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Research Article

Availability Analysis of a Cooking Oil Production Line

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Abstract

Availability and reliability of a manufacturing system are the most common indicators in the reliability engineering area to assess the quality and on-time deliveries of the products they produce. The purpose of this paper is to analyze the availability, reliability. failure metrics such as MTBF and MTTF, and also steady-state availability of a cooking oil production line using a Markov approach. The product line works in three consecutive shifts 24 hours a day, for which five main subsystems are identified for the analysis. The results show that the first shift has the best performance in terms of reliability while the second shift has the worst performance. To improve the reliability of the production line, a corrective maintenance policy is used. First, the critical components of the subsystems are identified using the Pareto charts, and then, by increasing the repair rates, the availability of the production line in all three shifts is increased.

Keywords: Reliability; Availability; Markov process; Corrective maintenance; Pareto diagram.

1. Introduction

The world in the 21st century has become a very complicated realm in many dimensions including the technological scope (Blischke & Prabhakar Murthy, 2003). With the advent of new technologies in different industries, one of the important goals of all companies is to raise or maintain their market shares in order to be competitive (Sharifi et al, 2014). To this aim, they need to provide high-quality products delivered to the customers on time. This becomes possible with the help of professional engineers and technical managers who are responsible for planning, operating, and designing systems (Billinton & Allen, 1992). They must design, plan, and operate appropriate systems with high maintainability and reliability. As many devices and machines are repairable in real-world systems, the most important tasks of professional engineers and technical managers are to design reliable repairable systems and to provide a proper plan for their maintenance, in order to produce products with desired qualities and to deliver them on time to the customers.

Availability and reliability (A&R) are the most important parameters in assessing the quality of a product (Blischke & Prabhakar Murthy, 2003). The main basis of A&R calculations is the extraction of statistical properties of empirical data obtained from the maintenance sector. Then, modeling and analysis of the desired system come to the picture.

Reliability analysis was initiated from the aerospace industry and military systems. Then it was expanded in many other industries such as the nuclear industry, power systems, manufacturing industries, and so forth (Billinton & Allen, 1992). For a non-repairable component or system, the reliability is defined as the probability that a component or a system operates successfully in a given period. The mean time to failure (MTTF) is another measure to assess the reliability of a component, a subsystem, or a system composing of some subsystems. However, for a repairable component or system, there is another parameter called availability. It is defined as the probability that a component or a system will be operational at any specific time (Rausand & Høyland, 2004). As determining the availability function of a system is very difficult most of the time, an alternative index called the steady-state availability is used instead. Steady-state availability is the horizontal asymptote of the availability function. Another measure to assess the availability of a repairable system is called the mean time between failures (MTBF).

The major part of the current work that is carried out to analyze the availability and reliability of a system is the use of the Markov process. This well-known approach has been frequently employed in the literature to illustrate the concepts of repairability and accessibility. For example, Gupta & Tewari (2011) determined the long-term availability of a power system in a thermal power plant. Kumar & Ram (2013) investigated a coal transportation system in a thermal power plant in terms of its reliability using the Markov model. Amiri & Přenosil (2014) calculated the meantime to failure and availability of an N-modular redundancy (NMR) system using Markov models. Using the Markov model, Zaidi & Goyal (2014) examined the availability of a pulping system in the paper

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industry. Gupta et al. (2015) performed A&R analyses on a lathe machining system using Markov models. Khalilnejad et al. (2016) examined the long-run reliability of the components of a photovoltaic system. Tan & Feng (2016) calculated the reliability of an unmanned aerial vehicle using the Markov degradation process. Zare (2016) assessed the availability and reliability of a nuclear energy-based combined cycle power plant with the Markov method. Wang et al. (2017) computed the reliability of a 6-component star system via the Markov processes. Zhou et al. (2018) assessed the reliability of a flight control system under two redundancy modes using homogeneous Markov processes. Wang et al. (2018) examined the availability and reliability of a biomass combined cooling, heating, and power (CCHP) system using the Markov approach. Yang and Tsao (2019) evaluated the availability and reliability of a repairable system using the Markov method. Wang et al. (2019) reported the availability and reliability of a hybrid cooling system using the Markov method. In line with the above work, availability and reliability of a cooking oil production line are studied using the Markov method. Furthermore, the critical components of the production line are determined by a Pareto diagram in order to employ a proper corrective maintenance plan to increase system availability in different shifts.

The paper is organized as follows. In the next section, the production line is specified, the problem is defined, the notations are given, and the assumptions are stated. The Markov method is employed in Section 3 to analyze the reliability and availability of the production line. A&R analysis of the considered production line is presented in Section 4. Experimental results are presented in Section 5 to demonstrate the applicability of the methodology. Finally, the paper is concluded in Section 6, where some future research recommendations are provided.

2. Problem Statement

This section is devoted to defining the production line under investigation, the problem, the notations used throughout the paper alongside the assumptions made. Figure 1 shows the steps involved to solve the problem at hand.

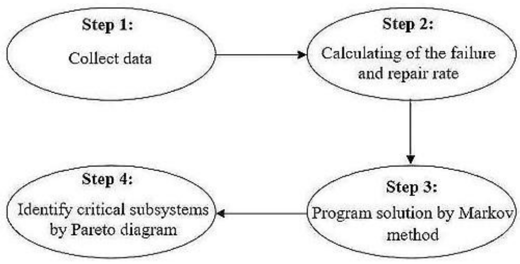


Fig. 1. The solution steps

2.1. The cooking oil production line

The production line consists of five subsystems including Posimat (X_1), Filler (X_2), Labellers (X_3), Shirring (X_4), and Pallet (X_5) that are arranged in a series configuration operating three shifts a day, each 8 working hours. In case each of the subsystems fails to operate properly, the whole line stops producing oil. In this production system, empty bottles stored in a container are first fed to the Posimat by a conveyor belt that brings them in a line. Then, the empty bottles go to the Filler through an air tunnel. In this subsystem, oil in a specific volume is fed into empty bottles. The bottles are capped at the end of this subsystem. Next, the filled-in bottles go to the Labeling machine by a conveyor belt where they are labeled. Afterward, the labeled bottles go to X_4 where they are packed in a set of 12. Finally, the packs go to X_5 by a conveyor belt, where they are gathered in pallets using robots and go to the storage area by lift-trucks. As this production line had some problems in terms of the reliability and on-time delivery of demands in the past, the reliability and availability of this system are analyzed in this paper using available historical data.

