Original Research

# A heuristic algorithm for solving bi-level programming problems with application to large-scale location-arc routing in urban traffic lights maintenance

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**Abstract**
The present study developed a bi-level mathematical model to determine optimal routes for repair teams in charge of inspecting urban traffic lights. In this model, the municipality as the leader locates the construction sites of urban spare parts warehouses and repair centers to minimize the costs of constructing the facilities. At the follower level, the contractor determines the optimal routes for the repair teams. Bi-level models are strongly NP-hard in type. A heuristic algorithm is therefore developed to solve numerical examples, in which the leader first determines different decision-making strategies for the follower through generating a set of justified solutions. In response to the leader's set of strategies, the follower presents a set of corresponding solutions. The individual solutions of the follower are then entered into the leader model and the corresponding values of the objective function are calculated. A solution with the optimal numerical value for the leader is ultimately selected as the Stackelberg equilibrium. The efficiency of the proposed model and algorithm was evaluated by presenting the computational results obtained from solving several random numerical examples of small, medium and large dimensions through generating the Stackelberg equilibrium and establishing a relationship between the leader and follower levels. The present findings are recommended to be used as a management tool by policymakers in the urban management sector.

**Keywords-** Bi-level programming; Heuristic; Support center location; Location arc routing; Urban services

## INTRODUCTION

Managing urban services, especially those related to traffic control, are influenced by many factors, including timely maintenance and repair (El Hatri and Boumhidi, 2017). Managers can directly integrate proper plans for delivering this type of services with the development of urban management (Stoilov and Stoilova, 2016). As an optimization tool, operational research is widely used for solving different problems, including managing urban services such as traffic control (Cruciol, Weigang, Clarke and Li, 2015), transportation network design (Skabardonis, 2020), traffic light scheduling (Göttlich, Herty and Ziegler, 2015), managing the maintenance and repair of street networks (Pérez-

Ocón, Rubiño, Pozo and Rabaza, 2013) as mathematical models (Eiselt and Marianov, 2015), optimization algorithms (Rabbani, Heidari, Farrokhi-Asl and Rahimi, 2018) and multi-criteria decision-making methods (Soltani, Hewage, Reza and Sadiq, 2015). Traffic management requires that traffic lights be repaired within two hours of their failure (Behura, 2007), which has been turned into an operational plan in the municipality of Tehran, Iran. The maintenance and repair involve two general categories of measures: (1) repairing failed traffic lights, and (2) taking preventive maintenance measures. Operational teams are required to be established to implement both these programs and effectively perform the necessary actions. Providing the teams with the necessary repair equipment is therefore essential. Nowadays, the teams formed in Tehran carry lots of equipment in modified buses to perform the necessary actions, which is costly and causes traffic congestion. To solve this problem, the managers decided to provide specific sites constructed in different areas of the city with technical equipment and workforce. The repair and inspection of traffic lights were then performed using the manpower and equipment provided through laying proper plans. Maintenance and repair of traffic lights have rarely been addressed as an optimization problem in literature. In fact, a large body of literature is devoted to laying plans for implementing maintenance and repair from a technical perspective rather than managing operational teams, which itself significantly affects the system costs and quality of services. Ozbek et al, (2010) used data envelopment analysis to develop and implement a comprehensive framework and investigate the information and modeling problems associated with measuring the overall efficiency of the repair and maintenance of street equipment.

## LITERATURE REVIREW

Gao and Zhang, (2013) investigated vehicle operating costs and shipping costs as a major component of user costs associated with street repair and maintenance. They developed a multi-objective Markov model to minimize repair and maintenance, and user costs provided average annual budget constraints and operational requirements were satisfied. L. Chen, Hà, Langevin and Gendreau, 2014, investigated routing optimization methods in the daily repair and maintenance of a street network considering the times of service and travel in road sections the random variables and ultimately solving the problem using an adaptive large neighborhood search. Ouma et al. 2015 evaluated a multi-attribute decision making approach and prioritized the repair and maintenance of street equipment by comparing a fuzzy AHP method with fuzzy TOPSIS. Moretti et al. (2016) presented a life-cycle cost analysis to compare construction, repair, maintenance and lighting costs between highway tunnels. The results of their case study highlighted the importance of pavement surface materials given their potential for reflecting light and reducing energy consumption.

Khan et al, 2017 proposed a novel framework for developing the repair and maintenance strategies of roads after floods. They also discussed two other strategies to maximize economic benefits and optimize the budget. According to the research literature, the traffic light repair management problem has not yet been considered as a location-routing problem. In fact, previous research has not examined the logistics structure of traffic lights management problem, while the logistics management problem in the urban environment has a wide range of applications. Huber, 2016 developed a decision support tool for managing urban logistics. They solved a bi-objective locating-arc routing problem by simultaneously determining the facilities and routes. The first objective was to minimize the total cost, including the fixed costs of vehicles and equipment commissioning and the distance travelled. The second objective was related to a service delivery dimension, i.e. the total delivery time for required services.

A locating-inventory-routing model proposed by Hiassat et al, 2017 was used to determine the location and number of warehouses required, the size of inventory of every retailer and the routes traveled by individual vehicles in an urban environment. To provide better results for the supply chain, they put the model into practice by integrating strategic decisions with tactical and operational decisions through adding the locating decisions to the inventory-routing problem. A genetic algorithm was developed to solve this NP-hard problem and achieve the best near-optimal solutions in a reasonable time. They ultimately examined the effectiveness of the proposed algorithm. Rayat et al, 2017 introduced a reliable multi-product and multi-period locating-inventory-routing model considering failure risks. They assumed that random demand in the inventory system randomly disrupted the supply of products in distribution centers. They therefore proposed a bi-objective mixed-integer nonlinear programming model. The first objective function minimized comprised the costs of locating, routing and shipping and the inventory comprised the costs of orders, maintenance and delayed orders. The second objective function minimized comprised the total failure costs associated with disruptions to distribution centers as an unreliability index of the supply chain network.

Norouzi et al, 2017 proposed a multi-objective mathematical model to solve arc routing and used metaheuristic algorithms to solve large-scale problems. Their calculation results showed that the modified particle swarm optimization algorithm significantly outperformed the other algorithms. An extended version of genetic algorithm used by Çetinkaya et al, 2018 was found more efficient than the original version in solving arc routing. A numerical example was solved by Tavakkoli-Moghaddam et al, 2018 using an integrated mathematical model of locating-are routing as an extended version proposed by Doulabi and Seifi, 2013. The numerical results obtained from the metaheuristic algorithms used by Wøhlk et al. and Han et al, 2018 to solve the arc routing problem showed that the population and particle swarm algorithms outperformed the solution-based algorithms. Fernández et al, 2019 presented a mathematical model based on a metaheuristic local search to solve a capacitated arc routing problem as the minimization of an objective function comprising the costs of the routes travelled. A hybrid model proposed by Wøhlk and Laporte, 2019 based on an iterative heuristic algorithm to block demographic areas and solve the arc routing problem was tested in a case study on an urban structure design in Denmark. Tirkolaee et al, 2019 investigated the capacitated arc routing problem as a mathematical programming model based on a metaheuristic local search that outperformed genetic algorithm.

