Original Research

Proposed Space-Time Multivariate Capability Indices applied to Manufacturing Processes

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Abstract
This paper proposes two methods of evaluating the Space and Time Characteristics of the product within the Process Capability Analysis framework with the introduction of two Multivariate Capability Indices that incorporate the product lifetime estimation based on accelerated Time to Failure data. The measures proposed were applied to the manufacturing process of LED luminaires to provide a numerical example for the proposed methodology. Light-Emitting-Diode technology has been increasingly adopted by end-users over the past few decades because of its durability and efficacy, which result in low cost of ownership and higher energy savings. However, the research on LED technology reliability and expected lifetime has been limited due to the development of robust failure mechanism for this product. The results indicate that the integration of both technical specification compliance and product reliability into a global index can be beneficial for manufacturing process assessment since it provides a new insight into process capability. The product reliability dimension of the product supports decision-making and process optimization, which subsequently increases customer satisfaction.

Keywords – LED technology; Multivariate Capability Analysis; Process Capability; Product Reliability; Quality.

INTRODUCTION

LED-based technologies have become ubiquitous over the past couple of decades due to their efficacy and environment-friendly characteristics. Additionally, this technology is more durable than conventional products, such as incandescent or halogen lighting, which translates into a lower cost of ownership that, in turn, drives increasing adoption rates. The combination of these factors has led manufacturers to focus on the quality of LED solutions, revealing that despite the current advancements, there is room for further improvement in both the efficacy and functionality of LED products [1].

Since its conception, reliability has been a common feature of LED technology. According to Pattison, the average usable life of the top performing LED products is approximately 45,714 hours, compared to the 12,628 hours for conventional lighting solutions. However, product reliability became a consequential characteristic of the product due to the focus on failure mechanisms development. Therefore, methods of evaluation and lifetime prediction have not been neither standardized nor widely implemented for LED systems [2].

Many authors have proposed different methods to analyze this important factor of LED technologies. The methodologies range from data-driven solutions to accelerated degradation experiments that contribute to the estimation of lifetime length and comparison with manufacturer's warranty promises. In this paper, the reliability of LED luminaires was analyzed using accelerated Time to Failure (TTF) data with a model based on the Rayleigh distribution, which considers an increasing function of time to estimate the product reliability factor.

This estimation was integrated into the process capability analysis of the LED luminaire manufacturing process. The process performance was evaluated through three important quality characteristics of the luminaire manufacturing: length, diameter and brightness. This required the implementation of multivariate analysis techniques that considered the correlation between the variables and summarized the result in a global index. Therefore, two different proposals of Multivariate Capability Indices are introduced, based on traditional multivariate analysis methods, such as the Principal Component Analysis and Modified Tolerance Region ratio.

BACKGROUND

I. Multivariate Capability Analysis

Process Capability Analysis is a highly effective tool to evaluate process performance and its ability to meet specific criteria determined by its various stakeholders [3]. One of the most common expressions of the Process Capability Analysis is the Process Capability Index (PCI), which is a numeric value that indicates if the products comply with quality requirements presented by manufacturers and customers [4]. Univariate PCIs are utilized when processes are being evaluated in relation to only one quality characteristic and several authors have introduced their measures for normal processes such as C_p, C_{pk}, C_{pm} and C_{pmk}. On the other hand, Multivariate Capability Indices (MPCIs) are engaged when assessing process variation in a multivariate setting i.e., in cases when a process is dependent on multiple quality variables.

Since manufacturing processes are rarely focused on meeting quality standards in only one of the features of their products, MPCIs have been widely implemented to present a comprehensive summary of all relevant product characteristics. In the past, researchers and manufacturers have applied univariate PCIs to all quality variables in the process and then analyzed them separately in the final product, however this method becomes inadequate when one or two of the variables are correlated [5]. Consequently, MPCIs are not only a convenient compilation of all quality variables into one global numeric tool but also present the advantage of exploring quality characteristic correlation in the process. This is especially significant in products with quality variables that are correlated and share a causal relationship, where the performance of one or two variables directly affects the results of a third one.

As [6] explain, multivariate capability measures, such as the MPCIs, constitute a statistical indicator that quantify how adequately a multivariate process performs. This method analyses the variability of multiple product characteristic measures compared to the accepted quality standard of the same measures. However, not every Multivariate PCI follow the same strategy and the results and application vary depending on the needs of researchers or stakeholders.

There are several approaches to obtaining a MPCI for a particular process, some of them do not consider the correlation between product characteristics as they are a direct derivation of existing Univariate PCIs. On the other hand, MPCIs that do consider the correlation between product characteristics can be classified into three different categories: MPCIs based on principal component analysis, MPCIs based on the relation between Process Region and Specification Region and MPCIs based on the inverse function of the cumulative distribution function [6].

This paper is focused on two proposed Multivariate Process Capability Indices, each one with a different approach to multivariate analysis. The first index is based on the Principal Component Analysis technique, which consists of transforming a number of related variables into a set of linear functions that are, in contrast, uncorrelated and derived from the original measurements. The principal components linearly combine the original variables and the variation among them. The second proposed index belongs to the category of MPCIs that are based on the relation between the Process Region (PR) and Specification Region (SR). Within this category, four distinct cases can be identified according to their approach to the PR and SR comparison: original PR and original SR ratio, original PR and modified SR ratio, modified PR and original SR ratio and modified PR and modified SR ratio.

