

# Designing a model of product arrangement and multi-warehouse location-routing problem

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Received: 14 Oct 2022/ Accepted: 25 Feb 2023/ Published online: 25 Feb 2023

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## Abstract

This paper presents a model of product arrangement and multi-warehouse location-routing problem under uncertainty. In the first stage of the designed model, the main goal is to arrange products in packages that can be sent to the customer. The second stage aims to locate potential warehouses and optimally route vehicles. Therefore, the objective function of the first stage includes the sum of the optimal dimensions of the packages, and the objective functions of the second stage include the simultaneous optimization of location-routing costs and the amount of greenhouse gas emissions. Due to the uncertainty of the designed model and considering the demand parameters and transmission costs in the form of trapezoidal fuzzy numbers, in this paper, the robust fuzzy optimization method is used to control the uncertainty parameters. Due to the NP-Hard nature of the designed model, the invasive weed optimization is used to determine the optimal dimensions of the packages and the arrangement of products in the packages in the first stage. In addition, the NSGA II is used to find efficient solutions to the problem in the second stage. The results of model implementation in a real case study in Safir Company have shown ten efficient solutions. By reviewing the efficient solutions, the most appropriate efficient solution for the management of Safir Company in Iran is presented. In this article, a two-stage model of product packaging and vehicle routing with simultaneous pickup and delivery under uncertainty is presented, which uses the robust fuzzy optimization method to control uncertainty and different methods to solve the problem.

**Keywords-** Multi-Warehouse Vehicle Routing; Product Arrangement; Invasive Weed Optimization; Robust Fuzzy Optimization Method

## INTRODUCTION

Economic and industrial growth are occurring more quickly than in the past in the modern world. Concerns about social and environmental issues have led to increased attention from governments and organizations to take social and environmental elements as well as financial ones into account when making choices in tandem with these changes in the economic environment (Wang et al., 2015). Environmental aspects include minimizing air pollution and the

use of natural resources, as well as social elements like the creation of jobs, the application of labor regulations, customer and employee happiness, and related issues (Casazza et al., 2021). Quality of labor or inspector productivity has the potential to contribute to the development of inspection performance and efficiency (Namdari et al., 2017). The earliest definitions of the term "sustainable development" were offered by the World Commission on Environment and Development in 1987. This concept states that sustainable development is growth that satisfies present demands without jeopardizing the ability of future generations to satiate their own needs. The release of greenhouse gases, particularly carbon dioxide, is one of the supply chain's (SC) most significant environmental consequences. Production, transportation, recycling, and the majority of other SC network operations all produce CO<sub>2</sub> emissions (Ghahremani-Nahr et al., 2023). The greatest significant impact on the emission of these gases, however, is now being had by transportation-related activities. As a result, many businesses have recently attempted to implement ways to lower pollution from transportation-related operations. German and British industrial businesses update and switch to electric and hybrid cars for their transportation fleets. Such changes typically necessitate very large investment expenditures spread across multiple businesses in a variety of ways. Operations choices are made to minimize fuel consumption and pollution as a result of changes to their transportation fleet (Shi et al., 2020). Therefore, rather than concentrating on the traditional aims of transportation concerns, which are mostly centered on transportation costs, many academics have sought to limit fuel consumption in the transportation process in recent years. Pollutant emissions are closely correlated with fuel use (Ahkamiraad et al., 2018).

Combining the two concerns of the traveling salesman (unlimited consideration of vehicle capacity) and box packing (zero consideration for freight costs on the routes) attempts to optimally design a set of routes for the transport fleet in a specific number of customer services provided and has a number of side effects. Since there are so many different versions of this issue, classifying them and expressing the additional states in which it might occur is difficult and time-consuming. Numerous adaptations have been developed since its start in the 1960s depending on their various real-world uses. As a result, there are now different versions available, including heterogeneous type (Yousefikhoshbakht and Sedighpour, 2012), simultaneous reception and delivery (Yousefikhoshbakht and Khorram, 2012), open kind of problem (Yousefi et al., 2012), and more. Vehicle routing problem (VRP) applications include the transportation of numerous items, such as various types of oil (Lahyani et al., 2015), various types of milk (Nozari et al., 2015), various types of gasoline (Avella et al., 2004), or frozen foods (Derigs et al., 2011). Some distribution businesses also have numerous warehouses. The expenditures of the total transportation system can be decreased by strategically locating and increasing the number of warehouses while concurrently routing cars to distribute items to customers. In order to fulfill the need of a group of demand nodes or clients, multiple warehouses distribute or store items so that they may be transported by vehicles (Mohammadi et al., 2015). Determining the sort of vehicle to distribute and deliver first-hand things and pick up items in the SC network is part of the problem of vehicle routing with simultaneous pickup and delivery (VRPSPD). In this research, model of product arrangement and multi-warehouse location-routing problem with simultaneous pickup and delivery (PA-MDLRPSPD). On the other hand, uncertainty in the network can increase the costs and emissions of greenhouse gases. Therefore, developing methods to deal with uncertainty is one of the necessities of every research.

The article has the following structure. The second section presents the background of the relevant research. PA-MDLRPSPD are provided in the third section of the mathematical model. Methods for solving model and definitions of chromosomes are provided in the fourth part. The research's conclusion and the outputs' presentation are discussed in the fifth part. The study's conclusions and suggested next steps are discussed in the sixth part.

## LITERATURE REVIEW

Jin et al. offered a two-step solution to the problem of multi-depot VRP using a complicated integer linear mathematical model. Allocation and routing were included in the first technique's division of the issue into two additional small issues, whereas the second method treated the issue as a whole. The findings of the problem-solving demonstrated the second approach's superior efficiency over the first way (Jin et al., 2004). In terms of confidence, Liu et al. proposed a linear mathematical model of a complex number. They offered a model that was based on a single cycle and took into account a product that employed an original approach to address the issue and route warehouses to demand places (Liu et al., 2010). In order to reduce fuel consumption, Xiao et al. addressed the problem of capacity routing in the distribution of commodities when a SA was used. Another research highlighted the open VRP while contrasting the CPLEX solution with the SA employing a cross-warehouse (Xiao et al., 2012). By imposing additional restrictions on earlier works, Lalla-Ruiz et al. established a novel mathematical model for the open multi-warehouse routing issue. The mathematical model they developed was highly effective, as seen by the

computational results from the sample issues (Lalla-Ruiz et al., 2016). To reduce the risk of predicted transportation when preparing hazardous items and moving goods from various warehouses to clients, Du et al. developed a fuzzy linear programming model. The issue was resolved using PSO, GA, SA, and ACO, four meta-heuristic algorithms. The suggested methods were then compared by means of numerical examples (Du et al., 2017).

Under the uncertainty of demand, Majidi et al. handled the vehicle's path using the same delivery and pickup. They took into account the flexible time period for item delivery and collection. The fuzzy approach was employed in this study to regulate the parameters (Majidi et al., 2017). In a closed-loop SC network, Ghahremani-Nahr et al. established a robust fuzzy optimization (RFO), which included finding prospective facilities and efficiently allocating product flows. They employed the WOA to solve their model and demonstrated that the suggested approach is more efficient than the currently used algorithms (Ghahremani-Nahr et al., 2019). Through GA, Qin et al. modeled a VRP to reduce total expenditures and take the environment into account (Qin et al., 2020). A VRP with the two goals of reducing the number of cars and distance was given by Shi et al. To resolve the issue, they employed the BSTS hybrid algorithm. The outcomes demonstrated the great efficacy of their suggested approach to problem-solving (Shi et al., 2020). A VRPSPD was controlled by Golsefidi et al. using a robust optimization approach. To reduce the transfer cost objective function, they employed GA and SA (Golsefidi et al., 2020). Banders' analytical approach was used by Casazza et al. to resolve a channel routing problem involving the simultaneous pickup and delivery of a single product. This paper's main goal was to reduce the total cost (Casazza et al., 2021). Öztaş and Tuş used a hybrid meta-heuristic algorithm to solve the VRPSPD. The proposed algorithm consists of low complexity components and has only one parameter (Öztaş and Tuş, 2022). Yu et al. presented a model for VRPSPD, in which the delivery time window was also considered. SA algorithm was also used to solve the mathematical model (Yu et al., 2022). Bouanane et al. reviewed SPDVRP papers from 1989. They classified 191 papers and categorized the trends in the VRPSPD literature (Bouanane et al., 2022). Bathaee et al., designed a closed loop supply chain with VRPSPD under uncertainty. They used RFO to control the uncertainty parameters and MOPSO, NSGA II to solved the model (Bathaee et al., 2023). The most important studies that have been done in the area of multi-warehouse VRP and product arrangement are summarized in Table 1.