Arakaki and Usberti, 2019 developed a multi-batch algorithm and used the shortest path concept to solve a capacitated arc routing problem. They investigated the algorithm performance using standard examples. A bi-objective programming model proposed by Parvasi et al, 2019 based on bi-level programming to determine the route of city buses assigned the leader level to locating the stations and maximizing the coverage level. At the second level, the follower routed the buses to reduce fuel costs. Tong et al, 2019 modeled the railroad design using bi-level programming and a structure based on the Karush-Kuhn-Tucker conditions. The genetic algorithm used in an optimization model by Yurtseven and Gökçe, 2019 to solve urban management planning based on determining the routes of electric taxis was found to outperform the other metaheuristic algorithms. Y. Chen et al, 2020 used a bi-level model based on heuristic algorithms to generate appropriate solutions in a reasonable time and determine the routes of urban buses as a mathematical optimization problem. Cvokic and Stanimirovic, 2020 introduced a novel problem of locating and allocating capacity-unconstrained hubs. Their objective was to maximize profit and achieve optimal pricing. To accomplish this, they proposed a nonlinear programming model and solved it using a fuzzy approach.

Wu et al, 2022 investigated a multi-depot routing and hub location problem to design an intra-city express service system, where flows of letters and packages between service provider branch offices were exchanged through local tours. The hubs in this program simultaneously performed receiving and delivery processes. Additionally, hub and vehicle capacities were considered in the model. They proposed a mixed-integer mathematical model for this problem and subsequently provided a heuristic algorithm for solving the model. Jayaswal and Vidyarthi, 2023 presented a hub network design problem in the context of a third-party logistics service provider. A third-party service provider typically caters to different classes of shipments with varying service level requirements, such as two-day delivery, next-day delivery, etc. They examined the problem under random demand from two classes of shipments. To address this, they proposed two models for designing a capacitated hub network with service level constraints, defined by the time spent at hubs for each shipment class. The models aimed to design a hub network with minimum total cost, including the overall fixed cost of equipping open hubs with sufficient processing capacity and variable transportation costs. Finally, they suggested an exact solution approach based on branch-and-cut for solving it.

Miao and Qin, 2023 introduced a capacitated hybrid routing model in the waste management domain with multiple objectives. They tackled the problem using the epsilon-constraint algorithm and addressed it at various scales. Ultimately, they designed a set of instances at different scales to be solved in numerical experiments, determining the model's solving scale. This review of literature suggests the problem of urban service management for traffic lights maintenance has not yet been considered as a mathematical optimization model. Also, other relevant studies have only addressed the technical aspect and did not provide results for making optimal decisions in the logistics management sector. In addition, optimally locating the construction sites of facilities and laying plans for determining the routes of operational staff can reduce both the costs and the time required for implementing the measures.

The present study therefore addressed this problem by focusing on the application of bi-level programming models. The potential problems caused by making operational decisions at two different levels are carefully addressed in this study. At the first level, the municipality as the leader decides on the location of the main and support warehouses. At the follower level, the contractor seeks to reduce the implementation costs based on optimized routing using the facility locations. The two different decision-making levels can be explained by the fact that the municipality does not directly involve itself in executing operational projects and performs its intended operations by concluding cooperation agreements with external contractors. Given that the municipality only seeks to reduce its construction costs of the facilities, the contractor is in charge of making all the executive decisions in an optimized and cost-effective manner. On the other hand, management problems may arise if the decisions made by the municipality to reduce the costs of facilities results in conflict of interests in terms of the contractors' access to the facilities, so they will decline to execute the intended decisions. To avoid this conflict, the municipality and the contractor seek to interactively make an optimal win-win decision. This joint decision-making is considered the Stackelberg equilibrium in operations research.

TABLE 1
SELECTED RECENT STUDIES IN THE FIELD OF PAPER

| Ref. | Problem type | | Model type | | Objective type | | Application | Case study |
|---|---|---|---|---|---|---|---|---|
| | Routing | Location-routing | Single level | Multi-level | Single objective | Multi-objective | | |
| Ozbek et al. (2010) | □ | | □ | | □ | | street equipment maintenance | □ |
| Gao and Zhang, (2013) | □ | | □ | | □ | | street repair and maintenance | □ |
| L. Chen, Hà, Langevin and Gendreau, 2014 | □ | | □ | | □ | | Street network daily repair and maintenance | □ |
| Ouma et al. 2015 | □ | | □ | | | □ | repair and maintenance of street equipment | □ |
| Moretti et al. 2016 | □ | | □ | | | □ | repair, maintenance and lighting in highway | □ |
| Khan et al, 2017 | □ | | □ | | | □ | repair and maintenance strategies of roads | □ |
| Huber, 2016 | | □ | □ | | | □ | managing urban logistics | □ |
| Hiassat et al, 2017 | | □ | □ | | □ | | Network design | □ |
| Rayat et al, 2017 | | □ | □ | | | □ | Urban network design | □ |
| Norouzi et al, 2017 | □ | | | □ | | | Network design | □ |
| Çetinkaya et al, 2018 | | □ | | □ | | | Network design | |
| Tavakkoli-Moghaddam et al, 2018 | □ | | □ | | | | Network design | □ |
| Tirkolaee et al, 2019 | | □ | | □ | | | Network design | |
| Arakaki and Usberti, 2019 | □ | | | □ | | | Network design | □ |
| Parvasi et al, 2019 | □ | | | □ | □ | | managing urban logistics | □ |
| Tong et al, 2019 | □ | | | □ | □ | | Urban network design | |
| Yurtseven and Gökçe, 2019 | □ | | | □ | □ | | Urban network design | □ |
| Y. Chen et al, 2020 | □ | | | □ | □ | | managing urban logistics | |
| Cvokic and Stanimirovic, 2020 | | □ | □ | | | □ | Network design | |
| Wu et al, 2022 | | □ | | □ | | □ | Urban network design | □ |
| Jayaswal and Vidyarthi, 2023 | | □ | | □ | □ | | Network design | |
| Miao and Qin, 2023 | □ | | □ | | | □ | Network design | □ |
| This paper | | □ | | | □ | | street equipment maintenance | □ |

This study developed a bi-level programming model in which the municipality as the leader seeks to locate the facilities to minimize construction costs in a way that certain constraints on the number of the main and support warehouses are satisfied. At the follower level, the model's objective function includes the costs of routing and implementing repair measures based on constraints on routing, coordination between constructed locations as the starting point and meeting the demand for traffic light repair. Given the location arc routing as a bi-level programming problem of a strongly NP-hard type, the present study proposed a heuristic algorithm based on full enumeration to obtain effective combinations of leader strategies and accordingly solve the follower problem for the individual strategies of the leader and ultimately achieve the Stackelberg equilibrium. All the leader-level strategies generated by this algorithm using an iterated neighborhood search based on simulated annealing are solved by the follower model. The optimal follower solutions are then considered a legitimate space for the leader, from which the leader can evaluate and select the best set of optimal solutions according to the objective function value. The rest of this article was organized as follows: Section 2 details the research problem and the mathematical model. Section 3 structurally explains the algorithms. Section 4 presents the calculation results and performs analyses. Section 5 summarizes the results and makes recommendations for future research.

## PROBLEM STATEMENT AND MATHEMATICAL MODEL

The routing problem was defined on $G = (V, A)$ as a graph of $|V|$ vertices and $|A|$ edges. This graph should an oriented digraph (without loop). Vertices played the role of intersections and edges (arcs) as streets or roads connected vertices to one another. A non-negative traversing cost $c_a$ and a non-negative demand $d_a$ were assigned to arc $a = (i, j)$ in the arc set $A$. In contrast to the classical capacitated arc routing problem, the set of arcs with a positive demand as members of $R \subseteq A$ could be supplied by several different tours. The journey of all the k vehicles in a fleet each with a capacity of $Q_k$ passing along a specific route began and ended at the same dedicated base. Overhead with a cost of $z_a$referred to moving from end $i$ of a demand arc to beginning $j$ of another demand arc without serving the demand or non-demand arcs traversed. The problem was to locate and establish an unknown number of bases at $J$ and allocate a number of tours to individual depots in a way that every tour could serve a set of users and the total fixed and traversing costs of serving the demand arcs be minimized. To the best of the authors' knowledge, arc routing has not been yet addressed in literature using bi-level programming. This section investigates the mathematical model of the problem as a bi-level programming model considering time windows. According to this model, the leader as the first level determined the support locations based on the cost criteria. The contractor in charge of traffic lights maintenance accordingly performed routing to minimize the total cost.

