Original Research

Performance assessment of a complex repairable system with k-out-of-n: G operational scheme and copula repair approach

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Abstract

The present study is a complex system with three subsystems connected in a series configuration. The first subsystem has n identical units and is ascribed to be operable by a k-out-of-n: G scheme, and the second subsystem, which has three indistinguishable parts, is capable of being operational if at least two of them are operable. The third subsystem has an independent unit and all subsystems are connected in a series in the sequence of subsystem 1, subsystem 2, and subsystem 3. The units of subsystem 1 and subsystem 2 are controlled by the auto changeover switches and the switch failure led to the complete failed state. The units' failure rates of subsystems are constant and follow an exponential distribution, however, there are two types of repair facilities: general and copula repair. Minor repair dealt with the general repair but complete failure needs copula repair. Traditional reliability measures have been studied for different values of failure and repair through supplementary variable and copula approaches.

Keywords – Availability; Cost Analysis; Gumbel- Hougaard family copula; MTTF, Reliability.

INTRODUCTION:

Every research project has a research methodology, research technique, and research output through analysis and for future implementations. The researchers have provided innovative research with their specific alternatives for improving the system performance, however, further work is needed to increase the system's performance. Transient availability modeling is an important process used to evaluate and improve the effectiveness of any industrial system where most units are set up as repairable. The involvedness of modern industrial systems, besides the need for genuine consideration, when modeling their availability purifies analytical methods. Redundancy is a collective and essential technique to improve the system's performance, availability, and reliability. Numerous applications of redundancy have been observed in systems like aerospace, nuclear plants, automotive, streetlight system automatics parking systems, and many more. Further, the standby redundancy is typically categorized as active redundancy, worm and cold redundancy among the active redundancy is always ready to function the task whenever the primary unit got damaged. A system configuration k-out-of-n: G is a particular type of redundancy in which k units out of n need to be active all the time to be a system in operative. Furthermore, failure is a natural phenomenon in system reliability analysis, and the nature of the failure and repair rate may be constant and variable also. If the system failure and repair rates both are constant and follow exponential distribution then Champmen Kolmogorov equation methodology is implemented to assess the system performance but if the system failure rates are constant and the repair rates are variable then the system performance is studied using the Markova method supplementary variable approach. A wide range of literature has been studied using Markova methods and Laplace transform implications. Researchers in the early 1990s assessed the performance of a repairable system with the idea of a single repair, which was not appropriate when the system be in completely broken down. Whenever the system is in complete shutdown mode it stops production which leads to a huge loss of manufacturers and the organization also may lose its market reputation hence it is necessary to restore the failed system as soon as possible. In many realistic situations, the complete damage state needs to be restored as soon as be possible, whenever such type of situation observed the system state must be repaired by employing copula,

Nelson, R. B. (2006). To cite a few works of literature with k-out-of-n: G/F operational scheme Singh & Rawal, (2011), Singh et al (2012, 2013), Monika et al. (2018, 2020), Poonia et al. (2020, 2021), and others studied the system performance under different types of failure and multi repair strategies. Singh and Ram (2014) investigated a three-state system with two subsystems in series under distinctive types of failure and two types of restoration. G. Gokhan et al. (2016) developed a new technique for estimating the reliability of consecutive k-out-of-n: F systems using a logical approach. Ibrahim Yusuf et al. (2018) investigated the operational reliability metrics of linear sequential 2-out-of-4 systems coupled to a 2-out-of-4 supporting device. Singh et al. (2020) analyzed a complex system in a degraded state by the use of Laplace transform via supplementary variables, and the traditional reliability measures were computed for different types of failures and copula repair approaches. A computer-based test (CBT) model system was studied by Singh et al. (2020), via a copula linguistics repair approach thru computing performance for different values of system parameters. Praveen, P. K, (2021), examined the performance of multistate computer network systems in series configuration employing copula repair.

Dhruv Raghav et al. (2021) examined reliability measures of the complex system in combinations of two subsystems in a series configuration and a copula repair scheme. Singh et al. (2022) have analyzed a complex system with n degraded states with different types of failure and copula repair. Niaki and Yaghoubi (2021) used an exact technique and a closed-form to forecast the reliability and mean time to failure (MTTF) of a 1-out-of-n: G, cold standby system with imperfect switching. Recently Abbas Bin Jibril et al. (2022), and Singh et al. (2022) studied the performance of a complex system in combination with subsystems under the k-out-of-n: G scheme and copula repair approach. It has been shown analytically that when copula repair was employed the system performance was found better than general repair.

- *Description of the model*

There is widespread literature on system performance evaluations thru the traditional measures for repairable systems, in which most of the five units are taken into the studies. Treating the above discussion in view, in this paper, the authors have examined the performance of a complex system having three subsystems in a series arrangement. The first subsystem consists of n units and employs the k-out-of-n: G policy. The second subsystem comprises three indistinguishable units and operates on a 2-out-of-3: G scheme, whereas the third subsystem only has one unit. A switching device controls the first two subsystem units, and a switching failure is viewed as a complete failure. There will be four different types of states in the system: perfect, minor degraded, major degraded, and entirely failed. To restore failed states, two distinct types of repairs are used: general repair together with copula repair. This paper is divided up into six sections for organization purposes.

In the first section of the paper, referenced "Introduction," the related research work and background knowledge on the k-out-of-n: G/F systems, as well as the state transition assumptions and notations used for mathematical formulation, are reviewed. Section two is mathematical modeling and solution of the system by use of supplementary variable approach. Third section of this paper is analytical part in which availability and reliability measures are deliberated. Fourth section is cost analysis and the fifth section is MTTF assessment corresponding to failure rates of the subsystem. The final section six encompasses a conclusion, results discussion and behavior of system for future studies.

