A Non-radial Approach for Setting Integer-valued Targets in Data Envelopment Analysis

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Abstract

Data Envelopment Analysis (DEA) has been widely studied in the literature since its inception with Charnes, Cooper and Rhodes work in 1978. The methodology behind the classical DEA method is to determine how much improvements in the outputs (inputs) dimensions is necessary in order to render them efficient. One of the underlying assumptions of this methodology is that the units consume and produce real valued data. This paper deals with the extension of this methodology for the case of integer-valued data. Based on an additive DEA model, a mixed integer linear programming model is proposed for setting integer-valued targets. An empirical example illustrates the approach.

Keywords: Data envelopment analysis (DEA); Efficiency; Mixed integer linear programming (MILP); Target setting.

1. Introduction

Data envelopment analysis (DEA) is a mathematical programming approach used for performance analysis of observed decision making units (DMUs). One of the main purposes of this analysis is to determine the best performers along with guidelines for improving the rest. Typically, these guidelines stem from benchmarking against a set of good performers in order for a poor performer to improve. As we know from the literature, when utilizing DEA to evaluate a set of decision making units (DMUs), an efficient frontier is created that determines which DMUs are performing well (efficient) and, which are not (inefficient), or, in Pareto-Koopmans' sense, which are non-dominate and, which are dominate.

One of the underlying assumptions of conventional DEA is that the units consume and produce real valued data. However, there are many occasions in which some inputs and/or outputs must only take integer values. In the former DEA literature, Integer-valued DEA (IDEA) has been recently studied from different points of view. The most of these approaches deal with the integer-valued data as categorical or ordinal data (see for example [1]; [5]; [9]; among others). One can find a few works focused on the integer numbers. Lozano and Villa [8] were among the first scholars to address a DEA based MILP model to guarantee the required integrality of the computed targets with integer valued inputs and outputs. The approach introduced by these authors is fully analyzed by Kuosmanen and Kazemi Matin [6, 7].

They developed and generalized the axiomatic foundation for DEA models that assumes subsets of input and output variables to be integer-valued. In these papers, it is shown that the conventional axioms of production technologies fail if DMUs are restricted to operate with integer valued input and output quantities and new refinements of the classical axioms consistence with integer environment are introduced. It is also shown that the production possibility set proposed by Lozano and Villa (2006) satisfies the minimal extrapolation principle under the new set of axioms. Kauosmanen and Kazemi Matin [6, 7] made modifications to Farrell's efficiency measure to take into account the possibility of integer valued inputs and outputs and presented a MILP formulation for computing it. Here, due to space limitations, we will not examine the details of their axiomatic approach. For details of the results see [6, 7].

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In this paper, a computational point of view is followed and by using the radial model introduced in [8] we will discuss in details the influences of the integrality assumption over its introduced targets.

Lozano and Villa's approach, based on a modified CCR model [3], examines the smallest scale to assess an efficiency index and projects the unit under evaluation on an integer-valued target point.

This paper goes beyond the axiomatic foundation of their model to show that the results of the introduced radial DEA model may be unstable and extremely sensitive to small variations of data and, hence, can lead to some inconsistencies. Accordingly, a modified additive model will be introduced later for setting integer-valued targets which may overcome this problem.

The rest of the paper is organized as follows. In section 2, Lozano and Villa's radial DEA model is introduced. A simple numerical example shows our motivation and reflects the influences of data perturbation on the introduced targets of their radial model. Section 3 is devoted to introducing our modified additive model for target setting. An empirical example on 42 university departments of Karaj Islamic Azad University (KIAU) is presented in section 4 to illustrate the approach further. Section 5 concludes the paper.

2. A Radial Approach

Following the most standard DEA notation, for DMU_j let x_{ij} and y_{rj} show the amounts of its i th input and r th output respectively, where $i=1,\cdots,m$, $r=1,\cdots,s$ and $j=1,\cdots,n$.

