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Research Article

Nurse Scheduling Problem by Considering Fuzzy Modeling Approach to Treat Uncertainty on Nurses' Preferences for Working Shifts and Weekends off

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Abstract

Nowadays, the nurse scheduling problem (NSP) has attracted a great amount of attentions. In this problem, the nurses are scheduled to be assigned to the shifts by considering the required nurses for each day during the planning horizon. In the current study, a bi-objective mathematical model is formulated in order to maximize the preferences of the nurses to work on the shifts in addition to be off on the weekends. In real-world problems, higher quality schedules are provided considering the uncertainty. In this point of view, we investigate the uncertainty on the preferences of the nurses for the working shifts and the weekends off. In fact, a compensatory fuzzy approach based on the *Werners' fuzzy and operator* is proposed to investigate the effects of the uncertainty on the considered research problem. Then, several sample problems are generated to support the efficiency of the developed fuzzy model. Finally, a sensitivity analysis is implemented to determine the effects of the changes of the parameters on the obtained results.

***Keywords*:** Health systems; Healthcare management; Nurse scheduling problem; Mathematical programming model; Fuzzy modeling approach

1. Introduction

Recently, the nurse scheduling problem (NSP) has attracted considerable attentions (Jafari and Salmasi 2016). In this problem, the number of the required nurses on each day has been given and the goal is to provide a schedule that assigs the nurses to the shifts satisfying the demands during the planning horizon. Several factors like the hospital managers' policies and the workload laws must be considered to provide an effective schedule (Demirbilek et al. 2018; El Adoly et al. 2018).

In the nurse scheduling problem, days are divided into some time slots that the number of the required nurses is determined for them. These time slots are called *scheduling periods*. For example, morning, evening, and night can be the scheduling periods on days. Days contents some *shifts* that the nurses work at them. For example, morning, evening, night, and long (morning-evening) can be the shifts on days. A nurse is *off* on a day, if no shift is scheduled to be assigned to her on that day. Also, a *leave day* is requested by a nurse to be off on it.

The nurse scheduling problem has been investigated by considering the various constraints and objectives. Furthermore, different mathematical programming models and meta-heuristic algorithms have been developed in order to solve the problem optimally or approximately.

The various mathematical programming models have been proposed for the nurse scheduling problem that some of them are addressed here. Al-Yakoob and Sherali (2007) developed a mixed integer programming model to consider the fairness concept in the generated schedules. Bard and Purnomo (2007) applied an algorithm based on the Lagrangian approach, while Bard and Purnomo (2005), Belien and Demeulemeester (2008), and Maenhout and Vanhoucke (2013) applied the branch-and-price approach to solve the mathematical programming models presented in their research problems. Furthermore, Aktunc et al. (2018) formulated a goal programming model to find a good quality solution for a multi-objective nurse scheduling problem.

Several two-stage mathematical programming models have been also developed for the NSP. Arthur and Ravindran (1981) proposed a two-stage mathematical approach in which the working days of the nurses are determined using the goal programming model at the first stage and the working shifts are assigned to them at the second stage. A two-stage mathematical programming model was applied by Valouxis et al. (2012) in which the nurses' workloads are determined at the first stage, whereas the nurses are scheduled to be assigned to the shifts at the second stage. Moreover, Leaven et al. (2018) developed an approach to minimize the total deviations from the nurses' preferences. A feasible schedule is initially generated and then this schedule is improved.

Osogami and Imai (2000) proved that the nurse scheduling problem is NP-hard. In this point of view, the meta-heuristic algorithms have been widely proposed for the NSP to provide the good quality schedules in a reasonable time. Dowsland and Thompson (2000) developed a tabu-search algorithm to establish a non-

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cyclical schedule. Hertz and Kobler (2000) applied the local search algorithm combined with the genetic algorithm. Majumdar and Bhunia (2007) used the genetic algorithm, whereas Gutjahr and Rauner (2007) applied the ant colony optimization algorithm to solve the research problem. Jafari and Salmasi (2015) developed a binary mathematical programming model for their research problem and then proposed a meta-heuristic approach based on the simulated annealing algorithm to maximize the nurses' preferences in a hospital in Iran. Moreover, Liu et al. (2018), Doerner et al. (2018), and Lin et al. (2018) have proposed the simulated annealing algorithm for their research problem.

