(D-q)^{+}] = \begin{cases} \dfrac{(2q-b)^2}{2(b-a)} & a \le q \le \dfrac{b}{2} \\ 0 & \dfrac{b}{2} \le q \le b. \end{cases} \quad (8)$$

**Remark 1**. Observe that the proof of Lemma 2 depends on the position of $q$ in the interval $[a, b]$, especially on the value of $\frac{b}{2}$. The lemma says we expect no lost sales when the order exceeds $\frac{b}{2}$,. This result is still valid when $a \ge \frac{b}{2}$. This situation occurs when $a$ is close enough to $b$. It means we do not expect a lost sale when the uncertainty interval is tight enough. Clearly speaking, when $a \ge \frac{b}{2}$, we have $E[(D-q)^{+}] = 0$. The following theorem presents a closed form of the expected profit for the retailer when the demand is a linear uncertain variable.

**Theorem 1**. Consider the linear uncertain demand $D$ with $a \le \frac{b}{2}$. The expected profit of the retailer is

$$\pi_r(q) = q(p - c_r - w) + \frac{(a-q)^2(v-p) - g_r(b-2q)^2}{2(b-a)} \qquad \text{if} \quad a \le q \le \frac{b}{2}, \quad (9)$$

$$\pi_r(q) = q(p - c_r - w) + \frac{(q-a)^2}{2(b-a)}(v-p) \qquad \text{if} \quad \frac{b}{2} \le q \le b. \quad (10)$$

The following claim corresponds to the case when $a \ge \frac{b}{2}$. It is an immediate consequence of the discussion in Remark 1 and Eq. (3).

**Corollary 1**. For the linear uncertain demand $D$ with $\frac{b}{2} \le a$, the expected profit of the retailer is

$$\pi_r(q) = \frac{(q-a)^2}{2(b-a)}(v-p) + q(p - c_r - w).$$

Analogous to Theorem 1, one can obtain a closed form for the supplier's expected profit. The proof is similar and omitted.

**Theorem 2**. For the linear uncertain demand $D$ with $a \le \frac{b}{2}$, the expected profit of the supplier is

$$\pi_s(q) = q(w - c_s) - \frac{(2q-b)^2}{2(b-a)} g_s, \quad \text{if} \quad a \le q \le \frac{b}{2} \quad (11)$$

$$\pi_s(q) = q(w - c_s), \qquad \text{if} \quad \frac{b}{2} \le q \le b. \quad (12)$$

**Corollary 2**. For the linear uncertain demand $D$ with $a \ge \frac{b}{2}$, the expected profit of the supplier is $\pi_s(q) = q(w - c_s)$. The following corollaries are direct consequences of Theorems 1 and 2. Proofs are straightforward and omitted.

**Corollary 3.** Consider the uncertain linear demand $D$, with $a \le \frac{b}{2}$. Then, the expected profit of the coordinated channel is

$$\pi(q) = -\frac{(a-q)^2(p-v) + g(b-2q)^2}{2(b-a)} + q(p-c), \qquad \text{if } a \le q \le \frac{b}{2}, \quad (13)$$

$$\pi(q) = -\frac{(q-a)^2}{2(b-a)}(p-v) + q(p-c), \qquad \text{if} \quad \frac{b}{2} \le q \le b. \quad (14)$$

**Corollary 4**. Consider the linear uncertain demand $D$ with $\frac{b}{2} \le a$. The expected profit of the coordinated channel is

$$\pi(q) = q(p-c) - \frac{(q-a)^2}{2(b-a)}(p-v).$$

One can find the optimal quantity order by considering Theorems 1 and 2 and Corollary 3. Since the uncertain demand is linear $L(a, b)$, we are entirely sure that the demand is not less than $a$ and not more than $b$. Thus, the optimizer of the profit function for all parties must be in the interval $[a, b]$. Recall that the optimizer of a function over a closed interval is at its critical points.

These points include the endpoints of the interval and the points where the function has no derivative. Moreover, they include zeros of the derivative, provided that they are inside the interval.

