Original Research

# Optimizing warranty cost in a three-stage non-renewing warranty policy

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**Abstract**

Attracting customers and keeping market share have become more important for manufacturers in today's competitive market. Offering warranty for their products is one of the techniques that manufacturers use to attract customers. There are different types of warranty policies. Most research in this area has focused on two-stage warranties and little attention has been paid to three-stage warranties. Hence, in this study, a three-stage non-renewing warranty is investigated in which, the length of warranty period is divided into three phases. In the first phase, if the product fails, the cost of repair is fully covered by the manufacturer. In the next phase, if failure happens, most of the repair cost is still covered by the manufacturer while the customer must pay for the rest of it. Finally, in the third phase, if the product faces any failure, most of the repair cost is paid by the customer as the product has been used for quite some time by then. In this study, a repairable system under this type of warranty policy is investigated and warranty cost during this period is derived. To minimize the cost of warranty period, the optimum length of each phase is obtained. Numerical examples are provided to verify the proper performance and applicability of the proposed model.

**Keywords-** Warranty Cost; Three-stage warranty; Optimization

## INTRODUCTION

Warranty is a contract between buyer and manufacturer that has a vital role in many trades [1]. Warranty contract is to assure the buyer that the product will perform satisfactorily for a specified period and, if the product fails during this period, the manufacturer will be responsible for repair or replacement of the failed item based on the warranty conditions. Considering technological improvements and complicated industrial systems development, consumers are worried about reliability of the products. On the other hand, manufacturers are looking for increasing the number of customers as well as increasing their profit. The warranty offer is one of the many factors that affect the competitive market. Although warranty offer imposes extra costs for manufacturer, they still accept it for increasing customer satisfaction and staying in the market [2]. In the recent years, offering warranty has become an important tool for manufacturers to gain a competitive advantage and advertise their products. Increasing competition between manufacturers has given consumers many choices. Hence, consumers have this chance to select the best one among them. One of the main factors in consumer choice, where other conditions (performance, price, payment conditions, etc.) are almost equal, is an attractive warranty offer [3]. A fair warranty assures the customers of their choice and decreases their risks. Warranty, particularly for new products, is a powerful tool to gain market share from experienced and capable competitors. Thus, manufacturers are looking for offering attractive warranties to satisfy the customers and assure the quality of their products as well as to provide desired profit. For this purpose, different types of warranty policies have been offered from manufacturers to their customers during recent years. Some of them are one-dimensional and two-dimensional warranty policies and renewing and non-renewing warranty policies. In parallel with defining different types of warranty policies, researchers are trying to assess and analyze them to estimate manufacturers and consumers profits in different conditions [4]. One of the initial researches about designing an optimal system considering maintenance and warranty was done by Monga and Zuo [5]. They studied designing a series-parallel system based on its reliability by considering three phases in product life cycle; burn-in period, useful life or warranty period, and wear out or after warranty period. They considered a bathtub shaped failure rate curve for this system and their model contained different types of cost such as burn-in test cost, installation cost, minimal repair cost, and preventive repair cost. They utilized a genetic algorithm-based optimization method to find optimal values for burn-in test period, time between preventive maintenance and replacement time.

One of the major groups of warranty policies is renewing warranty. In this warranty policy, if a component fails, the buyer receives warranty service and the period of warranty starts over. For instance, if the warranty period for a product is $W$, and this product fails at time $t$ ($t<W$), after repair/ replacement of the failed product, the new period of warranty will be $t+W$ (the warranty period of length $W$ starts at time $t$). One of the works that has been done in comparing such warranties is the research that has been done by Mi [6]. In his model, he considered random costs for early breakdowns and renewable warranty and obtained an average warranty cost. Finally, he compared the average warranty cost for different types of warranties like free replacement and combined renewable warranty. He also compared average costs for different types of lifetime density function. Generally, products are repairable and non-repairable. For non-repairable products, warranty service contains replacing of failed product. For repairable products, warranty service contains either repair or replacement of failed product. Iskandar *et. al*. [7] presented a new warranty strategy for repair and replacement of sold items under two-dimensional warranty. In their paper, they considered a new warranty strategy for sold items in which, if an item fails for a second time, it will be replaced by a new item. They obtained optimal values to minimize the total warranty cost.

