Multiple Batch Sizing through Batch Size Smoothing

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Received 3 Nov., 2008; revised 20 Jan., 2009; accepted 29 Jan. 2009

Abstract

Batch sizing in different planning period is categorized as a classical problem in production planning, that so many exact & heuristic methods have been proposed to solve this problem, each of which considering various aspects of the original problem. The solution obtained from majority – e.g. MRP – is in this format that there may be some periods of idleness or each period should produce as needed in different adjacent periods. If there are more the one final independent product to be produced in a factory, this makes the production planning experience strong variations in batch sizes for different periods, which production managers are opposed to these proposed production plans. In this paper, some of the models are proposed to solve this shortcoming of the production plan to smooth the variation of batch sizes and consequently to meet the managers ideal. Finally all of the proposed models are used in a real case problem and the best model is introduced in that case.

Keywords:Ideal Batch Size, Material Requirement Planning (MRP), Batch Sizing, Production Smoothing;

1. Introduction

Production managers usually desire to produce in a monotone rate for different periods [1,7,9]. One of the pitfalls of the common production models is that do not pay attention to this willing as well [2,7,14]. Consequently their optimal solutions would be with lots of variations in batch size of different periods. Most of the times, these variations make mangers not to accept the production plan. In recent years, various models are proposed to solve this problem and to satisfy managers. Most of these models are seeking a way to determine an ideal production level that variations of batch size would be as little as possible in a narrow band around this ideal level [5,10,13]. This band is usually labeled as ideal production band. Some of the models in the literature attempt to force a /some dummy objective(s) to the classic batch size models, so because of the conflicts between different goals of the model, those models are to be solved by goal programming /game theory, etc [6,8,11,12]. Others tried to obtain the narrowest ideal band as possible [1,3,4]. Few of them aimed to force their models to obtain an ideal production band limited to maximum production capacity. Figure 1 shows an example of what is meant as ideal level and ideal production band.

The dashed line in the ideal band is the ideal production level. In figure (1), the forecasted demand (D_t), batch size (x_t) and smoothed batch size (x_t^*) of t= 1, 2, ... , 12 periods are plotted. It is noteworthy that a fundamental assumption is that values of demands in each period is given or forecasted by a good method such as exponential smoothing as well. The upper dot line indicates the maximum production capacity. Another fundamental assumption is that the maximum production capacity of all periods is constant.

1.1. *Production Smoothing and JIT*

One of the topics in literature is the relation between JIT and batch size smoothing approaches. For example it is questioned whether or not to use batch size smoothing models via production in the framework of JIT? If so, how this should be done not to exceed from the philosophy of JIT? In short answer, it should be considered that JIT needs to have data of demands of different periods to be forecasted in a reasonable interval such as week/ day/ hour, or to be taken from customers themselves. Now if demand data have strong variations from one period to another

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period, or at least one data exceeds the maximum production capacity, how should the production manager plan the batch sizes? For example, consider the data set of 200, 300, 800, 15, 2, 350, 238, 645, 700, 46, 582 and 212 as the demands of 12 production periods by the maximum production capacity of 550. As we know, one of the requirements to produce in JIT framework is not to have strong variations in market demands. Now for solving this dilemma and nearing to the philosophy of JIT, some of the tricks could be taken. For example, customers' demand could be met in maximum one period delay. In the other words, if the demand of a period is less than the maximum production capacity, that period should produce as JIT, but in other cases, the difference between demand and maximum production capacity could be produced in the adjacent period. And the aim should be not to use the later method as possible. All of these are because of high slackness costs and low holding costs conditions. The high slackness costs would be for the importance/ attentions paid to customers. So these scenarios would depend on holding/ slackness/ delay/… costs, which will be considered. Further more, factories not producing in the framework of JIT, should not be worried about using the proposed models. These problematices makes us to use mathematical models instead of decision making based on simple conditions to consider different scenarios of production planning.