- ***Sets***

| | |
|---|---|
| $V$ | Set of all the vertices |
| $A = \{(i, j)\|i, j \in V, i < j\}$ | Set of all the arcs with an initial vertex of i and an end vertex of j |
| $R \subseteq A$ | Set of all the demand arcs |
| $J$ | Set of all the candidate points for establishing facilities |
| $K$ | Set of the vehicles |
| $C$ | Set of the support warehouses |
| $H$ | Set of the demand points (traffic lights) |

- ***Parameters***

| | |
|---|---|
| $c_a$ | Cost of serving arc $a \in R$ |
| $d_a$ | Demand of arc $a \in R$ |
| $f_j$ | Establishment cost of candidate base $j \in J$ |
| $G_k$ | Cost of using tour $k$ (vehicle) |
| $Q_k$ | Capacity of tour $k$ |
| $Cap_j$ | Capacity of candidate base j$\in J$ |
| $L$ | Maximum number of established bases |
| $TC_c$ | Cost of establishing support warehouse c |
| $TP_{jch}$ | Cost of transporting goods from depot m to support warehouse c incurred by end user h |
| $CD$ | Maximum number of support warehouses established |
| $DE_h$ | Demand of arc h |
| $UT_a$ | Maximum acceptable duration of serving arc $a \in A$ |
| $LT_a$ | Minimum acceptable duration of serving arc $a \in A$ |
| $Dis_a$ | Duration of traversing arc $a \in A$ |
| $M$ | A very large positive number |

- ***Decision variables***

| | |
|---|---|
| $Z_j$ | Equals 1 if base $m$ is established; otherwise equals 0 |
| $H_{kj}$ | Equals 1 if tour $k$ is assigned to base $m$; otherwise equals 0 |
| $Dem_{ak}$ | Demand of arc $a \in R$ supplied by tour $k$ |
| $x_{ak}$ | Equals 1 if demand arc $a \in R$ is served by tour $k$; otherwise equals 0 |
| $\delta^+(S)$ | Set of outgoing arcs of $S$ |
| $\delta^-(S)$ | Set of incoming arcs of $S$ |

| | |
|---|---|
| $L(S)$ | Set of arcs beginning and ending in $S$ |
| $\delta^+(v)$ replaces $\delta^+(\{v\})$ to represent $S$ with a single member $v$ | |
| $F_{jch}$ | Equipment transported from depot m to support warehouse c for end user h |
| $E_c$ | Equals 1 if support center c is established |
| $Time_{ak}$ | Time taken by vehicle in tour $k$ to reach arc $a \in A$ |

- ***Leader level model***

$$Min\ Z1 = \sum_{j \in J} f_j Z_j + \sum_{c \in C} E_c TC_c \quad (1)$$

$s.t.$

$$\sum_{j \in J} Z_j \leq L \quad (2)$$

$$\sum_{c \in C} E_c \leq CD \quad (3)$$

**Follower level model**

$$Min\ W = \sum_{a \in R} \sum_{k \in K} x_{ak} c_a + \sum_{k \in K} \sum_{j \in J} H_{kj} G_k + \sum_{j \in J} \sum_{c \in C} \sum_{h \in H} TP_{jch} F_{jch} \quad (4)$$

$$\sum_{a \in \delta^+(i)} x_{ak} - \sum_{a \in \delta^-(i)} x_{ak} = 0 \qquad \forall\, i \in V, \forall\, k \in K \quad (5)$$

$$\sum_{a \in R} Dem_{ak} \leq Q_k \qquad \forall\, k \in K \quad (6)$$

$$\sum_{k \in K} Dem_{ak} = d_a \qquad \forall\, a \in R \quad (7)$$

$$x_{ak} d_a \geq Dem_{ak} \qquad \forall\, a \in R, \forall\, k \in K \quad (9)$$

$$\sum_{a \in \delta^+(S)} x_{ak} + \sum_{a \in \delta^+(S)} x_{ak} \leq 1 + H_{km} \qquad \forall\, k \in K,\ \forall (p,q) \in A,\ \forall\, S \subseteq V/\{m\} \quad (10)$$

$$\sum_{k \in K} x_{ak} \geq 1 \qquad \forall\, a \in R \quad (11)$$

$$\sum_{a \in R} \sum_{k \in K} Dem_{ak} \leq Cap_j Z_j \qquad m \in J \quad (12)$$

$$H_{kj} \leq Z_j \qquad \forall\, j \in J, k \in K \quad (13)$$

$$Z_j \leq M \sum_{k \in K} H_{kj} \qquad \forall\, j \in J \quad (14)$$

$$\sum_{a \in R} x_{ak} \leq M \sum_{k \in K} H_{kj} \qquad \forall\, k \in K \quad (15)$$

$$M \sum_{a \in R} x_{ak} \geq \sum_{k \in K} H_{kj} \qquad \forall\, k \in K \quad (16)$$

$$\sum_{m \in J} H_{kj} \leq 1 \qquad \forall\, k \in K \quad (18)$$

$$\sum_{a \in \delta^+(m)} x_{ak} \geq 1 - M\left(1 - H_{kj}\right) \qquad \forall\, k \in K, \forall\, m \in J \quad (19)$$

$$\sum_{c \in C} E_c \leq CD \quad (20)$$

$$\sum_{j \in J} \sum_{c \in C} F_{jch} = DE_h \qquad h \in H \quad (21)$$

| | | |
|---|---|---|
| $\sum_{j\in J}\sum_{h\in H} F_{jch} \le ME_c$ | $c \in C$ | 22 |
| $E_c \le \sum_{m\in J}\sum_{h\in H} F_{jch}$ | $c \in C$ | 23 |
| $Time_{ak} \ge Time_{a'k} + Dis_a - M(1 - x_{ak})$ | $\forall\, k \in K, a \in \delta^+(i), a' \in \delta^-(i)$ | 24 |
| $Time_{ak} \le Time_{a'k} + Dis_a + M(1 - x_{ak})$ | $\forall\, k \in K, a \in \delta^+(i), a' \in \delta^-(i)$ | 25 |
| $LT_a \le Time_{ak} \le UT_a$ | $\forall\, k \in K, a \in A$ | 26 |

At the leader level, objective function (1) expressed the costs of traversing the arcs and establishing the centers. Serving the demand arcs corresponded to the fixed cost of establishing the bases and cost of using individual tours. Constraint (2) determined the maximum number of established main centers and constraint (3) that of established support centers. At the follower level, objective function (4) expressed the costs of routing, including traversing and equipment transfer. Given the different constraint categories of arc routing and locating-arc routing problems, the present study presented their logical combination. Continuity in the arc routing problem expressed as constraint 5 required that the input and output degrees of every vertex be the same. In other words, the number of arcs entering a vertex had to equal the number of outgoing arcs. According to constraint 6, the total demand supplied by every tour did not exceed the permissible level as the capacity of the vehicles. A demand arc could not be served unless traversed. Given the features of arc routing problems, and according to constraint 8, a demand arc had to be traversed at least once. Designing the trips as closed routes in a way that there were no trips involving user arcs without any bases made the problem more complicated. According to constraint 10, every trip started from and ended at the same base.