Though a vast amount of literature there exists on the nurse scheduling problem, studies on how to treat the uncertainty in this problem are still limited. The current study contributes to the literature by developing a fuzzy modeling approach to investigate the uncertainties on a real-word case of the nurse scheduling problem. First, a bi-objective model is formulated for maximizing the preferences of the nurses to work on the shifts in addition to be off on the weekends. Then, to treat the uncertainties in the preferences of the nurses, a fuzzy modelling approach is applied based on a compensatory fuzzy solution, i.e., *Werners' fuzzy and operator*. It can be stated that the developed fuzzy approach provides a more flexible solution for the considered research problem.

The rest of the paper is organized as follows: In Section 2, the detailed descriptions of the research problem are presented. A mathematical programming model is formulated in Section 3. The fuzzy modelling approach is proposed in Sections 4 to consider the uncertainty in the research problem. The computational results are summarized in Section 5. Finally, the conclusions are provided in Section 6.

2. Description of the Research Problem

In this section, the detailed descriptions of the research problem including the considered assumptions, constraints, and objectives are provided.

Here, the considered assumptions are presented: The planning horizon is two weeks, i.e., 14 days and Monday is the first day of each week. Each day is separated into three scheduling periods which the number of the required nurses has been specified for each of them, i.e., morning from 8:00 AM to 2:00 PM, evening from 2:00 PM to 8:00 PM, and night from 8:00 PM to 8:00 AM. Each day has three shifts which the nurses can work at them, i.e., morning (M) from 8:00 AM to 2:00 PM (6 hours), evening (E) from 2:00 PM to 8:00 PM (6 hours), and night (N) from 8:00 PM to 8:00 AM (12 hours). In fact, the shifts are assumed to be compatible with the scheduling periods. The preferences of the nurses to work on the shifts in addition to be off on the weekends are also considered. In each week, the nurses determine a number for each shift and a number for each weekend (i.e., Sunday) as their preferences to be scheduled at that shift and to be off on that weekend, respectively. Numbers 3, 2, and 1 indicate high, middle, and low preferences, respectively.

Now, the constraints are presented: If the nurses work at the night shift on a specific day, they should be off on the next day. The nurses work at most on three consecutive working days. The nurses can work between 60 to 80 hours during the planning horizon. The number of the required nurses for each scheduling period must be met. Moreover, the nurses are off on the leave days requested by them.

Moreover, the objective is to maximize the preferences of the nurses to work on the shifts in addition to be off on the weekends.

An illustrative example for the problem containing six nurses is provided in Table 1. The number of the required nurses for the morning, evening, and night periods on each day respectively are 2, 1, and 1. Symbols M, E, N, L, and – respectively denote the morning shift, evening shift, night shift, leave days, and off days. The total working time of each nurse and the total number of the assigned nurses to each scheduling period respectively are presented in the Working Time column and in the Scheduling Period row.

3. Bi-objective Mathematical Model

Below, a bi-objective model is formulated for the considered problem.

Indices and parameters are defined as follows:

n	Total number of the nurses
i	Index of the nurses ($i = 1, 2, \ldots, n$)
t	Index of the weeks ($t = 1, 2$)
j	Index of the days ($j = 1, 2, \ldots, 14$)
α_1	Weight of the preferences of the nurses to work on the shifts
α_2	Weight of the preferences of the nurses to be off on the weekends
md	Number of the nurses required for the morning period on each day
ed	Number of the nurses required for the evening period on each day
nd	Number of the nurses required for the night period on each day
$L_{i,j}$	= 1 if nurse *i* requests to be off on day *j*, and = 0 otherwise
$fp_{i,t}$	= 3, 2, and 1 if the preference of nurse *i* for weekends *t* respectively is high, middle, and low

$mp_{i,t}$ = 3, 2, and 1 if the preference of nurse *i* for the morning shift in week *t* respectively is high, middle, and low

$ep_{i,t}$ = 3, 2, and 1 if the preference of nurse *i* for the evening shift in week *t* respectively is high, middle, and low

$np_{i,t}$ = 3, 2, and 1 if the preference of nurse *i* for the night shift in week *t* respectively is high, middle, and low

Decision variables are defined as follows:

$fd_{i,j}$ = 1 if nurse *i* is off on day *j*, = 0 otherwise

$ms_{i,j}$ = 1 if nurse *i* works at the morning shift on day *j*, and = 0 otherwise

