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Hub Covering Location Problem Considering Queuing and Capacity Constraints

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Abstract

In this paper, a hub covering location problem is considered. Hubs, which are the most congested part of a network, are modeled as M/M/C queuing system and located in places where the entrance flows are more than a predetermined value. A fuzzy constraint is considered in order to limit the transportation time between all origin-destination pairs in the network. On modeling, a nonlinear mathematical program is presented. Then, the nonlinear constraints are converted to linear ones. Due to the computational complexity of the problem, genetic algorithm (GA), particle swarm optimization (PSO) based heuristics, and improved hybrid PSO are developed to solve the problem. Since the performance of the given heuristics is affected by the corresponding parameters of each, Taguchi method is applied in order to tune the parameters. Finally, the efficiency of the proposed heuristics is studied while designing a number of test problems with different sizes. The computational results indicated the greater efficiency of the heuristic GA compared to the other methods for solving the problem.

Keywords: Hub covering location, Queuing system, Congestion, Genetic algorithm, Hybrid particle swarm optimization algorithm.

1. Introduction

Every day, large amounts of goods are transported from different origins to a set of destinations. Most of the times, it is impossible to establish a direct link between the origins and destinations. In this situation, a group of locations are considered as hubs. Hubs are used to merge, redirect, and distribute the flow of goods to hub or spoke nodes. In a given network, some of the nodes are selected as hubs, while other nodes allocated to the hub nodes are considered as spokes. General assumptions about hub location problem are as being a direct link between each of the two hubs, the lack of link between spoke nodes, lower cost of transportation between hub nodes than spoke nodes, and the dependence of costs on distance.

The major contributions in the current research can be given as follows:

- Queuing and time constraints from origin to destination are considered simultaneously.
- The times when flows are transported from origin to hub, from hub to hub, and from hub to destination are considered as fuzzy numbers.
- Hubs are located in places where the entrance flows to be more than a known value of Γ for each hub.
- Some heuristics are developed in order to solve the hub covering problem.

2. Literature review

The basic idea for hub-spoke networks was proposed by Goldman (1969). The first mathematical formulation for hub location problem was given by O'kelly (1987). The first computational results for the single allocation hub covering problem was presented by Kara and Tansel (2003). They proved that these types of problems were Np-hard. Marianov and serra (2003) modeled the hub location problem considering M/D/C queuing system. A formula was derived for the probability of a number of customers being in the system in order to be used in a capacity constraint. A new modeling of single and multi-allocation for hub covering problem was given by Wagner (2004). Ernst and Krishnamoorthy (2005) proposed the uncapacitated single and multiple allocation hub covering problem. Rodriguez et al. (2007) presented a model for hub location problem in cargo transportation networks with limited capacity hubs. They modeled each hub as an M/M/1 queuing system. Calik et al. (2009) studied the single allocation hub covering problem with incomplete

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communications in hubs. Han (2010) developed an integer programming (IP) formulation for the problem and developed some valid inequalities which provided a tight lower bound for the problem. Gelare and Nickle (2011) proposed a 4-index formulation for the uncapacitated multiple allocation hub location problem tailored for urban transport and liner shipping network design. Mohamadi et al. (2011) considered a hub-and-spoke network problem with crowdedness or congestion in the system. A hub cannot serve all trucks simultaneously, and it has some restrictions like capacity and service time. They modeled it as M/M/c queuing systems. Alumur et al. (2012) introduced multi-modal hub location and hub network design problem and studied the hub location problem from a network design perspective. Alumur et al. (2012) addressed several aspects concerning hub location problems under uncertainty. They considered two sources of uncertainties: the set-up costs for the hubs and the demands to be transported between the nodes.