2.2. Notation

The notations used throughout the paper are as follows:

λ_i:	The failure rate of i^{th} subsystem
μ_i:	Repair rate of i^{th} subsystem
X_i	i^{th} Subsystem
$\underline{\lambda}$:	Vector of failure rates, $\underline{\lambda} = [\lambda_1, \lambda_2, \dots, \lambda_n]$
$\underline{\mu}$:	Vector of repair rates, $\underline{\mu} = [\mu_1, \mu_2, \dots, \mu_n]$
$P_i(t)$	Probability of the system being in the i^{th} state at time t
$\dot{P}_i(t)$	Derivative of $P_i(t)$
$P_i(s)$	Laplace transform of P_i
s	Laplace transform variable
$A_{Sys.}$	System availability
$R_{Sys.}$	System reliability
a_{ss}	Steady-state subsystem availability
A_{SS}	Steady-state system availability
$MTBF$	Mean time between failure
$MTTF$	Mean time to failure

2.3. Assumptions

In the considered oil production line, the subsystems

- can be repaired during their missions
- are independent (i.e., their failures do not affect each other)
- are binary (operational or failing)
- have constant failure and repair rates.

In the next section, the Markov method is used to assess the long-run availability of the system under investigation.

3. Markov Method

As a well-known, useful, and common method, the Markov approach has been widely used by analysts and researchers in the field of system reliability, particularly for the analysis of repairable systems. More details on this method can be found in many books such as (Billinton & Allen, 1992 and Rausand & Høyland, 2004 as well as in many works like Gupta et al., 2015; Saidi-Mehrabad et al., 2015; Khalilnejad et al., 2016; Zhou et al., 2018 and Yaghoubi et al., 2020).

The first stage to apply the Markov method is to determine the failure and the repair rates of the subsystems. The failure rate for each subsystem is equal to the number of its failures divided by its total operating time. Meanwhile, the repair rate means the number of repairs on a subsystem, divided by its total repair time (Billinton & Allen, 1992). Mathematically speaking, Equations (1) and (2) are used to determine the failure and the repair rates of subsystem *i*.

$$\lambda_i = (MTTF_i)^{-1} \quad (1)$$

$$\mu_i = (MTTR_i)^{-1} \quad (2)$$

where, λ_i and μ_i are the failure and repair rate of the i^{th} subsystems, respectively. Besides, $MTTF_i$ and $MTTR_i$ are the mean time to failure and the mean time to repair of the i^{th} subsystems, respectively.

Having $N(t)$ as the state of a system with five independent binary (operational and failing) subsystems at time t, $\{N(t);\ t \geq 0\}$ is a continuous-time Markov process with state space $\boldsymbol{X} = \{1,2,\dots,2^5\}$. Therefore, the number of total possible states for this system is 32, which are arranged in Table 1.

Table 1
Possible states of the production line

# of failures in the system	$\boldsymbol{X}$	Possible states of the system	System states
0	1	$X_1X_2X_3X_4X_5$	G*
1	2	$\bar{X}_1X_2X_3X_4X_5$	B**
1	3	$X_1\bar{X}_2X_3X_4X_5$	B
1	4	$X_1X_2\bar{X}_3X_4X_5$	B
1	5	$X_1X_2X_3\bar{X}_4X_5$	B
1	6	$X_1X_2X_3X_4\bar{X}_5$	B
2	7	$\bar{X}_1\bar{X}_2X_3X_4X_5$	B
2	8	$\bar{X}_1X_2\bar{X}_3X_4X_5$	B
2	9	$\bar{X}_1X_2X_3\bar{X}_4X_5$	B
2	10	$\bar{X}_1X_2X_3X_4\bar{X}_5$	B
2	11	$X_1\bar{X}_2\bar{X}_3X_4X_5$	B
2	12	$X_1\bar{X}_2X_3\bar{X}_4X_5$	B
2	13	$X_1\bar{X}_2X_3X_4\bar{X}_5$	B
2	14	$X_1X_2\bar{X}_3\bar{X}_4X_5$	B
2	15	$X_1X_2\bar{X}_3X_4\bar{X}_5$	B
2	16	$X_1X_2X_3\bar{X}_4\bar{X}_5$	B
3	17	$\bar{X}_1\bar{X}_2\bar{X}_3X_4X_5$	B
3	18	$\bar{X}_1\bar{X}_2X_3\bar{X}_4X_5$	B
3	19	$\bar{X}_1\bar{X}_2X_3X_4\bar{X}_5$	B
3	20	$\bar{X}_1X_2\bar{X}_3\bar{X}_4X_5$	B
3	21	$\bar{X}_1X_2\bar{X}_3X_4\bar{X}_5$	B
3	22	$\bar{X}_1X_2X_3\bar{X}_4\bar{X}_5$	B
3	23	$X_1\bar{X}_2\bar{X}_3\bar{X}_4X_5$	B
3	24	$X_1\bar{X}_2\bar{X}_3X_4\bar{X}_5$	B
3	25	$X_1\bar{X}_2X_3\bar{X}_4\bar{X}_5$	B
3	26	$X_1X_2\bar{X}_3\bar{X}_4\bar{X}_5$	B
4	27	$\bar{X}_1\bar{X}_2\bar{X}_3\bar{X}_4X_5$	B
4	28	$\bar{X}_1\bar{X}_2\bar{X}_3X_4\bar{X}_5$	B
4	29	$\bar{X}_1\bar{X}_2X_3\bar{X}_4\bar{X}_5$	B
4	30	$\bar{X}_1X_2\bar{X}_3\bar{X}_4\bar{X}_5$	B
4	31	$X_1\bar{X}_2\bar{X}_3\bar{X}_4\bar{X}_5$	B
5	32	$\bar{X}_1\bar{X}_2\bar{X}_3\bar{X}_4\bar{X}_5$	B