$es_{i,j}$ = 1 if nurse *i* works at the evening shift on day *j*, and = 0 otherwise

$ns_{i,j}$ = 1 if nurse *i* works at the night shift on day *j*, and = 0 otherwise

The bi-objective mathematical model is formulated as follows:

$$\text{Maximize } \alpha_1\left(\sum_{i=1}^{n}\sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(mp_{i,t}ms_{i,j}+ep_{i,t}es_{i,j}+np_{i,t}ns_{i,j}\right)\right)+\alpha_2\left(\sum_{i=1}^{n}\sum_{t=1}^{2} fp_{i,t}fd_{i,7t}\right) \quad (1)$$

Subject to:

$$ms_{i,j}+es_{i,j}+ns_{i,j}+fd_{i,j}=1 \qquad i=1,2,\dots,n, j=1,2,\dots,14 \quad (2)$$

$$ns_{i,j}\le fd_{i,j+1} \qquad i=1,2,\dots,n, j=1,2,\dots,13 \quad (3)$$

$$fd_{i,j}+fd_{i,j+1}+fd_{i,j+2}+fd_{i,j+3}\ge 1 \qquad i=1,2,\dots,n, j=1,2,\dots,11 \quad (4)$$

$$\sum_{j=1}^{14}\left(6\,ms_{i,j}+6\,es_{i,j}+12\,ns_{i,j}\right)\ge 60 \qquad i=1,2,\dots,n \quad (5)$$

$$\sum_{j=1}^{14}\left(6\,ms_{i,j}+6\,es_{i,j}+12\,ns_{i,j}\right)\le 80 \qquad i=1,2,\dots,n \quad (6)$$

$$\sum_{i=1}^{n} ms_{i,j}\ge md \qquad j=1,2,\dots,14 \quad (7)$$

$$\sum_{i=1}^{n} es_{i,j}\ge ed \qquad j=1,2,\dots,14 \quad (8)$$

$$\sum_{i=1}^{n} ns_{i,j}\ge nd \qquad j=1,2,\dots,14 \quad (9)$$

$$fd_{i,j}\ge L_{i,j} \qquad i=1,2,\dots,n, j=1,2,\dots,14 \quad (10)$$

$$ms_{i,j}, es_{i,j}, ns_{i,j}, fd_{i,j}\in\{0,1\} \qquad i=1,2,\dots,n, j=1,2,\dots,14 \quad (11)$$

The first and the second parts of the objective function (1) respectively denote the total sum of the preferences of the nurses to work on the shifts in addition to be off on the weekends. Regarding relation (2), none of the nurses can work at more than one shift on each day. Relation (3) guarantees that when the nurses work at the night shifts, they are off on the next day. Relation (4) ensures that the nurses can work at most on three consecutive days. Considering relations (5) and (6), each nurse works between 60 to 80 hours during the planning horizon. Relations (7), (8) and (9) ensure that the number of the required nurses respectively for the morning, evening, and night periods are met. Moreover, regarding relation (10), the nurses are off on the leave days requested by them.

Table 1
An example for the problem containing six nurses

The nurses required for the morning scheduling period									2							Number of the nurses: 6
The nurses required for the evening scheduling period									1							
The nurses required for the night scheduling period									1							
		Days														Working Time
		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Nurse	1	E	N	-	M	M	M	-	E	N	-	M	E	N	-	78
	2	N	-	E	E	N	-	M	M	E	-	M	-	M	M	72
	3	M	-	M	N	L	E	E	N	-	M	E	N	-	E	78
	4	M	M	M	-	E	M	N	-	M	E	-	M	M	N	78
	5	M	M	-	M	M	N	-	M	M	N	-	M	M	L	72
	6	M	E	N	L	M	E	M	-	E	M	N	-	E	M	78
Scheduling Period	Morning	4	2	2	2	3	2	2	2	2	2	2	2	3	2	
	Evening	1	1	1	1	1	2	1	1	2	1	1	1	1	1	
	Night	1	1	1	1	1	1	1	1	1	1	1	1	1	1	