Extracting optimal time for burn-in test period, warranty period, and maintenance period (useful lifetime after warranty period) was a research that was done by Moghimihadji and Rangan [8]. They considered a bathtub shaped function for product failure rate. In their model, a system consisting of some similar components was considered. During burn-in test, if any failure happened, it would be replaced by a new system. If one system could pass burn-in test successfully, it would sell in the market with a warranty offer. Moghimihadji and Rangan considered different types of costs in their model such as buying cost, installation cost, operation cost, and replacement cost. The considered type of warranty policy was a renewing warranty. Until the end of the warranty period, all the related costs should be covered by the manufacturer and after that, repair and replacement cost should be paid by the customer. During the lifetime period, they considered three types of repair to solve the product problem: minimal repair, replacement, and general repair. Finally, they solved the proposed model to minimize the average total cost during these three periods. In their model, they also considered some constraints such as maximum available volume and minimum level of required reliability.

In another study, Chang and Lin [9] investigated maintenance policy and length of extended warranty from seller's point of view for repairable products in which seller performs minimal repair or imperfect preventive maintenance based on the age of the product when product fails. Chen and his team of researchers proposed a complete warranty cost model by considering three different phases for repairable product life cycle, namely; burn-in, free replacement warranty, and pro-rata warranty along with two different types of failure; minimal and major failure, to optimize warranty length [10]. Su and Wang [11] worked on optimizing the model of two-stage preventive maintenance combined with a two-dimensional extended warranty. In their research, they assumed that a customer has bought a two-dimensional extended warranty. Then, they worked on optimizing an incomplete preventive maintenance for repairable items from manufacturer perspective. Long time warranty, after the main warranty period is over, is more attractive for customers when systems become more complicated. Jung [12] assessed a model of long-time warranty under minimal repair in his essay. Marshall *et. al.* [13] modeled warranty servicing cost under non-renewing and renewing free repair warranties. Unlike many similar researches, they assumed nonzero increasing repair times with the warranty cost depending on the length of the repair time. They derived analytical results for a generalized alternating renewal process with a finite horizon. They used simulation study to demonstrate the properties of their model. Manufacturers always look for providing attractive warranty services to their customers. Sometimes, making changes in warranty conditions results in offering a new warranty policy. Comparing between traditional extended warranties with a new warranty policy was a research that has done by Bian *et. al.* [14]. They compared a new trade policy in service with traditional repair or replacement service in extended warranty and reached the conclusion that the total cost of their new policy would never be more than traditional policies. When a product comes to the last phase of its lifetime cycle, some of its parts become phased out and no supplier will supply them anymore. Yet, manufacturers have to keep enough inventories to provide suitable product support for their customers. Shi [15] has studied spare parts inventory control problems in his work. Although warranty offers have some benefits for both sides (manufacturer and customer), offering unlimited warranty is not practical for manufacturer. Nevertheless, researches in this area are focused on competitive pricing. In fact, there is a competition in price and after sale services for durable products. Fang [3] in his recent paper, proposed a model for durable products in a bipolar market. He showed that after the markets reached a balance, manufacturer could increase their profit by increasing the price or extending the warranty period.

In a recent study done by Moghimihadji [16], optimal lengths of burn-in period and warranty period considering different types of repairs was obtained. In his paper, he studied the total cost during burn-in and warranty periods from manufacturer point of view. He considered different types of repair for a product with bathtub shaped failure rate curve. In his model, during the burn-in period, the product faces minor failures with probability $P_1$. The failed product is fixed with minimal repair and the burn-in test continues. During this period, the product faces major failures with probability $1-P_1$. In this case, the manufacturer must replace it and the burn-in test starts over. After this period, the warranty period begins in which the product faces minor failures with probability $P_2$ that requires minimal repair by the manufacturer. During same period, the product faces catastrophic failures with probability $1-P_2$ that requires a general repair from the manufacturer. Repair and replacement times are neglected in this model. The output of this model is the optimal length of burn-in and warranty periods. The layout of this article is as follows. In section 2, we extract the related costs functions for three deferent periods for a product with a non-renewable warranty of length W. Section 3 presents two numerical illustrations for two deferent types of failure rate functions to show applicability of the proposed method. The last section contains some concluding remarks.

## MATHEMATICAL MODEL

In this warranty policy, the entire warranty period is divided into three parts, in which the amount of compensation to be paid to the buyer decreases with the increase of the warranty period. Repair/replacement cost to both manufacturer and buyer of an item that fails when covered by this warranty policy are shown in the following figure.