In the classic arc routing problem, the demand of every arc should be supplied by at least one vehicle, as expressed in constraint 11. Constraints 7 and 9 also ensured the adequacy of the equipment delivered to every demand point. In contrast to previous studies, the present research modified the assumption of unlimited number of bases. As is the case in real-world situations and according to constraint 12, the arc demand supplied by a tour should not exceed the capacity of the base. Constraints 13-19 show the relationships between establishing a base and its allocated tours and between allocating tours to the bases and allocating the demand arcs to tours. Constraint 20 determined the maximum number of support warehouses, and constraint 21 ensured that all the transported goods equaled the amount of user demand. Moreover, constraints 22-23 ensured that support warehouses could be used only if they were established. Constraints 24-26 ultimately calculated the time of delivering goods to the demand points.

## PROPOSED HEURISTIC ALGORITHM

No comprehensive methods have been yet proposed to solve multilevel programming as a complicated problem and achieve globally-optimal solutions in operational research. In fact, bi-level programming problems with discrete variables at the follower level have always been considered an open problem in operational research; nevertheless, researchers have reported reliable numerical solutions by using different heuristic algorithms depending on the problem structure, simulating the Karush-Kuhn-Tucker conditions and converting the bi-level problem to a single-level one. The present research developed a heuristic algorithm based on full enumeration to obtain numerical solutions. According to this algorithm, all the potential solutions provided by the leader level were considered an initial set of solution. The follower level then stored sets of new solutions, with each set containing the solutions obtained by solving the problem at this level for the individual solutions at the leader level. Afterwards, the objection function values were calculated at the leader level by inserting all the new solutions into the model. The solution with the best numerical value was ultimately considered the optimal solution to the problem. The following diagram explains the implementation steps of algorithm 1.

Generating all the potential solutions at the leader level is, however, practically impossible in large-scale numerical models and examples. Certain solutions were therefore examined in practice to ensure the proper performance of this algorithm. In fact, validating the algorithm performance requires careful investigations. In a small-scale problem, the proposed algorithm can be compared with full enumeration by determining the closeness of its estimated solution to the globally-optimal solution obtained using the latter. Determining an appropriate set of strategies is, however, challenging and essential for the performance evaluation of the algorithm given the risk of losing part of the solution space in which the optimal solution may lie. An appropriate policy is therefore required for generating quality strategies. The present study implemented Steps 2 to 6 after generating proper strategies using a solution-based local search.

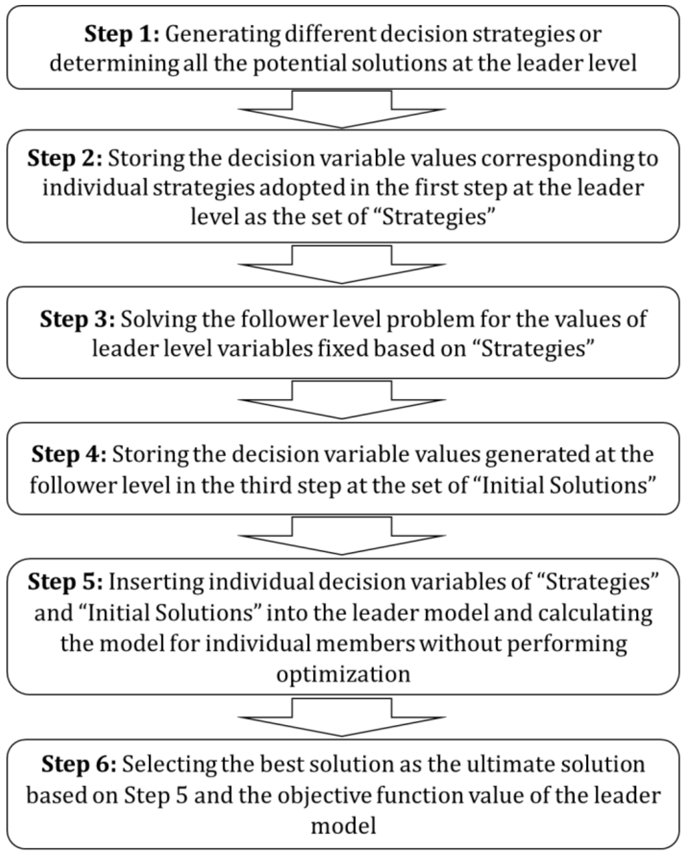


FIGURE 1
DIAGRAM OF HEURISTIC

### *I. Local search algorithm for generating leader strategies*

A heuristic algorithm integrated with full enumeration based on simulated annealing was used to solve the present bi-level model. The leader was also enabled to make decisions on the location of depots and subordinate warehouses using a fake knapsack problem. The solutions obtained from solving the fake problem were iteratively inserted into the follower problem, which was then solved with a decision obtained from the leader problem. After storing solutions to the follower problem, the values of its variables were iteratively inserted into the leader objective function and the solution resulting in the minimum value of the leader objective function was selected as the optimal. The fake knapsack problem employed is explained as follows:

$$\max \mathrm{Z} = Benfit_m\, Z_m + Benfit_c\, E_c \tag{27}$$

$$\sum_{m \in J} Z_m \leq L \tag{28}$$

$$\sum_{c \in C} E_c \leq CD \tag{29}$$

, where $\mathrm{Benfit}_\mathrm{m}$ and $\mathrm{Benfit}_\mathrm{c}$ respectively represent the selected value of individual depots and subordinate warehouses. Diverse solutions were obtained by generating the values of these parameters in different conditions.

### *II. Implementation procedure of the algorithm*

**Step 1:** Inserting the variables associated with the establishment of depots and subordinate warehouses quantified in every iteration using the fake knapsack problem into the follower problem
**Step 2:** Solving the follower problem according to the performed locating and storing the optimal solutions obtained as parameters corresponding to its variables
**Step 3:** Obtaining neighboring solutions using the following equation:

$$Benfit\_New_m = Benfit\_Initialized_m + \alpha \times D\_Par_m \quad (30)$$

$$Benfit\_New_c = Benfit\_Initialized_c + \alpha \times D\_Par_c \quad (31)$$

**Step 4:** Iteratively quantifying the leader objective function using the solutions to the follower problem
**Step 5:** Using the logic of simulated annealing and accepting worse solutions with a specific probability
**Step 6:** Minimizing the leader objective function through different iterations
The proposed algorithm was implemented in a programming language environment, and appropriate methods were used to solve the model.