4. Fuzzy Modeling Approach

Fuzzy theory has been introduced by Zadeh (Klir 1965). The various fuzzy operators have been defined in the literature (Zimmerman 1993; Jun and Liu 2017; Abbasi et al. 2016). In the current study, a fuzzy modelling approach is proposed to treat the uncertainty in the nurse scheduling problem. In this point of view, the Werners' (1970) *fuzzy and operator* is applied to provide high quality schedules for the problem. The compensatory or non-compensatory nature of the applied fuzziness operator is one of the most important characteristic of a fuzzy modelling approach (Topaloglu and Selim 2010; Khorrami et al. 2012; Nezamabadi et al. 2011; Moghari et al. 2011). Nonetheless, more decisions in real-world problems are not completely non-compensatory or fully compensatory (Zimmermann and Zysno 1980). Zimmermann and Zysno (1983) developed some hybrid compensatory operators. In this setting, the *fuzzy and operator* proposed by Werner (1970) combines the *weighted averaging operator* (Yager 1988) and the *minimum operator* (Zimmermann 1978).

Assume that X is the universal set and J is the number of the fuzzy membership functions. Moreover, consider parameter w_j as the weight of the membership function μ_j and parameter λ as the compensatory coefficient specified by the decision maker. Then, the fuzzy solution set $\tilde{F}$ concerning the *fuzzy and operator* is introduced as follows:

$$\tilde{F} = \{(x\,, \mu_{\tilde{F}}(x)) \big| x \in X\} \quad \text{s.t.} \quad \mu_{\tilde{F}}(x) = \lambda \min_j \big(\mu_j(x)\big) + (1-\lambda) \sum_{j=1}^{J} w_j\, \mu_j(x) \qquad (12)$$

In this situation, the best solution is calculated when the fuzzy membership function $\mu_{\tilde{F}}(x)$ is maximized on set *X*. By introducing variables β and β_j ($j = 1, 2, \dots, J$), this problem is reformulated to the ordinary linear programming problem as follows. In other words, these variables are contributed into the model to can transform the problem to a customary linear programming problem.

$$\text{Maximize } \beta + (1-\lambda) \sum_{j=1}^{J} w_j\, \beta_j$$

Subject to:

$$\beta + \beta_j \leq \mu_j(x) \qquad j = 1, 2, \dots, J$$
$$\beta, \beta_j \in [0,1] \qquad j = 1, 2, \dots, J$$

Considering other constraints of system (13)

As stated in the previous sections, we consider the uncertainty on the preferences of the nurses to work on the shifts in addition to be off on the weekends. In this point of view, the fuzzy membership functions are:

$\mu 1_i$ Fuzzy membership function concerning the preference of nurse *i* to work at the shifts

$\mu 2_i$ Fuzzy membership function concerning the preference of nurse *i* to be of on the weekends

Appling the triangular membership function, we have:

$$\mu 1_i = \begin{cases} 1 & if\ WSmax \le WS_i \\ \dfrac{WS_i - WSmin}{WSmax - WSmin} & if\ WSmin \le WS_i \le WSmax \\ 0 & if\ WSmin \ge WS_i \end{cases} \quad (14)$$

$$\mu 2_i = \begin{cases} 1 & if\ WOmax \le WO_i \\ \dfrac{WO_i - WOmin}{WOmax - WOmin} & if\ WOmin \le WO_i \le WOmax \\ 0 & if\ WOmin \ge WO_i \end{cases} \quad (15)$$

Where $WS_i = \sum_{t=1}^{2}\sum_{j=7t-6}^{7t}(mp_{i,t}ms_{i,j} + ep_{i,t}es_{i,j} + np_{i,t}ns_{i,j})$ and $WO_i = \sum_{t=1}^{2} fp_{i,t}fd_{i,7t}$ respectively refer to the total preferences of nurse *i* to work on the shifts and to be off on the weekends during the planning horizon. $WSmin$ is the minimum value of the preferences and $WSmax$ is the maximum value of the preferences that a nurse can obtain at the working shifts. Moreover, $WOmin$ is the minimum value of the preferences and $WOmax$ is the maximum value of the preferences that a nurse can obtain on the weekends. Regarding the assumptions and constraints considered in Section 2, the minimum and maximum values of the working shifts for each nurse during the planning horizon are respectively equal to 5 and 11. Therefore, according to the values assigned to the preferences, $WSmin$ and $WSmax$ are respectively equal to 5×1 = 5 and 11×3 = 33. Similarly, $WOmin$ and $WOmax$ are respectively equal to 0×1 = 0 and 2×3 = 6. The fuzzy membership functions $\mu 1_i$ and $\mu 2_i$ are shown in Fig. 1 and Fig. 2, respectively.