In the univariate case, the PR is defined as the interval that encompasses 99.73% of values in a normal distribution centered on the mean value of the measured product characteristics, which can be represented at its lowest point by $\mu - 3\sigma$ and at its highest point by $\mu + 3\sigma$. The SR, on the other hand, is the interval between the lower and upper specification limits (LSL and USL, respectively). A traditional univariate PCI, such as the C_p index, compares the width of the SR (USL – LSL) and the width of the PR (6σ) with the formula $C_p = (USL - LSL)/6\sigma$.

In multivariate settings, the PR is defined by surface that includes 99.73% of the values drawn from a normal distribution and which is centered on the mean vector of the data sample [6]. This PR surface has a v-dimensional shape, where v is the number of product characteristics taken into consideration. The process region can be mathematically represented by $(X -$

$\mu)' \Sigma^{\wedge}(-1)\ (X - \mu) = c^2$, where μ and Σ are the mean vector and variance-covariance matrix and c^2 is defined by the number of product characteristics. It is generally accepted that this PR equation follows a χ_v^2 distribution with v degrees of freedom. Figure 1 is an example of the PR defined for a $v = 2$ multivariate process.

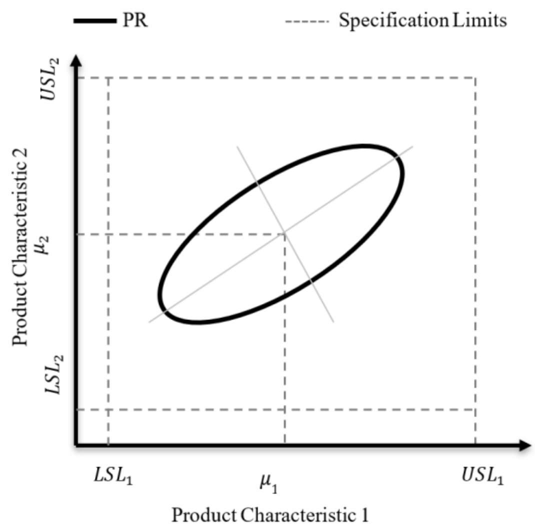

FIGURE 1
MULTIVARIATE PR DEFINITION

It is important to note that the definition of the SR and PR need to be extended and adapted to multivariate processes in order to correctly assess process capability based on these parameters. The Multivariate Capability Indices proposed in this paper analyse the ratio of these regions with two different approaches to develop global indices that allow the integration of space and time product characteristics.

II. LED Luminaires and Product Reliability

Light-emitting-diodes (LEDs) have been increasingly adopted over the last decade as a more efficient lighting solution than traditional incandescent or fluorescent lighting. This shift has progressed rapidly in the lighting market due to the technological advances in LED efficacy and decreasing cost of LED packages, which has allowed manufacturers to improve colour performance and compete with traditional lighting technologies. In fact, the efficacy of LED packages for cool white light has improved from 25 lm/W (lumens per Watt) to 160 lm/W in the last decade. These factors have contributed to low manufacturing costs and high performance for end-users, who not only make a lower initial investment when acquiring the product but also see the benefits in long-term usage for significantly lower energy consumption [1].

Another factor that has led to the increasing interest of LED lighting solutions is their longer lifetime. Product reliability is one of the most important aspects of manufacturing engineering and customer service. Users need the assurance that a product will perform adequately and efficiently throughout its useful life in order to make a purchasing decision [7]. This is especially important for LED solutions, where brand recognition and product design are not as prominent as with other manufactured goods. Despite its importance, LED technology reliability has not been a topic of particular interest among manufacturers, resulting in a lack of standardized method to predict or evaluate this aspect of LED manufacturing [2].

Wang and Chu [8] proposed a procedure to accelerate the degradation of LED-based lights in order to predict its lifetime. In their research, forward current and temperature were considered the two major contributing variables to LED degradation. This was tested exposing LED-based light bars to different conditions by manipulating the two variables and measuring light output at the various stress points. The results showed that a degradation model based on an exponential function can effectively predict the lumen depreciation of a light bar with an estimate of 11,571 hours, which is congruent with the 10,000 hours promised by the LED supplier's warranty.

Other authors have proposed data-driven degradation tests to predict LED technology lifetime. Fan et al. [2] analyzed the lumen maintenance data of High-Power White Light-Emitting Diodes (HPWLEDs) by applying a degradation-data-driven method, which encompasses the approximation, analytical and two-stage methods. The study focused on the Mean Time to Failure (MTTF) to predict product reliability. The two-stage method, which is based on the Weibull distribution and the simulation of different failure times, predicts LED lifetime with 95% accuracy. The estimation result using this method was a MTTF of 101,763 hours.

In this paper, a similar model based on the Rayleigh distribution was utilized to include the reliability factor in the Capability Analysis of LED luminaire manufacturing. The Rayleigh distribution is a special case of the Weibull distribution and is related to the chi-square and extreme value distributions. The adequacy of this method to analyze accelerated lifetime of a product is due to its consideration of an increasing function of time, which means than when failure times are distributed according to the Rayleigh principles the equipment undergoes an intense process of aging [10].