- *State Transition Diagram of Model:*

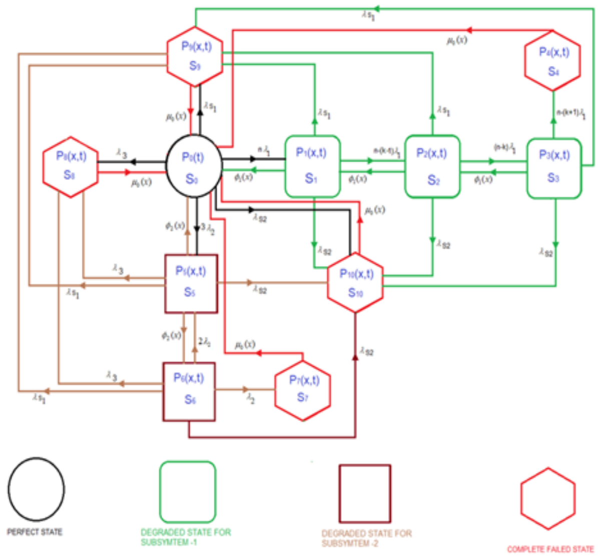


FIGURE 1
STATE CONVERSION DIAGRAM OF THE MODE.

By the probability arguments, the following states are possible as; {S_0, S_1, S_2, S_3, ..., S_{10}, presented in the state transition diagram, can be categorized as the set of states A, B, C, D, denoted as;
A= Set of perfect states = "S_0",
B= Set of minor degraded states = {S_1, S_2, S_5},
C= Set of major degraded states= {S_3, S_6},
D= complete failed states= {S_4, S_7, S_8, S_9, S_{10}}.

State Description in detail

State	State description
S_0	The state S_0 is a perfect state in which the system is in operational mode with all subsystems and switching devices in a perfectly working state with probability $P_0(0)$ =1.
S_1	The state S_0 is a minor degraded state with good working efficiency due to the working policy imposed on it.
S_2	In-state S_2 the system is also a minor degraded state after failure of (k-1) unit of subsystem 1, as work policy is k-out-of-n: G have imposed on it. The state is under repair via general repair.
S_3	This state is a major degraded state of system operation due to the failure of the k unit of subsystem 1. General repair is employed to repair the failed unit.
S_4	Due to the failure of the (k+1) unit in subsystem 1, it is in a completely failed state. The copula repair is used to fix the problem.
S_5	Under the operational scheme implemented as 2-out-of-3: G, this signifies a minor degraded state due to the failure of one unit in subsystem 2.
S_6	It is a major degraded state due to the failure of two units of subsystem 2 under the work policy for the subsystem.
S_7	Due to the complete failure of subsystem 2, state S_7 is a complete failure state. The system is being repaired, and the Gumbel-Hougaard family copula repair policy is being used to do it.
S_8	Due to the complete failure of subsystem 3, state S8 is a failed state.
S_9& S_{10}	Due to switch failure in subsystem 1 and subsystem 2, the states S_9 and S_{10} completely failed.

- *Assumptions used for the study of the model:*

The following under-mentioned assumption is taken during the study of the model.
1. The system is firstly in the state S0, in which all subsystems and switching devices are in pristine condition, with probability $P_0(0)=1$.
2. The partial failure degrades the proficiency of the system but did not stop functioning until it goes beyond working policy k-out-of-n: G
3. A complete failure state stops the system's operation.
4. General distribution is used to repair partially failed states, but copula distribution is used to repair complete failure.
5. All failure rate is constant and follows an exponential distribution.
6. At least k units are required to be functioning to be the subsystem 1 operative.
7. Repaired system units work like new and repairing does not damage anything.

TABLE 1
NOMENCLATURE OF SYSTEM VARIABLES AND NOTATIONS

t	On the time scale, there is a time variable.
s	Laplace transform variable.
$\lambda_1/\lambda_2/\lambda_3/\lambda_{S1}/\lambda_{S2}$	Failure rates for subsystem 1/2/3/ switch subsystem1/ switch subsystem 2
$\varphi_1(x)$/ $\varphi_2(x)$/	Repair rates of units of subsystem1/ subsystem 2.
$\mu_0(x)$:	Repair rate for all complete failed states of the system, i. e., $S_4, S_7, S_8, S_9, S_{10}$.
$P_0(t)$:	The probability that the system is in the perfect state is S_0.
$\bar{P}(s)$:	It is a notation of Laplace transformation of state transition P (t).
$P_i(x,t)$:	For i=1 to 10, the probability that a system is in state Si; the system is under repair and the elapsed repair time is x, t.
$E_p(t)$:	This represents the predictable profit thru the interval [0, t).
K_1/K_2:	In the time interval [0, t) respectively, K_1/K_2 revenue/service cost per unit time.
$\mu_0(x)$:	According to the Gumbel-Hougaard family copula, joint probability function (from failed state S j to good state S_0) is defined as; for $1\le\theta\le\infty, C_\theta(u_1(x), u_2(x)) = \exp[x^\theta + \{\log\varphi(x)\}^\theta]^{1/\theta}$, $u_1=\phi(x)$, and $u_2=e^{-\lambda x}$, θ is a parameter.