TABLE 1  
THE MOST IMPORTANT STUDIES THAT HAVE BEEN DONE IN THE AREA OF MULTI-WAREHOUSE VRP AND PRODUCT ARRANGEMENT

Paper	Economic aspect	Environmental aspect	Social aspect	Selecting warehouse	arrangement items	The solution method	Uncertainty
Zhou et al. (2017)	*	-	-	-	-	TS	-
Majidi et al. (2017)	*	*	-	*	-	Cplex	Fuzzy
Hu et al. (2018)	*	-	-	*	-	GA	Sustainable
Li et al. (2019)	*	-	*	*	-	FA	-
Qin et al. (2019)	*	*	-	*	-	GA	-
Shi et al. (2020)	*	-	*	-	-	BSTS	-
Golsefidi et al. (2020)	*	-	-	*	-	GA-SA	استوار
Casazza et al. (2021)	*	-	-	*	-	Banders' analysis	-
Ghobadi et al. (2021)	*	-	-	*	-	SA	Fuzzy
Öztaş and Tuş (2022)	*	-	-	*	-	GA	-
Yu et al. (2022)	*	-	-	*	-	SA	-
Bathaee et al. (2023)	*	*	*	*	-	NSGA II MOPSO	RFO
Current study	*	*	-	*	*	NSGA II IWO	RFO

The review of the literature shows that so far a comprehensive model of PA-MDLRSPD under uncertainty has not been provided. Based on the existing research gaps, this article has presented a new model of the problem, which can be summarized in the following points:

- Integration of product placement and multi-warehouse vehicle location-routing problems
- Designing efficient solutions to achieve optimal values at each stage

- Considering sustainability in the mathematical model (economic and environmental)
- The use of RFO method in controlling uncertainty parameters

**PROBLEM DEFINITION AND MODELING**

A PA-MDLRSPD are presented in this paper. The problem under investigation has two optimization stages. The main goal of the first stage (FSt) is to arrange goods into mobile packaging via trucks. Identifying the quantity and location of warehouses as well as the vehicle route to deliver packaged goods to clients are the main objectives of the second stage (SSt). Three customers are taken into account to organize the products in Figure 1(a), each of whom has a requirement for three distinct things. Each product that a customer orders has a specific length, breadth, and height that must be packaged in the best possible way before being delivered by trucks. As a result, the FSt's objective is to arrange goods that can be provided to customers in various packages. The SSt reduces total costs and greenhouse gas emissions through the distribution of these packages to customers. Table 2 also displays the product dimensions that customers are requesting to solve the issue. Figure 1(b) depicts the ideal placement of the products that customers have requested in the appropriate packages in accordance with the aforementioned table.

TABLE 2  
DIMENSIONS OF PRODUCTS REQUESTED BY CUSTOMERS (W, L, H)

Customer	Product 1	Product 2	Product 3
A	(30,40,50)	(10,40,20)	(30,10,10)
B	(20,40,10)	(20,40,30)	(30,40,50)
C	(20,20,20)	(30,40,30)	(30,10,40)

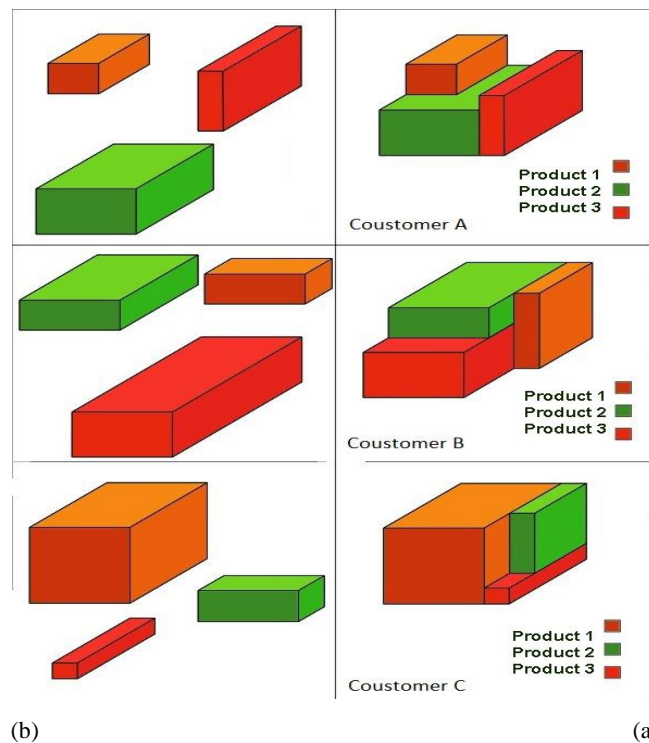


FIGURE 1  
OPTIMAL ARRANGEMENT OF CUSTOMER PRODUCTS IN SUPPLIED PACKAGES

Be aware that consumer demand is erratic, which might cause changes in packing proportions. In this study, demand and transportation costs are controlled as unknown parameters using the RFO approach. In the SSt, choices must be

taken on the placement of warehouses and the vehicle routing for distributing goods to clients after the best possible packing of products in packages. Figure 2 depicts the ideal route for authorized vehicles to use while transporting goods that have been packed in the first step. The restriction of vehicle dimensions is also taken into consideration in this article because each vehicle has certain dimensions in terms of package arrangement.

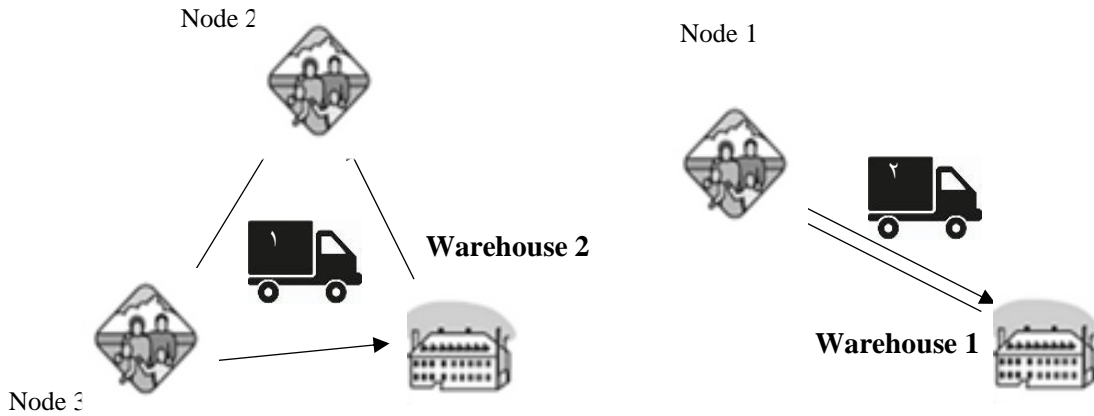


FIGURE 2. ROUTING THE VEHICLE TO DISTRIBUTE PACKAGES TO CUSTOMERS

The following are the presumptions underlying the PA-MDLRSPD:

- The subject at hand involves a number of items,
- Vehicles feature a variety of trucks for carrying various goods,
- The size of the trucks used to move cargo is already known,
- The bundles of items to be transferred are predetermined in terms of size.
- There are unknown numbers and locations of warehouses, and
- Demand and transfer cost variables are regarded as uncertain.

The main goal of this article is to design a PA-MDLRSPD based on the assumptions established. In order to do this, the modeling for this work was done in two stages:

- Optimal product arrangement within packaging,
- Choosing the best route for vehicles to take while transporting items to clients.

**The sets**

$DR$	Nodes $dr, dr' = \{R \cup D\}$	$D$	Warehouses $d = \{1, 2, \dots, D\}$
$R$	Customers $r, r' = \{1, 2, \dots, R\}$	$k$	Vehicle $k = \{1, 2, \dots, K\}$
$I$	Products $i, j = \{1, 2, \dots, I\}$		

**Parameters**

$FixD_d$	Fixed cost of established warehouse at node $d$
$FixK_k$	Fixed cost of vehicle $k$
$Dis_{dr, dr'}$	The distance from node $dr$ to node $dr'$
$\widetilde{Tr}_{dr, dr'}$	Transportation cost from node $dr$ to node $dr'$
$Ti_{dr, dr'}$	Transportation time from node $dr$ to node $dr'$
$[A_k, B_k]$	The time window for the vehicle $k$ to arrive at each customer

$Co_{dr,dr'}$	Co2 emitted by the vehicle from node $dr$ to node $dr'$
$CapK_k$	Maximum capacity of vehicles $k$
$CapD_{d,i}$	Maximum capacity of warehouse $d$ for $i$ -th product
$DQ_d$	Maximum number of vehicles in the warehouse $d$
$\widetilde{Dem}_r$	Demand of the product $i$ for customer $r$
$Wmax_r, Lmax_r, Hmax_r$	The package dimensions determined for the customer $r$
$WK_k, LK_k, HK_k$	Dimensions of vehicle $k$
$M$	Big number
$W_{r,i}, L_{r,i}, H_{r,i}$	The dimensions of product $i$ for customer $r$