**Remark 2**. For the linear uncertain demand $D$ with $a \leq \frac{b}{2}$, let the wholesale price be fixed at $w$. Critical points of the retailer's profit are $a$ and $b$ as the endpoints of the valid interval. Further, the retailer's expected profit, presented by (9) and (10), is not differentiable at $q = \frac{b}{2}$. The function (10) has its maximum value at

$$q_1 = \frac{(a-b)(c_r+w)+b(2g_r+p)-av}{4g_r+p-v},$$

while the maximum value of (*9*) occurs at

$$q_2 = \frac{(a-b)(c_r+w)+bp-av}{p-v}.$$

**Corollary 5.** Consider the linear uncertain demand $D$, with $a \geq \frac{b}{2}$, and the fixed wholesale price $w$. The retailer's optimal order quantity is either $q_r^* = a$ or $q_r^* = b$, or $q_r^* = q_2$. The following theorem indicates the optimal order quantity for the supplier's objective function.

**Theorem 3.** Consider the linear uncertain demand $D$ with $a \leq \frac{b}{2}$ and the fixed wholesale price $w \geq c_s$. The optimal order quantity of the supplier is $q_s^* = b$.

**Corollary 6**. Consider the uncertain demand $D$ with the linear distribution and the fixed wholesale price $w$. Further, let $a \geq \frac{b}{2}$. Then, the optimal order quantity of the supplier's profit function is $q_s^* = b$.

**Remark 3**. The result of Theorem 3 and the consequent corollary say that the highest profit for the supplier is guaranteed when the retailer orders as much as possible, i.e., $q_r = q_s^* = b$. This conclusion coincides with the intuitive desire of the supplier to sell more for any possible uncertainty of the demand. To coordinate the channel, one has to find the maximizer of (13) and (14). It can be applied then to settle the wholesale price $w$.

**Remark 4.** Consider the linear uncertain demand $D$ with $a \leq \frac{b}{2}$ and fixed wholesale price $w$. Critical points of the whole channel are some of the points $a, \frac{b}{2}, b$ or the zeros of the derivative of (13) and (14) as

$$q_4 = \frac{b(p+2g-c)+a(c-v)}{4g+p-v}, \quad \text{if} \quad a \leq q \leq \frac{b}{2}$$
$$q_5 = \frac{c(a-b)+bp-av}{p-v}, \quad \text{if} \quad \frac{b}{2} \leq q \leq b.$$

From the elementary calculus, the following theorem provides the optimal order quantity.

**Theorem 4**. Consider the linear uncertain demand $D$ with $a \geq \frac{b}{2}$ and the fixed wholesale price $w$. Then, the optimal order quantity of the channel is $a$,$b$, or $q_5$ provided that it is in the interval $[a, b]$.

### WHOLESALE PRICE IDENTIFICATION

Recall that the supplier would aim to adjust the price $w$ in the wholesale price contract before the deal is ultimately settled. This contract coordinates the channel when the supplier and retailer benefit not less than a predefined level. While the supplier prefers a higher wholesale price, the retailer prefers the opposite. Therefore, the wholesale price contract usually seems not to be a coordinating one. It is practically used, meaning the parties finally compromise on some price. The main feature of this contract is then its easy-to-operate characteristics. Here, we discuss the negotiation of supplier and retailer on the wholesale price. Let both parties be informed of the distribution of the uncertain demand and $a \leq \frac{b}{2}$. The results of the case $a \geq \frac{b}{2}$ is identical to the situation $\frac{b}{2} \leq q \leq b$. First, it is reasonable to assume that the retailer and the supplier set a minimum profit for themselves, denoted by $u_r$ and $u_s$, respectively. A reasonable option for these values would be a percentage of their specific costs. Therefore, the wholesale price should be in a range that satisfies $\pi_r(q) \geq u_r$ and $\pi_s(q) \geq u_s$. Thus, for the retailer, when the channel's optimal order quantity $q^*$ is in $[a, \frac{b}{2}]$, we must have $\mathcal{S}(q^*)(p-v) + q^*(v-c_r-w) - g_r\left(\frac{(2q^*-b)^2}{2(b-a)}\right) \geq u_r$, and when $\frac{b}{2} \leq q^* \leq b$, we must have $\mathcal{S}(q^*)(p-v)+q^*(v-c_r-w) \geq u_r$. After simplification, the following upper bound is obtained for the retailer's acceptable wholesale price,

$$w \leq \frac{(a-q^*)^2(v-p)-g_r(b-2q^*)^2}{2q^*(b-a)} + (p-c_r) - \frac{u_r}{q^*}, \quad \text{if} \quad a \leq q^* \leq \frac{b}{2}$$

$$w \leq \frac{(a-q^*)^2(v-p)}{2q^*(b-a)} + (p-c_r) - \frac{u_r}{q^*}, \quad \text{if} \quad \frac{b}{2} \leq q^* \leq b.$$