*G=Good (Operational) state; **B=Bad (Failed) state; $\bar{X}_i$: i^{th} failed subsystem

According to Table 1, the state transition diagram of the production line is illustrated in Figure 2. Besides, historical maintenance data on the number of stops (each stop due to one failure), on the uptimes and the downtimes of the line for 12 months in three consecutive shifts of a day are given in Tables 2-4.

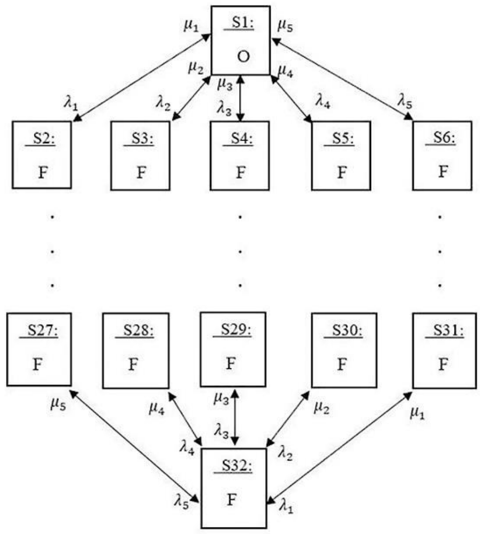


Fig. 2. State transition diagram of the production line

Table 2
The data on the first shift of the production line

Subsystem	No. of Failures	Uptime (Min.)	Downtime (Min.)
X_1	419	154,119	1,006
X_2	1,117	149,687	5,438
X_3	1,045	149,639	5,486
X_4	1,825	149,134	5,991
X_5	336	153,524	1,601

Table 3
The data on the second shift of the production line

Subsystem	No. of Failures	Uptime (Min.)	Downtime (Min.)
X_1	468	142,979	1,196
X_2	1,247	138,054	6,121
X_3	1,204	138,171	6,004
X_4	2,056	137,538	6,637
X_5	420	142,458	1,717

Table 4
The data on the third shift of the production line

Subsystem	No. Failures	Uptime (Min.)	Downtime (Min.)
X_1	445	146,798	1,027
X_2	1,286	142,149	5,676
X_3	1,122	142,380	5,445
X_4	2,182	141,432	6,393
X_5	361	146,075	1,750

According to the data shown in Tables 2 to 4, the failure and repair rate of each subsystem are determined using Equations (1) and (2) for three shifts. Table 5 contains the results.

Table 5

Estimated failure and repair rates of the subsystems in different shifts

Shift	Subsystem (X_i)	Failure rate (λ_i) (per minute)	Repair rate (μ_i) (per minute)
First	X_1	2.72×10^{-3}	4.17×10^{-1}
	X_2	7.46×10^{-3}	2.05×10^{-1}
	X_3	6.98×10^{-3}	1.90×10^{-1}
	X_4	1.22×10^{-2}	3.05×10^{-1}
	X_5	2.19×10^{-3}	2.10×10^{-1}
Second	X_1	3.27×10^{-3}	3.91×10^{-1}
	X_2	9.03×10^{-3}	2.04×10^{-1}
	X_3	8.71×10^{-3}	2.01×10^{-1}
	X_4	1.49×10^{-2}	3.10×10^{-1}
	X_5	2.95×10^{-3}	2.45×10^{-1}
Third	X_1	3.03×10^{-3}	4.33×10^{-1}
	X_2	9.05×10^{-3}	2.27×10^{-1}
	X_3	7.88×10^{-3}	2.06×10^{-1}
	X_4	1.54×10^{-2}	3.41×10^{-1}
	X_5	2.47×10^{-3}	2.06×10^{-1}

In the Markov method, the first-order differential equations of the system can be first derived based on Table 1 and Figure 2. Then, using the Laplace and inverse Laplace transforms, all of the states of the system are determined (Billinton & Allen, 1992). However, given the serial configuration of the production line under investigation, State 1 is the sole operational state of the production line. Hence, the corresponding differential-difference equation for State 1 is:

$$P_1(t+\Delta t) = P_1(t)\left[1-\left(\sum_{i=1}^{5}\lambda_i\right)\Delta t\right] + P_2(t)\mu_1\Delta t + P_3(t)\mu_2\Delta t + P_4(t)\mu_3\Delta t + P_5(t)\mu_4\Delta t + P_6(t)\mu_5\Delta t \quad (3)$$

Similarly, the other 31 equations are derived for the other 31 possible states, based on their first-order differential equations shown in Appendix A.