MATHEMATICAL MODELLING OF THE SYSTEM

If the system is in the state S_0 at any time t and will remain in that state in time [t, t + Δt], it must not move to any other state, and if it is in another failed state, it must return to S_0 after repair. With the current mathematical model in state transition figure 1, the following state equations can be derived via probability constraints: figure 1:

$$S_0:\left(\frac{\partial}{\partial t}+n\lambda_1+\lambda_{S_1}+\lambda_{S2}+3\lambda_2+\lambda_3\right)P_0(t)=\int_0^\infty\varphi_1(x)P_1(x,t)dx+\int_0^\infty\varphi_2(x)P_5(x,t)dx++\int_0^\infty\mu_0(x)P_7(x,t)dx+\int_0^\infty\mu_0(x)P_8(x,t)dx+\int_0^\infty\mu_0(x)P_9(x,t)dx+\int_0^\infty\mu_0(x)P_{10}(x,t)dx \quad (1)$$

$$S_1:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\left(n-(k-1)\right)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)P_1(x,t)=0 \quad (2)$$

$$S_2:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)P_2(x,t)=0 \quad (3)$$

$$S_3:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+(n-(k+1))\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)P_3(x,t)=0 \quad (4)$$

$$S_4:\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_7(x,t)=0 \quad (5)$$

$S_5: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\varphi_2(x)\right) \quad P_5(x,t)=0$ (6)

$$S_6: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\varphi_2(x)\right)P_6(x,t)=0 \quad (7)$$

$S_7: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_7(x,t)=0$ (8)

$S_8: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_8(x,t)=0$ (9)

$S_9: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_9(x,t)=0$ (10)

$S_{10}: \left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right)P_{10}(x,t)=0$ (11)

Equations for Boundary conditions:

$P_1(0,t)=n\lambda_1P_0(t), P_2(0,t)=n(n+1-k)\lambda_1^2P_0(t), P_3(0,t)=n(n+1-k)(n-k)\lambda_1^3P_0(t),$ $P_4(0,t)=n(n+1-k)(n-k)(n-1-k)\lambda_1^4P_0(t), P_5(0,t)=3\lambda_2P_0(t),$ $P_6(0,t)=6\lambda_2^2P_0(t), P_7(0,t)=6\lambda_2^3P_0(t),$ $P_8(0,t)=\lambda_3(1+3\lambda_2+6\lambda_2^2)P_0(t),$ $P_9(0,t)=\lambda_{S_1}[(1+n\lambda_1+n(n+1-k)\lambda_1^2+n(n+1-k)(n-k)\lambda_1^3+(3\lambda_2+6\lambda_2^2)]P_0(t), P_{10}(0,t)=\lambda_{S_2}[(1+n\lambda_1+n(n+1-k)\lambda_1^2+n(n+1-k)(n-k)\lambda_1^3+(3\lambda_2+6\lambda_2^2)]P_0(t)$ (12)

Initial condition: $P_0(0)=1$, $P_j(x,0=0$ for $j=1,2,\ldots.10)$

- *Solution of the Model*

Taking the Laplace transform of the equation (1) – (11) with the help of the Initial condition i.e., $P_0(0)=1$ and other state transition probability at $t=0$ are zero, and using Laplace transform as; $L[P_0(t)]=\bar{P}_0(s)$ and $L[\acute{P}_0(t)]=s\bar{P}_0(s)-P_0(0)$ one can obtain the equations as;

$(s+n\lambda_1+\lambda_{S_1}+\lambda_{S2}+3\lambda_2+\lambda_3)\bar{P}_0(s)\int_0^\infty\varphi_1(x)\bar{P}_1(x,s)dx+\int_0^\infty\varphi_2(x)\bar{P}_5(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_4(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_7(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_8(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_9(x,s)dx+\int_0^\infty\mu_0(x)\bar{P}_{10}(x,s)dx$ (13)

$$\left(s+\frac{\partial}{\partial x}+(n-k+1)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)\bar{P}_1(x,s)=0 \quad (14)$$

$$\left(s+\frac{\partial}{\partial x}+(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)\bar{P}_2(x,s)=0 \quad (15)$$

$$\left(s+\frac{\partial}{\partial x}+(n-k-1)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1(x)\right)\bar{P}_3(x,s)=0 \quad (16)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\bar{P}_4(x,s)=0 \quad (17)$$

$$\left(s+\frac{\partial}{\partial x}+2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\varphi_2(x)\right)\bar{P}_6(x,s) \quad (18)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\varphi_2(x)\right)\bar{P}_6(x,s) \quad (19)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\bar{P}_7(x,s)=0 \quad (20)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\bar{P}_8(x,s)=0 \quad (21)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{P}_9(x,s)=0 \quad (22)$$

$$\left(s+\frac{\partial}{\partial x}+\mu_0(x)\right)\overline{P}_{10}(x,s)=0 \quad (23)$$

Laplace transform of Boundary conditions:

$$\overline{P}_1(0,s)=n\lambda_1\overline{P}_0(s),\ \overline{P}_2(0,s)=n(n+1-k)\lambda_1^2\overline{P}_0(s),$$
$$\overline{P}_3(0,s)=n(n+1-k)(n-k)\lambda_1^3\overline{P}_0(s),$$
$$\overline{P}_4(0,s)=n(n+1-k)(n-k)(n-1-k)\lambda_1^4\overline{P}_0(s),$$
$$\overline{P}_5(0,s)=3\lambda_2\overline{P}_0(s),\ \overline{P}_6(0,s)=6\lambda_2^2\overline{P}_0(s),\overline{P}_7(0,s)=6\lambda_2^3\overline{P}_0(s),\overline{P}_8(0,s)=\lambda_3(1+3\lambda_2+6\lambda_2^2)\overline{P}_0(s)$$
$$\overline{P}_9(0,s)=\lambda_{S_1}[(1+n\lambda_1+n(n+1-k)\lambda_1^2+n(n+1-k)(n-k)\lambda_1^3+(3\lambda_2+6\lambda_2^2)]\overline{P}_0(s),$$
$$\overline{P}_{10}(0,s)=\lambda_{S_2}[(1+n\lambda_1+n(n+1-k)\lambda_1^2+\ n(n+1-k)(n-k)\lambda_1^3+(3\lambda_2+6\lambda_2^2)]\overline{P}_0(s) \quad (24)$$

Solving equation (11)- (24) with implication of equation (24) and notations.