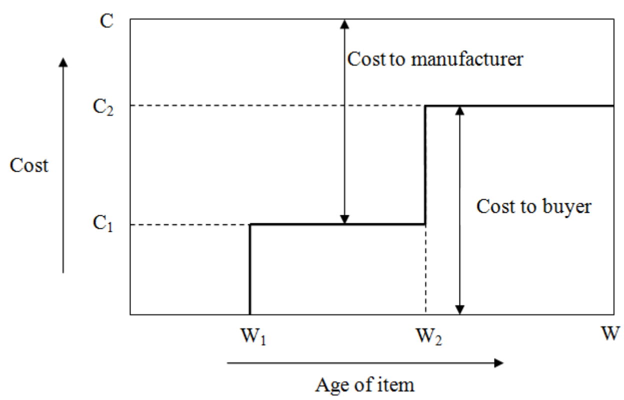


FIGURE 1
REPAIR/REPLACEMENT COSTS UNDER A THREE STAGE WARRANTY

Examples of products covered by this type of warranty are television sets and home appliances that are fully covered for an initial period and only partially covered for subsequent periods. Here, a product with a non-renewable warranty of length $W$ is considered. This length is divided into three periods: $(0 - w_1)$: from starting time to $w_1$, during this period the total repair cost of failed product is covered by the manufacturer. Since the product is new and repair cost should be paid by the manufacturer, they repair the failed product minimally to decrease the cost. It means that only the failed component will be repaired (if it is repairable) or replaced (if it is not repairable).

$(w_1 - w_2)$: from $w_1$ to $w_2$, if the product fails in this interval, a smaller portion of the repair cost (name it α, generally less than 50 percent) should be paid by the buyer while the bigger portion of the repair cost should be paid by the manufacturer. Again, since the product is not too old, manufacturer will do minimal repair to fix the failed item. $(w_2 - W)$: from $w_2$ up to the end of the warranty period, $W$, if the product fails during this period, a larger portion of the repair cost (name it β, generally more than 50 percent) is paid by the buyer. Based on the failure type (minor or major), manufacturer does a general repair with probability $p$ or minimal repair with probability $1\text{-}p$. The aim of this research is to obtain optimum length of each period by minimizing the average total cost during the entire warranty period.

- *Related costs during* $(0 - w_1]$

Suppose that a new product is bought by a customer at time zero. The selling price of the new product is $c_0$. The installation cost of this product is $c_1$ and its operation cost is $c_2$ per unit time. If this product fails during this period, manufacture must uninstall, repair (minimal repair), and reinstall it. The repair cost for manufacturer is $c_3$ during this period. Since the product is repaired minimally, at the end of this period the age of the product is $w_1$. Hence, the total cost function during this period is

$$C_{w_1} = (c_1 + c_3).N(w_1) \tag{1}$$

In this equation, $c_1$and $c_3$ are predefined coefficients but $N(w_1)$ (number of failures during this period) should be defined. Suppose that $f(t)$ is the failure time density function, then the failure rate function, $h(t)$is given by:

$$h(t) = {f(t)}\Big/{\bar{F}(t)} \tag{2}$$

where $\bar{F}(t) = 1 - F(t)$ is the survival function. Thus, the number of failures during this period is

$$N(w_1) = \int_0^{w_1} h(t)dt = \int_0^{w_1} \frac{f(t)}{\bar{F}(t)}dt = \int_0^{w_1} \frac{f(t)}{\int_t^{\infty} f(x)dx}dt \tag{3}$$

Thus, total cost function in the first period is

$$C_{w_1} = (c_1 + c_3).\int_0^{w_1} \frac{f(t)}{\int_t^{\infty} f(x)dx}dt \tag{4}$$

It is worth mentioning that, the age of the system is $w_1$ at the end of this period.