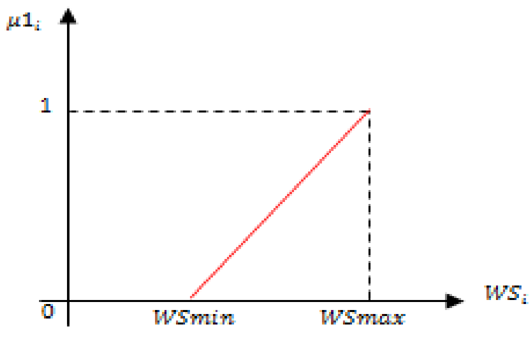

Fig. 1. Membership function concerning the preference of nurse *i* to work at the shifts

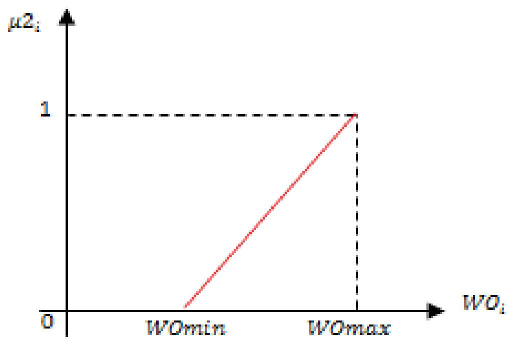

Fig. 2. Membership function concerning the preference of nurse *i* to be off on the weekends

Based on the above explanations and the developed mathematical programming model, the fuzzy mathematical model related to the *fuzzy and operator* for the research problem is formulated as follows:

$$\text{Maximize } ObjFuz = \beta + (1-\lambda)\left[\frac{\alpha_1}{n}\left(\sum_{i=1}^{n}\beta 1_i\right) + \frac{\alpha_2}{n}\left(\sum_{i=1}^{n}\beta 2_i\right)\right]$$

Subject to:

$$(WSmax - WSmin)(\beta + \beta 1_i) \le \sum_{t=1}^{2}\sum_{j=7t-6}^{7t}\left(mp_{i,t}ms_{i,j} + ep_{i,t}es_{i,j} + np_{i,t}ns_{i,j}\right) - WSmin \qquad i = 1,2,\dots,n$$

$$(WOmax - WOmin)(\beta + \beta 2_i) \le \sum_{t=1}^{2} fp_{i,t}fd_{i,7t} - WOmin \qquad i = 1,2,\dots,n$$

$$\beta, \beta 1_i, \beta 2_i \in [0,1]$$

Considering the constraints of system (2)-(11) (16)

5. Results

In this section, the computational results obtained from the proposed fuzzy model are presented. For this reason, 10 random test problems have been generated using the discrete uniform (*DU*) distribution that the generation intervals of the parameters are indicated in Table 2. Note the generation intervals of the parameters have been obtained inspiring from our observations in Milad hospital in Iran. To solve test problems by the developed fuzzy model, IBM ILOG CPLEX software version 12.2 has been applied. The results have been summarized in Table 3. Please note that, to solve test problems by the fuzzy model, the value of the compensatory coefficient has been assumed to be equal to $\lambda = 0.2$. Moreover, the weights of the objective functions have been considered to be equal to $\alpha_1 = \alpha = 0.8$ and $\alpha_2 = 1 - \alpha = 0.2$.

Table 2
The generation intervals of the parameters

Parameter	Generation interval
$\boldsymbol{n}$	$DU[2,40]$
$\boldsymbol{md}, \boldsymbol{ed}, \boldsymbol{nd}$	$DU[1,10]$
$\boldsymbol{L_{i,j}}$	$DU[0,1]$
$\boldsymbol{fp_{i,t}}, \boldsymbol{mp_{i,t}}, \boldsymbol{ep_{i,t}}, \boldsymbol{np_{i,t}}$	$DU[1,3]$

Table 3
The results obtained from the developed fuzzy model for the random test problems