$$S_\varphi(x)=\varphi(x)e^{-\int_0^\infty\varphi(x)}\ ,\bar{S}_\varphi(s)=\int_0^\infty e^{-sx}S_\varphi(x)dx$$

$$\overline{P}_0(s)=\frac{1}{D(s)} \quad (25)$$

$$\overline{P}_1(s)=\frac{n\lambda_1}{D(s)}\left[\frac{1-S_{\varphi_1}(s+(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}{(s+(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}\right] \quad (26)$$

$$\overline{P}_2(s)=\frac{n(n+1-k)\lambda_1^2}{D(s)}\left[\frac{1-S_{\varphi_1}(s+(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}{(s+(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}\right] \quad (27)$$

$$\overline{P}_3(s)=\frac{n(n+1-k)(n-k)\lambda_1^3}{D(s)}\left[\frac{1-S_{\varphi_1}(s+(n-1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}{(s+(n-1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})}\right] \quad (28)$$

$$\bar{P}_4(s)=\frac{n(n+1-k)(n-k)(n-1-k)}{D(s)}\left(\frac{1-\bar{S}_{\mu_0}(s)}{s}\right) \quad (29)$$

$$\bar{P}_5(s)=\frac{3\lambda_2}{D(s)}\left(\frac{1-\bar{S}_{\varphi_1}(s+2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2})}{(s+2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2})}\right) \quad (30)$$

$$\bar{P}_6(s)=\frac{6\lambda_2^2}{D(s)}\left(\frac{1-\bar{S}_{\varphi_1}(s+\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2})}{(s+\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2})}\right) \quad (31)$$

$$\bar{P}_7(s)=\frac{6\lambda_2^3}{D(s)}\left(\frac{1-\bar{S}_{\mu_0}(s)}{s}\right) \quad (32)$$

$$\bar{P}_8(s)=\frac{(1+3\lambda_2+6\lambda_2^2)}{D(s)}\left(\frac{1-\bar{S}_{\mu_0}(s)}{s}\right) \quad (33)$$

$$\bar{P}_9(s)=\frac{\lambda_{S_1}A}{D(s)}\left(\frac{1-\bar{S}_{\mu_0}(s)}{s}\right) \quad (34)$$

$$\bar{P}_{10}(s)=\frac{\lambda_{S_2}A}{D(s)}\left(\frac{1-\bar{S}_{\mu_0}(s)}{s}\right) \quad (35)$$

here,

$$A=[1+n\lambda_1+n(n+1-k)\lambda_1^2+n(n+1-k)(n-k)\lambda_1^3+(3\lambda_2+6\lambda_2^2)]$$

The sum of Laplace transforms the state transition probabilities where the system is in an operational state i.e. S_1,S_2,S_3,S_4, $S_4,S_5,\ S_6$

$\bar{P}_{up}(s)=\sum\bar{P}_i(s),\ \mathrm{i}=0,\ 1,2,3,4,5,6$

$$\overline{P}_{up}(s) = \frac{1}{D(s)}\begin{bmatrix} 1 + \frac{n\lambda_1}{s+(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1} + \\ \frac{n(n+1-k)\lambda_1^2}{s+(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1} + \\ \frac{n(n+1-k)(n-k)\lambda_1^3}{s+(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2}+\varphi_1} + \\ \frac{3\lambda_2}{s+2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2}+\varphi_2} \end{bmatrix} \quad (36)$$

ANALYTIC STUDY OF THE MODEL

- *Availability analysis for copula repair:*

When the repairs follow a copula distribution.

Setting: $\overline{S}_{\mu_0}(s) = \frac{exp[x^{\theta}+\{log\,\varphi(x)\}^{\theta}]^{1/\theta}}{s+exp[x^{\theta}+\{log\,\varphi(x)\}^{\theta}]^{1/\theta}}$, $\overline{S}_{\phi_i}(s) = \frac{\phi_i}{s+\phi_i}$, i= 1, 2 and using the following values for failure and repair rates; $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_{s_1} = 0.03,\ \ \lambda_{s_2} = 0.025,\ \varphi_1 = 1,\ \varphi_2 = 1,\ \mu_0 = 2.7183$ in equation (36) one can obtain the different expressions of system performance of repairable system by using invers Laplace transform.

The system performance via Availability analysis for copula repair for different configurations for subsystem 1.

A= Case 1. (For n=50, k=20)

Availability $=0.03654668e^{-2.83376991t} - 0.0794794e^{-1.765916379t} + 0.002167158x10^{-3}\ e^{-1.145599662t} + 1.0413024e^{-0.108013972t)} - 0.195467243x10^{-4}\ e^{-1.345000000t} - 0.00435345e^{-1.105000000t} - 0.0009880893828e^{-1.355000000t}$ (37a)

B=Case 2. (For n=50, k=30)

Availability$= -0.35742045x10^{-3}e^{-1.1050000t} + 0.03711598e^{-2.83250103t} - 0.06770939481e^{-1.698305734t} + 0.9089538204x10^{-3}e^{-1.138137934t} + 1.030143323e^{-0.08435523964t} - 0.1980816780x10^{-5}e^{-1.245000000t} - 0.994202898x10^{-4}\ e^{-1.255000000t}$ (37b)

Case 3. (For n=50, k=40)

Availability$=-0.201289705x10^{-3}e^{-1.1050000t} - 0.22091558x10^{-5}\ e^{-1.1450000t} + 0.03759025170e^{-2.831402377t} - 0.053774778e^{-1.634510832t} + 0.1227352268x10^{-3}e^{-1.129123034t} + 1.016368080e^{-0.058263750t} - 0.102787745510^{-3}e^{-1.1550000t}$ (37c)

We get various values of availability as presented in Table 2a & Figure 2a for different values of time-variable from the expressions 37a, 37b, and 37c respectively