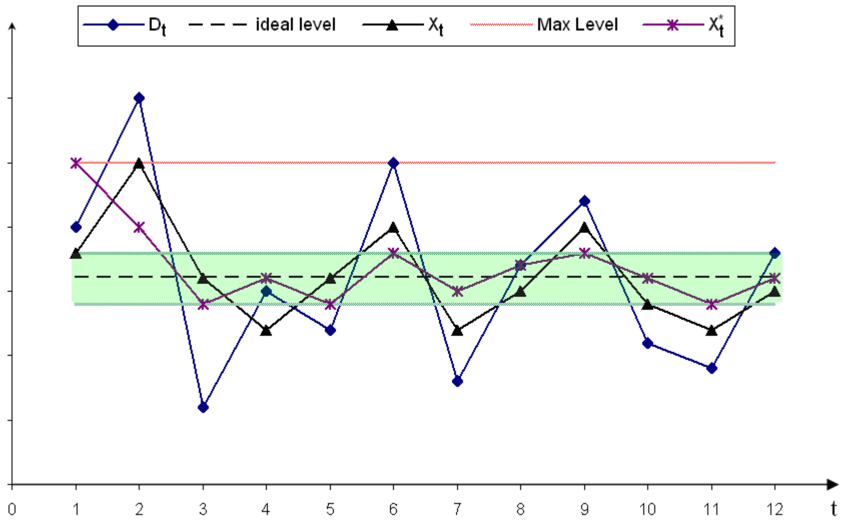

Fig. 1. Ideal Production Band

2. The Proposed Models

Because of independency of the final products, as it could be imagined a plant working with parallel production lines, it is proposed to smooth batch sizes of each product independently. Usually it has been observed that this procedure makes the final cumulative production plan to be as smooth as possible. It should be noted that before using each model for production smoothing, its efficiency should be considered by various criteria and then a model which is the most favorite by considering all of criteria should be used. So if some of the models have weakness & strength in different criteria, a good multiple criteria decision making method should be used to determine the fittest model to demand data. Model (1) is as the basis of the others that could be used even as a complement for classic batch sizing models. Parameters & variables of Model (1) are as follows:

x_t	variable of batch size in period *t*
Δ	variable of ideal band length
t	planning period ($t = 1, 2, \ldots, T$)
C	maximum production capacity
D_t	demand of period *t*

$$\begin{aligned} &Min \quad \Delta \\ &s.t. \\ &\quad x_t \le C \\ &\quad |x_{t+1} - x_t| \le \Delta \\ &\quad \sum_t (x_t - D_t) = 0 \\ &\quad x_t \in \text{int}^+ \cup \{0\} \end{aligned} \qquad \text{Model (1)}$$

One fundamental assumption in all of models of this paper is that summations of all demands should be equal to the summations of all productions. In the other words, the constraint

$$\sum (x_t - D_t) = 0$$

Indicates this idea as well. Most of the times if this constraint is not considered in the model, the result would be of not finding any ideal production band, because there may be little lost sales costs that not producing will be the best action. Model (1) could be converted to Model (2) in order to maximizing the number of smoothed batch sizes in the optimality.

$$
\begin{aligned}
& Max \quad \sum_t (1-\lambda_t) \\
& s.t. \\
& x_t \le C \\
& |x_{t+1} - x_t| \le \Delta + M\lambda_t \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\},\ \lambda_t \in \{0,1\}
\end{aligned}
\qquad \text{Model (2)}
$$

In Model (2), M is a big positive number and λ_t is a binary variable that if the batch size of period t exceeds from the ideal production band, it will be 1, and if else it will be zero. In Model (2) it is assumed that Δ is as a parameter. An interesting work is to solve Model (1) in order to obtain Δ^* and using that value as a parameter in Model (2). Authors have experimentally observed that this procedure makes narrower ideal band when dealing with deviational D_t. Model (3) is trying to minimize the maximum deviation of:

1. The difference of demand and production of each period, and
2. The difference of production of neighbor periods.

One of the most critical aspects in using Model (3) is that no common software package is able to solve it because of its first constraints, but it could be easily converted to Model (3-1) by introducing new variables and constraints:

$$
\begin{aligned}
& Min \quad \Delta \\
& s.t. \\
& Max \{ |x_{t+1} - x_t|, |x_t - D_t| \} \le \Delta \\
& x_t \le C \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\}
\end{aligned}
\qquad \text{Model (3)}
$$

$$
\begin{aligned}
& Min \quad \Delta \\
& s.t. \\
& x_t \le C \\
& y_t \le \Delta \qquad ; t = 1,2,\cdots,T \\
& |x_t - D_t| - |x_{t+1} - x_t| \le M(1-\lambda_t) \\
& |x_t - D_t| - |x_{t+1} - x_t| \ge -M\lambda_t \\
& y_t = (1-\lambda_t)|x_t - D_t| + \lambda_t |x_{t+1} - x_t| \\
& \sum_t (x_t - D_t) = 0 \\
& y_t \ge 0,\ x_t \in \text{int}^+ \cup \{0\}, \lambda_t \in \{0,1\}
\end{aligned}
\qquad \text{Model (3-1)}
$$

Another model that could be constructed according to Model (3) is as Model (4) that is not directly seeking to obtain the width of the ideal band but is leveling deviations of the product level in all of periods from ideal level.

$$
\begin{aligned}
& Min \{ Max (|x_{t+1} - x_t|) \} \\
& s.t. \\
& x_t \le C \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\}
\end{aligned}
\qquad \text{Model (4)}
$$

This model could be converted to the following Model (4-1):

$$
\begin{aligned}
& Min \quad \Delta \\
& s.t. \\
& |x_{t+1} - x_t| \le \Delta \\
& x_t \le C \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\}
\end{aligned}
\qquad \text{Model (4-1)}
$$

Model (5) is the generalized form of Model (3), (4) which needs less number of constraints/ variables:

$$
\begin{aligned}
& Min \{ Max (|x_{t+1} - x_t|, |x_t - D_t|) \} \\
& s.t. \\
& x_t \le C \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\}
\end{aligned}
\qquad \text{Model (5)}
$$

This model could be converted easily to Model (5-1):

$$
\begin{aligned}
& Min \quad \Delta \\
& s.t. \\
& |x_{t+1} - x_t| \le \Delta \\
& |x_t - D_t| \le \Delta \\
& x_t \le C \\
& \sum_t (x_t - D_t) = 0 \\
& x_t \in \text{int}^+ \cup \{0\}
\end{aligned}
\qquad \text{Model (5-1)}
$$

Model (6) is a mixture of previous ones and trying to maximize the 2 followings simultaneously:
1. The number of times that the difference of production in two adjacent periods is less than the width of the ideal band, and
2. The number of times that the difference of production and demand of each period is less than the width of the ideal band.

$$Max \ \sum_r \sum_s (1-\lambda_r)(1-\lambda_s)$$
$$s.t.$$
$$x_t \le C$$
$$|x_{t+1} - x_t| \le \gamma + M\lambda_r$$
$$|x_t - D_t| \le \gamma + M\lambda_s \qquad \text{Model (6)}$$
$$\gamma \le \Delta + M(\lambda_r + \lambda_s)$$
$$\sum_t (x_t - D_t) = 0$$
$$x_t \in \text{int}^+ \cup \{0\}; \lambda_r, \lambda_s \in \{0,1\}$$
$$\gamma, \Delta \ge 0$$

Model (6) could be solved in another ways: (a) assigning values to γ, Δ before running the model and the solving the model, (b) assigning an arbitrary value to γ and assigning the best obtained value of Δ by solving Model (1) to (5).

After solving each/all of Model (1) to (6) using data of demands of periods, there would be some of production plans by different width of ideal bands. If *b* is the unit cost of shortage and *h* is considered as the unit holding cost, the total cost (*TC*) of each production plan could be elaborated as follows:

$$C_t = \begin{cases} |D_t - x_t| b & if \ x_t \le D_t \\ |x_t - D_t| h & else \end{cases}$$
$$TC = \sum_t C_t$$

In above-mentioned formulas C_t is the corresponding cost of period t if there was no set up costs. Now if elaborating the standard deviation of each plan, there would be a multiple-criteria decision making problem in order to find the best production plan, where choices are the plans obtained from solving the models and criteria are:

- Standard deviation of λ_t in each plan, which the less standard deviation, the less deviational plan and the better plan,
- Width of production band, which the narrowest band is the most wanted,
- Total cost of each production plan, which the minimal is the best.