### *III. A heuristic method for generating $D\_Par_m$*

This method can be used to select highly-effective combinations of all the possible establishment scenarios of centers and subordinate warehouses from numerical results. Using this method effectively reduces the number of feasible initial solutions to the follower problem and increases the quality of eventually-achieved optimal local solutions. An identity matrix formed as a two-dimensional combination of a $1 \times$ L string assigned to combinations of depot locations with a $1 \times$ CD string assigned to combinations of subordinate warehouses was quantified for all the possible combinations of the two strings to determine the minimum and maximum costs as two ends of an interval divided into G subintervals. Moreover, 100 was assigned to the narrowest subinterval and -100 to the widest, with the other subintervals being associated with a value depending on G. All the potential depots were then placed in one of the G subintervals depending on construction cost $f_m$. This classification assisted in prioritizing low establishment cost centers and their cost-effective combinations. Different values assigned to α in each iteration also resulted in different combinations of locations of depots and subordinate warehouses based on the obtained values of $Benfit\_New_m$ and $Benfit\_New_c$. Only the effective combinations were therefore used to achieve the final solution to the problem; for instance, considering 3 out of 5 potential locations for depots and 2 out of 3 for subordinate warehouses, all the possible decision-making combinations or leader strategies included:

- *Leader strategies for locating the establishment sites of depots*

$[(1,2,3), (1,2,4), (1,2,5), (1,3,4), (1,3,5), (1,4,5), (2,3,4), (2,3,5), (2,4,5), (3,4,5)]$

- *Leader strategies for locating the establishment sites of subordinate warehouses*

$[(1,2), (1,3), (2,3)]$

A total of $10 \times 3 = 30$ scenarios are required to be investigated for a very small-scale problem. The following costs were assumed for establishing these centers:

TABLE 2
ESTABLISHMENT COST OF DEPOTS AND SUBORDINATE WAREHOUSES

| Depo center 1 | Depo center 2 | Depo center 3 | Depo center 4 | Depo center 5 | Subordinate warehouse 1 | Subordinate warehouse 2 | Subordinate warehouse 3 |
|---|---|---|---|---|---|---|---|
| 1000 | 1200 | 1500 | 800 | 900 | 500 | 600 | 400 |

The cost of individual combinations was therefore as follows:

TABLE 3
COMBINATION OF DIFFEERENT ENTABLISHMNT SITES OF DEPOS AND SUBORDINATE WAREHOUSES

| Subordinate warehouses | | | Depo Centers | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (2,3) | (1,3) | (1,2) | (3,4,5) | (2,4,5) | (2,3,5) | (2,3,4) | (1,4,5) | (1,3,5) | (1,3,4) | (1,2,5) | (1,2,4) | (1,2,3) |
| 1000 | 900 | 1100 | 3200 | 2900 | 3600 | 3500 | 2700 | 3400 | 3300 | 3100 | 3000 | 3700 |
| $Max = 1100, Min = 900$ | | | $Max = 3700, Min = 2700$ | | | | | | | | | |

To perform calculations for the proposed algorithm, the establishment costs of depots were categorized as follows with the number of categories given as G = 5.

$\text{Min( Depot centers)} = 2700$
$\text{Max(Depot centers )} = 3700$
$\text{step} = \text{Round}({}^{\text{Max} - \text{Min}}/_{5})$
$\text{if Min} \leq f_m \leq \text{Min} + \text{step} \quad \Longrightarrow \quad D\_Par_m = 100$
$\text{if Min} + \text{step} < f_m \leq \text{Min} + 2 \times \text{step} \quad \Longrightarrow \quad D\_Par_m = 50$
$\text{if Min} + 2 \times \text{step} < f_m \leq \text{Min} + 3 \times \text{step} \quad \Longrightarrow D\_Par_m = 0$
$\text{if Min} + 3 \times \text{step} < f_m \leq \text{Min} + 4 \times \text{step} \quad \Longrightarrow D\_Par_m = -50$
$\text{if Min} + 4 \times \text{step} < f_m \leq \text{Min} + 5 \times \text{step} \quad \Longrightarrow D\_Par_m = -100$

The establishment costs of subordinate warehouses were also categorized as follows:
$\text{Min(Subordinate warehous)} = 900$
$\text{Max(Subordinate warehous)} = 1100$
$\text{step} = \text{Round}({}^{\text{Max} - \text{Min}}/_{5})$
$\text{if Min} \leq f_m \leq \text{Min} + \text{step} \quad \Longrightarrow \quad D\_Par_m = 100$
$\text{if Min} + \text{step} < f_m \leq \text{Min} + 2 \times \text{step} \quad \Longrightarrow \quad D\_Par_m = 50$
$\text{if Min} + 2 \times \text{step} < f_m \leq \text{Min} + 3 \times \text{step} \quad \Longrightarrow D\_Par_m = 0$
$\text{if Min} + 3 \times \text{step} < f_m \leq \text{Min} + 4 \times \text{step} \quad \Longrightarrow D\_Par_m = -50$
$\text{if Min} + 4 \times \text{step} < f_m \leq \text{Min} + 5 \times \text{step} \quad \Longrightarrow D\_Par_m = -100$

Table 4 presents the scores of different combinations of depots and subordinate warehouses.

TABLE 4
SCORES OF DIFFERENT COMBINATIONS OF DEPOTS AND SUBORDINATE WAREHOUSES

| *Subordinate warehouses* | | | *Depot centers* | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (2,3) | (1,3) | (1,2) | (3,4,5) | (2,4,5) | (2,3,5) | (2,3,4) | (1,4,5) | (1,3,5) | (1,3,4) | (1,2,5) | (1,2,4) | (1,2,3) |
| *1000* | *900* | *1100* | *3200* | *2900* | *3600* | *3500* | *2700* | *3400* | *3300* | *3100* | *3000* | *3700* |
| *0* | *100* | *-100* | *0* | *100* | *-100* | *-50* | *100* | *-50* | *0* | *50* | *50* | *-100* |
| □ | □ | | □ | □ | | | □ | | □ | □ | □ | |

The combinations were limited to effective ones with a score of over 0. In fact, thirty initial scenarios were reduced to 12 effective scenarios in the solution space by considering 6 location alternatives for depots and 2 for subordinate warehouses. $\text{Benfit\_Initialized}_m$ and $\text{Benfit\_Initialized}_c$ were initialized by calculating the scores of individual depots and subordinate warehouses through averaging the scores of combinations in which they were present; for instance, given that the potential establishment location of depot 1 was present in combinations with scores 50, 50, 0 and 100, the score of the potential establishment site of depot 1 equaled 40. The same scoring procedure was applied to the other depots.

TABLE 5
Final establishment score of depots and subordinate warehouses

| $Benfit\_Initialized_m$ | | | | | $Benfit\_Initialized_c$ | | |
|---|---|---|---|---|---|---|---|
| *Depo Center 1* | *Depo Center 2* | *Depo Center 3* | *Depo Center 4* | *Depo Center 5* | *Subordinate Warehouse 1* | *Subordinate Warehouse 2* | *Subordinate Warehouse 3* |
| *40* | *66.7* | *0* | *50* | *62.5* | *100* | *0* | *500* |

These values were used to solve the fake knapsack problem and achieve the final leader strategies. In accordance with the structure provided for the heuristic algorithm, the numerical results are described in the next section.

### *IV. Computational results*

This section verifies the performance of the proposed model by presenting an average-dimension numerical example containing 72 arcs and 46 vertices, which include five potential sites for establishing depots and five potential sites for establishing subordinate warehouses (support centers). Figure 2 shows the basic graphical structure of this example.

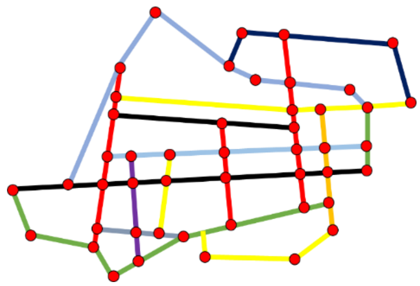

FIGURE 2
URBAN STRUCTRE AND STREETS OF BIRJAND, IRAN

Different colors in figure 2 show the main streets, each of which includes a number of traffic lights. The two-way urban structure of all the streets allowed the vehicles to travel in both directions. It is worth mentioning that 5 potential centers for depot and 5 potential locations for subordinate (support) warehouses have been considered for the implementation of the measures. Solving the problem using the proposed algorithm generated 9 scenarios at the leader level. Table 6 presents these scenarios, including the establishment sites of the main and subordinate warehouses.