### Decision variables

$Wmax_r, Lmax_r, Hmax_r$	Optimal dimensions of package for customer $r$
$X_{r,i}, Y_{r,i}, Z_{r,i}$	The starting point of arrangement the product $i$ for customer $r$ in the length-width-height of the package
$a_{r,i,j}, b_{r,i,j}, c_{r,i,j}$	1; If product $i$ is placed in front-right-top of product $j$ in package $r$ -0; Otherwise.
$Xl_{r,i}, Yw_{r,i}, Zh_{r,i}$	1; If the length-width-height of product $i$ is parallel to the X-Y-Z-axis for package $r$ -0; Otherwise.
$Zl_{r,i}$	1; If the length of product $i$ is parallel to the Z-axis for package $r$ -0; Otherwise.
$Al_{r,k}$	1; If package $r$ is allocated to vehicle $k$ -0; Otherwise.
$Xl1_r$	1; If the length of the package $r$ is parallel to the X-axis-0; Otherwise.
$Yl1_r$	1; If the length of the package $r$ is parallel to the Y-axis-0; Otherwise.
$Xw1_r$	1; If the width of the package $r$ is parallel to the X-axis-0; Otherwise.
$Yw1_r$	1; If the width of the package $r$ is parallel to the Y-axis-0; Otherwise.
$U_k$	1; If vehicle $k$ is used-0; Otherwise.
$Depo_d$	1; If warehouse is established to node $d$ -0; Otherwise.
$X'_{dr,dr',k}$	1; If the vehicle $k$ is traveled from node $dr$ to node $dr'$ -0; Otherwise.
$Y'_{r,i,k}$	1; If the product set of the customer $r$ is transported by vehicle $k$ -0; Otherwise.
$X1_r, Y1_r, Z1_r$	The starting point for the arrangement of the package $r$ in the length- width-height of the vehicle.
$ST_{dr,k}$	Auxiliary variable
$a'_{r,r'}, b'_{r,r'}, c'_{r,r'}$	1; If package $r$ is placed to the left- right- behind of the package $r'$ -0; Otherwise.
$d'_{r,r'}, e'_{r,r'}, f'_{r,r'}$	1; If package $r$ is placed in front- below- above of the package $r'$ -0; Otherwise.

$$MinZ = \sum_{r=1} (Wmax_r + Hmax_r + Lmax_r) \tag{1}$$

s. t.:

$$X_{r,i} + L_{r,i} \cdot Xl_{r,i} + W_{r,i} \cdot (Zl_{r,i} - Yw_{r,i} + Zh_{r,i}) + H_{r,i} \cdot (1 - Xl_{r,i} - Zl_{r,i} + Yw_{r,i} - Zh_{r,i}) \leq X_{r,j} + M \cdot (1 - a_{r,i,j}), \quad \forall r, i \neq j \tag{2}$$

$$Y_{r,i} + W_{r,i} \cdot Yw_{r,i} + L_{r,i} \cdot (1 - Xl_{r,i} - Zl_{r,i}) + H_{r,i} \cdot (Xl_{r,i} + Zl_{r,i} - Yw_{r,i}) \leq Y_{r,j} + M \cdot (1 - b_{r,i,j}), \quad \forall r, i \neq j \tag{3}$$

$$Z_{r,i} + H_{r,i} \cdot Zh_{r,i} + W_{r,i} \cdot (1 - Zl_{r,i} - Zh_{r,i}) + L_{r,i} \cdot Zl_{r,i} \leq Z_{r,j} + M \cdot (1 - c_{r,i,j}), \quad \forall r, i \neq j \tag{4}$$

$$X_{r,i} + L_{r,i} \cdot Xl_{r,i} + W_{r,i} \cdot (Zl_{r,i} - Yw_{r,i} + Zh_{r,i}) + H_{r,i} \cdot (1 - Xl_{r,i} - Zl_{r,i} + Yw_{r,i} - Zh_{r,i}) \leq Lmax_r, \quad \forall r, i \tag{5}$$

$$Y_{r,i} + W_{r,i} \cdot Yw_{r,i} + L_{r,i} \cdot (1 - Xl_{r,i} - Zl_{r,i}) + H_{r,i} \cdot (Xl_{r,i} + Zl_{r,i} - Yw_{r,i}) \leq Wmax_r, \quad \forall r, i \tag{6}$$

$$Z_{r,i} + H_{r,i} \cdot Zh_{r,i} + W_{r,i} \cdot (1 - Zl_{r,i} - Zh_{r,i}) + L_{r,i} \cdot Zl_{r,i} \leq Hmax_r, \quad \forall r, i \tag{7}$$

$$a_{r,i,j} + a_{r,j,i} + b_{r,i,j} + b_{r,j,i} + c_{r,i,j} + c_{r,j,i} \geq 1, \quad \forall r, i \neq j \tag{8}$$

$$Xl_{r,i} + Zl_{r,i} \leq 1, \quad \forall r, i \tag{9}$$

$$Zl_{r,i} + Zh_{r,i} \leq 1, \quad \forall r, i \tag{10}$$

$$Zl_{r,i} - Yw_{r,i} + Zh_{r,i} \leq 1, \quad \forall r, i \tag{11}$$

$$Zl_{r,i} - Yw_{r,i} + Zh_{r,i} \geq 0, \quad \forall r, i \tag{12}$$

$$1 - Xl_{r,i} - Zl_{r,i} + Yw_{r,i} - Zh_{r,i} \leq 1, \quad \forall r, i \tag{13}$$

$$1 - Xl_{r,i} - Zl_{r,i} + Yw_{r,i} - Zh_{r,i} \geq 0, \quad \forall r, i \tag{14}$$

$$Xl_{r,i} + Zl_{r,i} - Yw_{r,i} \leq 1, \quad \forall r, i \tag{15}$$

$$Xl_{r,i} + Zl_{r,i} - Yw_{r,i} \geq 0, \quad \forall r, i \tag{16}$$

$$Wmax_r, Lmax_r, Hmax_r, X_{r,i}, Y_{r,i}, Z_{r,i} \geq 0, \quad \forall r, i \tag{17}$$

$$a_{r,i,j}, b_{r,i,j}, c_{r,i,j}, Xl_{r,i}, Zl_{r,i}, Yw_{r,i}, Zh_{r,i} \in \{0,1\}, \quad \forall i, j, r \tag{18}$$

$$MinW_1 = \sum_{dr=1} \sum_{dr'=1} \sum_{k=1} \bar{Tr}_{dr,dr'} \cdot Dis_{dr,dr'} \cdot X'_{dr,dr',k} + \sum_{k=1} FixK_k \cdot U_k + \sum_{d=1} FixD_d \cdot depod \tag{19}$$

$$MinW_2 = \sum_{dr=1} \sum_{dr'=1} \sum_{k=1} Co_{dr,dr'} \cdot Dis_{dr,dr'} \cdot X'_{dr,dr',k} \tag{20}$$

s. t.:

$$\sum_{d=1} \sum_{r=1} X'_{d,r,k} \leq U_k, \quad \forall k \tag{21}$$

$$\sum_{k=1} Y'_{r,i,k} = 1, \quad \forall r, i \tag{22}$$

$$\sum_{i=1} Y'_{r,i,k} \leq M * \sum_{dr=1} X'_{dr,r,k}, \quad \forall r, k \tag{23}$$

$$\sum_{dr=1} X'_{dr,r,k} \leq \sum_{i=1} Y'_{r,i,k}, \quad \forall r, k \tag{24}$$

$$\sum_{dr=1} X'_{dr,r,k} \leq 1, \quad \forall r, k \tag{25}$$

$$\sum_{dr=1} X'_{dr,dr',k} = \sum_{dr=1} X'_{dr',dr,k}, \quad \forall dr', k \tag{26}$$

$$\sum_{r=1} \overline{Dem}_{r,i} \cdot Y'_{r,i,k} \leq CapK_k, \quad \forall i, k \tag{27}$$

$$\sum_{k=1} \sum_{r=1} X'_{d,r,k} \leq DQ_d \cdot depo_d, \quad \forall d \tag{28}$$

$$\sum_{r=1} \sum_{k=1} \overline{Dem}_{r,i} \cdot X'_{d,r,k} \leq CapD_{d,i} \cdot Depo_d, \quad \forall d, i \tag{29}$$

$$\sum_{d=1} \sum_{k=1} ST_{d,k} = 0 \tag{30}$$

$$ST_{dr',k} + 1 \leq ST_{r,k} + M(1 - X'_{dr',r,k}), \quad \forall dr', r, k \tag{31}$$

$$A_k \leq \sum_{dr=1} \sum_{dr'=1} Ti_{dr,dr'} \cdot X'_{dr,dr',k} \leq B_k, \quad \forall k \tag{32}$$