In order to assess the relative performance of the existing DMU in the variable returns to scale (VRS) integer-valued scenario and based on a classical DEA models, Lozano and Villa (2006), introduced the following MILP model.

$$\begin{aligned}
Min \quad & \theta_k - \varepsilon\Big(\sum_{j=1}^{n} s_i^- + \sum_{j=1}^{n} s_r^+\Big) \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\lambda_j = x_i \qquad i=1,\cdots,m \\
& x_i = \theta_k x_{ik} - s_i^- \quad i=1,\cdots,m \\
& \sum_{j=1}^{n} y_{rj}\lambda_j = y_r \qquad r=1,\cdots,s \\
& y_r = y_{rk} + s_r^+ \quad r=1,\cdots,s \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& s_i^+, s_r^-, \lambda_j \ge 0, \forall i, \forall r, \ \forall j \\
& x_i, y_r \in Z_+, \forall i \in I, \forall r \in O
\end{aligned} \tag{1}$$

Where $I \subseteq \{1,2,\cdots,m\}$ and $O \subseteq \{1,2,\cdots,s\}$ show subsets of indices associated with integer-valued inputs and outputs, respectively. They used the optimal solution of this model for efficiency evaluation and setting targets by imposing the integrality assumption on the computed targets of DMU_k in the VRS DEA model.

Having acknowledged the authors for their novel approach in this area, here we attempted to examine their model from a computational point of view. In this paper, it is shown that the results are sensitive to the data errors which are likely to occur in the software packages, based on their designed algorithms for solving MILP models (e.g. cutting plane approaches).

2.1 Motivation

The model (1) was applied to sample data presented in Table 1. Four units are considered with two inputs and one constant output. All variables are assumed to take integer values. Table 1 shows the efficiency scores (θ_k^*), and computed targets of the model (1). Due to the results of the model (1), the integer valued units A, B and D are BCC ([2]) integer-efficient and unit C is inefficient. Note that C has the unit D as its target point.

Table 1
Sample data with two inputs and one output

DMU	x_1	x_2	y	θ_k^*	s_1^*	s_2^{-*}	s_1^{+*}	x_1^*	x_2^*	y^*
A	0	3	1	1	0	0	0	0	3	1
B	2	0	1	1	0	0	0	2	0	1
C	0	4	1	0.7	0	0	0	0	3	1
D	1	1	1	1	0	0	0	1	1	1

Now, we show that the above radial approach may be unstable in the presence of data perturbations. Consider the following table.

Table 2
Perturbed data and results of the model

DMU	x_1	x_2	Y	θ_k^*	s_1^*	s_2^{-*}	s_1^{+*}
A′	0.0001	3	1	10000	0	29999	0
B′	2	0.0001	1	10000	19999	0	0
C′	0.0001	4	1	10000	0	39999	0
D	1	1	1	1	0	0	0

The data are given by replacing the zeros in Table 1 with relatively small values. The new results show that the model (1) is quite sensitive to the small noises. In production theory, we are unable to interpret these results for the efficiency scores and slacks value in improving a unit. The results also lead us to a different set of targets. These inconsistencies could be even worse if we use smaller noises and the model could lead us to an unbounded solution. In the following section, we will reexamine this example with a new procedure.

In introducing a new procedure, our idea is similar to the classical techniques in solving MILP models and is based on the results obtained from the corresponding real-valued DEA model. If the coordinates of the computed target did not satisfy the integrality assumption(s) a non-dominated integer-valued target point will be computed by solving a modified additive-DEA model. Our approach can be easily implemented and rectifies the above-mentioned difficulties in applications.