As Taha explains, the Rayleigh distribution is a commonly used tool to model the performance of products with increasing failure rates, making it a simple yet effective solution for the analysis of lifetimes and the reliability of technical equipment. Consequently, the accelerated time to failure (TTF) data collected for this study was analyzed using the Rayleigh distribution function in order to evaluate the LED luminaire lifetime probability in the context of the multivariate capability indices proposed. Thus, the Process Capability Analysis developed in this research is a global measure that combines the technical standards of the product with the intangible value of product reliability while simultaneously encompassing the benefits of multivariate setting-based assessment.

PROPOSED APPROACH TO MULTIVARIATE CAPABILITY ANALYSIS

Given the importance of product reliability in the manufacturing industry, it is important to explore the possibility of including this factor in the process capability analysis methodology. Traditional univariate methods have limitations in this regard since product reliability is not considered within capability indices such as C_p or any of its variations. This extends to the multivariate capability analysis in case of multivariate measures that are derived from univariate techniques or simply disregard product reliability as a process capability matter.

Product reliability is often related to time and lifetime of the products, which can be challenging to include in capability analysis techniques that are centered on specifications and set acceptance criteria for these measures. If the product reliability factor were to be simply added as another quality characteristic in these methodologies, there would be inconsistencies related to data distribution discrepancies and estimation of adequate parameters for evaluation. The lifetime of a product, the time to failure of a device or time series in general behave differently than technical specifications, which complicates the mere addition of these characteristics to a traditional PCI due to incompatibility.

In order to solve this issue, the proposed method in this paper introduces two multivariate capability indices that separate these quality characteristics into different categories and implement more adequate analysis techniques for each subgroup. Thus, the Space Characteristics ¬are defined as the standard technical specifications that described the dimension of the product in relation to a give set of criteria that deem them acceptable. The Time Characteristics describe the behavior of the product over time and its ability to maintain functionality as well as the expected lifetime of the product.

The advantage of this approach, aside from solving the limitations described before, relies on the possibility to analyze the process in a new dimension, where other criteria such as product warranty and manufacturer's promises to the customer influence the process capability while simultaneously evaluating the performance of the process in the standard specified dimensions required. Additionally, the proposed MPCIs allow decision-makers to decide which of these sets of characteristics is more valuable in the global index, facilitating the customization of the measure according to the fluctuations of customer satisfaction needs, stakeholder's requirements, or market evolution.

II. Principal Component Space-Time Multivariate Capability Index ($MC_{PC:t}$)

Wang and Chen [11] introduced a multivariate capability index based on the Principal Component Analysis. The Principal Component Analysis is a methodology that can transform large sets of correlated data into a more accessible set of variables that still possess the characteristics of the original data. Using the PCA technique, it is possible to obtain the main axes of the Process Region and a description of the Specification Region represented as the eigenvectors and the diagonal variance-covariance matrix, respectively [6].

Wang and Chen [11] state that if the data collected from a multivariate process follow a normal distribution, a capability study using Principal Component Analysis can be applied to the process. The result of applying the PCA is a new set of variables (Principal Components) that are mutually independent and normally distributed as well. Thus, [11]'s capability index for the multivariate processes data is calculated as

$$\mathrm{MC}_{Cp} = \left[\prod_{\mathrm{i}=1}^{r} \left|\mathrm{C}_{\mathrm{p};PC_{\mathrm{i}}}\right|\right]^{\frac{1}{r}} \tag{1}$$

Where r is the number of selected principal components and $C_{p;PC_i}$ represents the univariate measure of process capability for the i_th principal component, with

$$\mathrm{C}_{\mathrm{pu}} = \frac{USL-\mu}{3\sigma} \tag{2}$$

and

$$\mathrm{C}_{\mathrm{pu}} = \frac{\mu-LSL}{3\sigma} \tag{3}$$

for the Upper and Lower Specifications Limits, respectively. Additionally, $\mathrm{C}_{\mathrm{p;PC_i}}$ in (1) can be replaced with $\mathrm{C}_{\mathrm{pk;PC_i}}$, $\mathrm{C}_{\mathrm{pm;PC_i}}$ and $\mathrm{C}_{\mathrm{pmk;PC_i}}$, obtaining similar measures for the more traditional univariate capability indices.

The *Principal Component Space-Time Multivariate Capability Index* based on the Principal Component Analysis is a modification of the index in equation (1). The new measure includes the value of the traditional PCA-based index and considers a new variable that depends on the lifetime of the product. Thus, the proposed index is determined by

$$\mathrm{MC}_{C_p;t} = \mathrm{w}_1\left[\prod_{\mathrm{i}=1}^{\mathrm{r}} \left|\mathrm{C_p}^{\mathrm{PCi}}\right|\right]^{\frac{1}{\mathrm{r}}} + \mathrm{w}_2 \ \left\{{\Phi^{-1}[T(\mathrm{t})]}/{3}\right\} \tag{4}$$

Where $T(t)$ is a probability function expressing the time, it takes for a product to fail i.e., the time before it stops performing as expected under normal conditions. For this model, the TTF was analyzed using the Rayleigh distribution, which adequately describes the linear increase failure rate over time. In the Rayleigh distribution, the accelerated TTF is determined by $h(t) = \lambda_s t$. Consider,

$$\lambda_s = \frac{2r}{\sum_{i=1}^{r} t_i^2 + \sum_{i=1}^{n-r} t_i^{2+}} \tag{5}$$

And

$$\lambda_0 = \frac{1}{(A_F)^2}\lambda_s \tag{6}$$

Thus, the proposed model considers the reliability of a given product as a probability expression of TTF, determined by

$$f_0(t) = \lambda_0 t e^{\frac{-\lambda_0 t^2}{2}} \tag{7}$$

A_F in (6) is the acceleration factor applied to the evaluated component. The Rayleigh distribution density function is

$$T(t) = e^{\frac{-\lambda_0 t^2}{2}} \tag{8}$$

On the other hand, $\sum_{i=1}^{2} w_i = 1$, where w_i (w_1 and w_2) is the assigned weight to the principal component and lifetime factors, respectively. It is important to note that the importance assigned to each aspect is subjective, dependent on the circumstances of the process and the relevance of each criterion in it.