To the best of our knowledge, a few papers have studied the pHCP and pHMP with uncertainty in flows, costs, and transportation time. Sim et al. (2009) introduced a stochastic pHCP (SpHCP) utilizing a chance-constraint method to model the minimum delivery service requirement by taking the variability in transportation times into account. Yang et al. (2013a) presented a new risk aversion pHCP with fuzzy travel times by adopting value-atrisk(VaR) criterion in the formulation of objective function. In order to solve and validate the model, they first turned the original VaRpHCP into its equivalent parametric mixed-integer programming problem, and then developed a hybrid algorithm by incorporating genetic algorithm and local search (GALS) to solve the parametric mixed-integer programming problem. Yang et al. (2013b) proposed a new pHCP with normal fuzzy travel time, in which the main goal is to maximize the credibility of fuzzy travel times, such that not exceeding a predetermined acceptable time point along all paths on a network. Due to complexity of the proposed model, they applied an approximation approach (AA) to discretize fuzzy travel times and reformulate the original problem as a mixed-integer programming problem subject to logical constraints. Next, they made use of the structural characteristics to develop a parametric decomposition method to divide the approximate pHCP into two mixed-integer programming subproblems. Finally, the authors developed an improved hybrid particle swarm optimization (PSO) algorithm by combining PSO with genetic operators and local search (LS) to update and improve particles for the subproblems. In another work, Yang et al. (2014) reduced the uncertainty embedded in the secondary possibility distribution of a type-2 fuzzy variable by fuzzy integral and applied the proposed reduction method to pHCP. They also developed a robust optimization method to take uncertainty in travel times into account by employing parametric possibility distributions. Mohammadi et al. (2013) developed a stochastic bi-objective multi-mode transportation model for hub covering problem. They considered the transportation time between each pair of nodes as an uncertain parameter that is also influenced by a risk factor in the network. Similar to Contreras et al. (2011), Adibi and Razmi (2015) developed a 2-stage stochastic programming for formulating stochastic uncapacitated multiple-allocation HLP. They considered three cases, wherein (1) flow is stochastic, (2) cost is stochastic, and (3) both flow and cost, are stochastic. Unlike Contreras et al. (2011), the authors concluded that considering uncertainty into formulation could result in different solutions.

The paper is structured as follows. Section2 presents a nonlinear mathematical model and its linearization. Section 3 describes the proposed solution algorithms. Section 4 presents computational results. Section 5 concludes the paper and presents further research directions.

3. Parameters and Variables

The parameters and decision variables of the model are as follows:

i,j,k,m= Index of nodes$i,j,k,m=\{1,\dots,n\}$

$$X_{ikmj}=\begin{cases}1 & \text{if traffic from node i to node j goes through hubs located at nodes k and m;}\\ 0 & \text{otherwise,}\end{cases}$$

$$X_{ik}=\begin{cases}1 & \text{if node i is allocated to hub at node k;}\\ 0 & \text{otherwise,}\end{cases}$$

C_{ikmj} : The transportation cost of each unit flow from node *i* to node *j* going through hubs located at nodes *k* and *m*.

f_k : The fixed cost of locating a hub at node *k*

r_k : The maximum cost between hub *k* and nodes allocated to hub *k*

$\theta_{q,k}$: Desired upper bound for the probability of an extra queue length at a hub *k*

b_k : Upper bound of queue length at hub *k*

T : Maximum authorized transportation time between any origin/destination pair

t_{ik} : Transportation time between nodes *i* and *j*

a_{ij} : The average flow which is required to be transported from node *i* to*j*

Γ_k : Minimum required demand for locating a hub at *k*

μ_k : Service rate of hub located in node *k*

4. Problem Formulation

In the problem under study, there is a set of n nodes in a given network. A number of the nodes should be selected as hubs, while the rest of the nodes, called spokes, are allocated to the hub nodes. There are some constraints for locating hub nodes, such as cost, entrance flow, time and capacity. The model is of single allocation type which means each node can only be allocated to an individual hub. The proposed model in this research, which is based on Mohamadi et al. (2011), can be stated as follows:

$$Min \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}\sum_{j=1}^{n} c_{ikmj} x_{ikmj} + \sum_{k=1}^{n} f_k x_{kk} \tag{1}$$

S.T:

$$\sum_{k=1}^{n}\sum_{m=1}^{n} x_{ikmj} = 1 \qquad \forall i, j \tag{2}$$

$$x_{ikmj} \le x_{jm} \quad \forall i, j, k, m \tag{3}$$

$$x_{ikmj} \le x_{ik} \quad \forall i, j, k, m \tag{4}$$

$$x_{ik} \le x_{kk} \quad \forall i, j, k, m \tag{5}$$

$$c_{ik} x_{ik} \le r_k \quad \forall i, k \tag{6}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ji} x_{ik} - \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\, x_{ik} x_{jk} \ge \Gamma_k x_{kk} \quad \forall k \tag{7}$$

$$P\{\text{length of queue at node k} > b_k\} \le \theta_{q,k} \forall k \tag{8}$$

$$x_{ikmj}\left(\tilde{t}_{ik} + \tilde{t}_{km} + \tilde{t}_{mj}\right) \le \tilde{T} \qquad \forall i, j, k, m \tag{9}$$