Analogously, minimum profit $u_s$ for the supplier defines the following lower bounds for the wholesale price $w$, depending on the value of $q^*$.

$$w \geq \frac{u_s}{q^*} + g_s\left(\frac{(2q^*-b)^2}{2q^*(b-a)}\right) + c_s, \quad \text{if} \quad a \leq q^* \leq \frac{b}{2}$$
$$w \geq \frac{u_s}{q} + c_s, \quad \text{if} \quad \frac{b}{2} \leq q^* \leq b. \qquad (15)$$

Observe that the first derivative of these lower bounds are

$$-\frac{u_s}{q^2} - \frac{g_s(b^2-4q^2)}{2q^2(b-a)} \leq 0 \quad \text{if} \quad a \leq q^* \leq \frac{b}{2}$$
$$-\frac{u_s}{q^2} \leq 0, \quad \text{if} \quad \frac{b}{2} \leq q^* \leq b,$$

which are negative. Therefore, the lower bound is a decreasing function w.r.t. the optimal order quantity. It means that by increasing the order quantity, the supplier can reduce the wholesale price to the retailer with the hope that he increases the order quantity to reach another coordinated order. Considering the above-mentioned lower bounds for the wholesale price, the deal process would be as follows. Being conservative, the supplier may assume that the retailer orders the lowest quantity, $q = a$. Thus, the lower bound for the wholesale price, in this case, is

$$w \geq \frac{(2a-b)^2}{2a(b-a)} g_s + \frac{u_s}{a} + c_s.$$

On the other hand, this lower bound is $w \geq c_s + \frac{u_s}{b}$ when the maximum order quantity is $q = b$, which obviously denotes the decreasing behavior of this lower bound. To coordinate, the wholesale price should respect these lower and upper bounds. Thus, for the quantity order $a \leq q \leq \frac{b}{2}$, the following inequality must hold,

$$\frac{u_s}{q} + g_s\left(\frac{(2q-b)^2}{2q(b-a)}\right) + c_s \leq \frac{(a-q)^2(v-p)-g_r(b-2q)^2}{2q(b-a)} + (p-c_r) - \frac{u_r}{q},$$

which leads to

$$u_r + u_s \leq -\frac{g(2q-b)^2 + (p-v)(a-q)^2}{2(b-a)} \qquad (16)$$

Analogously, when $\frac{b}{2} \leq q \leq b$, we must have

$$u_r + u_s \leq q(p-c) - \frac{(q-a)^2(p-v)}{2(b-a)}. \qquad (17)$$

As a result, the wholesale price is coordinating if the wholesale price satisfies these upper and lower bounds. Moreover, the minimum profits of the retailer and supplier must satisfy (16) and (17). Otherwise, both parties will revise their expected profit to initiate the wholesale contract.

Notice that this analysis asymptotically resembles the discount-quantity contract ( See [1] from the probability theory point of view). Here, we do not go into details and leave the wholesale price settlement to the supplier and retailer to agree based on their own bargaining power.

### A TOY EXAMPLE

For an illustration, let the parameters of a wholesale contract problem be as $p = 15, c_r = 2,\ g_r = 3, c_s = 6, g_s = 4, \nu = 4,$ $a = 10, b = 40.$ Given that the goal is to coordinate the total channel, we first calculate the optimal total value. Using Theorems 3 and 4, we have:

| $q$ | $\pi(q)$ |
|---|---|
| $a$ =10 | 23.33 |
| $b$ =40 | 115 |
| $b/2$ =20 | 121.67 |
| $q_4 = 22.56$ | out of the interval |
| $q_5 = 29.10$ | 136.82 |

Thus, the optimal value is $q^* = 29.10$. Recall that the payment transfer $T$ does not affect the channel's optimal order quantity. The absence of this value in evaluating optimal order quantity may be interpreted as follows. The payment $T$ is not exercised between the supplier and the retailer since they are a part of a unique distribution channel.