TABLE 6
LEADER SCENARIOS

| Scenario1 | | Scenario2 | | Scenario3 | | Scenario4 | | Scenario5 | |
|---|---|---|---|---|---|---|---|---|---|
| Depot centers | Support centers | Depot centers | Support centers | Depot centers | Support centers | Depot centers | Support centers | Depot centers | Support centers |
| 1.4 | 1.2 | 2.3 | 3.2 | 1.5 | 4.5 | 3.5 | 2.4 | 4.5 | 1.4 |
| Scenario6 | | Scenario7 | | Scenario8 | | Scenario9 | | | |
| Depot centers | Support centers | Depot centers | Support centers | Depot centers | Support centers | Depot centers | Support centers | | |
| 2.4 | 2.3 | 1.2 | 1.5 | 3.2 | 3.5 | 3.4 | 4.5 | | |

For the individual leader level scenarios, the optimal value of the objective function at the follower level was determined as follows after solving the follower level model. Given the large-scale numerical example at the follower level, the metaheuristic grey wolf optimization algorithm was used to obtain the solutions.

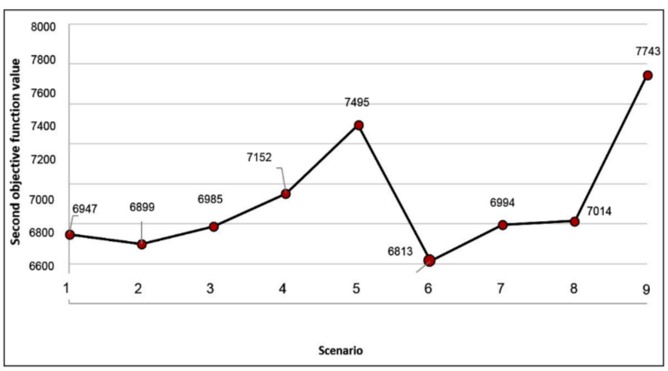


FIGURE 3
THE VALUE OF THE LEADER-LEVEL OBJECTIVE FUNCTION FOR THE FOLLOWER-LEVEL SCENARios

Inserting the numerical values of the variables derived from running the leader and follower models for individual scenarios into the leader level objective function revealed that Scenario 6 was the optimal scenario with the best value. Table 7 presents further details about the solution obtained through generating Scenario 6.

TABLE 7
RESULTS OF APPLYING THE GREY WOLF ALGORITHM

| Number of vertices | Number of arcs | Total cost (million IRR) | Cost of services (million IRR) | Cost of traversing (million IRR) | Establishment cost of the bases | Cost of tours (Million IRR) | Runtime (s) |
|---|---|---|---|---|---|---|---|
| 46 | 72 | 6813 | 1442 | 2846 | 1651 | 874 | 0:00:31.027 |

Figure 4 shows the convergence diagram of the follower level in Scenario 6, suggesting that the algorithm searched the solution space and improved the initial objective function value of 9998 through 300 iterations.

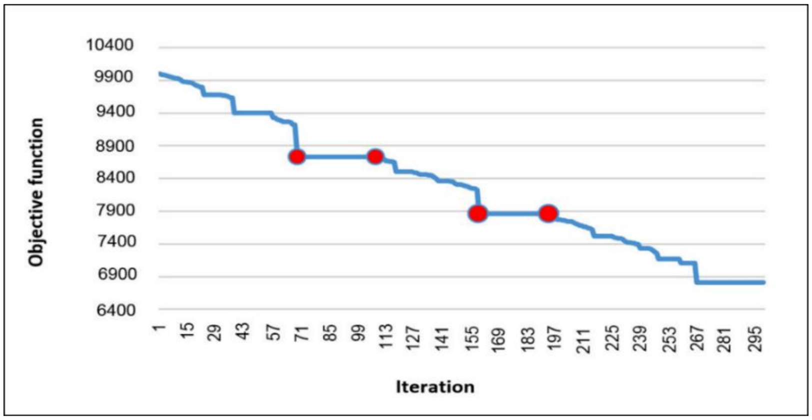


FIGURE 4
CONVERGENCE OF SOLVING SCENARIO 6 USING THE FOLLOWER LEVEL MODEL

The iterations highlighted with red circles show the temporary confinement of the algorithm in local optimal solutions. The algorithm avoided these traps and returned to the solution space to search for better results using its operators. The steady state value of 6813 obtained for the objective function in the final iterations was reported as the ultimate value of this function. The service structure model was ultimately presented as follows:

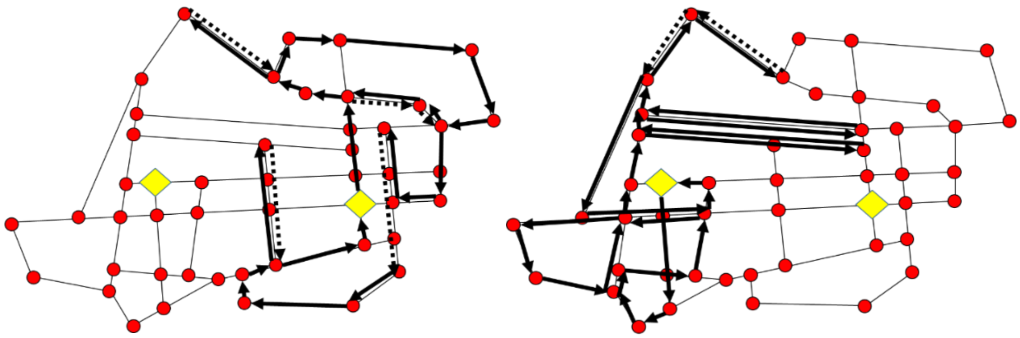

FIGURE 5
TOURS GENERATED TO SOLVE THE NUMERICAL EXAMPLE

Figure 4 confirms the potential of the proposed algorithm for generating acceptable numerical solutions, which are of high quality as well given the structure used for identifying the final solution. Further numerical examples were solved and necessary analyses performed to support the results discussed for the performance of the proposed algorithm.

### *V. Evaluating the proposed algorithm*

Different small, medium and large-scale numerical examples were designed and their numerical results explained in this section to verify the performance of the proposed model and algorithm. Given the random nature of the generated local search parameter in the proposed method, the algorithm was independently run five times for individual numerical examples and the results were recorded. Tables 8-9 present the results and the dimensions of the numerical examples. Moreover, the input parameters of the model were generated in specific numerical intervals with a uniform distribution.

TABLE 8
CHARACTERISTICS OF THE NUMERICAL EXAMPLES GENERATED TO VALIDATE THE PERFORMANCE OF THE PROPOSED ALGORITHM

| **Small scale** | | | **Medium scale** | | | **Large scale** | | |
|---|---|---|---|---|---|---|---|---|
| **Numerical example No.** | **Number of arcs** | **Potential location** | Numerical example No. | **Number of arcs** | **Potential location** | Numerical example No. | **Number of arcs** | **Potential location** |
| 1 | 5 | 3 | 11 | 25 | 10 | 21 | 60 | 12 |
| 2 | 7 | 3 | 12 | 30 | 10 | 22 | 65 | 15 |
| 3 | 9 | 3 | 13 | 32 | 12 | 23 | 70 | 15 |
| 4 | 10 | 4 | 14 | 35 | 15 | 24 | 75 | 18 |
| 5 | 11 | 4 | 15 | 37 | 17 | 25 | 80 | 18 |
| 6 | 12 | 4 | 16 | 40 | 17 | 26 | 85 | 20 |
| 7 | 15 | 5 | 17 | 42 | 18 | 27 | 90 | 20 |
| 8 | 17 | 5 | 18 | 45 | 20 | 28 | 95 | 20 |
| 9 | 19 | 7 | 19 | 47 | 20 | 29 | 100 | 20 |
| 10 | 20 | 8 | 20 | 50 | 20 | 30 | 110 | 25 |

Table 8 presents the objective function value obtained at the follower level after solving individual numerical examples of small, medium and large scales, suggesting a very small standard deviation for individual numerical examples as shown in figure 6.