Regarding $\dot{P}_1(t) = \lim_{\Delta t \to 0} \frac{P_1(t+\Delta t)-P_1(t)}{\Delta t}$, Equation (3) can be rewritten as follows

$$\dot{P}_1(t) = -\left(\sum_{i=1}^{5}\lambda_i\right)P_1(t) + \mu_1P_2(t) + \mu_2P_3(t) + \mu_3P_4(t) + \mu_4P_5(t) + \mu_5P_6(t) \quad (4)$$

Now, taking the Laplace transform and applying the initial condition $P_1(0) = 1$, Equation (4) is rewritten as

$$P_1(s) = \frac{1+\sum_{i=1}^{5}\mu_iP_{i+1}(s)}{s+\lambda_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5} \quad (5)$$

In Equations 3-5, $P_i(t)$ is the probability of the system being in the i^{th} state at time t, $\dot{P}_i(t)$ is the derivative of $P_i(t)$, $P_i(s)$ is the Laplace transform of P_i and s is the Laplace transform variable.

4. Availability and Reliability (A&R) Analysis

Applying the inverse Laplace transform on Equation (5), the system availability (the reliability of a repairable system) for the first possible state of the production line at time t is obtained as

$$A_{Sys.}\left(t;\underline{\lambda},\underline{\mu}\right) = \mathcal{L}^{-1}\{P_1(s)\} = P_1\left(t;\underline{\lambda},\underline{\mu}\right) \quad (6)$$

where, $\underline{\lambda}$ and $\underline{\mu}$ are the vectors of failure and repair rates, respectively, and $A_{Sys.}(.)$, is system availability. Moreover, $\mathcal{L}^{-1}$ is the inverse Laplace operator.

Another important measure is the steady-state availability defined as the horizontal asymptote of the availability function. Mathematically, it is the limit of the availability function when t approaches infinity (Grosh, 1989; Yaghoubi et al., 2020), i.e.

$$A_{SS} = \lim_{t\to\infty} A_{Sys.}\left(t;\underline{\lambda},\underline{\mu}\right) = A_{Sys.}\left(\infty;\underline{\lambda},\underline{\mu}\right) \quad (7)$$

According to Smith (2014), the long-run availability of a system with n independent subsystems in series is a function of the steady-state availabilities of its subsystems shown in Equation (8).

$$A_{SS} = \prod_{j=1}^{n} a_{ss.j} \ ; a_{ss.j} = \frac{\mu_j}{\lambda_j+\mu_j}, \quad (8)$$

where A_{SS} is the steady-state or long-run system availability and $a_{ss.j}$ is the steady-state availability of *j^th^* subsystem. Note that in Equation (6) when the vector $\underline{\mu} = \underline{0}$, the system availability is reduced to the reliability of non-repairable systems as

$$R_{Sys.}(t;\underline{\lambda}) = A_{Sys.}(t;\underline{\lambda},\underline{0}) = P_1(t;\underline{\lambda}) \quad (9)$$

in which $R_{Sys.}(.)$ is system reliability.

Other useful indicators to analyze the reliability of the production line are the meantime between failures (MTBF) and the meantime to failure (MTTF). MTBF is equal to the average time the production line is operational in its operating time interval (Gupta et al., 2015). In other words

$$MTBF = \int_0^\infty P_1\left(t;\underline{\lambda},\underline{\mu}\right)dt \quad (10)$$

Besides, MTTF is the average of the time intervals the production line is operational, i.e.

$$MTTF = \int_0^\infty P_1(t;\underline{\lambda})dt \quad (11)$$

In what follows in the next section, we investigate the applicability of the model.

5. Experimental Results

In this section, the availability of the considered cooking oil production line is analyzed using failure and repair data. Equations (6)-(11) alongside Table 5 are used for the reliability analysis of the considered cooking oil production line in all three shifts. The mission time for each shift is 6,000 minutes (100 hours). The reliability diagrams of the cooking oil production line under repairable and non-repairable conditions are drawn respectively in Figures 3 and 4 for each shift.