Problem	No. of nurses	Fuzzy objective value	Average of the membership functions		Average of the nurses' preferences	
			Working shifts	Weekends off	Working shifts	Weekends off
		$ObjFuz$	$\frac{1}{n}\sum_{i=1}^{n}\mu 1_i$	$\frac{1}{n}\sum_{i=1}^{n}\mu 2_i$	$\frac{1}{n}\sum_{i=1}^{n}WS_i$	$\frac{1}{n}\sum_{i=1}^{n}WO_i$
1	4	0.399	0.660	0.250	23.250	1.250
2	6	0.418	0.686	0.267	24.000	1.333
3	8	0.390	0.647	0.250	22.875	1.250
4	10	0.533	0.757	0.300	26.200	1.800
5	14	0.552	0.755	0.300	26.000	2.000
6	20	0.593	0.802	0.300	27.350	2.000
7	24	0.600	0.813	0.300	27.667	2.000
8	30	0.619	0.843	0.300	28.533	2.000
9	34	0.618	0.841	0.300	28.500	2.000
10	40	0.624	0.850	0.300	28.750	2.000

The fuzzy objective values and the averages related to the membership functions and the nurses' preferences to work on the shifts and to be off on the weekends have been respectively shown in Fig. 3, Fig. 4, and Fig. 5 under the generated random test problems.