II. Modified SR Space-Time Multivariate Capability Index ($\widehat{MC}_{Cp;t}$)

Taam et al. [5] define a multivariate capability index as a summary of the conditions of a multivariate process in relation to its specifications. They proposed that given the technical specifications of a product, the tolerance region can be modified according to the process distribution and compared to a scaled PR. Based on this principle, the multivariate capability index MC_{pm} is defined as

$$MC_{pm} = \frac{vol.(R_1)}{vol.(R_2)} \tag{9}$$

Where R_1 is the modified tolerance region and R_2 is a scaled 99.73% PR, which describes an elliptical region in the particular case of the normal distribution.

Taam et al. [5] defined his modified tolerance region as the largest ellipsoid centered at the target value within the original tolerance region. There are two main reasons for this particular assumption. Firstly, the multivariate normal distribution has an elliptical probability region and secondly, this method offers a shape correction factor for indices where the original tolerance region is not elliptical. Therefore, equation (9) can be modified to:

$$MC_{pm} = shape\ correction\ factor \times \frac{Vol.(Original\ TR)}{Vol.(R_2)}$$

$$= \frac{Vol.(R_1)}{Vol.(Original\ TR)} \times \frac{Vol.(Original\ TR)}{Vol.(R_2)} \tag{10}$$

Figure 2 is a representation of the modified tolerance region proposed by [5] for a $v = 2$ product characteristics. In this case, the original tolerance region is bound by the Upper and Lower specification limits and the modified region is the ellipsoid within this region. If the process data falls within the modified tolerance region and around the target value, then the process is considered capable. In contrast, if the data is scattered within this region or falls outside the ellipsoid, the multivariate capability index will reflect a lower value.

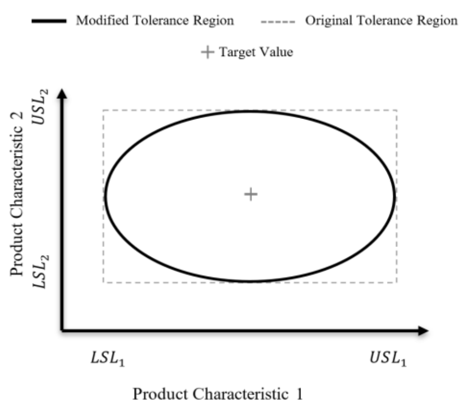

FIGURE 2
MODIFIED TOLERANCE REGION PROPOSED BY [12]

Given the considerations previously exposed, the second MPCI proposed in this paper follows the logic described by [5] with the introduction of a modified Specification Region B, approximated by the formula

$$B = \rho_{ij}\left(\frac{|\min(USL_i - \mu, \mu - LSL_i)|}{\sqrt{\chi^2_{v,0.9973}}}\right)\left(\frac{|\min(UPL_j - \mu, \mu - LSL_j)|}{\sqrt{\chi^2_{v,0.9973}}}\right) \tag{11}$$

$$\forall\, i,j = 1,2,3 \ldots v$$

Where ρ_{ij} is the correlation coefficient between the i^{th} and j^{th} quality characteristics and v is the total number of product characteristics.

Now, consider a diagonal variance-covariance matrix that describes the PR [11]. Let $|S|$ be the determinant of this PR variance-covariance matrix, then a ratio of the form $\frac{vol.(R_1)}{vol.(R_2)}$ in terms of the B matrix is calculated as

$$\left[\frac{|B|}{|S|}\right]^{\frac{1}{2}} \tag{12}$$

Note that in equation (12), the variance-covariance matrix and the sample mean are

$$S = (n-1)^{-1}\sum_{i=1}^{n}\left(X_i - \hat{X}\right)\left(X_j - \hat{X}\right)^r \text{ and } \hat{\mathrm{X}} = \mathrm{n}^{-1}\sum_{\mathrm{i}=1}^{\mathrm{n}}\mathrm{X}_\mathrm{I} \tag{13}$$

In this case, $|B|$ represents the modified tolerance region R_1 proposed in Taam's methodology, while the covariance matrix determines the standard process region R_2.

In order to consider the reliability factor in the second proposed index, equation (12) was modified to include the TTF analysis based on the Rayleigh distribution. Thus, the *Modified SR Space-Time Multivariate Capability Index* is determined by

$$\mathrm{MC}_{C_p;t} = \mathrm{w}_1\left[\frac{|\mathrm{B}|}{|\Sigma|}\right]^{\frac{1}{2}} + \mathrm{w}_2\ \left\{{\Phi^{-1}[T(\mathrm{t})]}/{3}\right\} \tag{14}$$

Where $T(\mathrm{t})$ is a probability function for the TTF of the product and $\sum_{i=1}^{2} w_i = 1$, where w_i (w_1 and w_2) is the assigned weight to the modified SR and lifetime factors, respectively.