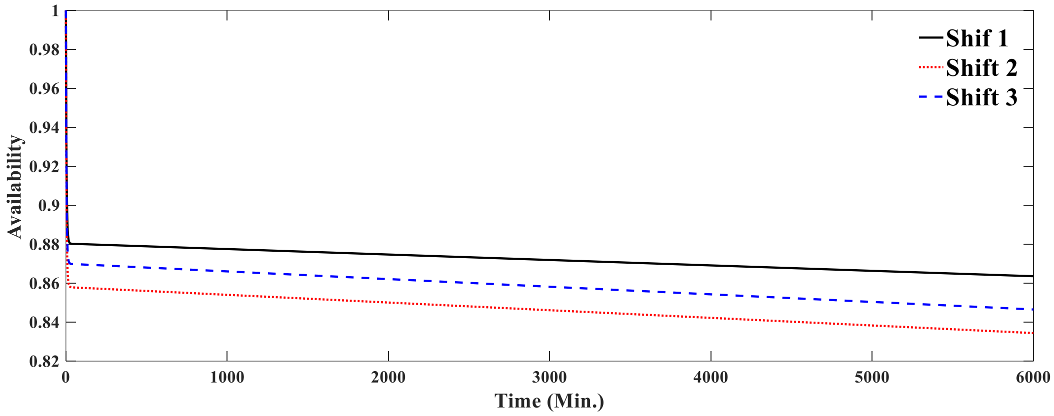


Fig. 3. Availability diagram of different shifts with repairable subsystems

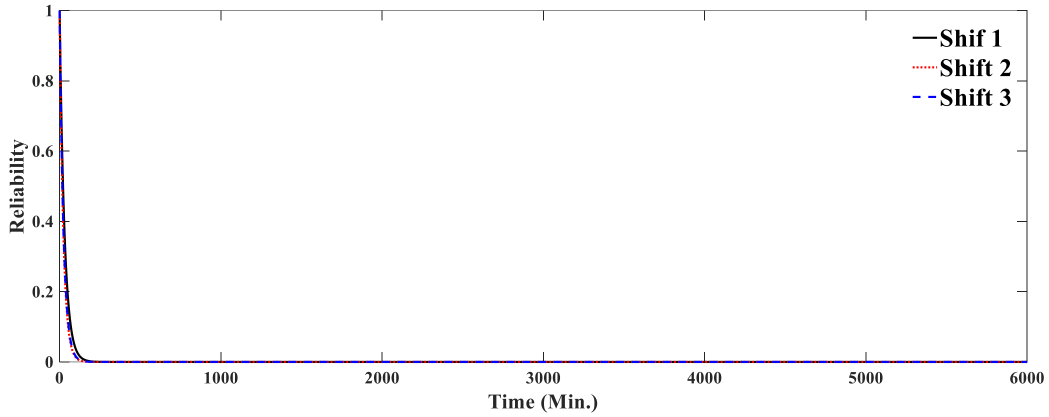


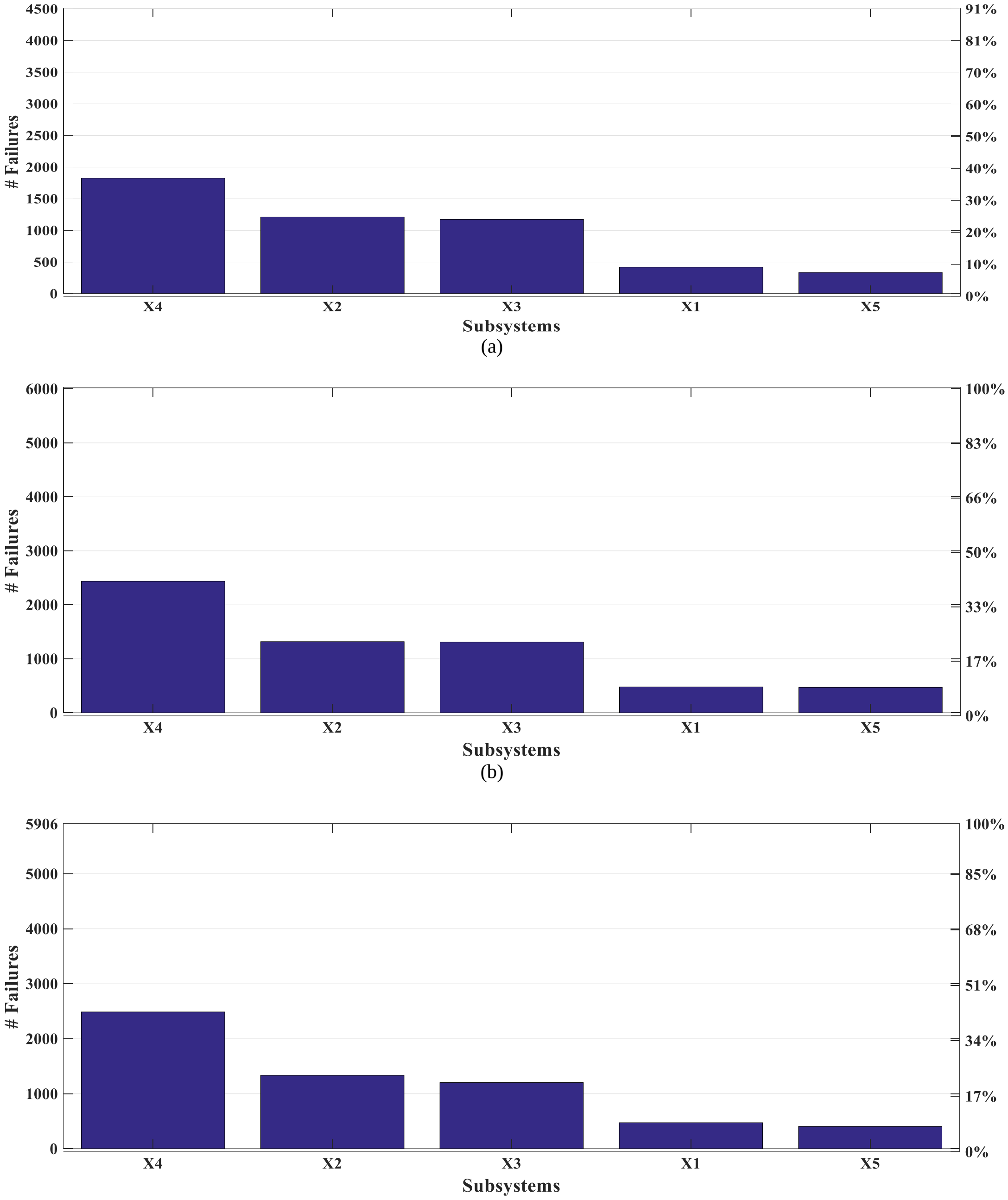
Fig. 4. Reliability diagram of different shifts with non-repairable subsystems

The long-run system availabilities (A_{SS}) of the production line are obtained using Equation (8) for the first, the second, and the third shift as 0.8799, 0.8582, and 0.8696, respectively. Besides, the MTBFs and MTTFs of the production line in each of the shifts are also calculated using Equations (10) and (11) and are arranged in Table 6.

Table 6
MTBF & MTTF in the different shifts

Shift	MTBF (Min.)	MTTF (Min.)
1	274,940	30.30
2	184,590	23.09
3	190,670	24.21

As seen in Figures 3 and 4, as well as in Table 6, it is clear that the production line in Shift 1 has the best performance, where Shift 2 performs the worst. Based on the results shown in Tables 2-4, the Pareto diagrams of the three shifts are shown in Figure 5 to identify the most critical subsystem in a shift. It can be seen from the diagrams that the subsystem with the most failures in the first shift is X_4, which represents 39.76% of the failures. In this shift, subsystems X_2 and X_3 with 22.92% and 22.09% of the failures, respectively, are the next. Thus, an overall 84.64% of the failures are related to the fourth, the second, and the third subsystems. In the second shift, the shares of the second to the fourth subsystems are 21.98%, 21.78%, 40.5% of total failures, respectively, that lead to a total of 85.17% of the system failures. In the third shift, 22.6% of the failures belong to the second subsystem, 20.37% failures belong to the third subsystem, and 42.13% of the failures come from the fourth subsystem and show a total of 85.10% of the failures. This implies that most of the failures in all shifts result from subsystems 2, 3, and 4.