- *Related costs during (w1 – w2]*

The manufacturer does a minimal repair in this period since the product is not old yet and they have to cover most of the repair cost. In this period, if the product fails, α percent (α<50%) of the repair cost should be paid by the buyer and the rest should be paid by the manufacturer. The main part of the repair cost in this period contains installation cost, $c_1$ and minimal repair cost, $c_4$ ($c_4 \geq c_3$). Thus, the cost function in the second period is

$$C_{w_2} = (1 - \alpha)(c_1 + c_4).N(w_1, w_2] \tag{5}$$

Since the age of the product at the end of first period is $w_1$, the number of failures in the interval $(w_1, w_2]$ is

$$N(w_1, w_2] = \int_{w_1}^{w_2} h(t)dt = \int_{w_1}^{w_2} \frac{f(t)}{\bar{F}(t)}dt = \int_{w_1}^{w_2} \frac{f(t)}{\int_t^{\infty} f(x)dx}dt \tag{6}$$

Thus, the manufacturer total cost during the second period is

$$C_{w_2} = (1 - \alpha)(c_1 + c_4).\int_{w_1}^{w_2} \frac{f(t)}{\int_t^{\infty} f(x)dx}dt \tag{7}$$

- *Related costs during (w2 – W]*

In the last period, most of the repair cost (β percent of the total repair cost) should be paid by the buyer ($\beta$>50%). Since this is the last period in the warranty period and the manufacturer wants to provide customer satisfaction, if the product faces any failure, manufacture does a thorough assessment first. The manufacturer will do a general repair only if it is necessary (with probability $p$).Otherwise, the manufacturer will repair the failed product minimally. In both cases, $\beta$ percent of the repair cost will be paid by the buyer. The main parts of the manufacturer cost are installation cost, $c_1$, minimal repair cost in this period, $c_5$ ($c_5 \geq c_4$) with probability $1-p$, and general repair cost, $c_6$ with probability $p$. If the product is repaired minimally, the related cost function is

$$C_m = (1-\beta)(1-p)(c_1 - c_5)\int_{w2}^{W} \frac{f(t)}{\int_t^{\infty} f(x)dx} dt \tag{8}$$

In the case of general repair, each maintenance action reduces the real age of the system by a factor δ, 0≤$\delta$≤1. Thus, if a maintenance action of type general repairs is performed on a system with age $x$, then this maintenance action reduces the age of the system to δ$x$, and δ is called rejuvenation factor (which is defined with repair degree). When $\delta$=0, the rejuvenation is perfect. It means that the age of product after repair returns to zero, just like a new product. In other words, it means replacement of the failed product with a new one. On the contrary, $\delta$=1 means that the repaired product is completely the same as the product just before failure. It means that only minimal repair has been done to the failed product.

When the rejuvenation factor, δ, is between 0 and 1 (0<$\delta$<1), it means that the repaired product is not as good as the new product and it is not in the situation just before the failure happened, but at a situation between these two situations. In this case, after the repair, the age of product will be decreased depending on the amount of δ. The expected number of failures $M_g(t)$ in an arbitrary time interval (0, t] when the system is maintained under general repair policy is known as g-renewal function which was first introduced by Kijima [17]. The g-renewal function can be derived as the solution of the g-renewal equation given by

$$M_g(t) = Q(t|0) + \int_0^t Q(t-x|x).m_g(x)dx \tag{9}$$

where Q(t|x) is defined as

$$Q(t|x) = \int_0^t q(y|x)dy = \int_0^t \frac{f(y+\delta x)}{\bar{F}(\delta x)} dy \quad , t, x \geq 0 \tag{10}$$

Thus, if the product in this period is repaired generally (with probability $p$), the manufacturer related cost is

$$C_g = (1-\beta).p.(c_1 + c_6)(1-\delta)M_g(W) \tag{11}$$

Where $M_g(W)$ is the expected number of failures during interval ($w2$, $W$] when general repair is done, thus, the total cost function in the third period, ($w_2$, $W$] is

$$C_W = (1-\beta)\left[(1-p)(c_1 + c_5)\int_{w2}^{W} \frac{f(t)}{\int_t^{\infty} f(x)dx} dt + p(c_1 + c_6)(1-\delta)M_g(W)\right] \tag{12}$$

When $\delta$ is between 0 and 1, it is not possible to find an explicit solution for equation (9). Hence, researchers have used different types of approximations to estimate it. In this research, we have used the approximation method introduced by Rangan and Moghimihadji [18]. Finally, the average total manufacturer cost during the warranty period is

$$E(TC) = \frac{C_{w_1} + C_{w_2} + C_W}{W} \tag{13}$$

MATLAB software is used to solve the above equation in the following numerical examples.

## NUMERICAL ILLUSTRATION

To demonstrate the performance and applicability of the proposed model, two numerical examples are presented in the following. Although it is possible to employ any type of failure rate function, a fixed failure rate function (failure rate = 0.05) is used in the first numerical example for simplicity. The cost coefficients are defined in the following table (Table 1).