So a good decision making method should be used to solve this problem. Authors themselves propose TOPSIS because of its simplicity for this purpose. After doing so, the ideal production band could be illustrated as follows:

Step 1. By considering the obtained results, elaborate the corresponding value of Δ,
Step 2. By averaging the batch sizes in different periods which are obtained from the best model -that is determined previously- and introduce this value as x_{ideal}.
Step 3. Plot a horizontal line of x_{ideal} and x_{ideal}+0.5Δ as the upper bound and x_{ideal}-0.5Δ as the lower bound of the ideal production band respectively.

3. Numerical Example

In Table (1), demand data of an automobile factory in an especial type from January to December 2009 are forecasted. In this table, *t* is as the indicator of planning period (*t*=1,2,…,*T*), D_t as the demand of period *t* and *Max Level* as the maximum production capacity. Then if it is assumed that *b*=3$ and *h*=1$ we would have a decision matrix as provided in table (2), where *SD* is the indicator of standard deviation and Δ is the indicator of the width of production band corresponding to each production plan. Model 2 and Model (6) are solved by different values of Δ that are shown as 2-1 to 2-12 and 6-1 to 6-6 in Tables (2) and (3) respectively.

Table 1
Data of planning periods

		Periods (*t*)											
		1	2	3	4	5	6	7	8	9	10	11	12
D_t	A	20	30	6	15	12	25	8	17	22	11	9	18
	B	20	15	18	28	45	30	12	6	33	0	0	15
	C	0	4	23	6	12	18	15	32	35	46	30	30
	D	5	5	5	17	26	38	21	47	42	4	18	5
	E	50	40	38	12	18	3	8	10	14	5	26	50
Max Level	A	25	25	25	25	25	25	25	25	25	25	25	25
	B	30	30	30	30	30	30	30	30	30	30	30	30
	C	35	35	35	35	35	35	35	35	35	35	35	35
	D	40	40	40	40	40	40	40	40	40	40	40	40
	E	45	45	45	45	45	45	45	45	45	45	45	45

Table 2
Decision matrix obtained from solving 6 smoothing models

		Models										
		1	2-1	2-2	2-3	2-4	2-5	3	4	5	6-1	6-2
TC	A	132	148	144	144	148	120	96	120	120	100	116
	B	244	276	308	256	248	264	264	244	228	308	288
	C	292	612	176	304	295	336	292	292	108	144	128
	D	296	316	316	316	308	336	316	296	228	248	348
	E	364	428	336	308	364	332	364	364	180	228	176
SD	A	0.668	0.793	0.668	1.621	1.311	2.065	4.100	5.434	5.434	3.058	5.035
	B	0.797	5.196	3.655	1.732	3.605	1.243	5.196	0.797	11.782	4.776	15.877
	C	0.668	14.932	14.932	3.175	1.403	3.654	4.209	0.668	11.484	11.681	14.939
	D	0.900	1.443	1.443	1.443	0.514	3.528	2.874	0.900	11.610	19.023	14.190
	E	0.389	17.507	2.886	6.350	0.389	3.069	4.839	0.389	11.424	16.921	13.868
Δ	A	0.999	0.8	0.85	0.9	0.999	2	8	8	8	1	8
	B	0.999	0.8	0.85	0.9	0.999	2	8	1	15	2	8
	C	0.999	0.8	0.85	0.9	0.999	2	8	1	11	2	8
	D	0.999	0.8	0.85	0.9	0.999	2	8	1	13	2	9
	E	0.999	0.8	0.85	0.9	0.999	2	9	1	12	4	8

Now for solving this multiple criteria decision making problem by TOPSIS, firstly the positive ideal solutions (*PIS*) and negative ideal solutions (*NIS*) should be determined from the decision matrix of table (2) and their coordinates could be elaborated easily as follows:

- Each coordinate of *PIS* is the best value of each criterion among production plans according to the originality of that attribute.
- Each coordinate of *NIS* is the worst value of each criterion among production plans according to the originality of that criterion.