$$\sum_{i=1} Y'_{r,i,k} \leq M * Al_{r,k}, \quad \forall r, k \tag{33}$$

$$X1_r + Lmax_r Xl1_r + Wmax_r(1 - Xl1_r) \leq X1_{r'} + M \cdot (1 - a'_{r,r'}), \quad \forall r < r' \tag{34}$$

$$X1_{r'} + Lmax_{r'} Xl1_{r'} + Wmax_{r'}(1 - Xl1_{r'}) \leq X1_r + M \cdot (1 - b'_{r,r'}), \quad \forall r < r' \tag{35}$$

$$Y1_r + Wmax_r Xl1_r + Lmax_r(1 - Xl1_r) \leq Y1_{r'} + M \cdot (1 - c'_{r,r'}), \quad \forall r < r' \tag{36}$$

$$Y1_{r'} + Wmax_{r'} Xl1_{r'} + Lmax_{r'}(1 - Xl1_{r'}) \leq Y1_r + M \cdot (1 - d'_{r,r'}), \quad \forall r < r' \tag{37}$$

$$Z1_r + Hmax_r \leq Z1_{r'} + M \cdot (1 - e'_{r,r'}), \quad \forall r < r' \tag{38}$$

$$Z1_{r'} + Hmax_{r'} \leq Z1_r + M \cdot (1 - f'_{r,r'}), \quad \forall r < r' \tag{39}$$

$$a'_{r,r'} + b'_{r,r'} + c'_{r,r'} + d'_{r,r'} + e'_{r,r'} + f'_{r,r'} \geq Al_{r,k} + Al_{r',k} - 1, \quad \forall k, r < r' \tag{40}$$

$$\sum_{k=1} Al_{r,k} = 1, \quad \forall r \tag{41}$$

$$X1_r + Lmax_r \cdot Xl1_r + Wmax_r \cdot Xw1_r \leq Lk_k + M \cdot (1 - Al_{r,k}), \quad \forall r, k \tag{42}$$

$$Y1_r + Wmax_r \cdot Yw1_r + Lmax_r \cdot Yl1_r \leq WK_k + M \cdot (1 - Al_{r,k}), \quad \forall r, k \tag{43}$$

$$Z1_r + Hmax_r \leq HK_k + M \cdot (1 - Al_{r,k}), \quad \forall r, k \tag{44}$$

$$\sum_{r=1} Al_{r,k} \leq M \cdot U_k, \quad \forall k \tag{45}$$

$$Yl1_r = 1 - Xl1_r, \quad \forall r \tag{46}$$

$$Xw1_r = 1 - Xl1_r, \quad \forall r \tag{47}$$

$$Yw1_r = Xl1_r, \quad \forall r \tag{48}$$

$$X1_r, Y1_r, Z1_r, ST_{dr,k} \geq 0 \tag{49}$$

$$Al_{r,k}, Yl1_r, Xl1_r, Xw1_r, Yw1_r, U_k, depo_d, X'_{dr,dr',k}, Y'_{rik}, a'_{r,r'}, b'_{r,r'}, \tag{50}$$

$$c'_{r,r'}, d'_{r,r'}, e'_{r,r'}, f'_{r,r'} \in \{0,1\}, \quad \forall r, k, d, dr, dr', r, r'$$



In the FSt, the problem's objective function is shown in Eq. 1, which comprises reducing the package dimensions created for each customer's product arrangement. The two products  $i$  and  $j$  are made sure they do not overlap using the Eqs. 2 to 4. Eqs. 5 to 7 keep the product sizes fixed and simply alter their orientation to get the optimal arrangement. Each product's location in relation to nearby items is shown in Eq. 8. The arrangement of each product in the package is kept consistent based on the required dimensions due to Eqs. 9 to 16. The type and gender of the choice variables are shown in Eqs. 17 and 18. The ideal length, breadth, and height of each box supplied to clients are established when the first phase is finished. The primary objective of the SSt is to organize each customer's items in the appropriate vehicles and the best vehicle routing.

The SSt of the problem's total expenses are displayed in Eq. 19. It covers the expense of setting up and running prospective warehouses, the price of operating vehicles, and the price of moving goods between nodes. Eq. 20 illustrates how the movement of items to consumers reduces greenhouse gas emissions. According to Eq. 21, if a vehicle is already chosen, the best transit route may be handled. Eq. 22 makes sure that each best-case transport route should only be traveled by one vehicle at a time. According to Eqs. 23 and 24, all items may be delivered to customers using a vehicle that is designed for them. Vehicles only need to visit each node once, according to Eq. 25. A vehicle will unavoidably depart a node if it enters one, according to Eq. 26. Eq. 27 makes sure that each vehicle's load does not go beyond the designated tank capacity for that product. According to Eq. 28, the most cars possible from each chosen warehouse may be employed. Each depot may distribute goods and offer services to the full extent of its product potential according to Eq. 29. Additional restrictions for the removal of the subtour are Eqs. 30 and 31. The situation pertaining to the delivery window to the client is shown in Eq. 32. The distribution of each package to each vehicle is determined by Eq. 33. The Eqs. 34 to 39 make sure that no packets overlap and cannot fit within. According to Eq. 40, each package may only be positioned adjacent to the other in one of the six possible orientations. Each package can only fit in one vehicle, as shown by Eq. 41. The dimensions of the packages cannot be more than those of the vehicle, as shown by Eqs. 42 to 44. The kind of vehicle loading is shown in Eq. 45. Eqs. 46 to 48 make sure that only the placement type changes during loading and that the package dimensions remain constant. The type and gender of choice variables are shown in Eqs. 49 and 50.

The controlled model of the issue utilizing the RFO approach is provided below since the SSt of the optimization problem involves unknown demand factors and transfer costs. Given below is a mathematical model for the RFO method of moving vehicles between different warehouses under controlled conditions.

$$MinW_1 = E[W_1] + \xi \left( E[W_1] - W_{1(\min)} \right) + \eta \left( \sum_{r=1} \sum_{i=1} (Dem_{ri}^4 - Dem_{ri}^3 - \rho(Dem_{ri}^4 - Dem_{ri}^3)) \right) \quad (51)$$

s. t.:

$$E[W_1] = \sum_{dr=1} \sum_{dr'=1} \sum_{k=1} \left( \frac{Tr_{dr,dr'}^1 + Tr_{dr,dr'}^2 + Tr_{dr,dr'}^3 + Tr_{dr,dr'}^4}{4} \right) \cdot Dis_{dr,dr'} \cdot X'_{dr,dr',k} + \sum_{k=1} FixK_k \cdot U_k + \sum_{d=1} FixD_d \cdot depo_d \quad (52)$$

$$W_{1(\min)} = \sum_{dr=1} \sum_{dr'=1} \sum_{k=1} Tr_{dr,dr'}^1 \cdot Dis_{dr,dr'} \cdot X'_{dr,dr',k} + \sum_{k=1} FixK_k \cdot U_k + \sum_{d=1} FixD_d \cdot depo_d \quad (53)$$

$$\sum_{r=1} [\rho \cdot Dem_{ri}^4 + (1 - \rho) \cdot Dem_{ri}^3] \cdot Y'_{r,i,k} \leq CapK_k, \quad \forall i, k \quad (54)$$

$$\sum_{r=1} \sum_{k=1} [\rho \cdot Dem_{ri}^4 + (1 - \rho) \cdot Dem_{ri}^3] \cdot X'_{d,r,k} \leq CapD_{d,i} \cdot Depo_d, \quad \forall d, i \quad (55)$$

Constraints or Eqs. 20-26, 28 and 30-50 (56)

**SOLUTION METHODS**

The techniques utilized to resolve the first and SSts of PA-MDLRPSPD are presented in this section. As previously said, the objective of the first step is to place the customer's desired items in the packages in the best possible order, and the objective of the SSt is to determine where to place the trucks to deliver the packages to the consumers. As a result, in the initial step of modeling, the IWO was utilized to determine the packages' ideal dimensions. The ideal package dimensions are the result of using this strategy. The NSGA II has been utilized to find effective solutions and

create the Pareto front because of the two objective functions of the VRPSPD in the SSt. The aforementioned algorithms, in addition to the major chromosome's design in each phase, are explained in the following subsections.

- **IWO**

A numerical optimization technique called the IWO was initially presented in 2006. This approach, which employs a weed propagation pattern, is now employed in a number of optimization-related applications, including SC network design and vehicle routing. This algorithm seeks to identify the location inside a given environment where seeds will thrive. By distributing the germs, the parent grasses hunt. The FSt identifies the elements of the environment that are potential candidates for the best place to live in, and the subsequent stages involve searching for regions with superior living conditions. The fundamental goal of the strategy is to broaden the scope and power of the study over time in a region where solutions are more likely to be found. The following succinct summary of this algorithm's implementation steps:

- Creation of the starting population: In the issue search space, a predetermined number of solutions are chosen by uniform distribution.
- Calculating the degree of competence: The competence function is used to assess the participants' level of proficiency.
- Calculating the number of new grasses: The number of new seeds around each grass is calculated based on the level of competence of each parent grass.
- Distribution of new seeds: A Gaussian distribution is used to spread similar seeds from new grasses around each (old) parent grass.
- Survival battle: At each step, the aptitude of each grass is assessed, and the old and new grasses are graded according to how well they perform. Additional replies with lower eligibility will be eliminated if the final population is more than the maximum population.
- Repeating steps: Go back to stage two and do it again till we have performed the most repetitions possible.