TABLE 1
REPAIR COST COEFFICIENTS DURING THE THREE PERIODS OF WARRANTY

| Cost title | Definition | Value |
|---|---|---|
| $c_1$ | Installation and setup cost | 1 |
| $c_3$ | Minimal repair cost in the first period, $(0, w_1]$ | 3 |
| $c_4$ | Minimal repair cost in the second period, $(w_1, w_2]$ | 6 |
| $c_5$ | Minimal repair cost in the third period, $(w_2, W]$ | 8 |
| $c_6$ | General repair cost in the third period, $(w_2, W]$ | 25 |

In the second period, $(w_1, w_2]$, $\alpha$ percent of the repair cost should be paid by the buyer ($\alpha$<0.5). However, in the third period, $(w_2, W]$, $\beta$ percent of the repair cost should be paid by the buyer ($\beta$>0.5). In the third period, with probability $p$, the failed product should get a general repair. These coefficients are defined in Table II.

TABLE 2
COEFFICIENTS OF CUSTOMER COST SHARE AND PROBABILITY OF DOING GENERAL REPAIR

| Coefficient name | Definition | Value |
|---|---|---|
| $\alpha$ | Customer cost share during second period, $(w_1, w_2]$ | 0.3 |
| $\beta$ | Customer cost share during third period, $(w_2, W]$ | 0.7 |
| $p$ | Probability of doing general repair during third period, $(w_2, W]$ | 0.2 |

In this case, the average lifetime is considered as the warranty length ($W$). Thus, based on the constant failure rate in this example, the warranty length will be equal to 10. The first period length $(0, w_1)$ is considered as 20 to 30 percent of the total warranty period. It is possible to consider shorter or longer length for this period. In this problem, three different options are considered for the length of the first period; $0.2W$, $0.25W$, and $0.3W$. The best option will be selected from these three options. The second period length $(w_1, w_2)$ is considered as 30 to 60 percent of the total warranty period. Similar to the first period, different options are considered for the length of the second period from$0.3W$ up to $0.6W$ with $0.05W$ increments. After solving the problem, the best option will be selected from these options. Different values from 0.5 up to 0.9 with 0.05 increments are considered for rejuvenation factor (δ). The general repair cost will increase by decreasing δ. Based on the above values, for the first case of Table III, the calculations will be as follows. Based on the equation number 4,

$$\boldsymbol{C_{w_1} = (1+3)\int_0^2 \frac{0.05}{\int_t^\infty 0.05\,dx}\,dt}$$

And based on the equation number 7,

$$\boldsymbol{C_{w_2} = (1-0.3)(1+6)\int_2^3 \frac{0.05}{\int_t^\infty 0.05\,dx}\,dt}$$

And finally according to the equation number 12,

$$\boldsymbol{C_W = (1-0.7)\left[(1-0.2)(1+8)\int_3^{10} \frac{0.05}{\int_t^\infty 0.05\,dx}\,dt + 0.2(1+25)(1-0.5)M_g(W)\right]}$$

Since there are number of such calculations should be done, we employed MATLAB software to do these calculations. The results of solving the model are shown in the following tables (Table 3 to Table 4).