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad \forall i \tag{10}$$

$$x_{ikmj} \in \{0,1\} \qquad \forall i, j, k, m \tag{11}$$

$$x_{ik} \in \{0,1\} \qquad \forall i, k \tag{12}$$

In the aforementioned single allocation model, the objective function in (1) minimizes the sum of transportation and fixed costs of locating the hubs. Constraint (2) ensures that all the flows between i and j are routed through a pair of hubs in m and k (perhaps a pair of k and k). Constraints (3) and (4) guarantee routing the flows between i and j through hubs k and m, involving allocating i and j to hubs k and m, respectively. Constraint (5) states that a node can be allocated only to a hub node. Constraint (6) declares that node i can only be allocated to hub k if the flow cost between i and k be less than r_k. Constraint (7) ensures forming a hub when the entrance flow to be more than the value of Γ_k . Constraint (8) forces the probability of more than $b_k$demand waiting at a queue to be less than or equal to $\theta_{q,k}$. Constraint (9) states that the travel time between all the origin-destination pairs in the network be less than $\tilde{T}$. Constraint (10) ensures that each node is assigned to exactly one hub. Constraints (11) and (12) give the status of the decision variables. As Mohamadi (2011) let Ps be the steady-state probability of s customers being in the system

$$\sum_{s=b_k+1+c}^{\infty} p_s \le \theta_{q,k} \quad or \quad 1-\sum_{s=0}^{b_k+c} p_s \le \theta_{q,k} \qquad (13)$$

An expression for the probabilities Ps is needed; this expression can be derived assuming an arrival rate of λ and a service rate of μ. Then, the service and arrival rates for any state can be given by Eqs. (14) and (15):

$$\lambda_n = \lambda \qquad (14)$$

with c servers; Constraint(8) can be stated as (13):

$$\mu_n = \begin{cases} n\mu & n \le c \\ c\mu & n > c \end{cases} \qquad (15)$$

p_0, as the probability of no demand in the system can be given by Eq.(16):

$$p_0 = \left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c \left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} \qquad (16)$$

The probability of being n demands in the system with c servers can be stated by Eq. (17) as Gross and Harris (1974).

$$p_n = \begin{cases} \dfrac{\lambda^n}{n!\mu^n} p_0 & n \le c \\ \dfrac{\lambda^n}{c^{n-c}c!\mu^n} p_0 & n \succ c \end{cases} \qquad (17)$$

And, the sum of $p_s$is given as in Eq. (18):

$$\sum_{s=0}^{c+b_k} p_s = \sum_{n=0}^{c}\frac{\lambda^n}{n!\mu^n} p_0 + \sum_{n=c+1}^{c+b_k}\frac{\lambda^n}{c^{n-c}c!\mu^n} p_0 \ge 1-\theta_{q,k} \qquad (18)$$

Eq.(13) can be rewritten as Eq. (19):

$$\sum_{n=0}^{c}\frac{\lambda^n}{n!\mu^n}\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1}$$

$$+\sum_{n=c+1}^{c+b_k}\frac{\lambda^n}{c^{n-c}c!\mu^n}\left[\sum_{n=0}^{c-1}\frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{c!}\left(\frac{\lambda}{\mu}\right)^c\left(\frac{c\mu}{c\mu-\lambda}\right)\right]^{-1} \ge 1-\theta_{q,k} \qquad (19)$$

Neither the locations of hubs nor the arrival rates to hubs are known before solving the problem. The locations of the hubs are given by the values of the variables X_{kk}. The arrival rate to a hub located at node k according to Mohamadi et al. (2011) can be obtained from (20)

$$\lambda_k = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} - \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i\ne j}}^{n} a_{ij} x_{ik} x_{jk} \quad \forall k \qquad (20)$$

According to Marianov and Serra (2003), Eq. (20) can be solved for variable λ and for finding the maximum value, λ_{max}. Once this value is found, any smaller value of λ will satisfy Eq. (18). This means that Eq. (18) is equivalent to $\lambda \le \lambda_{max}$. It can be rewritten as:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} - \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \neq j}}^{n} a_{ij} x_{ik} x_{jk} \leq \lambda_{max} \tag{21}$$

In the proposed model, constraint (9) is a fuzzy statement whose right-hand side ($\tilde{T}$) and the coefficient of variables($\tilde{t}_{ik}, \tilde{t}_{km}, \tilde{t}_{mj}$) are triangular fuzzy numbers. Each triangular fuzzy number, like A, can be presented by three real numbers (s,l,r) as in Fig.1(Zadeh, 1965).

