A Benders' Decomposition Method to Solve Stochastic Distribution Network Design Problem with Two Echelons and Inter-Depot Transportation

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Abstract

In many practical distribution networks, managers face significant uncertainties in demand, local price of building facilities, transportation cost, and macro and microeconomic parameters. This paper addresses design of distribution networks in a supply chain system which optimizes the performance of distribution networks subject to required service level. This service level, which is considered for each arbitrary request arriving at a distribution center (facility), has a (pre-specified) small probability of being lost. In this mathematical model, customer's demand is stochastic that follows uniform distribution. In this model, inter-depot transportation (transportation between distributions centers (DCs)), capacities of facilities, and coverage radius restrictions are considered. For this restriction, each DC cannot service all customers. The aim of this model is to select and optimize location of plants and DCs. Also, the best flow of products between DCs and from plants to DCs and from DCs to customers will be determined. The paper presents a mixed integer programming model and proposed an exact solution procedure in regard to Benders' decomposition method.

Keywords: Facility location, Distribution network, Benders' Decomposition, Coverage Radius, Uncertainty modeling, Inter-depot transportation.

1. Introduction

It is quite common nowadays to see manufacturers and retailers joining efforts to efficiently handle the flow of products and to closely coordinate the production and supply chain system. An important strategic aspect related to the design and operation of a physical distribution network in a supply chain system is to determine the best sites for intermediate warehouses, or distribution centers. The use of DCs provides a company with flexibility to respond to demand fluctuation in the marketplace and can result in significant cost savings due to economies of scale in transportation or shipping costs.

Many researchers have extensively studied facility location and demand allocation problems. Previous research studies are well reviewed by Francis, McGinnis, and White (1983), Aikens (1985), Brandeau and Chiu (1989), and Avella *et al*. (1998). More recently, Jayaraman (1998) considered the capacitated warehouse location problem that involves locating a given number of warehouses to satisfy customer demands for different products. Pirkul and Jayaraman (1998) developed the previous problem by allowing locating also a given number of plants. They formulated the problem as a

mixed integer model and developed a Lagrangean-based heuristic solution procedure.

Tragantalerngsak, Holt, and Ronnqvist (2000) considered a bi-echelon facility location problem in which the facilities in the first echelon are assumed to be uncapacitated but the facilities in the second echelon are capacitated. The aim of their effort is to determine the number and locations of facilities in both echelons in order to satisfy customer demand per each product. They proposed a Lagrangean relaxation based branch and bound algorithm to solve the problem. Gourdin, Labbe, and Laporte (2000) studied a different type of the uncapacitated facility location problem where two customers allocated to the same facility are matched. They developed several methods to solve the problem after obtaining valid inequalities, and optimality cuts for the problem.

Current, Daskin, and Schilling (2001) and Daskin and Owen (1999) overview both deterministic and stochastic facility location. For a further detailed study of facility location theory, see the contents by Daskin (2013), Drezner (1995), or Hurter and Martinich (1989). Sheppard (1974) was one of the first authors proposed a

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scenario approach to facility location. He suggests selecting facility locations to minimize expected cost;however he does not discuss the subject at length.

In any stochastic programming problem, one must determine which decision variables are first-stage and which are second-stage; that is, which variables must be set now and which may be set after the uncertainty has been resolved. In stochastic location modeling, locations are generally first-stage decisions while assignments of customers to facilities are second-stage decisions (Ghezvati, Jabal-Ameli, &Makui (2008) and Snyder, 2006). If both decisions occur in the first stage, most stochastic location problems can be reduced easily to deterministic problems.

Mirchandani, Oudjit, and Wong (1985) and Weaver and Church (1983) presented algorithms for a multi-scenario for P-median problem (PMP). Their algorithms effectively treat the problem as a deterministic PMP with $|I|\times|S|$ customers instead of $|I|$, where I is the set of customers and S is the set of scenarios. Louveaux (1986) introduced stochastic versions of the capacitated PMP and Capacitated facility location problem (CFLP) in which demand, production costs, and selling prices are arbitrary. Vidal and Goetschalckx (2000) discussed the importance of incorporating various types of uncertainty into global supply chain design decisions.