TABLE 2A
COPULA REPAIR CONCERNING TIME

Time	Availability n=50, k=20	Availability n=50, k=30	Availability n=50, k=40
0	1.00	1.000	1.000
1	0.92	0.94	0.95
2	0.84	0.87	0.90
3	0.75	0.80	0.85
4	0.68	0.74	0.81
5	0.61	0.68	0.76
6	0.55	0.62	0.72
7	0.49	0.57	0.68
8	0.44	0.53	0.64
9	0.39	0.48	0.60
10	0.35	0.44	0.57

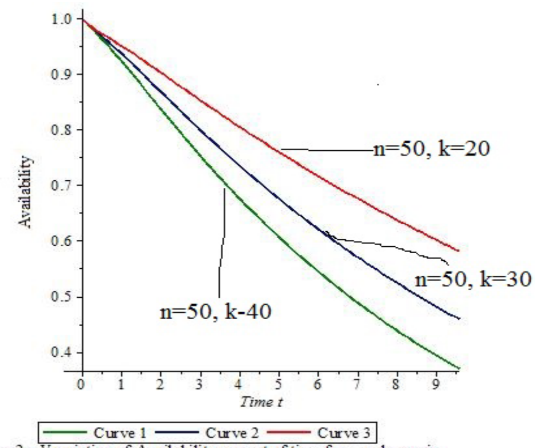


Figure 2a. Vaqriation of Availability respect of time for copula repair

- *Availability for the general repair*

When all repair follows general distribution keeping all failure rates as the same for copula repair one can obtain the following expressions presented in (38a, 38b& 38c);

Case 1. (For n=50, k=20) configuration

Availability= $-0.038102549e^{-1.849791754t}+ 0.92499186x10^{-2}e^{-1.1507647t}+0.03366445e^{-1.030607578t}+0.9958058773e^{-0.1038358801t} - 0.5161828687x10^{-3}e^{-1.105000000t} -0.19838109x10^{-5}e^{-1.345000t} -0.995378x10^{-4}e^{-1.355000t}$ (38a)

Case 2. (For n=50, k=30) configuration

Availability= $-0.410155582x10^{-3}e^{-1.105000000t} -0.100308133x10^{-3}e^{-1.255000000t} -0.01902305767e^{-1.788410401t} +0.3465143945x10^{-2}e^{1.140183652t}+0.02714107622^{-1.025127447t}+0.9889293185e^{-0.08127850010t} -0.2017299039x10^{-5}e^{-1.245000000t}$ (38b)

Case 3. (For n=50, k=40) configuration

Availability=$-0.217002676x10^{-3}e^{-1.105000000t} -0.104138308x10^{-3}e^{--1.155000000t} +0.212089482x10-^{2}e^{-1.731617561t}+0.33449849x10^{-3}e^{-1.129316329t}+0.01841740590e^{-1.017792478t}+ 0.9794506176e^{(-0.05627363150t)} -0.227595499x10^{-5}e^{-1.145000000t}$ (38c)

TABLE 2B
AVAILABILITY VARIATION RESPECT OF TIME FOR GENERAL REPAIR

Time	Availability n=50, k=20	Availability n=50, k=30	Availability n=50, k=40
0	1.00	1.00	1.00
1	0.91	0.92	0.93
2	0.81	0.84	0.88
3	0.73	0.78	0.83
4	0.66	0.72	0.78
5	0.59	0.66	0.74
6	0.53	0.61	0.70
7	0.48	0.56	0.66
8	0.43	0.52	0.62
9	0.39	0.48	0.59
10	0.35	0.44	0.56

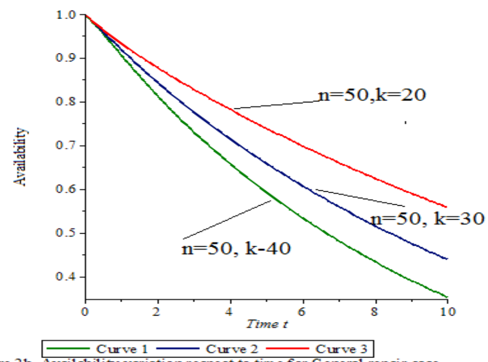


Figure 2b. Availability variation respect to time for General repair case

FIGURE 2B

AVAILABILITY VARIATION CORRESPONDS TO THE TIME FOR GENERAL REPAIR

- *Reliability*: When all repair is assumed to be zero then the system performance is proclaimed as a reliability metric. Taking all repairs in equation (36) to zero i.e., $\varphi_1 = \varphi_2 = 0\ and\ \mu_0(x) = 0$ and for some values of failure rates as; $\lambda_1 = 0.01,\ \lambda_2 = 0.02,\ \lambda_3 = 0.03,\ \lambda_{s_1} = 0.03,\ \lambda_{s_2} = 0.025$, we get distinct values of Reliability for different values of time t = 0,1,2,3,4,5,6,7,8,9,10 unit of time, as shown in table 2c and picture 2c.

Case 1. (For n=50, k=20)

$R_1(t)= 0.16981e^{-0.14500t} + 1.6129e^{-0.36500t} + 0.0096429e^{-0.11500t} - 0.80813e^{-0.67500t} + 0.015625e^{-0.35500t} + 0.00015152e^{-0.34500t}$ (39a)

Case 2. (n = 50, k = 30)

$R_2(t)=0.16981e^{-0.14500t} + 1.2195e^{-0.26500t} + 0.11628x10^{-3}e^{-0.24500)} + 0.96429\ 10^{-2}e^{-0.11500t} - 0.41099e^{-0.67500t} + 0.011905e^{-.25500t}$ (39b)