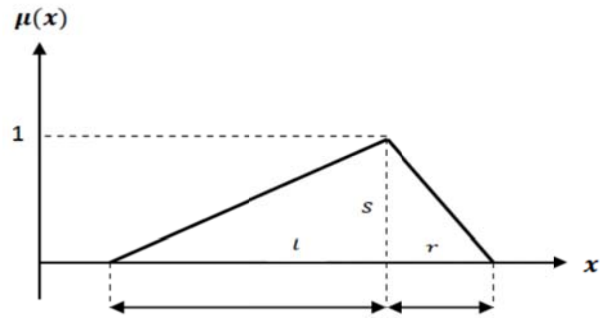

Fig. 1. Peresentation of a triangular fuzzy number

Consider two triangular fuzzy numbers as $\tilde{A} = (s_1, l_1, r_1)$ and $\tilde{B} = (s_2, l_2, r_2)$, then the constraint of $\tilde{A}X \leq \tilde{B}$ can be written as (22)-(24) using the given values of the fuzzy numbers.

$$s_1 + r_1 \leq s_2 + r_2 \tag{22}$$

$$s_1 - l_1 \leq s_2 - l_2 \tag{23}$$

$$s_1 \leq s_2 \tag{24}$$

Considering (22)-(24) and defining the triangular fuzzy numbers of Constraint (9) as $\tilde{t}_{ik} = (t1_{ik}, t2_{ik}, t3_{ik})$, $\tilde{t}_{km} = (t1_{km}, t2_{km}, t3_{km})$, $\tilde{t}_{mj} = (t1_{mj}, t2_{mj}, t3_{mj})$, $\tilde{T} = (T1, T_2, T_3)$,

Constraint(9) can be rewritten as in (25)-(26):

$$[(t1_{ik}, t2_{ik}, t3_{ik}) + (t1_{km}, t2_{km}, t3_{km}) + (t1_{mj}, t2_{mj}, t3_{mj})]x_{ikmj} \leq (T_1, T_2, T_3) \tag{25}$$

$$(t1_{ik} + t1_{km} + t1_{mj}, t2_{ik} + t2_{km} + t2_{mj}, t3_{ik} + t3_{km} + t3_{mj})x_{ikmj} \leq (T_1, T_2, T_3) \tag{26}$$

Considering (22) - (24), Constraint (30) can be stated as in (27) - (29):

$$(t1_{ik} + t1_{km} + t1_{mj})x_{ikmj} \leq T_1 \tag{27}$$

$$[(t1_{ik} + t1_{km} + t1_{mj}) - (t2_{ik} + t2_{km} + t2_{mj})]x_{ikmj} \leq (T_1 - T_2) \tag{28}$$

$$[(t1_{ik} + t1_{km} + t1_{mj}) + (t3_{ik} + t3_{km} + t3_{mj})]x_{ikmj} \leq (T_1 + T_3) \tag{29}$$

Therefore, the final model can be stated as in (30)-(43):

$$Min \sum_{i=1}^{n}\sum_{k=1}^{n}\sum_{m=1}^{n}\sum_{j=1}^{n} c_{ikmj} x_{ikmj} + \sum_{k=1}^{n} f_k x_{kk} \tag{30}$$

$$\sum_{k=1}^{n}\sum_{m=1}^{n} x_{ikmj} = 1 \qquad \forall i, j \tag{31}$$

$$x_{ikmj} \le x_{jm} \quad \forall i, j, k, m \tag{32}$$

$$x_{ikmj} \le x_{ik} \quad \forall i, j, k, m \tag{33}$$

$$x_{ik} \le x_{kk} \quad \forall i, j, k, m \tag{34}$$

$$c_{ik} x_{ik} \le r_k \quad \forall i, k \tag{35}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ji} x_{ik} - \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\, x_{ik} x_{jk} \ge \Gamma_k x_{kk} \quad \forall k \tag{36}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} x_{ik} - \sum_{i=1}^{n}\sum_{\substack{j=1 \\ i \ne j}}^{n} a_{ij} x_{ik} x_{jk} \le \lambda_{\max} \tag{37}$$