3. A Modified Additive Model for Setting Integer-Valued Targets

To set an integer-valued efficient target for DMU_k, first, we solve the associate DEA model (here we are using a variable returns to scale technology); If the computed target point $(\hat{x}_k, \hat{y}_k)$, does not satisfy the integrality assumption(s), we switch to a non-dominated point in a small neighborhood of this point based on the following approach. The possible input excesses and output shortfalls for $i \in I$ and $r \in O$ which may be caused in the rounding non-integer components of $(\hat{x}_k, \hat{y}_k)$ to its near feasible point in the production set, is discovered by solving the following MILP model.

$$
\begin{aligned}
\max \quad & \sum_{i\in I}\delta_i^- + \sum_{r\in O}\delta_r^+ \\
s.t. \quad & \sum_{j=1}^{n} x_{ij}\lambda_j \le \lceil \hat{x}_{ik} \rceil - \delta_i^- && \forall i \in I \\
& \sum_{j=1}^{n} x_{ij}\lambda_j \le \hat{x}_{ik} && \forall i \notin I \\
& \sum_{j=1}^{n} y_{rj}\lambda_j \ge \lfloor \hat{y}_{rk} \rfloor + \delta_r^+ && \forall r \in O \qquad (2)\\
& \sum_{j=1}^{n} y_{rj}\lambda_j \ge \hat{y}_{rk} && \forall r \notin O \\
& \sum_{j=1}^{n} \lambda_j = 1 \\
& \delta_i^+, \delta_r^-, \lambda_j \ge 0, \forall i, \forall r, \forall j \\
& \delta_i^-, \delta_r^+ \in Z, \qquad \forall i \in I, \forall r \in O
\end{aligned}
$$

Let $(\lambda^*, \delta^{-*}, \delta^{+*})$ as the optimal solution of the above model, then the following values give the components of computed target point for the unit k.

$$
x_{ik}^* = \begin{cases} \lceil \hat{x}_{ik} \rceil - \delta_i^{-*} & i \in I \\ \hat{x}_{ik} & i \notin I \end{cases}, \quad y_{rk}^* = \begin{cases} \lfloor \hat{y}_{rk} \rfloor + \delta_r^{+*} & r \in O \\ \hat{y}_{rk} & r \notin O \end{cases} \qquad (3)
$$

To incorporate decision makers preferences into the evaluation, one can replace the objective function in the model (2) with a non-negative weighted sum of slacks as $\sum_{i\in I} w_i^- \delta_i^- + \sum_{r\in O} w_r^+ \delta_r^+$.

By means of the following theorem, now we have a non-dominated integer-valued activity as a target point of the unit under evaluation.

Theorem 1. *The integer-valued activity* (x_k^*, y_k^*) *computed by (3) is a BCC-Integer efficient.*

Proof. Otherwise, there exists a feasible integer-valued activity like $(\tilde{x}, \tilde{y})$ in

$$
T'_{VRS} = \left\{ (x,y) \mid \exists(\lambda_1,\cdots,\lambda_n) \ge 0;\, x_i \ge \sum_j x_{ij}\lambda_j,\, y_r \le \sum_j y_{rj}\lambda_j, \right. \\
\left. \sum_j \lambda_j = 1, \quad x_i, y_r \in Z_+, \forall i \in I, \forall r \in O \right\}
$$

which dominates the introduced target, i.e. $\tilde{x}_i \le x_{ik}^*$ and $\tilde{y}_r \ge y_{rk}^*$ and the inequalities hold strictly at least in one index. Without loss of generality, we can assume that there exist $i' \in I$ with $\tilde{x}_{i'} < x_{i'k}^*$. Since $(\tilde{x}, \tilde{y}) \in T'_{VRS}$, then there exists a non-negative vector $(\lambda'_1, \cdots, \lambda'_n)$ such that

$$
\begin{aligned}
& \sum_j x_{i'j}\lambda'_j \le \tilde{x}_{i'} \\
& \sum_j x_{ij}\lambda'_j \le \tilde{x}_i \quad i = 1,\cdots,m, \,\&\, i \ne i' \qquad (4)\\
& \sum_j y_{rj}\lambda'_j \ge \tilde{y}_r \quad r = 1,\cdots,s \\
& \sum_j \lambda_j = 1 \qquad \tilde{x}_i, \tilde{y}_r \in Z_+, \forall i \in I, \forall r \in O
\end{aligned}
$$