(c)

Fig. 4. Pareto diagrams (a: first shift, b: second shift, c: third shift)

As mentioned previously, the parametric calculation of the availability function is typically very complicated. Therefore, the long-run availability can be used as a proper index for it. The A_{SS} of a system consisting of one subsystem is expressed as

$$A_{SS.j} = \frac{\mu_j}{\lambda_j + \mu_j}; \qquad j = 1,2,3,4,5 \tag{12}$$

Having $\rho = \lambda / \mu$, the Relation (12) is rewritten as

$$A_{SS.j} = \frac{1}{1 + \rho_j}; \qquad j = 1,2,3,4,5 \tag{13}$$

Hence, the A_{SS} of the production line, which consists of five subsystems in series is derived as

$$A_{SS} = \prod_{j=1}^{5} A_{SS.j}, \tag{14}$$

There are many ways to improve the performance of repairable systems. One of these ways is to implement a proper maintenance plan by the maintenance department. Maintenance refers to any kind of action taken for a system in order to maintain their functionality (Blischke & Prabhakar Murthy, 2003). There are two major types of maintenance policies for repairable systems called preventive maintenance and corrective maintenance. Preventive maintenance regularly checks system components and prevents them from failing. The corrective maintenance comes to the picture to repair a failed system component (Billinton & Allen, 1992; Blischke & Prabhakar Murthy, 2003). Here, the corrective maintenance policy is considered.

To enhance the availability of the production line, one may improve the performance of the corresponding repair facility by either increasing the number of repairmen or increasing its efficacy, both of which lead to an improvement in the repair rate. Now, according to Table 5, for the subsystems 2-4 with $\mu_i > 0.2; i = 2,3,4$, the A_{SS} diagram in terms of ρ (Eq. 14) is shown in Figure 6 for the three shifts.

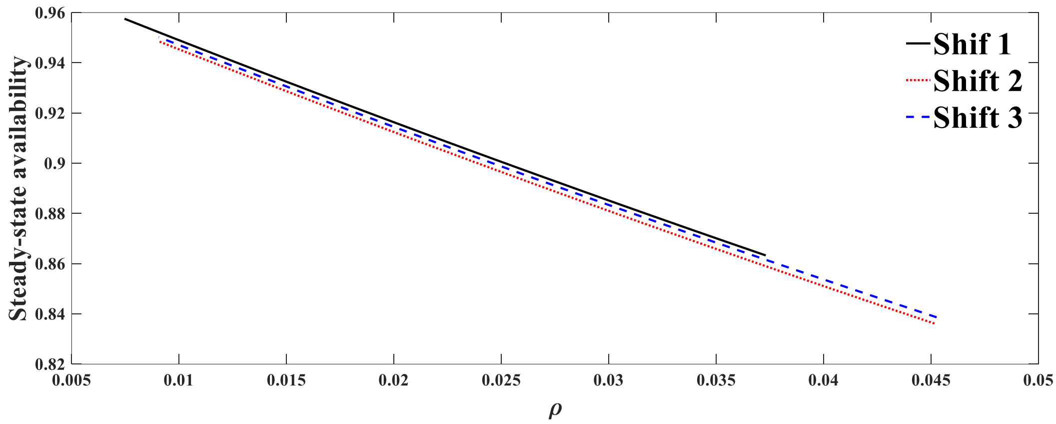


Fig. 6. Steady-state availability diagram of the shifts vs. ρ

As seen in Figure 6, when the repair rate increases for the second, third, and fourth subsystems, their A_{SS}s gradually increase, which in turn will increase the overall system availability. Thus, with the improvement in the technical and operational sections of the maintenance department, the performance of the production line also increases in terms of availability.

6. Conclusion

In this paper, the availability, reliability, and failure metrics such as MTBF and MTTF of a cooking oil production line in three consecutive shifts were analyzed using the Markov process. The results show that the first shift had the best performance in terms of reliability while the second shift had the worst performance. Employing the Pareto diagram in each of the shifts resulted in recognizing the most critical subsystems of the production line. This recognition helps the engineer and the practitioner to raise the reliability and availability of the production line by improving the maintenance policy used for the most critical subsystems. For instance, it was observed that when the repair rates of the critical subsystems increase, their availabilities in each of the three shifts also increase.

Further research is recommended to observe the actual reliability improvement of the line. This of course requires managerial agreement to implement a better preventive maintenance policy. Besides, having the system availability function, a cost analysis can be performed to investigate its effect on the reliability of the system.

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Appendix A

The other first-order differential-difference equations (31-equation) are derived as follows:

$$P_2(t+\Delta t) = P_2(t)[1-(\mu_1+\lambda_2+\lambda_3+\lambda_4+\lambda_5)\Delta t] + P_1(t)\lambda_1\Delta t + P_7(t)\mu_2\Delta t + P_9(t)\mu_4\Delta t + P_{10}(t)\mu_5\Delta t$$

$$P_3(t+\Delta t) = P_3(t)[1-(\lambda_1+\mu_2+\lambda_3+\lambda_4+\lambda_5)\Delta t] + P_1(t)\lambda_2\Delta t + P_7(t)\mu_1\Delta t + P_{12}(t)\mu_4\Delta t + P_{13}(t)\mu_5\Delta t$$