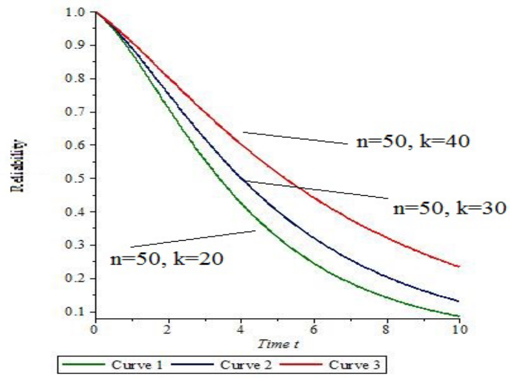
Case 3. (n = 50, k = 40)

$R_3(t)= 0.96154x10^{-2}\ e^{-0.15500t} + 0.16991e^{-0.14500t} + 0.9642x10^{-2}\ e^{-0.11500t} - 0.1695e^{-0.67500t} + 0.9803e^{-0.16500}$ (39c)

TABLE 2C

RELIABILITY TABLE FOR T= 0,1,2, 3…..10

Time t	$R_1(t)$ n=50, k=20	$R_2(t)$ =50, k=30	$R_3(t)$ n=50, k=40
0	1.00	1.00	1.00
1	0.87	0.89	0.91
2	0.71	0.75	0.80
3	0.56	0.62	0.70
4	0.43	0.50	0.60
5	0.32	0.40	0.52
6	0.24	0.32	0.44
7	0.19	0.26	0.38
8	0.14	0.20	0.32
9	0.11	0.16	0.27
10	0.08	0.13	0.23



- *Profit Analysis:*

If revenue generation K_1 and service cost K_2 are both per unit time in the interval [0, t], the expected profit E p (t) can be determined using the formula in equation (40). For the set of the parametric values of failure and repairs rates $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_{s_1} = 0.03,\ \lambda_{s_2} = 0.025,\ \varphi_1, = 1,\ \varphi_2 = 1,\ \mu_0 = 2.7183$ and assuming that the repair facility is always available then from equation (36), one can get the expression for an expected profit Ep (t) by operations of the system in interval $[0, t)$ as presented in the equation (41) from Maple Software output.

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \qquad (40)$$

- *Cost Analysis for copula repair*

Let's presume the repair follows two types of distributions: General and Gumbel Hougaard family copula distributions. Consequently, using fixed values of failure rates as in the availability analysis section, we get the following expression using equation (36) in the formula in the cost function Ep (t).

$$E_P(t) = K_1[0.18216\text{x}10^{-3}e^{-1.1050t} + 0.19294\text{x}10^{-5}e^{-1.1450t} - 0.013276e^{-2.8314t} + 0.032900e^{-1.6345t} - 0.10870\text{x}10^{-3}e^{-1.1291t} - 17.444\, e^{-0.058264t} + 0.88994\text{x}10^{-4}\, e^{-1.1550\,t} + 17.42 \quad ...(41)$$

Table 3a is obtained by setting K_1= 1 and K_2 = 0.6, 0.4, 0.2, and 0 and varying time t the profit variation concerning time t is depicted in Figure 3a.

TABLE 3A
EXPECTED PROFIT FOR COPULA REPAIR

Time t	Expected Profit K_1=1, and K_2			
	K_2=0.6	K_2=0.4	K_2=0.2	K_2=0
0	0	0	0	0
1	0.372	0.572	0.772	0.972
2	0.699	1.099	1.499	1.899
3	0.977	1.577	2.177	2.777
4	1.205	2.005	2.805	3.605
5	1.387	2.387	3.387	4.387
6	1.525	2.725	3.925	5.125
7	1.621	3.021	4.421	5.821
8	1.678	3.278	4.878	6.478
9	1.697	3.497	5.297	7.097
10	1.682	3.682	5.682	7.682

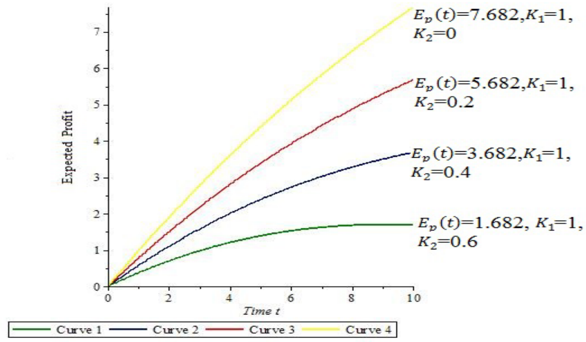


FIGURE 3A
EXPECTED PROFIT GRAPH FOR COPULA REPAIR

- *Cost Analysis for General repair*

Assuming the repair follows only general distribution than for fixed values of failure rates as are in availability analysis section via the use of equation (36) in the formula in cost function $E_p(t)$ we have following expression.

$E_P(t)= K_1[0.010637e^{-1.7884t} -0.0030391e^{-1.1402t} -0.026476e^{-1.0251t} -12.167e^{-0.081279t} +0.79927\times10^{-4}e^{-1.2550t} +0.16203\times10^{-5}e^{-1.2450t} + 0.37118\times10^{-3}e^{-1.1050t} +12.186] - K_2t$ (42)

Using different values of time t in equation (42) as; t = 0,1, 2, 3….10 one can obtain the values of expected profit presented in table 3b and the corresponding figure 3b

TABLE 3B
EXPECTED PROFIT IN [0, T), T = 0,1, 2, 3….10

Time t	Expected Profit K_1=1, and K_2			
	K_2=0.6	K_2=0.4	K_2=0.2	K_2=0
0	0	0	0	0
1	0.360	0.560	0.760	0.960
2	0.641	1.041	1.441	1.841
3	0.850	1.450	2.050	2.650
4	0.995	1.795	2.595	3.395
5	1.082	2.082	3.082	4.082
6	1.115	2.315	3.515	4.715
7	1.200	2.498	3.898	5.298
8	1.036	2.636	4.236	5.836
9	0.931	2.731	4.531	6.331
10	0.788	2.788	4.788	6.788