Combining (3) and (4) with the dominance assumption we get

$$
\begin{aligned}
& \sum_j x_{i'j}\lambda'_j \le x_{i'k}^* - 1 = \lceil \hat{x}_{i'k} \rceil - (\delta_{i'}^{-*} + 1) \\
& \sum_j x_{i'j}\lambda'_j \le x_{ik}^* = \lceil \hat{x}_{ik} \rceil - \delta_i^{-*} && i \in I \,\&\, i \ne i' \qquad (5)\\
& \sum_j x_{ij}\lambda'_j \le \hat{x}_{ik} && i \notin I \\
& \sum_j y_{rj}\lambda'_j \ge \lfloor \hat{y}_{rk} \rfloor + \delta_r^{-*} && r \in O \\
& \sum_j y_{rj}\lambda'_j \ge \hat{y}_{rk} && r \notin O \\
& \sum_j \lambda_j = 1 \\
& \forall j \quad \lambda'_j \ge 0, && \tilde{x}_i, \tilde{y}_r \in Z_+, \forall i \in I, \forall r \in O
\end{aligned}
$$

Now, $(\lambda', \delta^{'-}, \delta^{'+})$ is a feasible solution of the model (2) with

$$
\delta_i^{'-} = \begin{cases} \delta_{i'}^{-*} + 1 & i = i' \\ \delta_i^{-*} & i \in I \,\&\, i \ne i' \end{cases}, \quad \delta_r^{'+} = \delta_r^{+*} \quad r \in O
$$

Furthermore, we have $\sum_{i\in I}\delta_i^{-*} + \sum_{r\in O}\delta_r^{+*} < \sum_{i\in I}\delta_i^{'-} + \sum_{r\in O}\delta_r^{'+}$

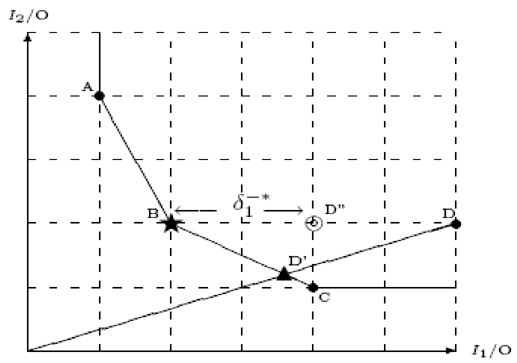

Fig. 1. Integer target setting constant output and two case

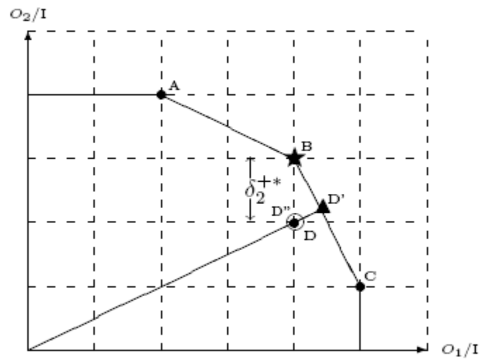

Fig. 2. Integer target setting constant input and two output case

which contradicts with the optimality condition of the vector $(\lambda^*, \delta^{-*}, \delta^{+*})$ for the model (2). This completes the proof.

Illustration

By starting from the computed target of a real-valued DEA model, the above MILP model moves toward the frontier of PPS with integer valued steps for possible improvements of selected integer-valued unit.

Figures 1 and 2 here are used to illustrate the procedure further. A typical PPS of integer-valued data with VRS technology in three dimensions (two inputs and one constant output) is depicted in Fig.1. First, the unit D' with non-integer coordinates is obtained by solving the radial BCC model in evaluating performance of the unit D. With our rounding method, we get D" which is a dominated-integer valued production plan. Now, by using the model (2) we can remove the integer input excesses of the first input with the value $\delta_1^{-*} = 2$. This leads to the non-dominated integer-valued target point B which is marked by $*$ in the figure. Fig. 3 also shows a similar situation in outputs space.

Note that the model (2) is obviously feasible and its objective value is bounded above by the amount $\sum_{i\in I}\lceil \hat{x}_{ik} \rceil + \sum_{r\in O} \max_j \{y_{rj}\}$. In addition, and because in the introduced non-radial approach, the slacks could just vary over a limited range within the production space, the results will be stable against data errors.