TABLE 3
AVERAGE TOTAL COST FOR W1=0.2W

| W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 0.5 | 0.3159 | 2 | 3.5 | 0.5 | 0.3155 | 2 | 4 | 0.5 | 0.3150 |
| 2 | 3 | 0.55 | 0.3134 | 2 | 3.5 | 0.55 | 0.3129 | 2 | 4 | 0.55 | 0.3125 |
| 2 | 3 | 0.6 | 0.3108 | 2 | 3.5 | 0.6 | 0.3104 | 2 | 4 | 0.6 | 0.3100 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 3 | 0.65 | 0.3083 | 2 | 3.5 | 0.65 | 0.3079 | 2 | 4 | 0.65 | 0.3074 |
| 2 | 3 | 0.7 | 0.3057 | 2 | 3.5 | 0.7 | 0.3053 | 2 | 4 | 0.7 | 0.3049 |
| 2 | 3 | 0.75 | 0.3032 | 2 | 3.5 | 0.75 | 0.3028 | 2 | 4 | 0.75 | 0.3024 |
| 2 | 3 | 0.8 | 0.3007 | 2 | 3.5 | 0.8 | 0.3003 | 2 | 4 | 0.8 | 0.2998 |
| 2 | 3 | 0.85 | 0.2981 | 2 | 3.5 | 0.85 | 0.2977 | 2 | 4 | 0.85 | 0.2973 |
| 2 | 3 | 0.9 | 0.2956 | 2 | 3.5 | 0.9 | 0.2952 | 2 | 4 | 0.9 | 0.2948 |
| 2 | 4.5 | 0.5 | 0.3146 | 2 | 5 | 0.5 | 0.3142 | 2 | 5.5 | 0.5 | 0.3138 |
| 2 | 4.5 | 0.55 | 0.3121 | 2 | 5 | 0.55 | 0.3117 | 2 | 5.5 | 0.55 | 0.3113 |
| 2 | 4.5 | 0.6 | 0.3096 | 2 | 5 | 0.6 | 0.3092 | 2 | 5.5 | 0.6 | 0.3088 |
| 2 | 4.5 | 0.65 | 0.3070 | 2 | 5 | 0.65 | 0.3066 | 2 | 5.5 | 0.65 | 0.3062 |
| 2 | 4.5 | 0.7 | 0.3045 | 2 | 5 | 0.7 | 0.3041 | 2 | 5.5 | 0.7 | 0.3037 |
| 2 | 4.5 | 0.75 | 0.3020 | 2 | 5 | 0.75 | 0.3016 | 2 | 5.5 | 0.75 | 0.3012 |
| 2 | 4.5 | 0.8 | 0.2994 | 2 | 5 | 0.8 | 0.2990 | 2 | 5.5 | 0.8 | 0.2986 |
| 2 | 4.5 | 0.85 | 0.2969 | 2 | 5 | 0.85 | 0.2965 | 2 | 5.5 | 0.85 | 0.2961 |
| 2 | 4.5 | 0.9 | 0.2944 | 2 | 5 | 0.9 | 0.2940 | 2 | 5.5 | 0.9 | 0.2936 |
| | | | | | | | | W1 | W2 | Delta | Average (TC) |
| | | | | | | | | 2 | 6 | 0.5 | 0.3134 |
| | | | | | | | | 2 | 6 | 0.55 | 0.3109 |
| | | | | | | | | 2 | 6 | 0.6 | 0.3084 |
| | | | | | | | | 2 | 6 | 0.65 | 0.3058 |
| | | | | | | | | 2 | 6 | 0.7 | 0.3033 |
| | | | | | | | | 2 | 6 | 0.75 | 0.3008 |
| | | | | | | | | 2 | 6 | 0.8 | 0.2982 |
| | | | | | | | | 2 | 6 | 0.85 | 0.2957 |
| | | | | | | | | 2 | 6 | 0.9 | **0.2932** |