### ANALYSIS OF THE WHOLESALE PRICE

Let us analyze the dependence of the wholesale price on the minimum expected profit for the retailer and supplier. As mentioned before, a sensible option for the values of $u_r$ and $u_s$ would be a percentage of each party's total cost. For example, one may consider $u_r = \beta_r(g_r E[(D-q)^+] + q(c_r + w)),$ and $u_s = \beta_s(c_s q + g_s E[(D-q)^+])$. Without loss of generality, we set $\beta_r = 0.05$ and $\beta_s = 0.1$. As mentioned earlier, the supplier initially considers the minimum order quantity proposed by the retailer and offers the initial wholesale price accordingly. The retailer then calculates its optimal order quantity subsequently. In this case, the supplier may revise the wholesale price and receive a fresh order from the retailer. This process will continue until an agreement is settled.

According to this process, the supplier first considers the lowest order amount $q = a = 10$, and by using (18) determines its minimum profit $u_s = 8.67$. Then, the lower bound for the wholesale price is calculated using (15) as $w \geq 9.54$. For example, the supplier may offer $w = 10$ to the retailer. Then, the retailer determines its optimal order amount as $q_r^* = 19.1$. As a result, the retailer's and the supplier's expected profits are $\pi_r(q_r^*) = 41.95$ and $\pi_s(q_r^*) = 76.18$, respectively. Considering this order quantity, the supplier has a new lower bound for the wholesale price as $w \geq 6.86$. Thus, he may offer a reduced wholesale price to the retailer, say $w = 9$. Consequently, the retailer may revise his order as $q_r^* = \frac{b}{2} = 20$. In this case, the expected profits are $\pi_r(q_r^*) = 61.67$ and $\pi_s(q_r^*) = 60$. With this new order quantity, the supplier has another lower bound for the wholesale as $w \geq 6.6$ and he may offer $w = 8$. In this case, the retailer orders $q_r^* = 23.6$, and expect profits will be $\pi_r(q_r^*) = 84.09, \pi_s(q_r^*) = 47.4$. The supplier may reduce the wholesale price more as its $u_s = 14.4, w \geq 6.6$, and offers $w = 7$. In this case, we have $q_r^* = 26.4, \pi_r(q_r^*) = 109.091$, and $\pi_s(q_r^*) = 26.4$. As this instance shows, reducing the wholesale price increases the retailer's order quantity and expected profit. However, the supplier's expected profit drops by decreasing the wholesale price. We believe this is not the case in general, and depending on the problem's parameters and the market situation, other results are expected.

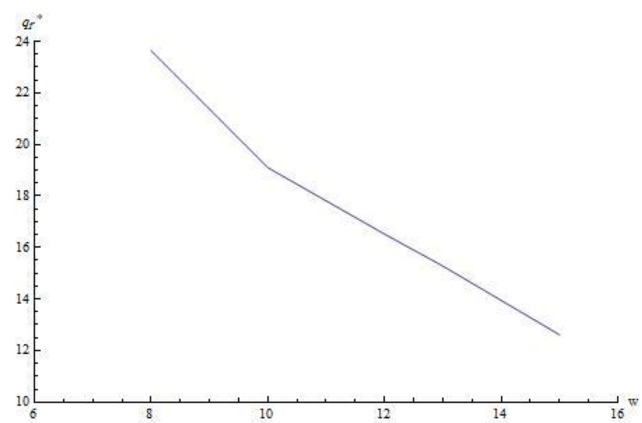


FIGURE 1
THE RELATIONSHIP BETWEEN WHOLESALE PRICE AND RETAILER'S OPTIMAL ORDER QUANTITY

As stated in the theorems above, the optimal production (order) amount for the supplier occurs at the end of the interval. This means that under uncertainty theory assumptions, the supplier prefers to sell more of his products. Therefore, the supplier's optimal decision at any wholesale price is $q^* = b$. On the other hand, due to the fact that $T = wq$ is absent in the profit of the channel, the optimal $q$ for the channel is independent of the value $w$, that is $q^* = 29.1$ and its expected profit is $136.82$. Here, the main discussion is to determine the retailer's order quantity according to the price proposed by the supplier. As seen in Figure 1, the retailer orders fewer quantities with the increase in the wholesale price. This conclusion aligns with the common sense of unknown markets for almost all newly introduced products. This figure also reveals that the inverse relation between the wholesale price and the optimal order quantity of the retailer is not linear; it is a piecewise decreasing linear function; in fact, the function is convex. Convexity means that before and after the breakpoint, the downward slope decreases, denoting that a higher wholesale price than the breakpoint reduces the willingness of the retailer to decrease the order amount.