More formal stochastic programming techniques are used by Alonso-Ayuso, Escudero, Garı´n, Ortun˜o, and Pe´rez (2003), to solve multi-echelon supply chain design problems.

Kang and Kim (2010) considered an integrating inventory replenishment model and delivery planning in a two-level supply chain consisting of a supplier and a retailer. Shen and Qi (2007) proposed a single-product, single-period LRI problem with an approximate routing cost and solved the LRI model by a Lagrangian relaxation based solution algorithm, their model was introduced as a modified inventory-location model given in Daskin, Coullard, and Shen (2002). Chanchan, Zujun, and Huajun (2008) formulated a dynamic LRI problem in a closed loop supply chain solved by a two-phase heuristic algorithm. Ahmadi-Javid and Azad (2010) developed the model presented by Shen and Qi (2007). Their model simultaneously optimizes location, inventory and routing decisions without approximation, and is solved by a heuristic method based on a hybridization of tabu search and simulated annealing.

Moin, Salhi, and Aziz (2011) addressed an inventory routing, many-to-many distribution network consisting of an assembly plant and many distinct suppliers where each supplies a distinct product. Hiassat and Diabat (2011) studied the LRI problem with perishable product, through a multi-period model.

In this paper, we define a notation of a supply chain system considering stochastic demand and inert-depot transportation. Also some restrictions such as coverage radius and inventory capacity are assumed in the thinking process.

The structure of this paper is as follows. In Section 2, we present the formulation of the problem. Section 3 proposes solution procedure regarded to Benders' decomposition method. We present computational results and sensitivity analysis in Section 4. In Section 5, we summarize our conclusions and discuss avenues for future research.

2. Model Formulation

Let N be the set of customers, which face uniform distributed demands that are independent among customers. Let M be the set of potential sites for distribution centers. The firm pays a fixed location cost for opening a DC, as well as a holding cost for inventory. In this model since customers' demand is stochastic, service level constraint is considered for that an arbitrary request arriving at a DC, should only have a (pre-specified) small probability of being lost.

The notice is that all distribution centers cannot service all of the customers due to considering special coverage restriction for each DC.Thus,if a customer cannot fall in the coverage radius,so the DC cannot service that customer. In this model,this point is considered which makes the model to be more realistic.

The other assumption applied in this study is inter-depot transportation. This point can enable DCs to have transportation products among each other. Due to stochastic demand which leads to inventory fluctuation in facilities, inter –depot transportation has risk-pooling effect on the supply chain network. Thus, holding inventory and shortage in the whole chain will be reduced. The sample of representation of a distribution network considering inter − depot transportation is illustrated in Figure 1.

In the previous location - inventory models the required order, for each DC's, is the sum of two parts. First part is the sum of mean demand, assigned to the DC and the second part is the safety stock. A new approach for this model in this article can be proposed without dividing the order to two parts. By combining the two parts, let MI_j be the amount of the order DC *j*. In this section, we aim to develop a chance constraint programming in order to prevent demand shortage for each customer that is assigned to a DC. As it was defined earlier, the demand for each customer follows uniform distribution within the interval (a_i, b_i). For this purpose, anamount of product that a DC orders to cover each customer's demand (Q_{ij}) must satisfy the following constraint. By this way, the probability that each customer faces to demand shortage is at most *a*%.
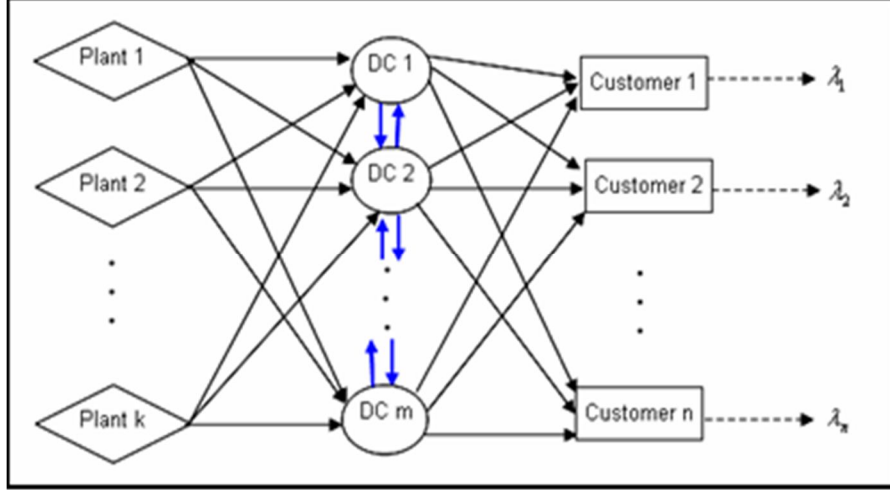
Fig. 1. Sample of distribution network considering inter-depot transportation