FIGURE 6
THE DIFFERENCE BETWEEN THE RESULTS OF THE INDEPENDENT RUNS AND THE MEAN VALUE

The match between the mean value of the objective function in every numerical example with almost all the runs as shown in figure 5 confirmed the algorithm performance in terms of the appropriateness of the solutions generated in different runs or the algorithm robustness in generating optimal solutions. Figure 7 shows the standard deviation for the five runs, suggesting a maximum deviation of merely 5%, which was negligible in this algorithm.

TABLE 9
NUMERICAL RESULTS OBTAINED FROM SOLVING THE SMALL, MEDIUM AND LARGE-SCALE EXAMPLES

| | Problem No. | Run 1 | Run 2 | Run 3 | Run 4 | Run 5 | Mean | SD |
|---|---|---|---|---|---|---|---|---|
| Small | 1 | 6006.9 | 6006.9 | 6006.9 | 6006.9 | 6006.9 | 6006.9 | 0 |
| | 2 | 7782.6 | 7782.6 | 8239.7 | 7782.6 | 7782.6 | 7874.02 | 182.8 |
| | 3 | 11781.2 | 11781.2 | 11781.2 | 12109.7 | 11781.2 | 11846.9 | 131.4 |
| | 4 | 10728.3 | 10658.7 | 10938.6 | 10728.3 | 9791.1 | 10569 | 400.1 |
| | 5 | 13894 | 13457.8 | 13421.8 | 13421.8 | 13457.8 | 13530.64 | 182.3 |
| | 6 | 9029.6 | 9396.6 | 9029.6 | 9128.1 | 9128.1 | 9142.4 | 134.5 |
| | 7 | 12694.2 | 11438 | 11438 | 12983.2 | 12750.5 | 12260.78 | 678.7 |
| | 8 | 16892.1 | 16428.3 | 16842 | 16765.2 | 16892.1 | 16763.94 | 174.1 |
| | 9 | 17203.1 | 18064.3 | 17281 | 17406.3 | 17345.7 | 17460.08 | 309.5 |
| Medium | 10 | 21593.4 | 21593.4 | 21593.4 | 21669 | 21669 | 21623.64 | 37 |
| | 11 | 21193.1 | 21172.1 | 21143.2 | 21143.2 | 21143.2 | 21158.96 | 20.4 |
| | 12 | 28144.4 | 27922.4 | 27640.9 | 27640.9 | 27640.9 | 27797.9 | 204.6 |
| | 13 | 30081.2 | 29230.6 | 29602.1 | 29170.1 | 29314.2 | 29479.64 | 335.2 |
| | 14 | 32927.6 | 32945.1 | 32809.3 | 32664.9 | 32169.9 | 32703.36 | 284.9 |
| | 15 | 27626.6 | 28570.1 | 29664 | 28153 | 27635.2 | 28329.78 | 754.4 |
| | 16 | 30429.8 | 29286.2 | 30375.1 | 30375.1 | 30933.4 | 30279.92 | 539.4 |
| | 17 | 32039.5 | 32028.8 | 31618.3 | 30719.3 | 30551.6 | 31391.5 | 637.9 |
| | 18 | 30120.3 | 29903.6 | 29903.6 | 29487.9 | 29903.6 | 29863.8 | 205.8 |
| | 19 | 35944.6 | 36330.6 | 36330.6 | 36087.5 | 37017.4 | 36342.14 | 368.5 |
| Large | 20 | 30034.4 | 31179 | 30683.2 | 31142.7 | 30117.9 | 30631.44 | 486.6 |
| | 21 | 203.976 | 29299.3 | 29725.2 | 29113.6 | 28922.7 | 29304.8 | 203.976 |
| | 22 | 396.112 | 38744.2 | 38744.3 | 37685 | 38743.9 | 38152.8 | 396.112 |
| | 23 | 398.08 | 43258.2 | 42607.9 | 43088.6 | 42420 | 42004.3 | 398.08 |
| | 24 | 475.152 | 39926.1 | 40331.1 | 39517.7 | 40578.3 | 39059.5 | 475.152 |
| | 25 | 736.616 | 39386.3 | 37453.5 | 37190.8 | 38772.4 | 38411.6 | 736.616 |
| | 26 | 467.408 | 39146.2 | 40427 | 38905.5 | 39769.6 | 39321.9 | 467.408 |
| | 27 | 652.52 | 49181.5 | 47839.6 | 48728.7 | 50097.9 | 49651.3 | 652.52 |
| | 28 | 708.528 | 40401.7 | 42351.8 | 40504.6 | 40687 | 41662.6 | 708.528 |
| | 29 | 501.768 | 39589.4 | 38810.8 | 39287.1 | 40559.4 | 39989.5 | 501.768 |
| | 30 | 51.64 | 47403.9 | 47422.3 | 47318.3 | 47403.9 | 47543.6 | 51.64 |

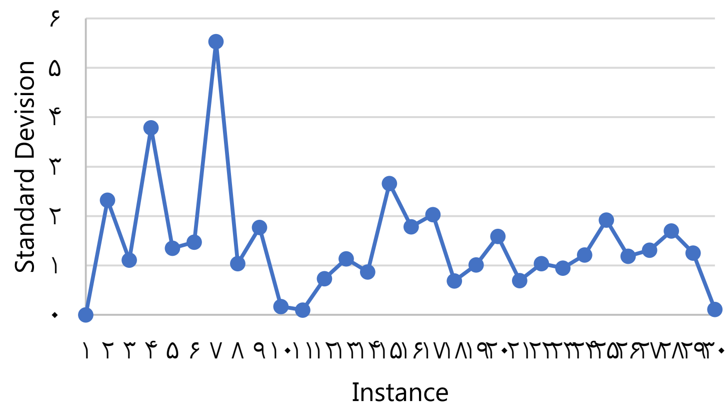


FIGURE 7
STANDARD DEVIATION IN PERCENTAGE OF THE SOLUTIONS GENERATED BY THE PROPOSED ALGORITHM

Overall, given the effectiveness of the algorithm developed in the present study in solving numerical examples, it can be applied to solving real-world problems.

### *VI. Sensitivity analysis*

Given that investigating a numerical example cannot accurately determine the accuracy of a model, the model sensitivity to certain input parameters at the leader level should be examined.

### VII. Using or not using support warehouses

Ten small-scale numerical examples designed in this section are solved in CPLEX to compare the use (CD>0) or non-use (CD=0) of support warehouses in terms of the objective function value; nevertheless, using a support warehouse is expected to significantly reduce the cost.

TABLE 10
ANALYZING THE MODEL SENSIVITY TO SUPPORT WAREHOUSES

| Row | Solution time (seconds) | Objective function value when not using support warehouses | Objective function value when using support warehouses | Improvement rate |
|---|---|---|---|---|
| 1 | 122.427 | 1573186000 | 1573186000 | - |
| 2 | 121.691 | 1631607000 | 1631607000 | - |
| 3 | 110.509 | 1189557000 | 1189557000 | - |
| 4 | 124.779 | 676042000 | 676042000 | - |
| 5 | 139.517 | 3350316000 | 3204793000 | 4.34 |
| 6 | 134.372 | 2360199000 | 2249228000 | 4.70 |
| 7 | 101.043 | 2931975000 | 2821164000 | 3.78 |
| 8 | 124.994 | 2378130000 | 2307649000 | 2.96 |
| 9 | 126.179 | 1948101000 | 1865599000 | 4.23 |
| 10 | 118.352 | 4716154000 | 4394350000 | 6.82 |

According to table 9, the zero rate of improvement in the objective function in examples 1-4 indicates the independency of the objective function value on using or not using support warehouses, which can be explained by the model's decision based on the problem input parameters to still use the main warehouse irrespective of the presence or absence of support warehouses. Given the different conditions in examples 5-10, the model decides to use support warehouses, resulting in a certain improvement in the objective function.