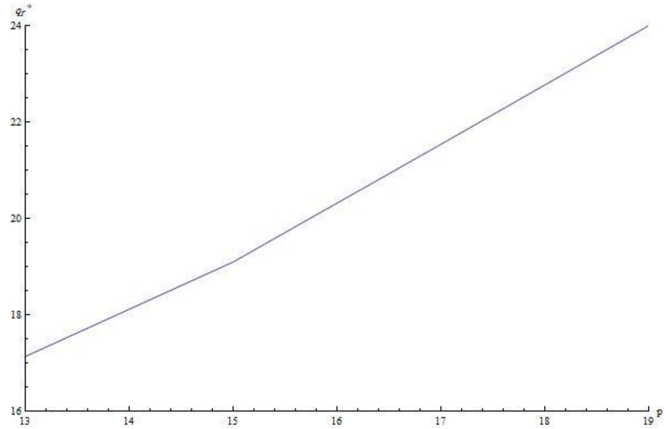


FIGURE 2
THE RELATIONSHIP BETWEEN RETAILER PRICE AND RETAILER'S OPTIMAL ORDER QUANTITY

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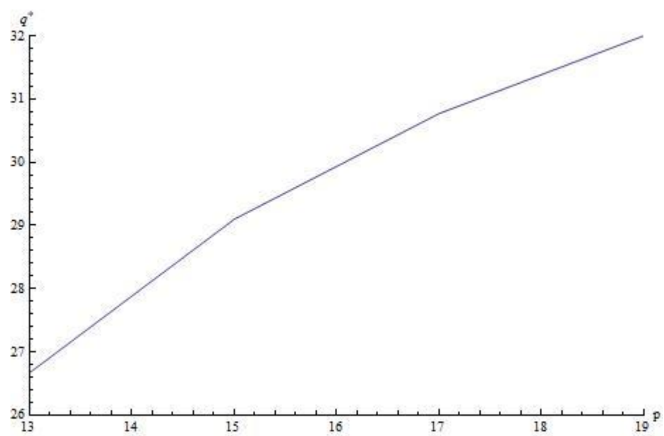


FIGURE 3
THE RELATIONSHIP BETWEEN THE RETAILER PRICE AND THE OPTIMAL ORDER QUANTITY OF THE WHOLE CHANNEL

Figures 2 and 3 depict a direct relation between the retail price and the optimal order quantity of the retailer and the whole channel. As the selling price increases, the retailer will be more inclined to order more. Again, it is observed that these relations are not linear. Clearly speaking, this function is convex for the retailer while it is concave for the whole channel. Convexity for the retailer means that by increasing the retail price before and after the breakpoint, eagerness (slope of the function) to order more, intensifies. It should also be noted that the retailer's price does not affect the supplier's profit. Therefore, this behaviour is not observed in the supplier's decision. In conclusion, this fact changes convexity to concavity for the whole channel. We also investigated the relationship between the various parameters of the problem and the optimal order quantity for the retailer and the whole channel. Figures 4 and 5 show that by increasing the salvage value for the retailer and the whole channel, the optimal order quantity rises, respectively. This phenomenon can be interpreted as follows. Increasing the order amount is less risky for the retailer when the salvage value increases. If some ordered items are not sold, he can still profit by scrapping them at the right price. However, any increase in the marginal cost for the retailer forces him to order less (See Figure 6). This conclusion is also valid in coordinating situations (See Figure 7). These results are in accord with the intuition in a real-world atmosphere.