TABLE 4
AVERAGE TOTAL COST FOR W1=0.25W

| W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.5 | 3 | 0.5 | 0.3165 | 2.5 | 3.5 | 0.5 | 0.3161 | 2.5 | 4 | 0.5 | 0.3156 |
| 2.5 | 3 | 0.55 | 0.3140 | 2.5 | 3.5 | 0.55 | 0.3135 | 2.5 | 4 | 0.55 | 0.3131 |
| 2.5 | 3 | 0.6 | 0.3114 | 2.5 | 3.5 | 0.6 | 0.3110 | 2.5 | 4 | 0.6 | 0.3106 |
| 2.5 | 3 | 0.65 | 0.3089 | 2.5 | 3.5 | 0.65 | 0.3085 | 2.5 | 4 | 0.65 | 0.3080 |
| 2.5 | 3 | 0.7 | 0.3063 | 2.5 | 3.5 | 0.7 | 0.3059 | 2.5 | 4 | 0.7 | 0.3055 |
| 2.5 | 3 | 0.75 | 0.3038 | 2.5 | 3.5 | 0.75 | 0.3034 | 2.5 | 4 | 0.75 | 0.3030 |
| 2.5 | 3 | 0.8 | 0.3013 | 2.5 | 3.5 | 0.8 | 0.3009 | 2.5 | 4 | 0.8 | 0.3004 |
| 2.5 | 3 | 0.85 | 0.2987 | 2.5 | 3.5 | 0.85 | 0.2983 | 2.5 | 4 | 0.85 | 0.2979 |
| 2.5 | 3 | 0.9 | 0.2962 | 2.5 | 3.5 | 0.9 | 0.2958 | 2.5 | 4 | 0.9 | 0.2954 |
| 2.5 | 4.5 | 0.5 | 0.3152 | 2.5 | 5 | 0.5 | 0.3148 | 2.5 | 5.5 | 0.5 | 0.3144 |
| 2.5 | 4.5 | 0.55 | 0.3127 | 2.5 | 5 | 0.55 | 0.3123 | 2.5 | 5.5 | 0.55 | 0.3119 |
| 2.5 | 4.5 | 0.6 | 0.3102 | 2.5 | 5 | 0.6 | 0.3098 | 2.5 | 5.5 | 0.6 | 0.3094 |
| 2.5 | 4.5 | 0.65 | 0.3076 | 2.5 | 5 | 0.65 | 0.3072 | 2.5 | 5.5 | 0.65 | 0.3068 |
| 2.5 | 4.5 | 0.7 | 0.3051 | 2.5 | 5 | 0.7 | 0.3047 | 2.5 | 5.5 | 0.7 | 0.3043 |
| 2.5 | 4.5 | 0.75 | 0.3026 | 2.5 | 5 | 0.75 | 0.3022 | 2.5 | 5.5 | 0.75 | 0.3018 |
| 2.5 | 4.5 | 0.8 | 0.3000 | 2.5 | 5 | 0.8 | 0.2996 | 2.5 | 5.5 | 0.8 | 0.2992 |
| 2.5 | 4.5 | 0.85 | 0.2975 | 2.5 | 5 | 0.85 | 0.2971 | 2.5 | 5.5 | 0.85 | 0.2997 |
| 2.5 | 4.5 | 0.9 | 0.2950 | 2.5 | 5 | 0.9 | 0.2946 | 2.5 | 5.5 | 0.9 | 0.2942 |
| | | | | | | | | W1 | W2 | Delta | Average (TC) |
| | | | | | | | | 2.5 | 6 | 0.5 | 0.3140 |
| | | | | | | | | 2.5 | 6 | 0.55 | 0.3115 |
| | | | | | | | | 2.5 | 6 | 0.6 | 0.3090 |
| | | | | | | | | 2.5 | 6 | 0.65 | 0.3064 |
| | | | | | | | | 2.5 | 6 | 0.7 | 0.3039 |
| | | | | | | | | 2.5 | 6 | 0.75 | 0.3014 |
| | | | | | | | | 2.5 | 6 | 0.85 | 0.2963 |
| | | | | | | | | 2.5 | 6 | 0.9 | 38 |