### *VIII.* Sensitivity of the solution time to input parameters

Investigating the effect of the value of input parameters on the solution time is crucial. In fact, different factors that potentially complicate the problem or change the solution time include changes in parameters and variables. These effective factors in the solution time are individually examined in this article. Given the problem model of this research as a linear integer problem, the larger the problem, the significantly longer the solution time. The effective factors in the solution time are as follows:

1. The demand
2. The number of main warehouses
3. The number of vehicles

After solving different numerical examples for different values of the demand, number of products and number of vehicles, the following diagrams were obtained to show the sensitivity of the solution time to these parameters.

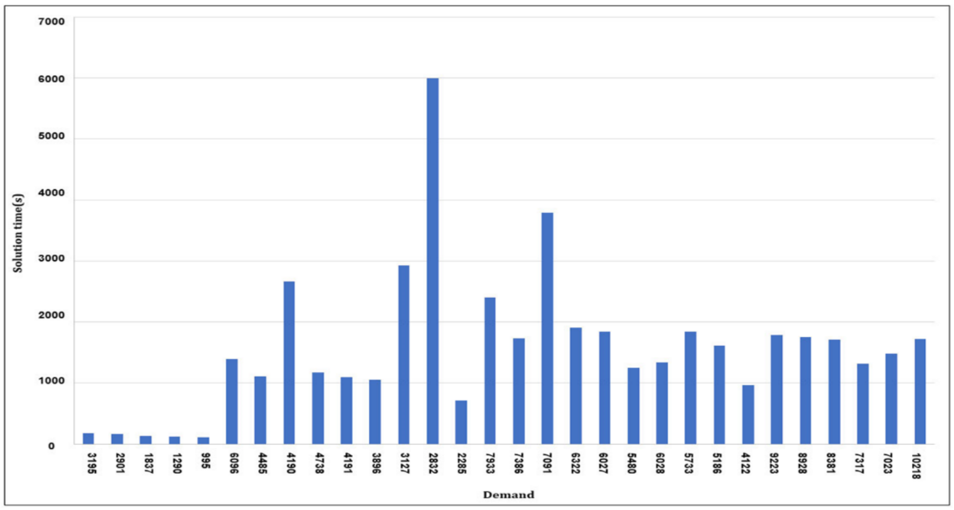


FIGURE 8
THE EFFECT OF DEMAND ON THE SOLUTION TIME

The increase observed in the problem size with the demand or the number of products obviously increases the solution time. In fact, the solution time ranges from instantaneous for small-scale problems to significantly-long durations for actual large problems with a high number of products. According to figure 7, the solution takes very long to achieve in certain problems mainly in the neighborhood of the desired gap, which are converted into outliers, despite the generally-upward trend observed.

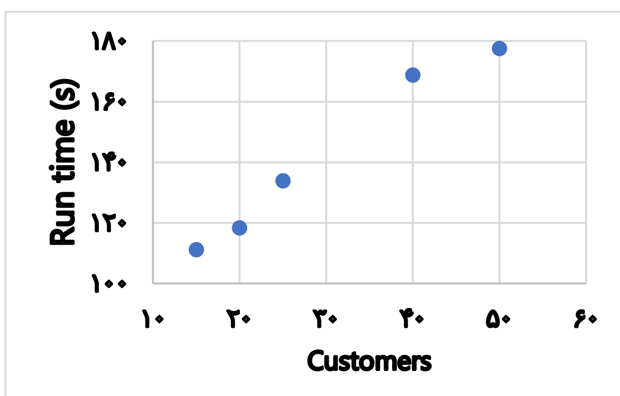


FIGURE 9
THE EFFECT OF THE NUMVER OF MAIN WAREHOUSESO ON THE SOLUTION TIME

According to figure 9, the solution time increases with the numbers of either the main warehouses or vehicles.

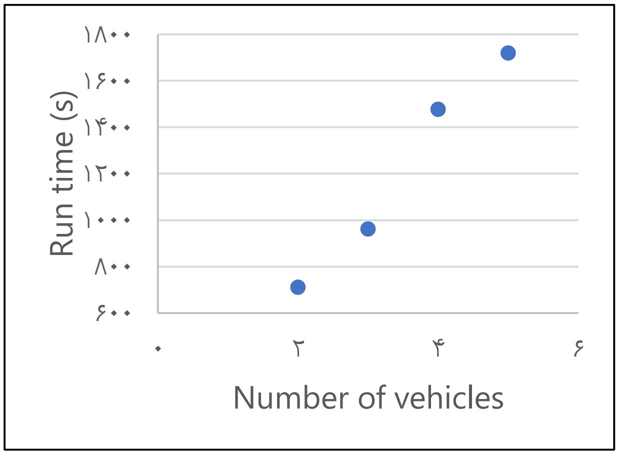


FIGURE 10
THE EFFECT OF THE NUMBER OF VEHICLES ON THE SOLUTION TIME

The demonstrated sensitivity of the proposed model to input parameters therefore confirms its proper performance.

## CONCLUSION AND RECOMMENDATIONS

The mathematical optimization model developed in the present research based on bi-level programming to optimally locate the establishment sites of support centers for the repair and maintenance of traffic lights and determine optimal routes for repair teams in charge of inspecting the lights was found to generate appropriate solutions as the final decisions. Given the outsourcing of the delivery of these urban management services to contractors in the private sector, executive decisions are not made by a single decision maker. The contractors should match their operational plans to the facilities provided based on specific criteria by municipalities, which causes a lack of decision coordination between municipality as the leader level and contractor as the follower level and the associated service delivery problems. Reaching a common and desirable final solution by both levels is therefore crucial for decision makers. The present study therefore employed bi-level programming in which the municipality as the leader level located the facilities and the contractors as the followers determined the optimal route of traffic light repair teams based on the establishment sites of the support facilities. No comprehensive methods have been yet proposed to solve multilevel programming as a complicated problem and achieve globally-optimal solutions in operational research. In fact, bi-level programming problems with discrete variables at the follower level have always been considered an open problem in operational research; nevertheless, researchers have reported reliable numerical solutions by using different heuristic algorithms depending on the problem structure, simulating the Karush-Kuhn-Tucker conditions and converting the bi-level problem to a single-level one.

The present research developed a heuristic algorithm based on full enumeration to obtain numerical solutions. According to this algorithm, all the potential solutions provided by the leader level were considered an initial set of solutions. The follower level then stored sets of new solutions, with each set containing the solutions obtained by solving the problem at this level for the individual solutions at the leader level. Afterwards, the objection function values were calculated at the leader level by inserting all the new solutions into the model. The solution with the best numerical value was ultimately considered the optimal solution to the problem. To evaluate the performance of the proposed algorithm, a medium-sized numerical example was solved and the algorithm steps were detailed. The numerical results confirmed the effectiveness of the proposed algorithm in solving numerical examples and its potential application to solving real-world problems. Future studies are recommended to address uncertainty in input parameters such as demand and to employ robust programming to solve this type of problems.

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