Finally, in comparison with the model (1), the new introduced MILP model has less $m+s$ variables and also less $2m+2s+1-(|I|+|O|)$ constraints, which could alleviate the computational burden. In the next section of results, a comparison between the number of branches and also computed projections of two models in an empirical study is shown.

Here, we come back to Table 2 and reexamine the effect of the small error on the results of the new model.
The reported results in the Table 3 shows that the introduced MILP model could easily handle the perturbed data and the computed targets are reasonable.

In the next section, we will apply both approaches in an empirical study to make a better comparison of their features.

Table 3
Perturbed data and target setting with the model 3.1

DMU	δ_1^{-*}	δ_2^{-*}	δ_1^{+*}	x_1^*	x_2^*	y^*
A′	0	2	0	1	1	1
B′	1	0	0	1	1	1
C′	0	2	0	1	1	1
D	0	0	0	1	1	1

4. An Empirical Application

To illustrate the above procedures and for comparison purposes, we apply models 1 and 2 to a data set of 42 departments of IAUK. The data set is available in Kuosmanen and Kazemi Matin [6].

In this evaluation, each department has post graduate students (x_1), bachelor students (x_2), masters students (x_3) as its inputs and graduated students (y_1), members for scholarship (y_2), research products (y_3) and manager satisfaction (y_4) as the outputs. Note that all coordinates have integer structure and y_4 is an ordinal data.

To make a comparison with reference results, the efficiency scores and targets computed by the BCC model are shown in Table 4 (Appendix 1.). Note that, except for the observed efficient DMUs, the rest of units have fractional valued target and we need to run the model (2) for these units. The results of the models (1) and (2) for the data setting are summarized in Table 5 (Appendix 1.).

The computed targets in our integer DEA model come very close to DEA BCC model, and it is notable to see differences between the results of mode (1) and BCC model; for example, see the targets for units 42, 41, 1 and 23. This is also true for benchmarks obtained for these two models. Moreover, there exist considerable differences between the computed targets in our integer DEA model and those obtained by the model (1).

As we can see, the computed targets based on model (1) contain some dominated activities, which are marked by × in Table 5 (Appendix 1.). This means that using model 1 can lead to suboptimal efficiency scores and performance targets.

Finally, the empirical study also highlights more discrimination power in the introduced approach in setting integer-valued targets in comparison with the other approaches in the literature.

5. Conclusion

In this paper, an additive model is presented for setting targets with integer-valued coordinates in activity analysis. It is shown that the results of our MILP formulation are stable against the data errors; the computed targets are non-dominated points of production set and very close to their targets with classical DEA models. An empirical application study on 42 university departments further illustrated the differences resulting from alternative approaches. The application also shows the computational advantage of our non-radial model as well as its higher discrimination power.

As suggestions for future study, it seems to be interesting for the upcoming researchers to adapt non-radial models like slack based measure ([10]) for the case of integer-valued data and define a new complete efficiency measurement model.

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Appendix 1.

Table 4
Efficiency scores and BCC projections [b]