$$P_4(t+\Delta t) = P_4(t)[1-(\lambda_1+\lambda_2+\mu_3+\lambda_4+\lambda_5)\Delta t] + P_1(t)\lambda_3\Delta t + P_8(t)\mu_1\Delta t + P_{14}(t)\mu_4\Delta t + P_{15}(t)\mu_5\Delta t$$

$$P_5(t+\Delta t) = P_5(t)[1-(\lambda_1+\lambda_2+\lambda_3+\mu_4+\lambda_5)\Delta t] + P_1(t)\lambda_4\Delta t + P_9(t)\mu_1\Delta t + P_{14}(t)\mu_3\Delta t + P_{16}(t)\mu_5\Delta t$$

$$P_6(t+\Delta t) = P_6(t)[1-(\lambda_1+\lambda_2+\lambda_3+\lambda_4+\mu_5)\Delta t] + P_1(t)\lambda_5\Delta t + P_{10}(t)\mu_1\Delta t + P_{15}(t)\mu_3\Delta t + P_{16}(t)\mu_4\Delta t$$

$$P_7(t+\Delta t) = P_7(t)[1-(\mu_1+\mu_2+\lambda_3+\lambda_4+\lambda_5)\Delta t] + P_2(t)\lambda_2\Delta t + P_3(t)\lambda_1\Delta t + P_{18}(t)\mu_4\Delta t + P_{19}(t)\mu_5\Delta t$$

$$P_8(t+\Delta t) = P_8(t)[1-(\mu_1+\lambda_2+\mu_3+\lambda_4+\lambda_5)\Delta t] + P_2(t)\lambda_3\Delta t + P_4(t)\lambda_1\Delta t + P_{20}(t)\mu_4\Delta t + P_{21}(t)\mu_5\Delta t$$

$$P_9(t+\Delta t) = P_9(t)[1-(\mu_1+\lambda_2+\lambda_3+\mu_4+\lambda_5)\Delta t] + P_2(t)\lambda_4\Delta t + P_5(t)\lambda_1\Delta t + P_{20}(t)\mu_3\Delta t + P_{22}(t)\mu_5\Delta t$$

$$P_{10}(t+\Delta t) = P_{10}(t)[1-(\mu_1+\lambda_2+\lambda_3+\lambda_4+\mu_5)\Delta t] + P_2(t)\lambda_5\Delta t + P_6(t)\lambda_1\Delta t + P_{19}(t)\mu_2\Delta t + P_{21}(t)\mu_3\Delta t + P_{22}(t)\mu_4\Delta t$$

$$P_{11}(t+\Delta t) = P_{11}(t)[1-(\lambda_1+\mu_2+\mu_3+\lambda_4+\lambda_5)\Delta t] + P_3(t)\lambda_3\Delta t + P_4(t)\lambda_2\Delta t + P_{17}(t)\mu_1\Delta t + P_{23}(t)\mu_4\Delta t + P_{24}(t)\mu_4\Delta t$$

$$P_{12}(t+\Delta t) = P_{12}(t)[1-(\lambda_1+\mu_2+\lambda_3+\mu_4+\lambda_5)\Delta t] + P_3(t)\lambda_4\Delta t + P_5(t)\lambda_2\Delta t + P_{18}(t)\mu_1\Delta t + P_{23}(t)\mu_3\Delta t + P_{25}(t)\mu_5\Delta t$$

$$P_{13}(t+\Delta t) = P_{13}(t)[1-(\lambda_1+\mu_2+\lambda_3+\lambda_4+\mu_5)\Delta t] + P_3(t)\lambda_5\Delta t + P_6(t)\lambda_2\Delta t + P_{19}(t)\mu_1\Delta t + P_{24}(t)\mu_3\Delta t + P_{25}(t)\mu_4\Delta t$$

$$P_{14}(t+\Delta t) = P_{14}(t)[1-(\lambda_1+\lambda_2+\mu_3+\mu_4+\lambda_5)\Delta t] + P_4(t)\lambda_4\Delta t + P_5(t)\lambda_3\Delta t + P_{20}(t)\mu_1\Delta t + P_{23}(t)\mu_2\Delta t + P_{26}(t)\mu_5\Delta t$$

$$P_{15}(t+\Delta t) = P_{15}(t)[1-(\lambda_1+\lambda_2+\mu_3+\lambda_4+\mu_5)\Delta t] + P_4(t)\lambda_5\Delta t + P_6(t)\lambda_3\Delta t + P_{21}(t)\mu_1\Delta t + P_{24}(t)\mu_2\Delta t + P_{26}(t)\mu_4\Delta t$$

$$P_{16}(t+\Delta t) = P_{16}(t)[1-(\lambda_1+\lambda_2+\lambda_3+\mu_4+\mu_5)\Delta t] + P_5(t)\lambda_5\Delta t + P_6(t)\lambda_4\Delta t + P_{12}(t)\lambda_1\Delta t + P_{27}(t)\mu_4\Delta t + P_{28}(t)\mu_5\Delta t$$

$$P_{17}(t+\Delta t) = P_{17}(t)[1-(\mu_1+\mu_2+\mu_3+\lambda_4+\lambda_5)\Delta t] + P_7(t)\lambda_3\Delta t + P_8(t)\lambda_2\Delta t + P_{11}(t)\lambda_1\Delta t + P_{27}(t)\mu_4\Delta t + P_{28}(t)\mu_5\Delta t$$

$$P_{18}(t+\Delta t) = P_{18}(t)[1-(\mu_1+\mu_2+\lambda_3+\mu_4+\lambda_5)\Delta t] + P_7(t)\lambda_4\Delta t + P_9(t)\lambda_2\Delta t + P_{12}(t)\lambda_1\Delta t + P_{27}(t)\mu_3\Delta t + P_{29}(t)\mu_5\Delta t$$