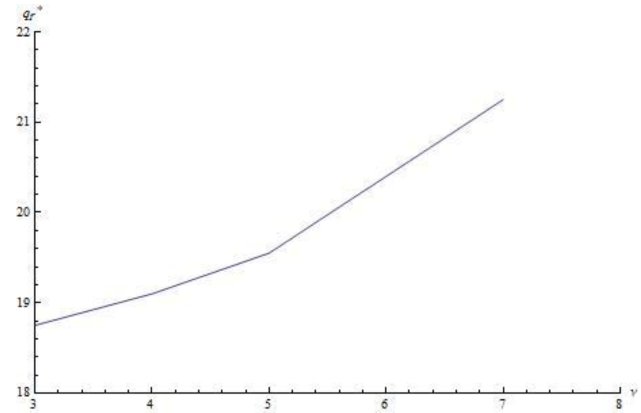


FIGURE 4
THE RELATIONSHIP BETWEEN SALVAGE VALUE AND RETAILER'S OPTIMAL ORDER QUANTITY

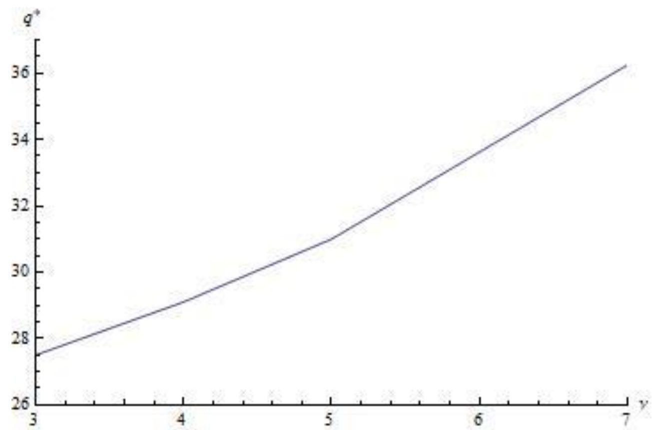


FIGURE 5
THE RELATIONSHIP BETWEEN SALVAGE VALUE AND OPTIMAL ORDER QUANTITY OF THE WHOLE CHANNEL

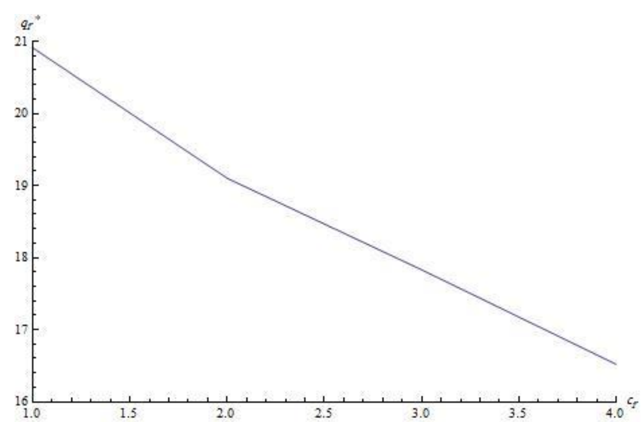


FIGURE 6
THE RELATIONSHIP BETWEEN RETAILER'S MARGINAL COST AND RETAILER'S OPTIMAL ORDER QUANTITY

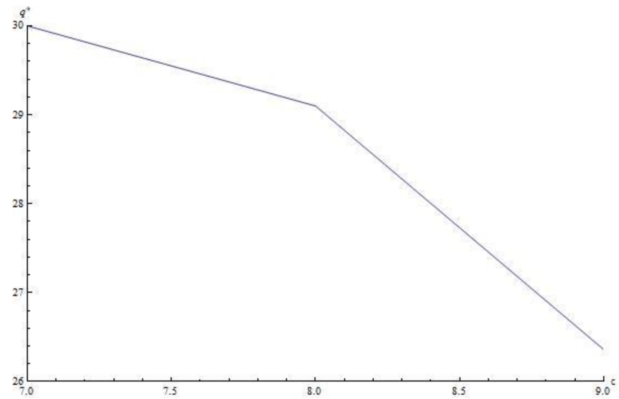


FIGURE 7
THE RELATIONSHIP BETWEEN PRODUCTION COST AND MARGINAL COST IN THE WHOLE CHANNEL WITH THE OPTIMAL ORDER QUANTITY OF THE CHANNEL

### COMPARISON WITH SCENARIO-BASED APPROACH

This section investigates the problem from the probability theory point of view. We will compare these results with the ones from the uncertainty theory standpoint. The results are depicted in Tables 2 and 3. Table 2 represents the results for the retailer, and Table 3 summarizes the results for the coordinated channel. We again assume that the demand is an amount in $[10, 40]$ and consider three sets of potential demands. Each set includes four different possibilities (scenarios). The first set of scenarios D1 includes uniform demands $\{10, 20, 30, 40\}$. The second set of scenarios D2 is $\{10, 30, 35, 40\}$ at which scenarios are closer to the right endpoint of the interval, while the third set of scenarios is $\{10, 15, 20, 40\}$ at which potential demands are closer to the left endpoint of the interval. For each set of scenarios, four potential probabilities are assigned. The first probability distribution is uniform and assesses equal probability to each scenario. Scenarios 2 and 3 are more probable in the second probability distribution than the others, so their probability is higher. The third probability distribution considers a slight probability to the two first scenarios than the others. In comparison, the last probability distribution soundly weighs the three first scenarios more than the last one (three times more probable) while they are equally likely to happen. Expected values of each potential demand with the probability mentioned above distributions are also depicted. Observe that the expected demands for the first set of scenarios D1 are almost close to the middle of the interval $[10, 40]$, while for the second set of scenarios D2, they are higher than the middle of the interval, and for the third set of scenarios D3, they are less than the middle of this interval. These facts are in accord with probability distributions for each set of scenarios.