DMU	θ_j	Benchmarks $\left(\lambda_j^* > 0\right)$	BCC Projections						
			$\hat{x}_{1j}$	$\hat{x}_{2j}$	$\hat{x}_{3j}$	$\hat{y}_{1j}$	$\hat{y}_{2j}$	$\hat{y}_{3j}$	$\hat{y}_{3j}$
1	0.8842	16(0.07)17(0.88) 18(0.05)	0	230.7762	0	225	6.59	1	3.72
2	1	-	0	170	56	213	2	0	3
3	1		0	281	70	326	2	0	3
4	0.9514	2(0.19) 14(0.61) 17(0.19) 18(0.11)	0	131.2932	31.3962	159	1	0.33	2.37
5	1		164	0	0	52	1	0	3
6	1	-	291	815	0	1014	2	2	2
7	1		0	0	61	50	0	0	4
8	0.7928	5(0.55) 17(0.45)	89.5864	75.316	0	103.72	3.72	0	3.45
9	1	-	0	727	0	675	3	0	3
10	0.9863	9(0.79) 18(0.11) 32(0.11)	0	762.4099	0	697	2.57	0.32	3
11	0.8333	7(0.78) 14(0.22)	0	0	54,9978	46	0	0	3.56
12	0.3402	5(0.39) 17(0.40) 31(0.20)	117.7092	67.0194	0	132	3.83	0	3.40
13	1	-	0	988	0	812	8	10	2
14	1	-	0	0	34	32	0	0	2
15	0.8609	13(0.04) 16(0.95) 18(0.01)	0	684.4155	0	601	6	11.82	2
16	1	-	0	627	0	591	6	12	2
17	1	-	0	166	0	166	7	0	4
18	1	-	0	761	0	761	0	3	2
19	1	-	193	124	0	293	0	0	3
20	1	-	484	0	0	361	0	0	1
21	0.8816	16(0.27) 17(0.47) 18)(0.26)	0	455.7872	0	434	4.92	4	2.95
22	0.8756	16(0.24) 17(0.38) 18(0.38)	0	.511.876	0	492	4.12	4	2.77
23	0.8404	16(0.10) 17(0.30) 18(0.60)	0	573.1528	0	565	2.7]	3	2.60
24	0.7589	16(0.07)17(0.55) 18(0.38)	0	428.7785	0	423	4.26	2	3.1
25	0.7406	16(0.17)17(0.50) 18(0.33)	0	446.5818	0	433	4.53	3	2.01
26	0.8937	16(0.02) 17(0.72) 18(0.27)	0	333.3501	0	332	5.11	1	3.43
27	0.9969	16(0.22) 17(0.66) 18(0.11)	0	345.9243	0	328	5.98	3	3.33
28	1	-	0	0	70	51	0	3	4
29	0.6181	J 7(0.87) 36(0.12)	0	202.7368	0	193.38	6.62	1	3.87
30	0.6217	17(1)	0	166	0	166	7	0	4
31	1	-	262	0	0	219	3	0	3
32	1	-	0	1023	0	794	2	0	4
33	1	-	223	0	535	232	14	6	4
34	0.9649	14(0.24) 16(0.13) 17(0.27) 28(0.09)	0	256.6634	14.4735	238	3.75	4	3
35	1	-	172	375	0	547	4	3	3
36	1	-	0	460	0	385	4.	8	3
37	1	-	223	0	535	232	14	6	4
38	1	-	0	1202	58	1158	12	0	3
39	0.3743	3(0.19) 17(0.41) 18(0.25) 28(0.08)	0	383.6575	22.8323	394	4	1	3.25
40	0.9710	7(0.33) 28(0.67)	0	0	66.999	50.67	0	2	4
41	0.8064	5(0.09) 31(0.91)	253.20960	0	0	204	2.82	0	3
42	0.7357	20(0.05) 31 (0.95)	272.9447	0	0	226	2.85	0	2.90

[b] The EMS software is used for these computations.

Table 5
5 Integer projections and comparisons between models 3.1 and 2.1[c]