NUMERICAL ANALYSIS

I. Case Study

Throughout the manufacturing process of a LED filament light bulb used for exterior areas, four main quality characteristics are considered: brightness, measured in lumens (lm), diameter (mm), length (mm) and lifetime expectancy, measured in hours. The collected data from the process is summarized in Table 1. In this study, the quality characteristics of the LED filament light bulb were classified into two categories. Firstly, the brightness, diameter, and length, corresponding to the technical specifications and deemed Space Characteristics and secondly, the reliability factor that corresponds to the Time to Failure (TTF) and lifetime of the product, which were categorized as the Time Characteristics.

TABLE I
QUALITY CHARACTERISTICS EVALUATED IN LED FILAMENT LIGHT BULBS MANUFACTURING

Unit	Brightness (lm)	Diameter (mm)	Length (mm)	Accelerated Time to Failure (hrs)[1]
1	49,92	39,96	69,87	2145
2	50,02	39,97	70,07	
3	49,99	39,99	69,99	4390
4	50,03	40,05	69,98	
5	49,94	40,00	70,01	5560
6	49,98	40,05	70,06	
7	49,96	39,98	69,93	8990
8	50,05	39,99	70,06	
9	49,97	39,97	70,01	10700
10	50,06	40,03	70,02	
11	49,96	40,01	69,98	13485
12	50,03	39,98	70,06	

[1] Censored data labeled with "+".

13	50,02	39,90	69,92	15000+
14	50,04	39,99	70,01	
15	50,03	39,92	69,98	15000+
16	50,02	40,00	70,00	
17	50,02	40,05	70,05	3215
18	49,94	40,04	70,03	
19	50,04	39,98	69,95	5360
20	50,02	39,90	69,98	
21	49,97	40,02	69,98	7590
22	50,03	40,05	70,04	
23	49,98	39,98	69,95	9840
24	49,97	40,05	70,09	
25	50,00	40,04	69,97	12300
26	49,96	40,01	69,95	
27	49,95	39,88	70,03	15000+
28	50,06	40,01	70,09	
29	50,05	39,90	69,98	15000+
30	50,04	39,90	69,93	

This information was then compared with the technical quality specifications for the LED filament light bulbs. The acceptance criteria for each variable are described in the table below.

TABLE II
ACCEPTANCE CRITERIA FOR LED QUALITY CHARACTERISTICS

Variable	Acceptance Criteria	Unit
Brightness	50 ± 2	lm
Diameter	40 ± 0.05	mm
Length	70 ± 0.05	mm
Reliability	10000	hrs

II. Applied Methodology

The Principal Component Analysis proposed by [11] was applied to the *Space Characteristics* of the LED filament light bulbs data presented in Table 1, i.e., the Brightness, Diameter and Length characteristics. First, the mean vector (μ) and variance-covariance matrix (S) were calculated.

$$\mu = \begin{bmatrix} 50.002 \\ 39.987 \\ 69.999 \end{bmatrix}$$

And

$$S = \begin{bmatrix} 0.001571 & -0.000097 & 0.000645 \\ -0.000097 & -0.002433 & 0.000911 \\ -0.000645 & 0.000911 & 0.002680 \end{bmatrix}$$

The eigenvalues and cumulated percentage were obtained through the application of the eigen-decomposition theorem.

TABLE III
EIGENVALUES AND VARIANCE PERCENTAGE PER PRINCIPAL COMPONENT

Component	Eigenvalue	Variance Percentage (%)	Cumulated Percentage (%)
1	1.4377	47.924	47.924
2	1.0911	36.368	84.292
3	0.4712	15.708	100.000

The eigenvalue results indicate that the first and second component account for 84% of the variability in the quality space characteristics evaluated for the product. The number of components to include in the analysis depends on the criteria of the researcher. Commonly used rules in this regard are including eigenvalues greater than 1 or the number of components that account for most of the variability of the sample, as shown in Figure 3. For the purposes on this investigation, all three components will be considered.

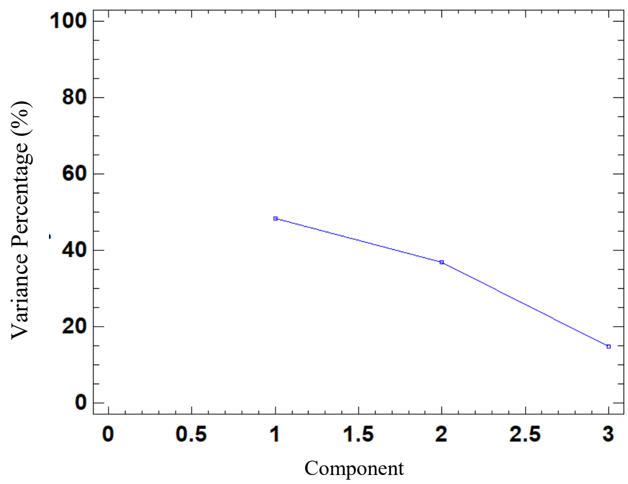

FIGURE 3
PRINCIPAL COMPONENT SEDIMENTATION

Table 4 describes the component coefficients for each variable. These indicate the magnitude and direction of each eigenvector derived from the matrix calculations between each of the variables.