The calculation denotes that the optimal order quantities for the retailer is independent from the probability distribution, while these values vary for different sets of scenarios. In all sets of scenarios, the optimal order quantity for the retailer is less than the expected values. On the other hand, though the optimal order values are independent of the probability distribution, optimal profits vary with no special pattern. In comparison with our proposed approach, for the wholesale price $w = 7$, optimal order of the retailer is $q_r^* = 26.4$ with the optimal profit more than 109. This means that our approach denotes much more higher profit than the probability theory approach in all 12 scenarios for the retailer even in non-coordination sitiation. This is also visible when the wholesale price is higher; when $w = 10$, the optimal profit for the retailer is about 42. This value is also higher than the expected amount in all 12 scenarios.

Let us analyze the results of two points of view when coordination is settled. In all three sets of demands, when the probability distribution is uniform, the model suggests ordering the most amount. i.e., $q^* = 40$, but with different optimal profit of the channel; 115, 156.3, and 73.75, respectively, for D1, D2, and D3. In contrast, in our approach, the value is about 29, with an optimal profit of more than 136. This is higher than the expected optimal profit for the scenarios set D1 and D2. Observe that the assumption in our approach for the demand distribution is linear. This is almost comparable with the scenarios set D1. In this case, our approach outperforms the probability theory approach. All optimal profits for scenarios in D1 is less than the resulting profit in our approach, while the optimal order is almost identical. In conclusion, we could conclude that in a coordinated channel, our approach suggests higher profit than the scenario-based approach.

TABLE 2
THE OPTIMAL ORDER QUANTITY FOR THE RETAILER IN DIFFERENT SCENARIOS

| | Scenarios | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | E.V. | $q_r^*$ | $\pi(q_r^*)$ |
| D1 | 10 | 20 | 30 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 25 | 20 | 10 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 24 | 20 | 20 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 23 | 20 | 9 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 22 | 20 | 30 |
| D2 | 10 | 30 | 35 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 28.75 | 30 | 23.75 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 29 | 30 | 37 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 27 | 30 | 15 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 26.5 | 30 | 16.5 |
| D3 | 10 | 15 | 20 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 21.25 | 15 | 8.75 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 18.5 | 15 | 20.5 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 19.5 | 15 | 10.5 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 17.5 | 15 | 16.5 |

## CONCLUSION

In this paper, the supply chain with the wholesale price contract was considered where the demand is a linear uncertain variable. Optimal order quantities of the retailer, supplier, and whole channel are determined. The retailer obtains its optimal order quantity according to the wholesale price determined by the supplier, and then each party achieves its desired profit. Lower and upper bounds for the wholesale price were proposed to satisfy the retailer's and supplier's desire to conduct the contract. An example shows the performance of findings numerically. It was observed that the wholesale price contract coordinates the members when indeterminacy is dealt with as an uncertain variable. Analogous analysis can be carried on over other coordinating contracts in SCM.

TABLE 3
THE OPTIMAL ORDER QUANTITY FOR THE CHANNEL IN DIFFERENT SCENARIOS

| | Scenarios | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | E.V. | $q^*$ | $\pi(q^*)$ |
| D1 | 10 | 20 | 30 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 25 | 40 | 115 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 24 | 30 | 126 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 23 | 30 | 97 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 22 | 30 | 104 |
| D1 | 10 | 30 | 35 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 28.75 | 40 | 156.3 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 29 | 35 | 170 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 27 | 35 | 139 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 26.5 | 35 | 142.5 |
| D2 | 10 | 15 | 20 | 40 | | | |
| Probabilities | 0.25 | 0.25 | 0.25 | 0.25 | 21.25 | 40 | 73.75 |
| | 0.2 | 0.3 | 0.4 | 0.1 | 18.5 | 20 | 87.5 |
| | 0.3 | 0.3 | 0.2 | 0.2 | 19.5 | 20 | 62.5 |
| | 0.3 | 0.3 | 0.3 | 0.1 | 17.5 | 20 | 76.5 |

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## Appendix

In this section, we present all the proofs needed.