In the initial stage of modeling, the IWO is utilized to obtain the packages' ideal dimensions.

- **NSGA II**

The first population of chromosomes produced by GAs is initially generated at random while fulfilling the constraints or boundaries of the challenge. In other words, each suggested value for the variables used to solve problems is represented by a chromosome, which is a string. The generation process, which involves repeated replications, is used to infer chromosomes. These chromosomes are assessed in each generation with the aim of optimizing them. Superior problem-solving chromosomes are more likely to spawn better problem-solving chromosomes. To speed the convergence of computations toward the overall best solution, it is crucial to design the chromosomal evaluation function. Each chromosome's evaluation function value has to be determined in the GA. Because of this, we frequently face a large number of chromosomes in situations, making it difficult in some cases to employ the GA due to the time-consuming computation of the evaluation function. Each string is given a fit number depending on the values of the goal function that were determined for the population of strings. For each discipline, the chance of selection will be based on this fitness number. A collection of strings is chosen initially based on this selection probability. By joining two chromosomes from one generation using the combination operator or by changing a chromosome using the mutation operator, new chromosomes, also known as offspring, are produced to produce the next generation. In order to maintain a constant number of threads over many computational iterations, new threads replace the threads of the initial population. Due to the stochastic dynamics that act on the selection and deletion of strands, more adapted strands have a higher probability to combine and form new strands and are more durable than other strands in the replacement phase. In an objective function-based competition, the population of sequences is completed over several generations and increases with the value of the objective function in the population of sequences, so that after a few years, the algorithm converges to the best chromosome. Ideally represents the optimal or suboptimal solution. In general, this algorithm examines the process of searching for regions of the space where the statistical mean of the objective function is greater, even if new points of the response space are searched by the genetic operators by the selection mechanism in each computational iteration. A new population that replaces an old population is often more suitable. This shows that the population is getting better with time. The search will be successful when we reach the oldest possible age, achieve convergence, or meet the stopping requirements. As a result, the best chromosome found in the last generation is chosen as the ideal answer for this issue.

- Displaying the solution and generating the initial population

*The FSt*

Think about an example where there are three items and two customers. The sizes of each item the customer ordered are listed in Table 3.

TABLE 3  
CUSTOMER PRODUCTS'S DEMENSIONS

Customer	Product		
	First	Second	Third
First	1 2 1	2 1 3	3 2 2
Second	3 1 3	2 1 2	2 1 3

The first part of the chromosome deals with the arrangement of the contents of each package, considering the position of the items in six different dimensions inside the package. As a result, each customer generates three random numbers (1, 2, and 3). The x-axis is represented by the number 1, the y-axis by the number 2, and the z-axis by the number 3. For example, the dimensions of product 2 for customer 1 are (2,1,3). That is, the product has two units of length, one unit of width and three units of height. Consider chromosome 3-1-2 randomly selected for this product. Since chromosome #1 is recorded on the y-axis, this means that the length of the product should be along that axis. Since chromosome number 2 is written on the x-axis, the width of the product must be along that axis. In addition, the height of the product must be along the z-axis. Consequently, Table 4 may be used to determine the chromosomally intended for Table 3.

TABLE 4  
FST INITIAL SOLUTION

Customer	Product		
	First	Second	Third
First	1 2 1	2 1 3	3 2 2
Second	3 1 3	2 1 2	2 1 3

↓ **Sorting**

First	1 2 1	1 2 3	2 2 3
Second	3 3 1	2 2 1	2 1 3

Solution decoding requires the same procedure. The items of each customer are first classified according to their maximum volume. Table 5 is an example of product volume sorting.

TABLE 5  
SORTING THE VOLUME OF PRODUCTS IN THE CHROMOSOME OF THE FST

Customer	Product		
	First	Second	Third
First	2	4	12
Second	9	4	6

↓ **Sorting**

First	3	2	1
Second	1	3	2

Following sorting, product 3, which is the priority of customer 1, is initially put in the lower-left corner of the box in accordance with the updated measurements in Table 4. Figure (3) demonstrates how to assemble package number 1 with the first item inside.

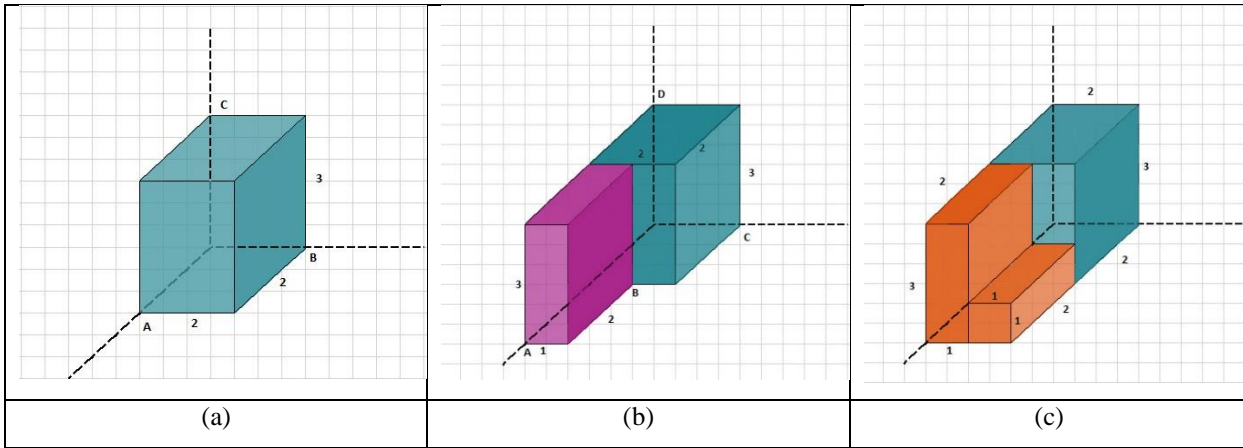


FIGURE 3  
FIRST PRODUCT ARRANGEMENT BASED ON FST INITIAL SOLUTION

Based on the priority in Table 5, it can be seen from Figure 3(a) that 3 points A, B and C are eligible for the next product, i.e. product number 2. As a result, the algorithm organizes the second product. Inside the package by choosing one of the valid locations randomly. The positioning of product 2 can be seen in Figure 3(b) (for example, the algorithm has selected point A). Four additional positions, A, B, C, and D, are eligible for the next product when the second product is located along the y-axis. The third product in this section is sorted using four acceptable points (B) selected by the random score method. The position of the third product in package #1 is shown in Figure 3(c). Ideal measurements for package length, width, and height are made when all items are placed inside. Figure 4 also shows the order of contents of package #2 for the second customer.

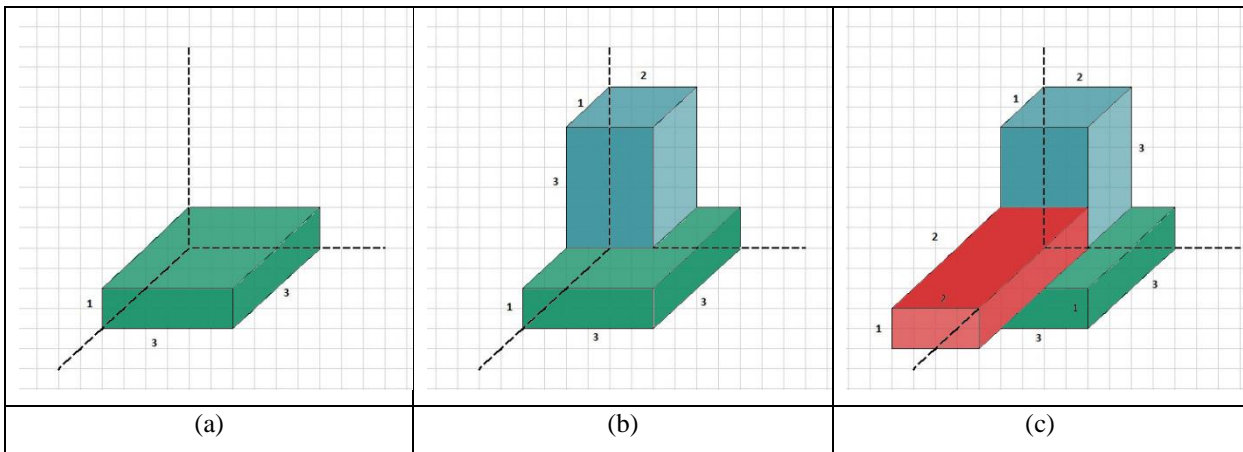


FIGURE 4.  
SECOND PRODUCT ARRANGEMENT BASED ON FST INITIAL SOLUTION

Additionally, it can be shown in Figure 4 that the Jupiter package's dimensions, 2, 4, 3, and 3, longitudinal units, are achieved. The IWO will make an effort to increase the size of each customer's parcels and plan out exactly how the contents will be arranged within each one.