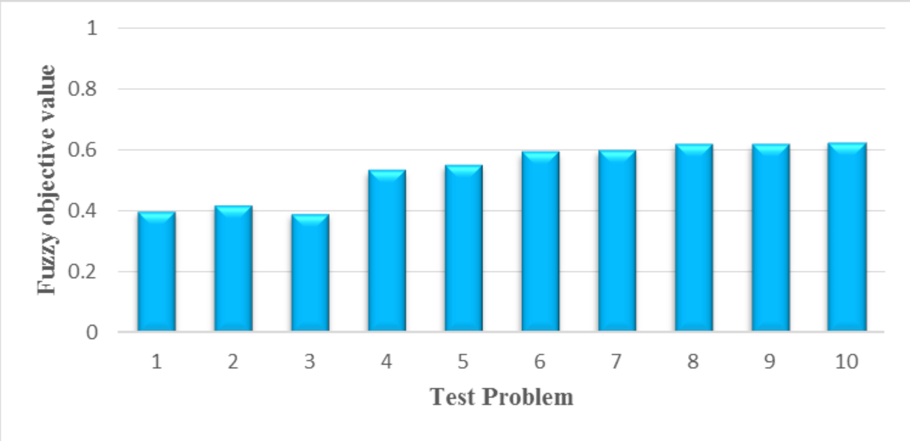

Fig. 3. The fuzzy objective values under the test problems

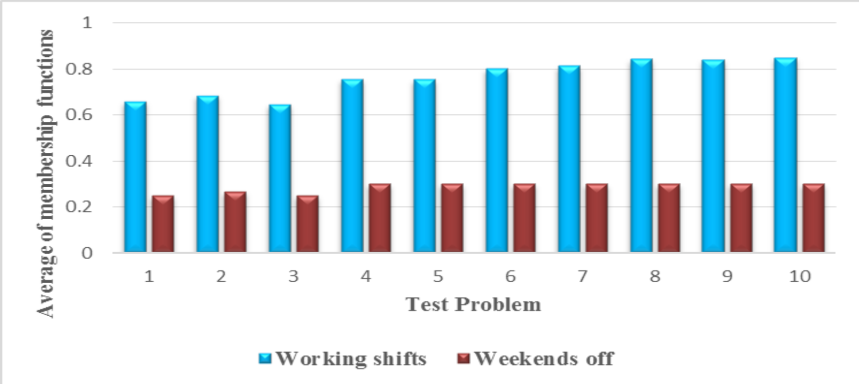

Fig. 4. The averages of the membership functions under the test problems

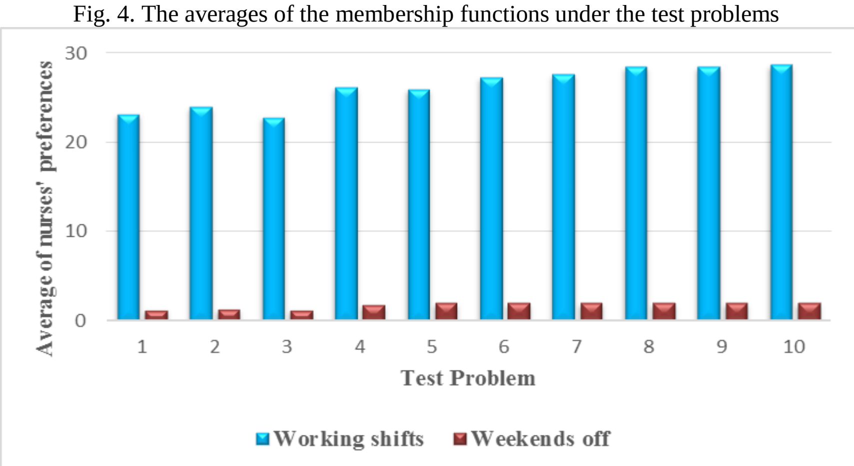

Fig. 5. The averages of the nurses' preferences under the test problems

Regarding the figures, the averages of the membership functions for the working shifts are higher than for the weekends off. Moreover, the averages of the nurses' preferences to work at the desirable shifts are significantly greater than to be off on the favorite weekends.

Here, to illustrate the proposed fuzzy approach, the test problem 4 with 10 nurses is investigated. The values of the parameters related to this problem have been presented in Table 4. The schedule generated by the proposed fuzzy model is also provided in Table 5. Regarding this schedule, all the assumptions and constraints considered in Section 2 have been met. Moreover, the nurses' preferences to work on the shifts and to be off on the weekends have been satisfied. For example, consider nurse 6. Regarding the values of the parameters presented in Table 4, nurse 6 prefers to be off in weekend 2. She prefers to work at the night shift in the first and the second weeks, as possible as. Moreover, she has requested for the leave days on days 2 and 12. According to the provided schedule, her preferences for the shifts as well as the weekends and her requests for the leave days have been met.

Table 4
The values of the parameters related to the test problem 4

n = 10									md = 3					ed = 2					nd = 2			
Nurse	$fp_{i,t}$		$mp_{i,t}$		$ep_{i,t}$		$np_{i,t}$		$L_{i,j}$													
	week		Week		week		week		day													
	1	2	1	2	1	2	1	2	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	3	1	3	3	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	3	3	3	1	1	1	2	0	0	0	0	1	0	0	0	1	0	0	0	0	0
3	2	3	3	3	1	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	3	3	2	1	3	1	1	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
5	3	1	3	1	1	1	1	3	1	0	0	0	0	0	0	0	0	0	0	0	1	0
6	1	3	1	1	1	1	3	3	0	1	0	0	0	0	0	0	0	0	0	1	0	0
7	3	2	1	2	3	3	1	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	3	2	2	3	2	1	2	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0
9	2	3	3	1	1	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1
10	3	1	3	3	3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 5
The schedule generated by the fuzzy model for the test problem 4

The nurses required for the morning scheduling period	3
The nurses required for the evening scheduling period	2
The nurses required for the night scheduling period	2
Number of the nurses : 10	

		Days														Working Time
		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday	
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Nurse	1	M	N	-	M	M	N	-	M	M	M	-	M	M	M	78
	2	-	M	M	N	L	M	M	M	L	M	M	N	-	N	78
	3	M	M	N	-	M	M	N	-	M	E	M	-	M	E	78
	4	E	E	E	-	E	E	N	-	N	-	N	-	N	-	78
	5	L	M	M	M	-	M	M	N	-	E	E	N	L	N	78
	6	N	L	N	-	N	-	E	-	N	-	N	L	N	-	78
	7	E	N	-	E	E	E	-	E	E	N	-	E	E	E	78
	8	N	-	E	N	L	E	E	N	-	M	M	M	-	M	78
	9	M	-	M	M	N	-	M	E	E	-	E	E	E	L	66
	10	M	E	-	E	M	N	-	M	M	N	-	M	M	M	78
Scheduling Period	Morning	4	3	3	3	3	3	3	3	3	3	3	3	3	3	
	Evening	2	2	2	2	2	3	2	2	2	2	2	2	2	2	
	Night	2	2	2	2	2	2	2	2	2	2	2	2	2	2	

Table 6
The results related to the sensitivity analysis of the compensatory coefficient