A new constraint must be added for this purpose and thus, we don't need to compute the safety stock.

$$P\left(\begin{array}{l}\text{an arbitrary request arrivng}\\ \text{by cumstomer i at DC j with inventory}\\ \text{level } Q_{ij} \text{ is not lost}\end{array}\right) \geq 1-\alpha \quad (1)$$

By computing above constraint, the following constraint will be considered in the mathematical model.

According to this, order quantity is greater than customers' mean demand and model determines optimum quantity so that customers' demand satisfies at least (1-α) %, so there is no necessity to hold safety stock. The constraint considered for this is

$$\Pr(x_i \leq Q_{ij}) \geq 1-\alpha \Rightarrow \int_{a_i}^{Q_{ii}} \frac{1}{b_i - a_i} d_x \geq 1-\alpha \Rightarrow$$

$$Q_{ij} \geq (1-\alpha) \times (b_i - a_i) + a_i$$

Amount of product that is found by above chance constraint is valid, once customer i is assigned to DC j. so, this constraint is modified as follows:

$$Q_{ij} \geq \left[(1-\alpha) \times (b_i - a_i) + a_i\right] \times X_{ij} \quad (2)$$

x_i : Random variable denoting

  demand distribution function for customer i .

Q_{ii} : Amount of product that DC j orders to satisfy demand of

  customer i with the probability at least 1-α .

Also, let η_{ij} be the probability that customer i which is assigned to DC j faces to product shortage. So, amount of η_{ij} can be computed as follows:

$$\Pr(x_i \leq Q_{ij}) + \eta_{ij} = 1 \Rightarrow \eta_{ij} = \frac{1 \times (b_i - a_i) - Q_{ij} + a_i}{b_i - a_i} \quad (3)$$

Once above decision variable (Q_{ij}) is determined, each DC can compute total amount of products that must be ordered to satisfy all assigned customers by the following equation:

MI_j : Decesion variable denoting total

  amount of product which DC j has and

    can satisfies customers' demand.

$$MI_j = \sum_i Q_{ij}. \quad (4)$$

Note: Above equation leads to risk pooling of the system since the demand of customers follows probabilistic structure and some customers request demand less than decision maker's estimation and some of them request demand more than decision maker's estimation.

2.1 Parameters and Decision Variables

Parameters

N index set of customer zones.

M index set of potential distributi on sites .

L index set of potential plant site.

C_{ij} Cost of supplying one unit of demand to customer zone i from DC at site j.

$\overline{C}_{jk}$ Cost of supplying one unit of demand to DC at site j from plant k.

F_j Fixed cost for opening DC at site j.

G_k Fixed cost for opening plant at site k.

b_j Capacity for the potential DC at site j.

e_k Capacity for the potential plant at site k.

$U_i \sim (a_i, b_i)$ Uniform distribution function for demand of customer i.

λ_i Mean of demand per unit time for customer i. ($\lambda_i = \dfrac{a_i + b_i}{2}$)

h_i Holding cost per unit in DC at site j.

$$Z_{ij} = \begin{cases} 1 & \text{if the DC at site } j \text{ covers the customer site } i. \\ 0 & \textit{Otherwise} \end{cases}$$

Decision variables:

$$X_{ij} = \begin{cases} 1 & \text{if demand of custome} r\ i \text{ satisfied by DC at site } j. \\ 0 & \text{Otherwise} \end{cases}$$

Q_{ii} : Amount of product that DC j orders to satisfy demand of customer i with the probability at least $1 - \alpha$.