**Proof of Lemma 1**. Considering the definition of expected value, we have

$$\mathcal{S}(q) = E[S(q)] = \int_0^{+\infty} 1 - \Phi(x)dx,$$

where $\Phi(x)$ is the uncertainty distribution of the demand defined as (2). Recall that the second integral in (1) is zero, since it is assumed that $a > 0$. Therefore,

$$\mathcal{S}(q) = \int_0^a dx + \int_a^q (1 - \frac{x-a}{b-a})dx = q - \frac{(q-a)^2}{2(b-a)}.$$

The proof is complete.

**Proof of Lemma 2**. Respecting the definition of expected value, we have:

$$E[(q-D)^+] = \int_0^\infty M\{(q-D)^+ \geq x\}dx = \int_0^q M\{q - D \geq x\}dx = \int_0^q M\{D \leq q - x\}dx = \int_0^q \Phi(q-x)dx.$$

With substitution $q - x = y$, we have

$$E[(q-D)^+] = \int_0^q \Phi(y)dy = \int_0^a \Phi(y)dy + \int_a^q \Phi(y)dy = \int_a^q \frac{y-a}{b-a}dy = \frac{(q-a)^2}{2(b-a)}.$$

Considering Eq. ( 6), (7) is proved. To prove (8), we have:

$$E[(D-q)^+] = \int_0^\infty M\{(D-q)^+ \geq x\}dx = \int_q^\infty M\{D - q \geq x\}dx = \int_q^\infty 1 - \Phi(x+q)dx = \int_q^{b-q} 1 - \Phi(x+q)dx.$$

The last equality results from the fact that $\Phi(x+q) = 1$ for $x \geq b - q$. With substitution $x + q = t$, we have

$$E[(D-q)^+] = \int_{2q}^b 1 - \Phi(t)\,dt.$$

Let us consider two situations. If $2q \leq b$ (or $q \leq \frac{b}{2}$), then

$$\int_{2q}^b 1 - \Phi(t)dt = \int_{2q}^b 1 - \frac{t-a}{b-a}\,dt = \frac{(2q-b)^2}{2(b-a)}.$$

On the other hand, when $b \leq 2q$ (or $q \geq \frac{b}{2}$), we have

$$\int_{2q}^b 1 - \Phi(t)dt = -\int_b^{2q} 1 - \Phi(t)dt = 0$$

The proof is complete.

**Proof of Theorem 1**. Respecting the fact $T = wq$, Lemmas 1 and 2, and the retailer profit function (3), when $a \leq q \leq \frac{b}{2}$ we have:

$$\pi_r(q) = pE[S(q)] + vE[(q-D)^+] - g_rE[(D-q)^+] - c_rq - T = p\mathcal{S}(q) + v\big(q - \mathcal{S}(q)\big) - c_rq - wq - g_r(\frac{(2q-b)^2}{2(b-a)})$$

$$= \mathcal{S}(q)(p-v) + q(v - c_r - w) - g_r(\frac{(2q-b)^2}{2(b-a)}) = \frac{(a-q)^2(v-p) - g_r(b-2q)^2}{2(b-a)} + q(p - c_r - w).$$

For $\frac{b}{2} \leq q \leq b$, we have

$$\pi_r(q) = pE[S(q)] + vE[(q-D)^+] - g_r E[(D-q)^+] - c_r q - T = p\mathcal{S}(q) + v\big(q - \mathcal{S}(q)\big) - c_r q - wq$$
$$= \mathcal{S}(q)(p-v) + q(v - c_r - w) = \frac{(q-a)^2}{2(b-a)}(v-p) + q(p - c_r - w).$$

The proof is complete.

**Proof of Theorem 3**. Observe that the maximum value of (11) is at

$$q_3 = \frac{(b-a)(w-c_s)}{4g_s} + \frac{1}{2}b.$$

Note that the practical assumption $w \geq c_s$ imposes $q_3 \geq \frac{b}{2}$. This means that (11) is an increasing function over $[a, \frac{b}{2}]$. On the other hand, (12) is an increasing function, since $w \geq c_s$. Thus $q_s^* = b$.