Problem	No. of nurses	Fuzzy objective value										
		Compensatory coefficient (λ)										
		0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
1	4	0.498	0.448	0.399	0.349	0.299	0.249	0.199	0.149	0.100	0.050	0.000
2	6	0.522	0.470	0.418	0.366	0.313	0.260	0.207	0.155	0.103	0.051	0.000
3	8	0.488	0.439	0.390	0.342	0.293	0.244	0.195	0.145	0.098	0.048	0.000
4	10	0.592	0.563	0.533	0.512	0.497	0.412	0.335	0.285	0.128	0.063	0.000
5	14	0.612	0.589	0.552	0.503	0.457	0.401	0.365	0.228	0.149	0.079	0.000
6	20	0.671	0.645	0.593	0.509	0.423	0.357	0.308	0.212	0.189	0.082	0.000
7	24	0.690	0.641	0.600	0.550	0.507	0.457	0.401	0.259	0.174	0.097	0.000
8	30	0.721	0.680	0.619	0.530	0.479	0.408	0.368	0.287	0.112	0.087	0.000
9	34	0.697	0.654	0.618	0.510	0.434	0.354	0.276	0.211	0.131	0.079	0.000
10	40	0.768	0.687	0.624	0.548	0.436	0.378	0.289	0.241	0.158	0.081	0.000

Now, a sensitivity analysis has been also implemented to investigate the effects of the changes of some considered parameters on the inferences given by the generated random problems.

First, the compensatory coefficient λ related to the developed fuzzy approach is changed from 0.0 to 1.0 in step sizes of 0.1. These results have been provided in Table 6.

Regarding the presented results, the higher compensatory coefficient λ, the lower fuzzy objective values. Note that the *fuzzy and* operator gives the greatest fuzzy objective values comparing to the others as the value of the compensatory coefficient is equal to $\lambda = 0.0$.

Next, the changes of the weight of the objective function α are investigated on the fuzzy objective values. For this reason, this parameter is changed from 0.0 to 1.0 in step sizes of 0.1. The obtained results have been summarized in Table 7.

Regarding the given results, more the weight of the objective function leads to higher the fuzzy objective values. Obviously, the highest fuzzy objective values are obtained when the value of α is equal to 1.0.

Table 7
The results related to the sensitivity analysis of the weights of the objective functions

Problem	No. of nurses	Fuzzy objective value										
		Weight of the first objective function (α)										
		0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
1	4	0.119	0.161	0.399	0.249	0.284	0.302	0.345	0.374	0.399	0.412	0.420
2	6	0.122	0.147	0.418	0.214	0.289	0.317	0.374	0.401	0.418	0.453	0.470
3	8	0.200	0.224	0.390	0.270	0.295	0.319	0.342	0.364	0.390	0.416	0.445
4	10	0.204	0.241	0.533	0.287	0.317	0.397	0.418	0.478	0.533	0.563	0.581
5	14	0.214	0.261	0.552	0.341	0.369	0.395	0.449	0.501	0.552	0.594	0.609
6	20	0.245	0.283	0.593	0.359	0.391	0.446	0.497	0.546	0.593	0.617	0.649
7	24	0.280	0.291	0.600	0.387	0.421	0.467	0.529	0.576	0.600	0.632	0.671
8	30	0.301	0.325	0.619	0.415	0.475	0.506	0.571	0.595	0.619	0.639	0.694
9	34	0.295	0.341	0.618	0.429	0.471	0.567	0.438	0.502	0.618	0.645	0.670
10	40	0.310	0.364	0.624	0.473	0.502	0.536	0.571	0.611	0.624	0.669	0.719

6. Conclusions

In this research, the nurse scheduling problem was investigated in order to maximize the preferences of the nurses to work on the shifts in addition to be off on the weekends. First, a bi-objective mathematical model was formulated by considering several assumptions and constraints taken into account in real world problems.

Considering the uncertainty has a significant effect on generating the good quality solutions. In this point of view, we investigated the uncertainty in this study on the preferences of the nurses to work on the shifts in addition to be off on the weekends. For this reason, a fuzzy modeling approach based on the *Werner's fuzzy and* was proposed.

Then, several instances were also generated to determine the performance of the developed fuzzy model. Regarding the results, the fuzzy membership functions for the working shifts are derived to be higher than for the weekends off. Furthermore, the preferences of the nurses for their shifts are significantly greater than for their weekends off.

Finally, we implemented a sensitivity analysis to investigate the effects of the changes of the compensatory coefficient and the weights of the objective functions on the calculated fuzzy objective values. One can derive that the greater compensatory coefficient, the greater fuzzy objective values. Moreover, the higher weight of the objective function leads to the higher fuzzy objective values.

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