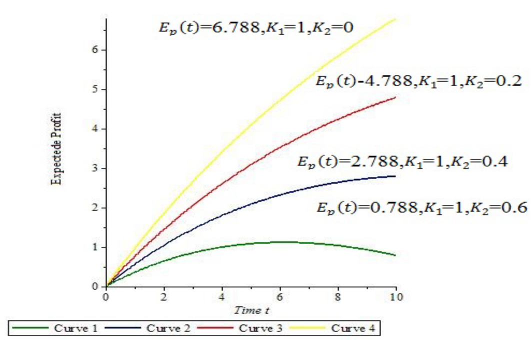


FIGURE 3B
EXPECTED PROFIT GRAPH FOR GENERAL REPAIR

- *Mean time to failure (MTTF) Analysis*

In system operation, the (MTTF) is a very crucial measure with predicts which subsystem or unit needs to be much more sensitive to take care of. It is the average time in which a system fails and depends on subsystem failure rates. Mathematically taking all repair rates i.e., ϕ_1, ϕ_2, and μ_0, to zero in equation (36) and taking the limit of s tend to zero one can get an expression of MTTF corresponding to failure rates. i.e., $F = \lim_{s\to 0} \bar{P}_{up}(s)$, with all repair zero.

$$MTTF = \frac{1}{K}\begin{bmatrix} 1 + \frac{n\lambda_1}{(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})} \\ + \frac{n(n+1-k)(n-k)\lambda_1^3}{(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})} + \\ \frac{3\lambda_2}{2\lambda_2+\lambda_3+\lambda_{S_1}+\lambda_{S_2})} + \\ \frac{6\lambda_2^2}{(n+1-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})} + \frac{n(n+1-k)\lambda_1^2}{(n-k)\lambda_1+\lambda_{S_1}+\lambda_{S_2})} \end{bmatrix} \quad (43)$$

Here, $K = n\lambda_1 + 3\lambda_2 + \lambda_3 + \lambda_{S_1} + \lambda_{S_2}$

We can find the values of the MTTF corresponding to the failure rate λ_1 by setting $\lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_{s_1} = 0.03,\ \ \lambda_{s_2} = 0.025$ and varying λ_1 from 0.01, with an increment of 0.01 in each next value up to reach 0.1, in equation (43) the variation of MTTF concerning failure λ_1 is perceived in column 2 of Table 4.

Setting, $\lambda_1 = 0.01, \lambda_3 = 0.03, \lambda_{s_1} = 0.03,\ \ \lambda_{s_2} = 0.025$ and varying λ_2 from 0.01, with an increment of 0.01 in each next value up to reach 0.1, in equation (43) one may obtain the column 3 of table 4 reveals fluctuations of MTTF concerning the failure rate λ_2.

Setting, $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_{s_1} = 0.03,\ \ \lambda_{s_2} = 0.025$ and varying λ_3 from 0.01, with an increment of 0.01 in each next value up to reach 0.1, in equation (43) one may obtain the values of the MTTF corresponding to the failure rate λ_3. The fluctuation of MTTF for the failure rate λ_3 is seen in column 4 of Table 4.

Setting, $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03,\ \ \lambda_{s_2} = 0.025$, and varying λ_{s_1} from 0.01, with increment of 0.01 in each next value up to reach 0.1, in equation (43) we can obtain the values of MTTF corresponding to the failure rate λ_{s_1}. Column 5 of Table 4 shows the variation of MTTF to the failure λ_{s_1}.

Setting, $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_2 = 0.03, \lambda_{s_1} = 0.03$, and varying $, \lambda_{s_2}$ from 0.01, with increment of 0.01 in each next value up to reach 0.1, in equation (43) one may obtain the values of the MTTF corresponding to the failure rate λ_{s_2}. Column 6 of Table 4 shows the variation of MTTF concerning the failureλ_{s_2}.

TABLE 4
THE VARIATION OF MTTF

Failure Rates	MTTF λ_1	MTTF λ_2	MTTF λ_3	MTTF λ_{s_1}	MTTF λ_{s_2}
0.01	7.079	7.081	7.460	8.137	7.844
0.02	4.558	7.079	7.260	7.571	7.317
0.03	3.362	7.008	7.079	7.079	6.856
0.04	2.670	6.904	6.913	6.646	6.449
0.05	2.220	6.782	6.761	6.263	6.087
0.06	1.905	6.653	6.619	5.920	5.763
0.07	1.672	6.523	6.486	5.613	5.471
0.08	1.493	6.394	6.362	5.336	5.207
0.09	1.351	6.268	6.244	5.085	4.967
0.1	1.236	6.147	6.132	4.855	4.748