TABLE 5
AVERAGE TOTAL COST FOR W1=0.3W

| W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) | W1 | W2 | Delta | Average (TC) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 0.5 | 0.3171 | 3 | 3.5 | 0.5 | 0.3167 | 3 | 4 | 0.5 | 0.3162 |
| 3 | 3 | 0.55 | 0.3146 | 3 | 3.5 | 0.55 | 0.3141 | 3 | 4 | 0.55 | 0.3137 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 3 | 0.6 | 0.3120 | 3 | 3.5 | 0.6 | 0.3116 | 3 | 4 | 0.6 | 0.3112 |
| 3 | 3 | 0.65 | 0.3095 | 3 | 3.5 | 0.65 | 0.3091 | 3 | 4 | 0.65 | 0.3086 |
| 3 | 3 | 0.7 | 0.3069 | 3 | 3.5 | 0.7 | 0.3065 | 3 | 4 | 0.7 | 0.3061 |
| 3 | 3 | 0.75 | 0.3044 | 3 | 3.5 | 0.75 | 0.3040 | 3 | 4 | 0.75 | 0.3036 |
| 3 | 3 | 0.8 | 0.3019 | 3 | 3.5 | 0.8 | 0.3015 | 3 | 4 | 0.8 | 0.3010 |
| 3 | 3 | 0.85 | 0.2993 | 3 | 3.5 | 0.85 | 0.2989 | 3 | 4 | 0.85 | 0.2985 |
| 3 | 3 | 0.9 | 0.2968 | 3 | 3.5 | 0.9 | 0.2964 | 3 | 4 | 0.9 | 0.2960 |
| 3 | 4.5 | 0.5 | 0.3158 | 3 | 5 | 0.5 | 0.3154 | 3 | 5.5 | 0.5 | 0.3150 |
| 3 | 4.5 | 0.55 | 0.3133 | 3 | 5 | 0.55 | 0.3129 | 3 | 5.5 | 0.55 | 0.3125 |
| 3 | 4.5 | 0.6 | 0.3108 | 3 | 5 | 0.6 | 0.3104 | 3 | 5.5 | 0.6 | 0.3100 |
| 3 | 4.5 | 0.65 | 0.3082 | 3 | 5 | 0.65 | 0.3078 | 3 | 5.5 | 0.65 | 0.3074 |
| 3 | 4.5 | 0.7 | 0.3057 | 3 | 5 | 0.7 | 0.3053 | 3 | 5.5 | 0.7 | 0.3049 |
| 3 | 4.5 | 0.75 | 0.3032 | 3 | 5 | 0.75 | 0.3028 | 3 | 5.5 | 0.75 | 0.3024 |
| 3 | 4.5 | 0.8 | 0.3006 | 3 | 5 | 0.8 | 0.3002 | 3 | 5.5 | 0.8 | 0.2998 |
| 3 | 4.5 | 0.85 | 0.2981 | 3 | 5 | 0.85 | 0.2977 | 3 | 5.5 | 0.85 | 0.2973 |
| 3 | 4.5 | 0.9 | 0.2958 | 3 | 5 | 0.9 | 0.2954 | 3 | 5.5 | 0.9 | 0.2948 |
| | | | | | | | | W1 | W2 | Delta | Average (TC) |
| | | | | | | | | 3 | 6 | 0.5 | 0.3146 |
| | | | | | | | | 3 | 6 | 0.55 | 0.3121 |
| | | | | | | | | 3 | 6 | 0.6 | 0.3096 |
| | | | | | | | | 3 | 6 | 0.65 | 0.3070 |
| | | | | | | | | 3 | 6 | 0.7 | 0.3045 |
| | | | | | | | | 3 | 6 | 0.75 | 0.3020 |
| | | | | | | | | 3 | 6 | 0.8 | 0.2994 |
| | | | | | | | | 3 | 6 | 0.85 | 0.2969 |
| | | | | | | | | 3 | 6 | 0.9 | 0.2944 |

The average lifetime of this product is equal to 10 units of time. As can be seen in the tables above, the minimum average total cost (0.2932) is obtained when the length of the first period is at its minimum amount (since all the repair cost during this period is totally covered by the manufacturer), the length of the second period is at its maximum amount (0.6W), and the amount of rejuvenation is at its minimum (i.e.,$\delta$=0.9). Changing the cost coefficients or the probability of doing a general repair will change the length of these periods and the average total cost during the warranty period. In the second numerical example, all cost coefficients and other parameters are kept the same as the first example and only the failure rate function is changed. The new failure rate function is

$$\boldsymbol{h(t) = 0.001t \qquad , t \geq 0} \tag{14}$$

Considering this failure rate density function, the average lifetime of this product is 39.633 units of time. By making adjustments in the program and running it, the following results are obtained:

$$\boldsymbol{w_1 = 0.3W = 11.88998}$$

$$\boldsymbol{w_2 = 0.6W = 23.77996}$$

$$\boldsymbol{w_3 = 0.1W = 3.96333}$$

It is not unexpected that the rejuvenation is at its minimum (i.e., $\delta$=0.9). Since increasing the rejuvenation (decreasing $\delta$) would increase the repair cost, the model always tries to minimize the rejuvenation. It is possible to define a value for $\delta$, before solving the problem.

## CONCLUDING REMARKS

In this research a short review on different types of warranty has been presented. Since there was not any research about three-stage non-renewing warranty in the literature, this type of warranty policy has been studied. To assess the repair cost during warranty period in this warranty policy, the cost function of repair has been derived for each period from the manufacturer point of view. To minimize the average total cost during the warranty period, the optimal length of each period and rejuvenation factor have been obtained. Two numerical examples have been studied to demonstrate the application and performance of the proposed method. Considering that the goal of the model is to achieve the minimum repair cost, thus the model tries to reduce the amount of rejuvenation in general repairs (in the last period). Therefore, it is better to define the amount of rejuvenation during general repairs before solving the model. As can be seen in these examples, because in the second period, the repairs are minimal (therefore not much) and some percentage of the repair cost is also paid by the buyer, the model tries to maximize the length of this period. Simplicity and practicality of the proposed model to be utilized by the manufacturers are its main advantages. In addition, it helps the manufacturers to estimate repair costs during the warranty period by defining cost coefficients and other parameters.

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