Y_{jk} Amount of product that is transported from plant k to DC j.

$$U_j = \begin{cases} 1 & \text{if a DC is located at site } j. \\ 0 & \text{Otherwise} \end{cases}$$

$$V_k = \begin{cases} 1 & \text{if a plant is located at site } k. \\ 0 & \text{Otherwise} \end{cases}$$

$IDT_{j,j'}$ Total product shipped from DC j to DC j'.

2.2 Proposed Model

$$MinZ = \sum_{j \in M} \sum_{k \in L} \overline{C}_{jk} \times Y_{jk} + \sum_{j \in M} \sum_{i \in N} \lambda_i \times X_{ij} \times C_{ij} + \sum_{j \in M} F_j \times U_j + \sum_{k \in L} G_k \times V_k +$$

$$\sum_{i \in N} \sum_{j \in M} \lambda_i \times X_{ij} \times LI \times \eta_{ij} \tag{5}$$

St:

$$\sum_{j \in M} X_{ij} = 1 \qquad \forall i \in N \tag{6}$$

$$X_{ij} \leq Z_{ij} \times U_j \qquad \forall i \in N, \forall j \in M \tag{7}$$

$$\sum_{k \in L} Y_{jk} \leq U_j \times b_j \qquad \forall j \in M \tag{8}$$

$$Q_{ij} \geq \left[(1-\alpha) \times (b_i - a_i) + a_i\right] \times X_{ij} \tag{9}$$

$$MI_j = \sum_i Q_{ij}. \tag{10}$$

$$\eta_{ij} = \frac{1 \times (b_i - a_i) - Q_{ij} + a_i}{b_i - a_i} \tag{11}$$

$$MI_j = \sum_{k \in L} Y_{jk} + \sum_{j' \in M, j' \neq j} IDT_{j',j} - \sum_{j' \in M, j' \neq j} IDT_{j,j'} \qquad \forall j \in M \tag{12}$$

$$\sum_{j' \in M, j' \neq j} IDT_{j,j'} \leq \sum_{k \in L} Y_{jk} \qquad \forall j \in M \tag{13}$$

$$\sum_{i \in N} \lambda_i \times X_{ij} \leq b_j \times U_j \qquad \forall j \in M \tag{14}$$

$$\sum_{j \in M} Y_{jk} \leq e_k \times V_k \qquad \forall j \in M \tag{15}$$

$$X_{ij}, U_j, V_k \in \{0,1\} \tag{16}$$

$$Y_{jk}, IDT_{j,j'}, MI_j, Q_{ij} \geq 0 \tag{17}$$

This model minimizes total expected cost made of: the costs to serve the demand of customers from the DCs, the costs of shipments from the plants to the DCs, the costs associated with opening and operating the DCs and plants, the expected costs of inventory which is lost in any opened DC (the expected cost = probability of being lost for demand × total expected demand), andtotal costs of ordering and holding inventory. Constraint set (6) ensures that all customers must be allocated to DCs. Constraint set (7) says that a customer can be allocated to a DC once the customer is in the coverage radius of DC and also, DC is opened. Constraint set (8) guarantees that the total product shipped from plants to DCs should be less than DC's capacity. Set constraint (9) ensures that service level for DCs would be at least (1-α) %. Set constraints (10) computes total products that each DC must order to cover all assigned customers subject to service level limitation. Constraint set (11) present total net inventory that DC has and can satisfies customers' demand and ships to the other DCs by that. Set constraint (12) computes the probability that each customer faces to product shortage. Constraint set (13) says that total product shipped from a DC to the customers should be less than total product received from plants and others depots. Constraint set (14) specifies that total averagedemand that a DC satisfies should beless than capacity of the DC. Constraint set (15) shows capacity constraint for plants. Constraints set (16), (17) determine type of the variables.