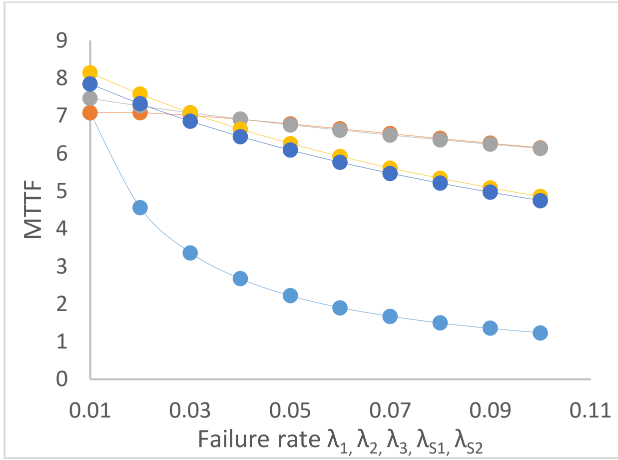


FIG 4
MTTF VARIATION CONCERNING FAILURE RATES

RESULT DISCUSSION AND CONCLUSIONS

When failure rates are fixed at different levels, Figure.4.1 demonstrates how the availability of the repairable system varies over time. For the specific presumed values of failure rates like; $\lambda_1 = 0.01, \lambda_2 = 0.02, \lambda_3 = 0.03, \lambda_{s_1} = 0.03, \quad \lambda_{s_2} = 0.025, \varphi_1, = 1, \varphi_2 = 1, \mu_0 = 2.7183$ the system's availability drops and eventually stabilizes at zero after a sufficient period. As a result, as evidenced by the graphical reflection of the model, one may reliably forecast the future behavior of the system at any time for any given set of parametric values. Availability of system for the configuration k-out-of-n: G for subsystem 1 has computed in tables 2a, 2b for the set of values n= 50 and k= 20, 30 & 40 for copula repair and general repair. It is observed from keeping the fixed value of n=50 and varying k as 20, 30 40 the availability decreases in both the cases of copula repair and general repair. It is also noticed that system performance is better when the repair follows two types of distribution.

The fluctuation in the reliability of a non-repairable system is shown in Figure 2c, and the system performance is relatively low when compared to a repairable system. When revenue cost per unit time K_1 is fixed at 1, service costs K=0.6, 0.4, 0.2, and 0 and t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are varied, revenue cost per unit time K_2=0.6, 0.4, 0.2, and 0 accordingly. Tables 3a and 3b show the predicted profit from the system's operation using the same set of system variables. It shows that the predicted profit grows over time. Finally, it's worth noting that when service expenses rise, profit falls. In general, when low service costs are compared to high service costs, the predicted profit is significant.

When the other parameters are kept constant, figure.4 depicts the system's mean-time-to-failure MTTF for variations in $\lambda_1, \lambda_2, \lambda_3, \lambda_{s_1},\ and\ \lambda_{s_2}$ respectively. The variation in MTTF corresponds to the failure rate corresponding to failure rateλ_1, the trend is decreasing, and after this value becomes constant for higher values of failure rate. The MTTF respects failure rate λ_2 &λ_3, and λ_{s_1}& λ_{s_2} very concomitantly.

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APPENDIX 1

Mathematical Modelling of the Model:

If the system at any time t is in the state S_0 and will remain in the state S_0 in time [t, t + Δ t] is that it must not move to any other state and if it is in other states after failing then it must approach to S_0 after repair. If the failure rate from S_0 to S_1 is $n\lambda_1 \Delta t$, then not going to S_1 state $(1 - n\lambda_1 \Delta t)$.

i.e., $P_0(t+\Delta t) = (1 - n\lambda_1 \Delta t)(1-\lambda_{s1}\Delta t)(1 - 3\lambda_2 \Delta t)$

$$\left(1 - \lambda_3 \Delta t\right)\left(1 - \lambda_{s_2} \Delta t\right) P_0(t) + \int_0^\infty \varphi_1(x)\, P_1(x,t)dx\, \Delta t + \int_0^\infty \varphi_2(x)\, P_5(x,t)dx\, \Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_8(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)\,dx\,\Delta$$

$P_0(t+\Delta t) = (1-(n\lambda_1+\lambda_{s_1}+\lambda_2+\lambda_3+\lambda_{s_2})\,P_0(t) +$ [(multiplication of two failure rates) (Δt) + (multiplication of three failure rates) $(\Delta t)^3$ + ... +........] $P_0(t) + \int_0^\infty \varphi_1(x)\, P_1(x,t)dx\, \Delta t + \int_0^\infty \varphi_2(x)\, P_5(x,t)dx\, \Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_8(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)\,dx\,\Delta t$

$\frac{P0(t+\Delta t) - P0(t)}{\Delta t} = -(n\lambda_1+\lambda_{s_1}+\lambda_2+\lambda_3+\lambda_{s_2})\,P_0(t) +$ [(multiplication of two failure rates) (Δt) + (multiplication of three failure rates) $(\Delta t)^3$ + . . .] $P_0(t) + \int_0^\infty \phi_1(x)P_1(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)dx\,\Delta t \int_o^\infty \mu_0(x)P_8(x,t)\,dx\,\Delta t + \int_0^\infty \varphi_2(x)P_5(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_{10}(x,t)\,dx\,\Delta t$

$\lim_{\Delta t\to 0} \frac{P0(t+\Delta t) - P0(t)}{\Delta t} = -(n\lambda_1+\lambda_{s_1}+\lambda_2+\lambda_3+\lambda_{s_1})\,P_0(t) +$ [(multiplication of two failure rates) $(\Delta t)^2$ + (multiplication of three failure rates) $(\Delta t)^3$ + ...] $P_0(t) + \int_0^\infty \varphi_1(x)\, P_1(x,t)dx\, \Delta t + \int_0^\infty \varphi_2(x)\, P_5(x,t)dx\, \Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_8(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)\,dx\,\Delta t$

$$\frac{\partial}{\partial t}Po(t) - \left(+n\lambda_1+\lambda_{s_1}+\lambda_{s_2}+3\lambda_2+\lambda_3\right)Po(t) = \int_0^\infty \varphi_1(x)\, P_1(x,t)dx\, \Delta t + \int_0^\infty \varphi_2(x)\, P_5(x,t)dx\, \Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_8(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)\,dx\,\Delta t$$

Finally, $\left(\frac{\partial}{\partial t} + n\lambda_1+\lambda_{s_1}+\lambda_{s_2}+3\lambda_2+\lambda_3\right)Po(t) = \int_0^\infty \varphi_1(x)\, P_1(x,t)dx\, \Delta t + \int_0^\infty \varphi_2(x)\, P_5(x,t)dx\, \Delta t + \int_o^\infty \mu_0(x)P_7(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_8(x,t)dx\,\Delta t + \int_o^\infty \mu_0(x)P_9(x,t)\,dx\,\Delta t$ (1)