TABLE IV
PRINCIPAL COMPONENT COEFFICIENTS

	Component 1	*Component 2*	*Component 3*
Brightness	0.291865	0.848688	0.441071
Diameter	0.611193	-0.520204	0.596515
Length	0.735702	0.0954777	-0.670542

Thus, these coefficients lead to the next step in the PCA methodology, which corresponds to the determination of the normalized eigenvectors and subsequent linear combinations calculation as follows,

$$\hat{u}_1 = (0.291865;\ 0.611193;\ 0.735702)$$

$$\hat{u}_2 = (0.848688;\ -0.520204; 0.095478)$$

$$\hat{u}_3 = (0.441071;\ 0.596515;\ -0.670542)$$

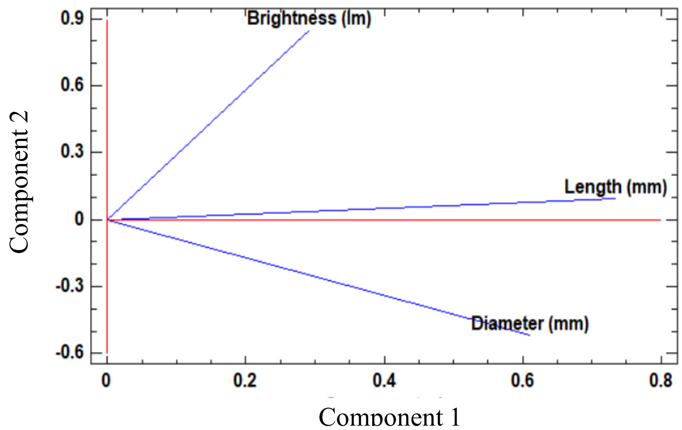

FIGURE 4
PRINCIPAL COMPONENT BIPLOT

Figure 4 is a representation of how each variable influences the components and the level of correlation between them. The further away a vector is from the origin of the component, the more it influences said component. In this case, Length and Diameter are the most influential in Principal Component 1, while Brightness influences Principal Component 2 the most. On the other hand, Length is positively correlated with the other two variables and Diameter and Brightness are the least correlated of the characteristics presented.

Following the PCA methodology, the specification criteria and the component weight coefficient are utilized to calculate the linear combinations for each component PC_i. For the first component, consider

LSL: $L_{PC_1} = (48 \times 0.291865 + 39.95 \times 0.611193 + 69.95 \times 0.735702) = 89.8890$
USL: $U_{PC_1} = (52 \times 0.291865 + 40.05 \times 0.611193 + 70.05 \times 0.735702) = 91.1912$

For the second and third components,

LSL: $L_{PC_2} = (48 \times 0.848688 + 39.95 \times (-0.520204) + 69.95 \times 0.0954777) = 26.6335$
USL: $U_{PC_2} = (52 \times 0.848688 + 40.05 \times (-0.520204) + 70.05 \times 0.0954777) = 29.9858$

LSL: $L_{PC_3} = (48 \times 0.441071 + 39.95 \times 0.596515 + 69.95 \times (-0.670542)) = -1.9022$
USL: $U_{PC_3} = (52 \times 0.441071 + 40.05 \times 0.596515 + 70.05 \times (-0.670542)) = -0.1453$

The capability indices of each component are calculated as

$$C_{pPC_i} = \frac{\left(USL_{PC_i} - LSL_{PC_i}\right)}{6\sigma} \tag{15}$$

Where σ is the square root of each corresponding component eigenvalue for and USL_{PC_i} and LSL_{PC_i} are the upper and lower specification limits calculated for the components. Thus,

$$C_p^{PC_1} = \left[\left|{(91.1912 - 89.8890)}\Big/{(6 \times 1.1990)}\right|\right] = 0.18099$$

$$C_p^{PC_2} = \left[\left|{(29.9858 - 26.6335)}\big/{6 \times 1.0446}\right|\right] = 0.53488$$

$$C_p^{PC_3} = \left[\left|{(-0.1453 - (-1.9022))}\big/{6 \times 0.6864}\right|\right] = 0.42657$$

Using (1) with $r = 3$, the traditional global multivariate capability index for the technical specifications is

$$MC_{Cp} = [0.18099 \times 0.53488 \times 0.42657]^{\frac{1}{3}} = 0.345653$$

II. Estimation of the Principal Component Space-Time Multivariate Capability Index ($MC_{C_p;t}$)

So far, the analysis has focused on the space characteristics capability through the Principal Component Analysis. The first proposed multivariate capability index is a variation of the traditional method previously exposed, which includes a time parameter based on the Time to Failure of the product. Let r be the number of non-censored data in the sample, then λ_s is determined with (5)

$$\lambda_s = \frac{2(11)}{776731575 + 9 \times 10^8} = 1.3121 \times 10^{-8}$$

With an acceleration factor $A_F = 15$, then λ_0 is calculated using (6) as

$$\lambda_0 = \frac{1}{(15)^2} \times 1.3121 \times 10^{-8} = 5.8315 \times 10^{-11}$$