2.3 Linearization of the proposed model

The last term in the objective function is appeared non-linear because a binary variable (X_{ij}) is multiplied to a continuous variable (η_{ij}). This term can be converted to a linear one by the algorithm that is introduced by Ghezavati and Saidi-Mehrabad (2011). To apply this method the following auxiliary constraints are added to the model.

$$LP_{ij} = X_{ij} \times \eta_{ij}$$
$$LP_{ij} \leq M \times X_{ij} \qquad LP_{ij} \leq \eta_{ij}$$
$$LP_{ij} \geq \eta_{ij} - M(1 - X_{ij})$$

3. Solution Procedure

All the decision variables except location variables are continuous for proposed model. The model without binary variables (W_j) is a linear programming solved easily. Experimental studies of our model, obtained by LINGO and CPLEX solver, demonstrate the running times to very long (more than 3 hours) for the instances that have large number (more than 30) of the binary variables. Also, Geoffrion and Powers (1995) show distribution systems design become extremely difficult for the associated large-scale models to solve optimality without the implementation of Benders' decomposition or factorization methods. For this characteristic, Benders (1962) presents a solution procedure called Benders' Decomposition algorithm. In this way, the problem decomposes into two distinct problems called as sub-problem and master problem. The master problem is established by part of original objective function that contains complicated variables as its objective function, and constraints of original problem includes only the complicated variables. The reminder of original problem makes sub-problem which its complicated variables are fixed by solution of master problem. Objective functions of the master problem and the sub-problem obtain respectively upper bound and lower bound of the original objective functions to maximize problem. The algorithm tries to close the gap between these bounds through using optimality cuts in the master problem. The optimality cuts are formulated by dual values of complicated constraints

in sub-problem. If sub-problem becomes infeasible, feasibility cut will be produced in the master problem. We adopt a solution procedure as Benders' Decomposition algorithm. In the proposed algorithm, master problem and sub-problem are formulated in regard to following indices and parameters.

Indices of algorithm:

$iter \in I$ Iterations of the algorithm

$p \subseteq I$ Set of iterations that sub-problem is feasible

$f \subseteq I$ Set of iterations that sub-problem is infeasible

Parameters of algorithm:

W_j Set of binary variables

$fix\_W_{j(iter)}$ Fixed value for W_j at iteration iter

$u_{j(iter)}$ Dual value in relation to constraints (8) and (9) at iteration iter

$objsub_{iter}$ Value of objective function for sub-problem at iteration iter

LB   Lower bound of problem

UB   Upper bound of problem

Sub-problem:

Max

$$u_1 \times (\sum_{k \in L} Y_{jk} - fix\_U_j \times b_j) +$$

$$u_2 \times ([(1-\alpha) \times (b_i - a_i) + a_i] \times fix\_X_{ij} - Q_{ij})$$

Subject to all constraints expect those ones contain binary variables.

$$\sum_{k \in L} Y_{jk} \le fix\_U_j \times b_j \qquad \forall j \in M$$

$$Q_{ij} \ge [(1-\alpha) \times (b_i - a_i) + a_i] \times fix\_X_{ij}$$

Master problem:

Min λ

$$\lambda \ge Z_{where\ bibary\ variables\ are\ fixed} +$$

$$u_1 \times (\sum_{k \in L} Y_{jk} - fix\_U_j \times b_j) +$$

$$u_2 \times ([(1-\alpha) \times (b_i - a_i) + a_i] \times fix\_X_{ij} - Q_{ij})$$

$$\sum_{j:fix\_W_j = 0} W_j + \sum_{j:fix\_W_j = 1} (1 - W_j) \ge 1 \qquad \forall f$$

In the objective function of master problem, α is a free variable that exists in optimality cuts. At first, master problem without α and cuts solves then optimal values are fixed in sub-problem. In turn, if the sub-problem is feasible, optimality cut will be placed in master problem or if it is infeasible, feasibility cut will be located in master problem. Consequently, master problem obtained by optimality cuts or feasibility cuts solves then optimal binary values are fixed again. This procedure continues until upper bound is equal to lower bound. Final solution

is a global solution for original problem. For very big dimensions, stopping criteria may be used to avoid extreme run time, such as maximum iteration and minimum thegap between lower and upper bounds. We present a pseudo-code for the proposed procedure as follows:

1. $iter = 0$ *and* $p, f = \varnothing$, $LB = -\infty$, $UB = \infty$

2. *solve master problem*

3. $iter = iter + 1$

4. $fixW_{j(iter)} = W_j^*$

5. *solve subproblem*

6. *if* *infeasible then* $f = f \cup \{iter\}$

7. *else if* $p = p \cup \{iter\}$ $u_{j(iter)} =$ *dual constra* int s (18) *and* (19)

8. $LB = \max \left\{ LB, obj_{iter} - \sum_j Lo\cos t_j fixW_j \right\}$

9. *solve master problem*

10. $UB = objmaster$

11. *if* $UB = LB$ *then* $obj^* = objmaster^*$

12. *else if* *go to step* 3

4. Computational Results

4.1 Benders' validation

We formulate the problem, and implement the proposed algorithm in GAMS 23.5 in relation with CPLEX® Solver version 12.2. In the first section, validity of GAMS codes are controlled via some test problems. All examples are generated randomly and they are solved by both regular GAMS and also Benders' decomposition method. From Table 1,we can see that there is no difference between all the objective functions. Therefore, it is obvious that there is no gap when different problems are solved. This implies that the proposed Benders' decomposition method as well as the regular GAMS solver is effective in solving the presented model.

Table 1
Validation of the proposed Benders' decomposition

|M|	|N|	|L|	No. of open DCs	No. of open plants	Total cost by Benders'	Total cost by GAMS
8	5	3	2	1	15221	15221
9	5	3	2	1	16044	16044
10	6	3	2	1	16215	16215
11	6	3	3	1	16899	16899
12	6	3	3	1	17451	17451

4.2 Sensitivity analysis

In this section, problems sets were generated randomly but systematically to capture a wide range of problems structures. The numbers of customer zones, potential DC and potential plants vary from 40 to 70, 6 to 12 and from

3 to 7. The demand mean requirements of customers were drawn from a uniform distribution between 10 and 100 which the demand has Poisson distribution function. If we let SUM represents total demand mean requirements, then the capacity of DCs were drawn from uniform distribution between 100 and SUM/2. And also the capacities of plants were drawn between SUM/2 and SUM+100 .

The parameters $C_{ij}, \overline{C}_{jk}, F_j, G_k$ and h_j follow uniform distributions which are in the below:

$C_{ij} \sim U(2, 10)$ $\overline{C}_{jk} \sim U(2, 8)$ $F_j \sim U(1500, 2500)$

$G_k \sim U(5000, 7000)$ $h_j \sim U(1, 8)$

Ordering cost was fixed to either 200 or 500. Any number in Z_{ij} matrix can be 1 with probability 0.2 and can be 0 with probability 0.8.

The results per first set of experiments testing problems are reported in Table 1. The results are described by providing the number of user nodes $(|N|)$, the number of potential DC sites $(|M|)$, the number of potential plants $(|L|)$, the number of opened DCs and plants, total cost, andtotal loss of inventory cost. Finally, we reported the CPU times in seconds.

As we see in the Table 1 CPU times is sensitive tothe number of customers and DCs. So, the more thenumber of customers and DCs, the more CPU time is needed to solve the problem.

Table 2
Experiments results for test problems

|M|	|N|	|L|	No. of open DCs	No. of open plants	Total Cost	Inventory lost cost	1-a	CPU time
40	6	3	4	1	75000.22	792.96	0.84	207
40	6	3	4	1	75104.06	789.12	0.86	215
40	6	3	4	1	75109.33	785.28	0.88	202
40	6	3	3	1	73283.48	779.01	0.91	217
40	6	3	4	1	75128.54	775.67	0.93	225
40	6	3	4	1	75209.07	769.40	0.97	238
40	6	3	4	1	75698.11	275.52	0.99	235
50	8	3	4	1	110168.80	745.47	0.95	241
50	8	3	4	1	109999.60	1025.44	0.95	314
50	10	5	4	2	120958.60	984.29	0.90	465
50	10	5	4	1	119436.20	950.76	0.95	421
50	10	5	5	1	120335.54	343.80	0.97	463
50	10	5	4	1	119437.30	952.67	0.99	472
50	10	5	7	1	119739.50	343.80	0.99	512
60	10	5	6	1	133802.60	428.28	0.99	551
70	12	5	6	2	131962.70	2389.01	0.95	622
70	12	7	6	3	128432.10	789.96	0.99	642