Then by considering values of *TC*, *SD* and Δ as the set of criteria, it is clear that all of *TC*, *SD* and Δ are of cost type. Therefore *PIS* would have the smallest coordinates, and *NIS* would have the largest coordinates among different production plans. Then it is clear that:

PIS(A) $=\{TC_{PIS}=96,\ SD_{PIS}=0.668,\ \Delta_{PIS}=0.8\}$
NIS(A)$=\{TC_{PIS}=148,\ SD_{NIS}=5.434,\ \Delta_{NIS}=8\}$

PIS(B)$=\{TC_{PIS}=228,\ SD_{PIS}=0.797,\ \Delta_{PIS}=0.8\}$
NIS(B)$=\{TC_{PIS}=308,\ SD_{NIS}=15.877,\ \Delta_{NIS}=15\}$

PIS(C)$=\{TC_{PIS}=108,\ SD_{PIS}=0.668,\ \Delta_{PIS}=0.8\}$
NIS(C)$=\{TC_{PIS}=612,SD_{NIS}=14.939,\ \Delta_{NIS}=11\}$

PIS(D)$=\{TC_{PIS}=228,\ SD_{PIS}=0.514,\ \Delta_{PIS}=0.8\}$
NIS(D)$=\{TC_{PIS}=348,SD_{NIS}=19.023,\ \Delta_{NIS}=13\}$

PIS(E)$=\{TC_{PIS}=176,\ SD_{PIS}=0.389,\ \Delta_{PIS}=0.8\}$
NIS(E)$=\{TC_{PIS}=428,\ SD_{NIS}=17.507,\ \Delta_{NIS}=12\}$

Now the production plan obtained from solving each model as table (2) could get its scores by CL_i^+, CL_i^- as being neighbor to PIS and *NIS* respectively, and finally could get an aggregate score as CL_i which is shown in table (3):

$$CL_i^+ = \sqrt{(TC_{PIS}-TC_i)^2+(SD_{PIS}-SD_i)^2+(\Delta_{PIS}-\Delta_i)^2}$$

$$CL_i^- = \sqrt{(TC_{NIS}-TC_i)^2+(SD_{NIS}-SD_i)^2+(\Delta_{NIS}-\Delta_i)^2}$$

$$CL_i = \frac{CL_i^-}{CL_i^- + CL_i^+}$$

It is clear that CL_i would aggregate CL_i^+, CL_i^- such that the more CL_i, the more favorite ness of the production plan *i*. So it could be observed from table (3) that the production plans obtained from model 6-1, 5, 6-2, 5 and 6-2 are the best ones for product A, B, C, D and E respectively as they have been selected by TOPSIS as the most favorite production plans, which their batch sizes are shown in table (4). Figures 2 to 7 show the graphical results of using the proposed approach for determining the most favorite batch size smoothing model.

Table 3
Values of *CL* of each product planned by each of 6 model

		Models										
		1	2-1	2-2	2-3	2-4	2-5	3	4	5	6-1	6-2
Products	A	0.335	0.141	0.165	0.158	0.135	0.545	0.867	0.523	0.523	0.912	0.596
	B	0.808	0.432	0.189	0.665	0.757	0.572	0.553	0.808	0.817	0.176	0.254
	C	0.635	0.020	0.863	0.611	0.629	0.548	0.635	0.635	0.971	0.926	0.950
	D	0.453	0.304	0.304	0.304	0.363	0.172	0.291	0.453	0.879	0.787	0.049
	E	0.263	0.042	0.370	0.478	0.263	0.385	0.258	0.263	0.939	0.786	0.943