*The SS<sub>t</sub>*

The major goal of the SS<sub>t</sub> of the problem is to organize each customer's items into the cars before determining the best position and route for the vehicles. The first part of this phase involves holding the IWO-derived ideal package dimensions for each customer. The ideal dimensions of the customer packages 1, 3 \* 4 \* 2, and 2, 4 \* 4 \* 3 were noticed to have been attained, according to the result that was supplied. The positioning of packets in six appropriate sizes is initially decided in this part. In this manner, exactly in accordance with Table 4, random numbers 1, 2, and 3

with the same idea are created for each box in order to establish the proper placement or arrangement of the item in the truck. This idea is demonstrated for customer packages in Table (6) of the preceding example.

TABLE 6  
DETERMINING THE APPROPRIATE DIMENSIONS OF PACKETS IN THE SST

Costomer	First Dimensions	SSt solution	Rotation operator
First	2 4 3	1 3 2	2 3 4
Second	3 4 4	2 1 3	4 3 4

The new dimensions of the customer's packages and their placement inside the cars are fixed in this way. Packages must be loaded onto vehicles in such a way that their size is kept within the available space. Consequently, as seen in Figure 5, a new chromosome is used at this point.

2	1	2	3	1	1	2
Selecting warehouse		Vehicle selection			Vehicle routing	

FIGURE 5  
SST SOLUTION

In the SSt solution in Figure 5, the above numbers are random. Customer 2 must be delivered before Client 1 in the first part of the chromosome, the VRPSPD segment. As a result, the package for customers 2 and 1 is placed in this car, provided that the dimensions of car number 1 are not greater than that (because the first number from the car selection section is 1). After visiting customer 2, this vehicle should next contact customer 1 and start routing from warehouse number 1 (since number 1 is from the selected area of warehouse 1).

- Tuning the algorithm parameter by Taguchi method

The Taguchi approach was used to alter the IWO and NSGA II parameters before to solving the problem. Tuning the parameters for meta-heuristic algorithms has as its major objective combining the best starting values of each method's parameters to maximize the effectiveness of that algorithm in the best problem search. The two-step nature of the problem and the use of two different algorithms has led to the consideration of three levels for each parameter of the IWO and NSGA II. We first calculate the value of each experiment before analyzing the data because the proposed model's single-objective function in the FSt and the dual-purpose model in the SSt. The unscaled value of each experiment (RPD), which is computed using Equations 57 for the FSt and 58 for the SSt, is then used to examine the design of the Taguchi experiment.

$$RPD = \frac{S_i - S_i^*}{S_i^*} \tag{57}$$

$$S_i = \frac{NPF + MSI + SM - Cpu\_tmie}{4} \tag{58}$$

$S_i$  and  $S_i^*$  are the result of each and the best value of the experiments performed.

TABLE 7  
TUNING THE PARAMETERS BY TAGUCHI METHOD

Solution	Parameter	L1	L2	L3	Optimal
----------	-----------	----	----	----	---------

NSGA II	Max it	50	100	<b>150</b>	150
	N pop	50	<b>100</b>	200	100
	Pm	0.3	<b>0.5</b>	0.7	0.3
	Pm	0.3	<b>0.5</b>	0.7	0.7
IWO	Max it	50	<b>100</b>	<b>150</b>	150
	N grass	50	<b>100</b>	200	50
	Max grass	30	20	<b>40</b>	40
	Min grass	10	<b>20</b>	30	20
	Mc	<b>3</b>	4	5	3

**RESULTS**

- Solving small sample problem with GAMS software

This part investigates a small example issue with six clients, three possible warehouses, four items, and six different types of heterogeneous vehicles after introducing the two-step model solution approaches. The investigation of the problem's output variables has been done using random data based on the uniform distribution function and Table 8 below due to the creation of the model.

TABLE 8  
THE RANGE OF PARAMETERS

Parameter	Range	Parameter	Range	Parameter	Range
$FixD_a$	(100000, 200000) ~U	$CapD_{a,i}$	(1600, 1850)~U	$Co_{dr,dr'}$	(5, 10) ~U
$FixK_k$	(5000, 10000)~U	$DQ_d$	2	$CapK_k$	$LK_k, LK_k, WK_k$
$Dis_{dr,dr'}$	(10, 30) ~U	$[A_k, B_k]$	(0, 200) ~U	$Hk_k$	(4, 10) ~U
$Ti_{dr,dr'}$	(10, 30) ~U	$WK_k$	(4, 10) ~U	$LK_k$	(4, 10) ~U
$Tr_{dr,dr'}$	(10, 20) ~U	(20, 30) ~U	(30, 40)~U	(40, 50)~U	
$Dem_{r,i}$	(50, 100) ~U	(100, 150)~U	(150, 200)~U	(200, 300)~U	

Each consumer (demand node) in this example has a desire for four distinct items, and six various vehicle kinds are also taken into account. The outputs of the problem-solving are taken to be 0.5 due to the uncertainty of the specified model. The ideal size of the packages sent to clients must be established at the first phase of the problem. In order to do this, the ideal dimensions of the small sample size problem have been determined using GAMS software. Table 9 displays the ideal measurements for each package that may be shipped to customers.

TABLE 9  
OPTIMAL DEMENSIONS OF EACH PACKAGE

Customer	Length	Width	Height
1	4.42	4.71	4.67
2	4.56	4.89	4.82
3	4.76	5.00	4.44
4	4.50	4.59	5.05
5	4.96	4.39	4.63
6	4.57	4.94	5.00

The ideal size of the products transported to clients by vehicles are decided upon before moving on to the second step, which involves two critical strategic and tactical considerations.

Making decisions in this area includes choosing the best quantity and location for the warehouse as well as the best route for delivering items to customers. The Epsilon constraint approach has been employed in order to find solutions since the created model serves two purposes. The effective solutions found after using the Epsilon Constraint approach to address the issue of small-size samples are displayed in Table 10.

TABLE 10  
EFFICIENT SOLUTIONS OBTAINED FROM SOLVING THE PROBLEM BY EPSILON CONSTRAINT METHOD

The solution is efficient	Overall cost	CO2 emissions
1	177698,76	898,10
2	178499,29	869,55
3	215731,41	808,81
4	219524,69	781,55
5	219665,37	753,67
The best value of the objective function	177674,17	749,80

The Epsilon constraint approach was used to solve the problem of small-size samples, and the findings are shown in Table 10 as five effective solutions. Figure 6 also depicts the Pareto front that was produced after the Epsilon approach was used to address the issue of small size samples. Based on the findings from Table 10 and Figure 6, it is evident that the quantity of greenhouse gas emissions has dropped as transmission costs have increased.

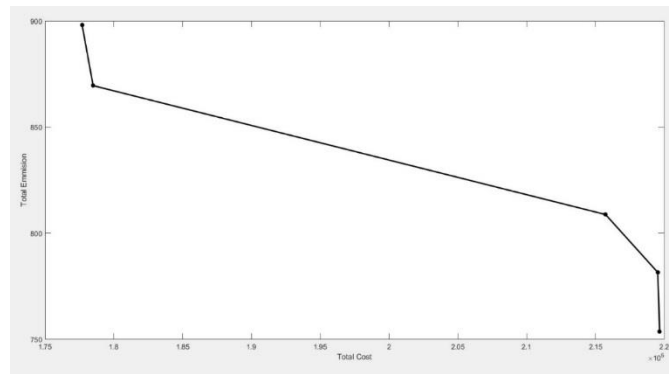


FIGURE 6  
PARETO FRONT OF SAMPLE PROBLEM

Warehouse number 2 has been determined to be the only one that is ideal for delivering goods to consumers based on the results of the study of the efficient solution number 1 and the outputs from the issue-solving process. Figure 7 displays the first effective solution and the ideal routing for packet transfer by vehicles in the small sample.