Inventory theory confirms that once value of service level (1-α) increases, the costs related to inventory lostdecrease,thisway customers' demand will be more satisfied and thus, the expected cost for inventory which is lost, decreases. As it can be found in Table 2, this concept will be verified clearly by local optimum solution. But the important notice is that by increasing (1-α), the transportation costs and holding costs will increase

(because to have more service level, we must order more products and this leads to increased transportation and holding costs). So a trade of between them is necessary to findoptimum value of (1- α). With comprehension above costs we can find best (1- α) with the minimum cost.
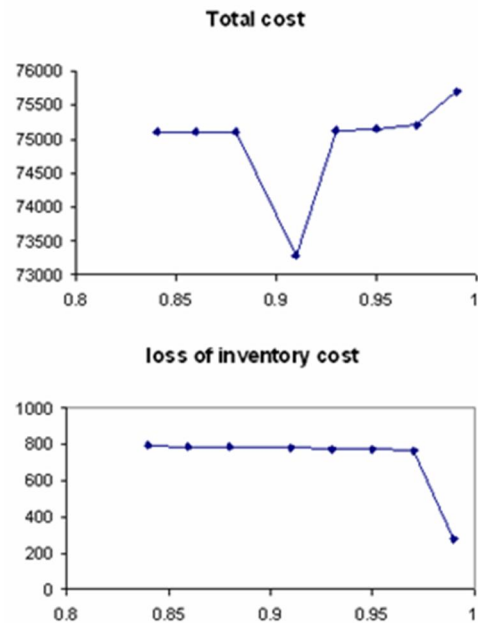

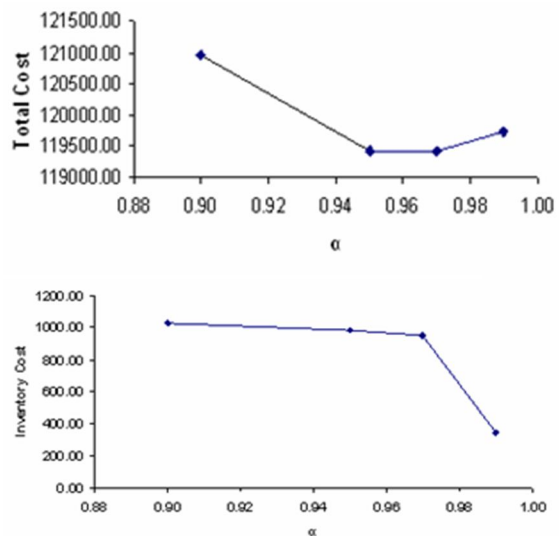Fig. 2. Sensitivity analysis for filling rate of the network

Fig. 3. Sensitivity analysis for filling rate of the network

Figure 2 and Figure 3illustrate the trend of total cost and loss of inventory cost for 2 problems with specific number of customers, DCs and plants regarding to the increasing (1-α). As it can be seen, loss of inventory cost decreases by increasing (1-α) but due to increasing transportation and holding cost with having a trade of between them, total cost may be increased or decreased.

5. Conclusion and Future Directions

In this paper, we introduced notation of distribution network in a supply chain system while we have service level constraint and customer's demand is stochastic with Poisson distribution function. In the proposed model, we assumed that DCs have coverage radius restriction and also, inert-depot transportation. We formulated the problem as a nonlinear integer program. The advantages of the proposed model are as follows: considering coverage radius which yields more flexibility for the model, considering service level constraint that byusing it the required inventory can be computed automatically, it prevents inventory lost and finally all types of cost for inventory are consider in the model. Also, we presented a mixed integer programming model and proposed an exact solution procedure in regard to Benders' decomposition method.

For future research we suggest three directions:

(a) We can study this model while Z_{ij} is probability and this parameter does not have deterministic value. For example, it has a distribution probability function and based on it, Z_{ij}gets value 0 or 1.

(b) Setting inventory capacity constraint is another development for this model.

(c) Applying the other ordering methods to calculate inventory costs in designing distribution networks.

(d) Aggregating this model with other assumption such as routing and considering time window can be an interesting development of the proposed model.

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