DMU	δ_{ij}^{+*}	δ_{rj}^{+*}	Number of branches	Integer BCC projections based on the model 3.1							Integer BCC projections based 01: the model 2.1							Number of branches
	(>0)	(>0)		x_{1j}^*	x_{2j}^*	x_{3j}^*	y_{1j}^*	y_{2j}^*	y_{3j}^*	y_{4j}^*	x_{1j}^*	x_{2j}^*	x_{3j}^*	y_{1j}^*	y_{2j}^*	y_{3j}^*	y_{4j}^*	
1	None	None	3	0	231	0	225	6	1	3	0	256	0	237	5	1	3	38
2*	-	-	-	0	170	56	213	2	0	3	0	170	56	213	2	0	3	-
3*	-	-	-	0	281	70	326	2	0	3	0	281	70	326	2	0	3	-
4	None	$\delta_2^{-*}=1$ $\delta_2^{-*}=1$	3	0	132	32	159	2	0	3	0	134	32	159	2	1	3	24
5*	-	-		164	0	0	52	1	0	3	164	0	0	52	I	0	3	-
6*	-	-		291	815	0	1014	2	2	2	291	815	0	1014	2	2	2	-
7*	-	-		0	0	61	50	0	0	4	0	0	61	50	0	0	4	-
8	None	$\delta_1^{-*}=3$	4	90	16	0	106	3	0	3	× 101	84	0	89	2	0	3	12
9*	-	-		0	727	0	675	3	0	3	0	727	0	675	3	0	3	-
10	None	None	1	0	163	0	697	2	0	3	× 0	7i3	0	697	2	0	3	22
11	None	None	1	0	0	55	46	0	0	3	× 0	0	61	50	0	0	4	18
12	None	0;-' = 2	4	118	68	0	135	3	0	3	× 127	72	0	132	3	0	3	31
13*	-	-		0	988	0	812	8	10	2	0	988	0	812	8	10	2	-
14*	-	-		0	0	34	32	0	0	2	0	0	34	32	0	0	2	-
15	None	None	6	0	685	0	601	6	11	2	× 0	690	0	601	6	11	2	52
16*	-	-		0	627	0	591	6	12	2	0	627	0	591	6	12	2	-
17*	-	-		0	166	0	166	7	0	4	0	166	0	166	7	0	4	-
I8*	-	-		0	761	0	761	0	3	2	0	761	0	761	0	3	2	-
19*	-	-		193	124	0	293	0	0	3	193	124	0	293	0	0	3	-
20*	-	-		484	0	0	361	0	0	1	484	0	0	361	0	0	I	-
21	None	None	3	0	456	0	434	4	4	2	0	463	0	434	5	4	3	74
22	None	None	1	0	512	0	492	4	4	2	× 0	523	0	492	2	4	2	23
23	None	None	3	0	514	0	565	2	3	2	0	590	0	565	2	4	2	28
24	None	None	1	0	429	0	423	4	2	3	× 0	431	0	423	3	2	3	67
25	None	$\delta_1^{-*}=2$	1	0	441	0	433	4	3	3	× 0	449	0	433	4	3	3	43
26	None	None	1	0	334	0	332	5	1	3	× 0	342	0	332	4	1	3	40
27	None	None	3	0	346	0	328	5	3	3	× 0	347	0	328	2	3	3	7
28*	-	-		0	0	70	51	0	3	4	0	0	70	51	0	3	4	
29	None	None	3	0	203	0	193	6	1	3	0	256	0	238	5	1	3	26
30*	-	-		0	166	0	166	7	0	4	0	166	0	166	7	0	4	-
31*	-	-		262	0	0	219	3	0	3	262	0	0	219	3	0	3	-
32*	-	-		0	1023	0	794	2	0	4	0	1023	0	794	2	0	4	-
33*	-	-		396	995	0	1111	2	2	3	396	995	0	1111	2	2	3	-
34	None	$\delta_4^{-*}=1$	1	0	251	15	240	3	4	3	0	260	14	238	3	4	3	28
35*	-	-		172	375	0	547	4	3	3	172	375	0	547	4	3	3	-
36*	-	-		0	460	0	385	4	8	3	0	460	0	385	4	8	3	-
37*	-	-		223	0	535	232	14	6	4	223	0	535	232	14	6	4	-
38*	-	-		0	1202	58	115&	12	0	3	0	1202	58	1158	12	0	3	-
39		None	I	0	384	22	394	4	1	3	× 0	386	23	394	4	1	3	8
40	None	None	0	0	0	61	50	0	2	4	0	0	61	50	0	2	4	3
41	None	$\delta_1^{-*}=1$	3	254	0	0	205	2	0	3	262	0	0	219	3	0	3	12
42	None	None	0	213	0	0	226	2	0	2	× 325	0	0	226	I	0	2	15

c The Lingo software used for these computations.