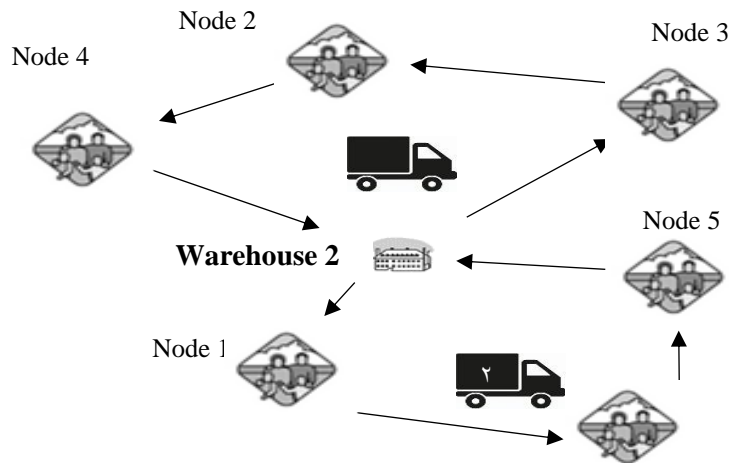


FIGURE 9  
OPTIMAL LOCATION-ROUTING OF SAMPLE

The issue's sensitivity and the impact of the uncertainty rate on the price and the volume of greenhouse gas emissions are further examined once the small sample size issue is looked at.

**PROBLEM SENSITIVITY ANALYSIS**

Investigation is done into how the uncertainty rate affects the values of the problem's first and second objective functions. In this case, it was assumed that the uncertainty rate in the preceding section was 0.5. The absence rate's value is raised from 0.1 to 0.9 in this section. Additionally, Table 11 displays the value of the relevant goal function in the first and SSs.

TABLE 11  
EFFECT OF UNCERTAINTY RATE ON THE VALUES OF OBJECTIVE FUNCTIONS

The uncertainty rate	The FSt objective function	Overall cost	CO2 emissions	Selected warehouses	Vehicles used
0.1	72.20	177437,75	898,81	2	3-2
0.2	75,32	177445,38	898,81	2	3-2
0.3	78,47	177519,15	898,81	2	2-1
0.4	81,63	177678,37	898,81	2	2-1
0.5	84,96	177698,76	898,80	2	2-1
0.6	88.31	178057,95	898,81	2	2-1
0.7	91,87	178101,40	898,81	2	4-2
0.8	95,35	178281,05	898,81	2	4-2
0.9	98,79	378418,14	825,35	3-2	3-2-1

Following the impact of the uncertainty rate on the values of the objective functions, it is possible to draw the conclusion that as the uncertainty rate has grown, so has the volume of demand and, as a result, the size of packages. As a result, in the first phase, which comprises the optimal sum of the ideal dimensions of the packages shipped to clients, the objective function's value is enhanced. Additionally, it has been shown that as demand has grown, transportation costs have risen in tandem with the uncertainty rate. Additionally, the quantity of greenhouse gas emissions has stayed constant across changes in the uncertainty rate since order size has no impact on the value of the second objective function. Due to the usage of two warehouses and the close proximity of consumers to warehouses, the distance of transportation and subsequently the volume of greenhouse gas emissions has decreased, although only at an uncertainty rate of 0.9. Figure 10 depicts the pattern of changing objective function values at various rates of uncertainty.

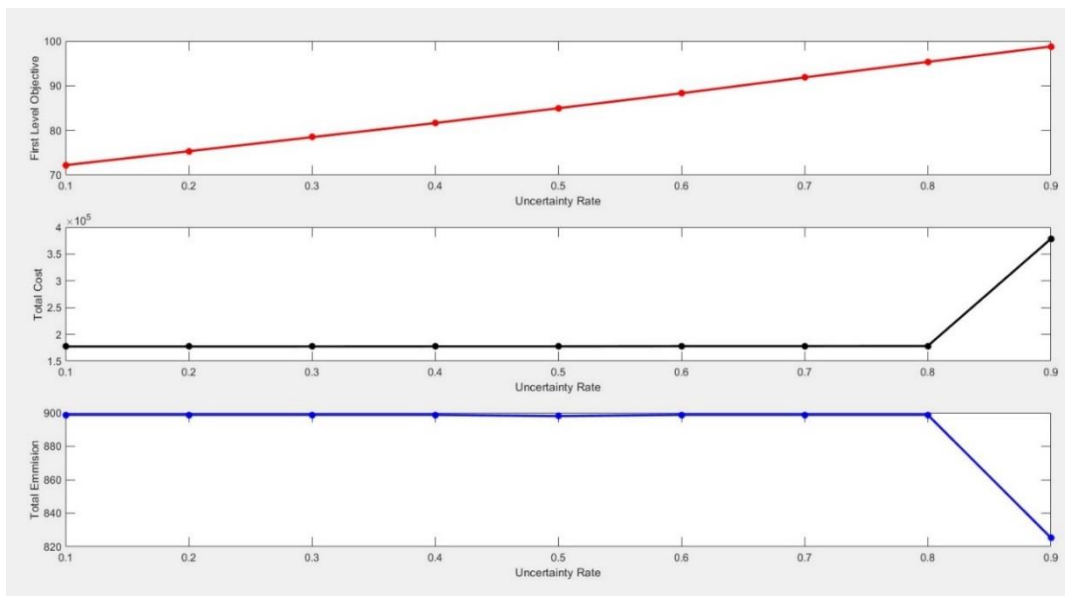


FIGURE 10  
THE TREND OF CHANGES IN THE VALUES OF OBJECTIVE FUNCTIONS AT DIFFERENT RATES OF UNCERTAINTY

- Solving the sample problem by meta-heuristics algorithm

The issue of a limited sample size is resolved in this part, which also studies the problem's output variables and assesses the solution identified in the Fst and SSts of the issue. As a result, the suggested meta-heuristic methods are used to resolve the problem of the small size sample created in the previous section. The outcomes of these two problem-solving techniques are contrasted below. Due to the two-stage structure of the created model, it is possible to produce effective solutions in the SSt (optimal placement and routing of transportation), using the NSGA II, after achieving the optimal dimensions of the packages in the FSt using the IWO. The ideal package dimensions made accessible to the customer by the IWO are shown in Table 12.



TABLE 12  
OPTIMAL DIMENSIONS OF EACH PACKAGE WITH IWO

Customer	Length	Width	Height
1	4.67	4.91	4.95
2	4.98	4.71	5.05
3	4.98	4.63	4.18
4	4.50	4.59	5.05
5	4.53	4.86	4.63
6	4.57	4.94	5.00

The ideal value of the objective function in the FSt was determined using GAMS software findings, and it was equivalent to 84.96 longitudinal units in the 126.47-second time model. While the IWO's objective function has a value of 85.73 longitudinal units, it is 4.56 seconds in the time model. As a consequence, the relative difference between the IWO outcomes of the GAMS program is just 0.9% when taking into account the values of the two objective functions. The Pareto front is formed using the NSGA II after the packages have been given their ideal dimensions. The effective solutions produced from the NSGA II and Pareto front comparisons of the Epsilon method constraint and the NSGA II for the small sample issue are shown in Table 13 and Figure 11.

TABLE 13  
EFFICIENT SOLUTIONS OF SAMPLE PROBLEM WITH NSGA II

Efficient solution	Total cost	CO2 emissions
1	178459,76	912,10
2	19126,34	890,14
3	183476,16	851,36
4	199347,46	833,81
5	218555,30	777,16
6	220346,74	758,02
7	223142,36	750,39

The results show that the NSGA II approach has produced seven efficient solutions whereas the Epsilon Constraint method has produced a limit of five. As a result, in order to compare the approved efficient solutions, comparison indicators have been employed owing to the two objective functions of the produced model. According to the results, there is no discernible difference between the NSGA II and the Epsilon constraint approach in terms of the means of the first and second objective functions. In order to get the mean values of the first objective function, the quantity of effective solutions, the maximum amplitude, and the computing time, the NSGA II outperformed the Epsilon technique. This leads to the conclusion that the NSGA II is a highly effective tool for solving the actual sample problem.

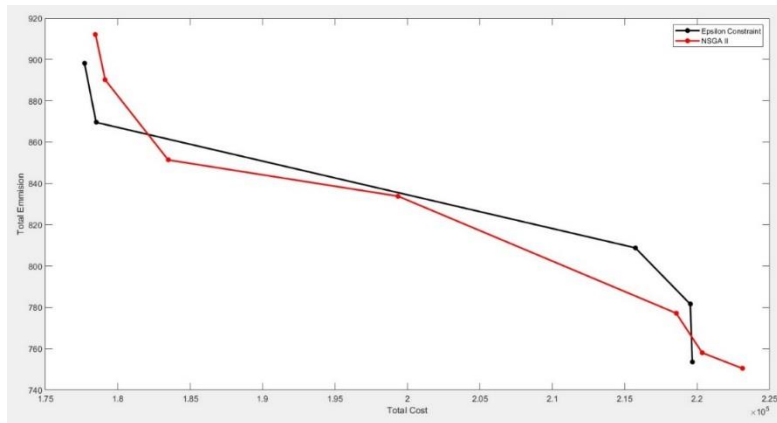


FIGURE 11  
COMPARISON OF PARETO FRONT OBTAINED WITH EC AND IWO

Additionally, Figure 12 displays the ideal transport routing for the first effective NSGA II solution in the SSSt of the issue.