Next, the reliability factor of the LED luminaires was calculated applying the Rayleigh distribution probability function. The manufacturer expects the product to maintain adequate functionality for at least 1000 hours. Thus, evaluating the probability function for an expected TTF of $t = 1000$ hours with (8), the result obtained is

$$T(1000) = e^{\frac{-(5.8315 \times 10^{-11})(1000)^2}{2}} = 0.99997$$

Indicating that there is a 99.99% probability that the product will outlast this threshold. Then, the Principal Component Space-Time Multivariate Capability Index $MC_{C_p;t}$ is then determined by (4), as follows

$$MC_{C_p;t} = (0.5)(0{,}345653) + (0{,}5)\left\{{\Phi^{-1}[0{,}99997]}\big/{3}\right\} = 0{,}8427$$

In this case, both the space and time aspects of the manufacturing process were given the same weight in the final result, corresponding to 50% for each type of variables. However, it is the responsibility of researchers and decision-makers of the manufacturing process to assign the respective importance to each area, depending on the goals of process and perceived quality assessments. In order to evaluate the result of the Capability Index, Tsai and Chen [12] propose an interval-based approach to determine the capability performance level of a given process.

TABLE V
PROCESS CAPABILITY PERFORMANCE CRITERIA

Capability Index Interval	Assessment
$C_p > 2.00$	Super Excellent

$1.67 \leq C_p \leq 2.00$	Excellent
$1.33 \leq C_p \leq 1.67$	Satisfactory
$1.00 \leq C_p \leq 1.33$	Capable
$0.67 \leq C_p \leq 1.00$	Inadequate
$C_p < 0.67$	Poor

Based on this appreciation and considering that the product reliability factor shows positive results for this particular product, the global value of the Principal Component Space-Time Multivariate Capability Index reflects that the LED manufacturing process is performing inadequately, i.e., the current process is not capable of producing LED luminaire units that comply with the product technical specifications, which has a direct impact on product lifetime it the expected value were to be higher that the $t = 1000$ hrs threshold.

II. Estimation of the Principal Component Space-Time Multivariate Capability Index $(MC_{C_p;t})$

As explained in previous sections, the second multivariate capability index is a variation of [12]'s proposal, which incorporates a tolerance region analysis as the space quality characteristics of the product and, similar to the first proposal, the time to failure factor related to the LED luminaire reliability. The covariance matrix for the given length, diameter, and brightness data in Table 1 is

$$S = \begin{bmatrix} 0.00157059 & -0.00009694 & 0.00064466 \\ -0.00009694 & 0.00243295 & 0.00091103 \\ 0.00064466 & 0.00091103 & 0.00267962 \end{bmatrix}$$

Next, the correlation matrix was calculated as

$$\rho_{ij} = \begin{bmatrix} 1.00000000 & -0.04958856 & 0.31423812 \\ -0.04958856 & 1.00000000 & 0.35680550 \\ 0.31423812 & 0.35680550 & 1.00000000 \end{bmatrix}$$

Let $C_{pkv} = \min\,(USL - \mu, \mu - LSL)$ for the v quality characteristics, then

$$\mu_{diameter} = 39.9867$$
$$\mu_{length} = 69.9990$$
$$\mu_{brightness} = 50.0017$$

Then, based on the upper and lower specification limits defined in Table 2, consider

$$C_{pk_{diameter}} = \min(40.0500 - 39.9867, 39.9867 - 39.9500) = 0.0367$$
$$C_{pk_{length}} = \min(70.0500 - 69.9990, 69.9990 - 69.9500) = 0.0490$$
$$C_{pk_{brigthness}} \min(52.0000 - 50.0017, 50.0017 - 48.0000) = 1.9983$$

With a significance level of 0.0027 and $v = 3$, the Chi-square value is $\chi^2_{v,0.9973} = 14.1563$. Thus, $\sqrt{\chi^2_{v,0.9973}} = 3.7624$ and matrix B is calculated using (10) as follows

$$B = \begin{bmatrix} 0,28208992 & 0,00025667 & 0,00217358 \\ 0,00025667 & 0,00009497 & 0,00004529 \\ 0,00217358 & 0,00004529 & 0,00016961 \end{bmatrix}$$

Subsequently, the determinant values for matrix $|B|$ and covariance matrix $|S|$ are

$|B| = 3.5561 \times 10^{-9}$
$|S| = 7.7856 \times 10^{-9}$

Thus, the proposed *Modified SR Ratio Space-Time Multivariate Capability Index* ($\widehat{MC}_{C_{p};t}$) is calculated as

$$\widehat{\mathrm{MC}}_{C_{p};t} = (0.5)\left[\frac{3.5561 \times 10^{-9}}{7.7856 \times 10^{-9}}\right]^{\frac{1}{2}} + (0.5)\left\{\Phi^{-1}[0.99997]\Big/_{3}\right\} = 1.0067$$

Based on the criteria described in Table 5, this result shows that the process is capable of producing LED luminaires that are technically acceptable, while maintaining product reliability in terms of TTF expectancy.

Concluding Remarks and Future Work

This paper presented two different MPCIs as a proposed approach to analyse the process capability of a standard LED luminaire manufacturing process. In addition to the traditional methods such as Principal Component Analysis and Tolerance Region Modification introduced by [5] and [11] respectively, these proposals considered the impact of product reliability and lifetime prediction on the capability assessment of the manufacturing process. The *Space Characteristics* were analysed with modified traditional techniques that highlight the correlation between the quality characteristics studied: Brightness, Diameter and Length of the luminaires. On the other hand, the *Time Characteristics* were studied with an analysis of the TTF failure of the luminaires to estimate product lifetime.