$$P_{19}(t+\Delta t) = P_{19}(t)[1-(\mu_1+\mu_2+\lambda_3+\lambda_4+\mu_5)\Delta t] + P_7(t)\lambda_5\Delta t + P_{10}(t)\lambda_2\Delta t + P_{13}(t)\lambda_1\Delta t + P_{28}(t)\mu_3\Delta t + P_{29}(t)\mu_4\Delta t$$

$$P_{20}(t+\Delta t) = P_{20}(t)[1-(\mu_1+\lambda_2+\mu_3+\mu_4+\lambda_5)\Delta t] + P_8(t)\lambda_4\Delta t + P_9(t)\lambda_3\Delta t + P_{14}(t)\lambda_1\Delta t + P_{27}(t)\mu_2\Delta t + P_{30}(t)\mu_5\Delta t$$

$$P_{21}(t+\Delta t) = P_{21}(t)[1-(\mu_1+\lambda_2+\mu_3+\lambda_4+\mu_5)\Delta t] + P_8(t)\lambda_5\Delta t + P_{10}(t)\lambda_3\Delta t + P_{15}(t)\lambda_1\Delta t + P_{28}(t)\mu_2\Delta t + P_{30}(t)\mu_4\Delta t$$

$$P_{22}(t+\Delta t) = P_{22}(t)[1-(\mu_1+\lambda_2+\lambda_3+\mu_4+\mu_5)\Delta t] + P_9(t)\lambda_5\Delta t + P_{10}(t)\lambda_4\Delta t + P_{16}(t)\lambda_1\Delta t + P_{29}(t)\mu_2\Delta t + P_{30}(t)\mu_3\Delta t$$

$$P_{23}(t+\Delta t) = P_{23}(t)[1-(\lambda_1+\mu_2+\mu_3+\mu_4+\lambda_5)\Delta t] + P_{11}(t)\lambda_4\Delta t + P_{12}(t)\lambda_3\Delta t + P_{14}(t)\lambda_2\Delta t + P_{27}(t)\mu_1\Delta t + P_{31}(t)\mu_5\Delta t$$

$$P_{24}(t+\Delta t) = P_{24}(t)[1-(\lambda_1+\mu_2+\mu_3+\lambda_4+\mu_5)\Delta t] + P_{11}(t)\lambda_5\Delta t + P_{13}(t)\lambda_3\Delta t + P_{15}(t)\lambda_2\Delta t + P_{28}(t)\mu_1\Delta t + P_{31}(t)\mu_4\Delta t$$

$$P_{25}(t+\Delta t) = P_{25}(t)[1-(\lambda_1+\mu_2+\lambda_3+\mu_4+\mu_5)\Delta t] + P_{12}(t)\lambda_5\Delta t + P_{13}(t)\lambda_4\Delta t + P_{16}(t)\lambda_2\Delta t + P_{29}(t)\mu_1\Delta t + P_{31}(t)\mu_3\Delta t$$

$$P_{26}(t+\Delta t) = P_{26}(t)[1-(\lambda_1+\lambda_2+\mu_3+\mu_4+\mu_5)\Delta t] + P_{14}(t)\lambda_5\Delta t + P_{15}(t)\lambda_4\Delta t + P_{16}(t)\lambda_3\Delta t + P_{30}(t)\mu_1\Delta t + P_{31}(t)\mu_2\Delta t$$

$$P_{27}(t+\Delta t) = P_{27}(t)[1-(\mu_1+\mu_2+\mu_3+\mu_4+\lambda_5)\Delta t] + P_{17}(t)\lambda_4\Delta t + P_{18}(t)\lambda_3\Delta t + P_{20}(t)\lambda_2\Delta t + P_{23}(t)\lambda_1\Delta t + P_{32}(t)\mu_5\Delta t$$

$$P_{28}(t+\Delta t) = P_{28}(t)[1-(\mu_1+\mu_2+\mu_3+\lambda_4+\mu_5)\Delta t] + P_{17}(t)\lambda_5\Delta t + P_{19}(t)\lambda_3\Delta t + P_{21}(t)\lambda_2\Delta t + P_{24}(t)\lambda_1\Delta t + P_{32}(t)\mu_4\Delta t$$

$$P_{29}(t+\Delta t) = P_{29}(t)[1-(\mu_1+\mu_2+\lambda_3+\mu_4+\mu_5)\Delta t] + P_{18}(t)\lambda_5\Delta t + P_{19}(t)\lambda_4\Delta t + P_{22}(t)\lambda_2\Delta t + P_{25}(t)\lambda_1\Delta t + P_{32}(t)\mu_3\Delta t$$

$$P_{30}(t+\Delta t) = P_{30}(t)[1-(\mu_1+\lambda_2+\mu_3+\mu_4+\mu_5)\Delta t] + P_{20}(t)\lambda_5\Delta t + P_{21}(t)\lambda_4\Delta t + P_{22}(t)\lambda_3\Delta t + P_{26}(t)\lambda_1\Delta t + P_{32}(t)\mu_2\Delta t$$

$$P_{31}(t+\Delta t) = P_{31}(t)[1-(\lambda_1+\mu_2+\mu_3+\mu_4+\mu_5)\Delta t] + P_{23}(t)\lambda_5\Delta t + P_{24}(t)\lambda_4\Delta t + P_{25}(t)\lambda_3\Delta t + P_{26}(t)\lambda_2\Delta t + P_{32}(t)\mu_1\Delta t$$

$$P_{32}(t+\Delta t) = P_{32}(t)[1-(\mu_1+\mu_2+\mu_3+\mu_4+\mu_5)\Delta t] + P_{27}(t)\lambda_5\Delta t + P_{28}(t)\lambda_4\Delta t + P_{29}(t)\lambda_3\Delta t + P_{30}(t)\lambda_2\Delta t + P_{31}(t)\lambda_1\Delta t$$