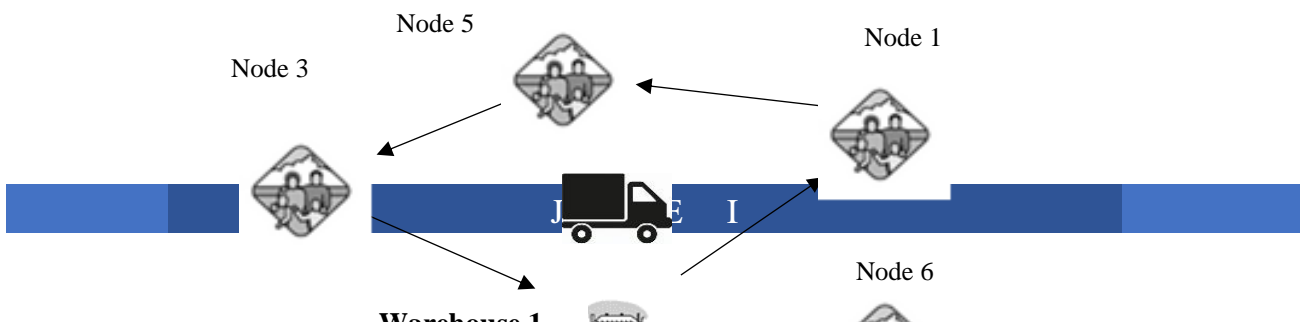


FIGURE 12  
VEHICLE ROUTING OBTAINED WITH NSGA II

- Solve sample problems in a larger size

The issue of real sample in Safir Broadcasting Company in Iran is examined in this part due to the high effectiveness of the IWO in acquiring the ideal package dimensions and the high effectiveness of the NSGA II in finding effective solutions in the SS<sub>t</sub>. One of Iran's biggest economic complexes, Safir Holding, is made up of a number of manufacturing and distribution businesses. Each of these businesses has a purpose, but Safir Broadcasting Company in Iran serves a crucial function as a conduit between holding merchants and manufacturing firms. The collection of processes associated to this firm are optimized in a thorough and trustworthy manner by executing the current study. This improvement will promote operational and situational clarity, which will eventually boost productivity and cut expenses. The results of the article can empower this company, which plays a decisive role in the whole complex due to the newness of this project in the whole complex, considering the current sensitive competitive conditions that are becoming more delicate day by day with the opening of economic borders. . The country is in a better situation. Give the company domestic and international competitors. The dataset required to solve the problem in a real-world case study is outlined below:

- There are 12 months from 2019 included in the total number of periods for settling the issue.
- There are 47 firms that supply goods to Safir Broadcasting Company.
- Twelve major centers are located in the nation's provinces, making up Safir Broadcasting Company's total number of distribution centers. Only Tehran's sales distribution hub has been considered in this concept.
- The demand regions are chosen in ten clusters that are adjacent to one another due to the restricted sales distribution hub.
- Due to the Safir Broadcasting Company's enormous product selection, 76 goods in 27 categories were chosen based on the highest sales (90%) and included in the model's solution.
- The firm owns 2 Isuzu vehicles, 2 Alvand vehicles, and has leased 6 Nissan vehicles, 1 Isuzu vehicle, and used them for product distribution and sales in the greater Tehran area.

The Pareto front is generated as the end outcome of utilizing meta-heuristic methods to solve the issue, as shown in Figure 13. The outcomes of Figure (14) have therefore been suggested to the company's management as the most desirable efficient alternative for employing vehicles after examining the effective solutions.

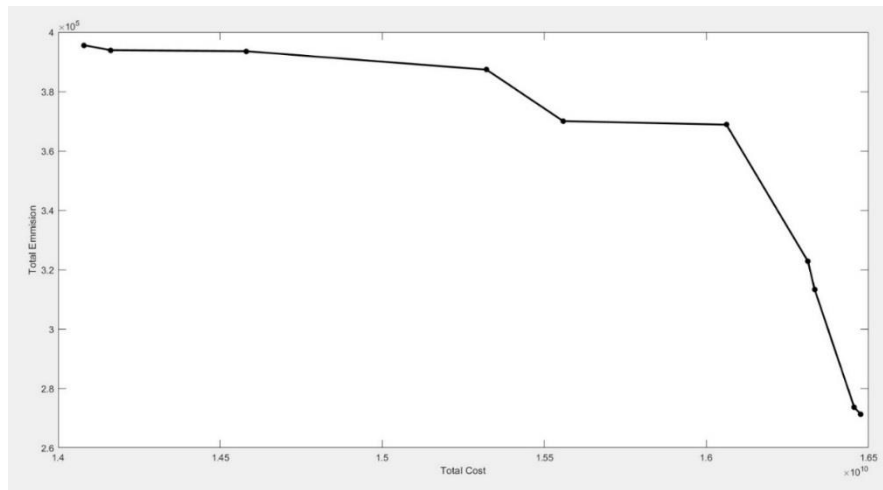


FIGURE 13  
 PARETO FRONT OBTAINED FROM SOLVING A REAL SAMPLE PROBLEM WITH NSGA II ALGORITHM

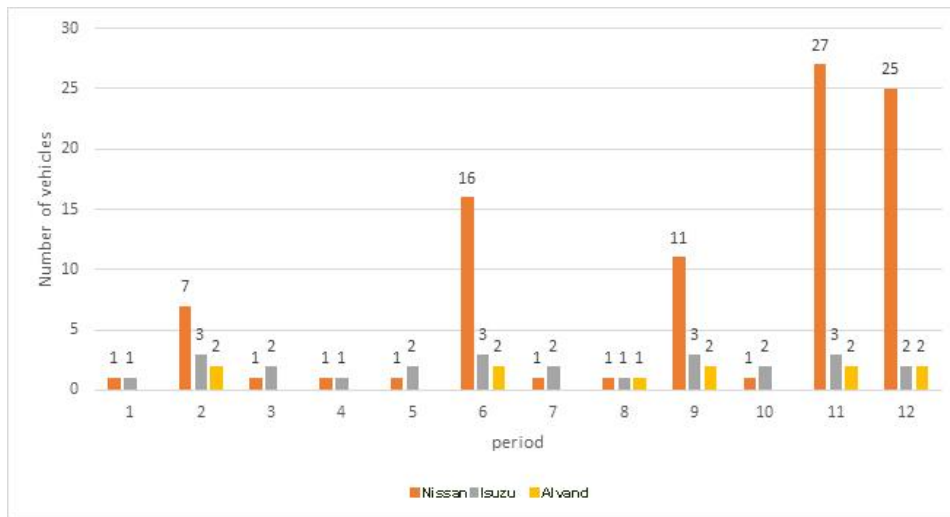


FIGURE 14  
 NUMBER OF VEHICLES USED TO SOLVE THE REAL SAMPLE PROBLEM

**CONCLUSION**

A PA-MDLRSPD under uncertainty is presented in this research. The primary objective of the developed model's initial step is to organize goods into packages that may be delivered to customers. Finding suitable warehouses and planning the best route for trucks are the key objectives of the SS<sub>t</sub>. As a result, the goal functions of the FS<sub>t</sub> and SS<sub>t</sub>s are the sum of the packages' ideal dimensions and the simultaneous optimization of location-routing costs and greenhouse gas emissions, respectively. In this study, the RFO approach is utilized to regulate the uncertainty parameters due to the uncertainty of the developed model and taking into account the demand parameters and transmission costs in the form of trapezoidal fuzzy numbers. Due to the NP-Hard nature of the intended model, the IWO is utilized in the first step to establish the ideal package dimensions and product placement, and the NSGA II is employed in the SS<sub>t</sub> to discover effective solutions to the problem. The experiment's findings demonstrate the IWO's excellent effectiveness in achieving the FS<sub>t</sub>'s ideal results (0.93% relative difference).

The remarkable efficiency of this approach for tackling issues involving large sample sizes was further demonstrated by the absence of substantial discrepancies between the effective solutions derived from the NSGA II with the Epsilon constraint technique. The model's use in an actual case study at Safir Company produced findings that indicated the availability of 10 effective solutions. The best suitable efficient solution for the management of Safir Company has been offered after an assessment of the effective alternatives. The following ideas are offered to enhance the article in light of the aforementioned research:

- Taking into account the social aspect's goal functions, such as the number of drivers

- Taking corruption into account using the study in issue modeling paradigm
- Using novel multi-objective meta-heuristic algorithms and contrasting different approaches to the problem at hand to arrive at the required efficient solutions.
- Modeling the issue under the grid's game title and using techniques for two-step to one-step problem conversion

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