Table 4

Batch sizes of each product through 12 planning periods

		Periods (*t*)											
		1	2	3	4	5	6	7	8	9	10	11	12
Products	A	20	21	14	14	13	14	18	18	18	12	13	18
	B	30	27	30	30	30	18	18	18	18	3	0	0
	C	3	0	7	15	9	17	19	27	35	43	38	38
	D	5	17	14	26	27	25	34	34	29	17	5	0
	E	50	42	36	30	12	12	10	10	10	18	18	26

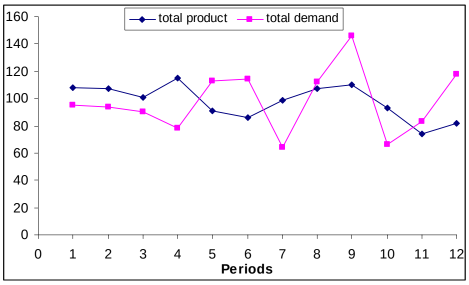

Fig. 2. Plot of total products and total demand in 12 planning periods

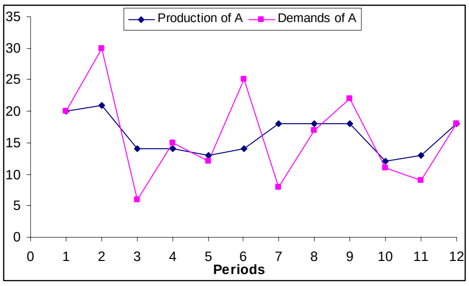

Fig. 3. Plot of production plan of *A* versus demands of *A* in 12 planning periods

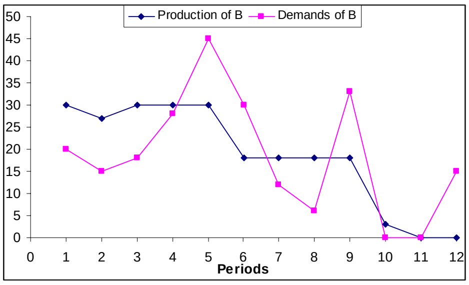

Fig. 4. Plot of production plan of *B* versus demands of *B* in 12 planning periods

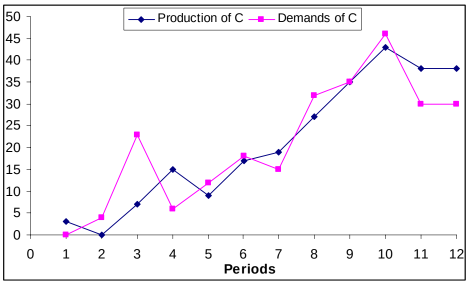

Fig. 5. Plot of production plan of *C* versus demands of *C* in 12 planning periods

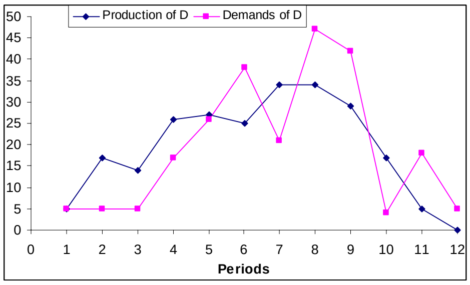

Fig. 6. Plot of production plan of *D* versus demands of *D* in 12 planning periods

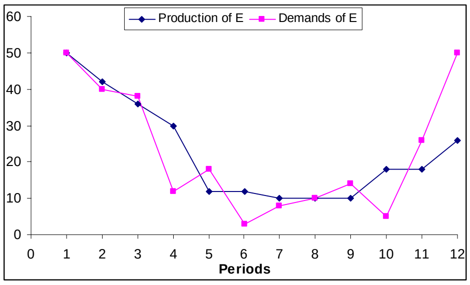

Fig. 7. Plot of production plan of *E* versus demands of *E* in 12 planning periods

4. Conclusion

As it has been shown in this paper, there are so many potential production plan obtained from solving numerous mathematical models, which are seeking different goals. The main goal of this paper was to model the willingness of production mangers to produce as monotone as possible. Furthermore as it became clear, these models could be used for dealing with seasonal demand data or they could be extended using fuzzy ideal band because of its vagueness embedded in the meaning of ideal.

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