The first proposed MPCI is the *Principal Component Space-Time Multivariate Capability Index* $MC_{C_{p};t}$, which combines the PCA approach with the lifetime estimation techniques. With a global result of $MC_{C_{p};t}$= 0.8427, this measure shows that the LED luminaire manufacturing process is not capable of producing units that comply with the technical specifications criteria, according to the scale presented by Tsai and Chen to assess process capability performance. Within this measure, the reliability aspect was tested with an expectation of 1000 hours of lifetime and 50% of weighted impact on the global result. It is important to note that while specifications compliance has been traditionally used as an indicator or process capability, the TTF analysis in this context must not be considered as a unilateral indicator of process performance, rather a complementary measure to help understand process behavior.

Similarly, the second index *Modified SR Space-Time Multivariate Capability Index* $\widehat{MC}_{C_{p};t}$ is similarly comprised of a space characteristics component and a TTF analysis. The reliability factor was not modified for this measure, which means that the main difference between the two MPCIs proposed is the approach to technical specification compliance assessment. Although both the PCA and [5]'s methods are a form of tolerance region modification, the latter relies more heavily of quality characteristics correlation with the inclusion of the correlation coefficient in the B matrix calculation. The global index value for $\widehat{MC}_{C_{p};t}$ is 1.0067, resulting in a less stringent measure than $MC_{C_{p};t}$ with an acceptable process performance, based on Tsai and Chen evaluation scale.

Comparing the MPCIs proposed with their traditional counterparts, the Principal Component Method resulted in a 0.3457 global index, showing poor process capability in LED luminaire manufacturing. The tolerance region ratio analysis had a similar result with an index value of 0.6758, which indicates an inadequate process performance level. Despite the fact that the proposed MPCIs seem to be more lenient than the traditional methods, they present the advantage of including product reliability into the same measure. This is especially significant if the parameters of the two indices are adjusted to better suit the needs of a particular process. Firstly, the weighted impact of the space and time characteristics can be modified to represent the needs of the process stakeholders and the aspects that they consider most relevant for a certain product to enter the market. Secondly, the lifetime expectations can be set higher than the 1000 hours considered in this experiment, depending on the specific type of product that are being evaluated and the warranty promised by the manufacturers. This consideration could then become an asset for any company whose end-users value reliability and plan to use it as a competitive advantage in any given market.

The main contribution of this paper is presenting a viable way to integrate product reliability within the process capability analysis, in order to gain insight into the impact of this variable on process performance. However, this methodology opens the door to other time-related variables that behave similarly or differently than product lifetime. By adjusting the distribution function and parameters of the *Time Characteristics* there is a possibility to evaluate process capability along with reliability elements such as Mean Time to Failure (MTTF), Mean Time to Repair (MTTR), Mean Time between Failures (MTBF) and Failure in Time (FIT).

REFERENCES

[1] Morgan Pattison, P., Hansen, M. and Tsao, J. (2018). LED lighting Efficacy: Status and directions. Comptes Rendus Physique 19(2018), 134-145. DOI: https://doi.org/10.1016/j.crhy.2017.10.013.

[2] Fan, J., Yung, K., and Pecht, M. (2012). Lifetime Estimation of High-Power White LED Using Degradation-Data-Driven Method. IEEE Transactions on Device and Materials Reliability 12(2), 470 – 477.

[3] Chen, K. S., Luang, M. L. and Yung, H. (2008) Process Capability Analysis Chart with the Application of C_{pm}. International Journal of Production Research. DOI: 10.1080/00207540600806422.

[4] Zainab, G., and Bahran, G. (2017). A new multivariate process capability index. Total Quality Management and Business Excellence. DOI: http://dx.doi.org/10.1080/14783363.2017.1315936.

[5] Taam W., Subbaiah, P., and Liddy, J. (1993). A note on multivariate capability indices. Journal of Applied Statistics 20(3), 339 – 351. DOI: http://dx.doi.org/10.1080/02664769300000035.

[6] De-Felipe, D., and Benedito, E. (2017). A Review of univariate and multivariate process capability indices. International Journal of Advanced Manufacturing Technology. DOI: 10.1007/s00170-017-0273-6.

[7] Murthy, D. (2007). Product reliability and warranty: an overview and future research. Produção 17(3), 426 – 434.

[8] Wang, F. K., and Chu, T. (2012). Lifetime predictions of LED-based light bars by accelerated degradation test. Microelectronics Reliability 52(2012), 1332 – 1336.

[9] Tohidi, H., Jabbari, M.M., (2012). “Important factors in determination of innovation type”. Procedia Technology, 1, 570-573. https:// DOI: 10.1016/j.protcy.2012.02.124

[10] Taha, T. (2015). On Reliability Estimation for the Rayleigh Distribution Based on Monte Carlo Simulation. International Journal of Science and Research. DOI: 10.21275/ART20176044.

[11] Wang, F. K., and Chen, J. (1998). Capability Index Using Principal Components Analysis. Quality Engineering 11(1): 21- 27. DOI: 10.1080/08982119808919208.

[12] Tsai, C., and Chen, C. (2006). Making decision to evaluate process capability index C_p with fuzzy numbers. International Journal of Advance Manufacturing Technology 30(3-4), 334 – 339. DOI: 10.1007/s00170-005-0052-7.