(38)

$$t1_{ik} x_{ikmj} + t1_{km} x_{ikmj} + t1_{mj} x_{ikmj} \le T_1 \tag{39}$$

$$(t1_{ik} - t2_{ik}) x_{ikmj} + (t1_{km} - t2_{km}) x_{ikmj} + (t1_{mj} - t2_{mj}) x_{ikmj} \le (T_1 - T_2) \tag{40}$$

$$(t1_{ik} + t3_{ik}) x_{ikmj} + (t1_{km} + t3_{km}) x_{ikmj} + (t1_{mj} + t3_{mj}) x_{ikmj} \le (T_1 + T_3) \tag{41}$$

$$\sum_{k=1}^{n} x_{ik} = 1 \qquad \forall i \tag{42}$$

$$x_{iklj} \in \{0,1\} \forall i, j, k, l \tag{43}$$

$$x_{ik} \in \{0,1\} \forall i, j, k, l$$

5. Solution Techniques and Numerical Rresults

In this section, the solution heuristics are explained consisting of a genetic algorithm(GA) based and a particle swarm optimization (PSO) based heuristics, as well as improved hybrid PSO.

5.1. Genetic algorithm

Holand (1975) proposed the idea of GA for optimizing a number of various types of problems. This algorithm has been used by many researchers interested in location problems including Topcuoglu (2005), Cunha and Silva (2007), and Mohamadi et al. (2011). Now, the steps of this heuristic, based on GA, are explained.

- **Representation of solution:** As the GA standard algorithm, the solutions are called by chromosomes. Here, the chromosome is composed of two parts: hub and assignment arrays. The length of the chromosome is equal to the number of nodes on the network. The first part (hub array) is a binary string. Value "1" indicates that the node is selected as a hub, while value "0" indicates that the node is just a spoke. The second part represents the assignment of each node to the corresponding hub as in Fig.2.

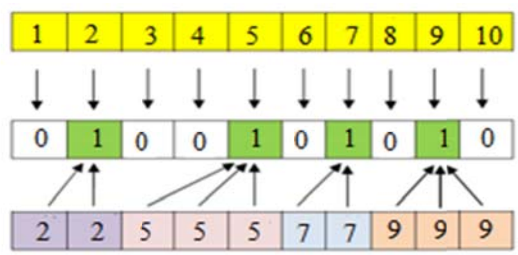

Fig.2. Representation of solution in genetic algorithm for an example with ten nodes.

- **Initial solution:** A random solution for this purpose is generated.
- **Fitness function:** The fitness function value is considered as the difference between the maximum value of the objective and the current objective functions.
- **Parents selection strategy:** The roulette wheel rule is used in this regard.
- **Crossover operator*:*** The single point and random key operator are used, so that each operator is selected with probability of $\frac{1}{2}$. Applying each operator, the generated child may need to be modified. Regarding the first part of the chromosome, if the generated offspring does not have any hub node, or all nodes be selected as hubs, the offspring will be rejected and reproduction is done; for the second part of the chromosome, if a hub node is allocated to another hub node, a modification is done.

- **Mutation operator:** The shift and movement operators of Topcuoglu et al. (2005) are used in this regard. If the generated solutions of this operator are feasible, they are conveyed to the next generation; otherwise, the operator runs again.
- **Stop criterion:** If no improvement occurs within a specific number of successive generations, the algorithm stops.

5.2. Particle Swarm optimization

This algorithm was initially introduced by Kenedy and Eberhart (1995) and was used by Yapicioglu et al. (2007) and Yang et al. (2013) for location problems. Now, the steps of this corresponding heuristic are explained.

- **Representation of the solution:** Representation of solution is just similar to that of GA's.
- **Initial solution:** The heuristic stars with a randomly generated initial solution.
- **Fitness function:** the fitness function is considered equal to the objective function, which means that the particle with less objective function value is of higher priority.

The rest of the conditions are based on the regular PSO. If no improvement happens after a predetermined number of iteration, the algorithm stops.

5.3. Improved Hybrid PSO

This heuristic is a combination of GA and PSO. The initial idea was given by Yang et al. (2013). The major characteristics of this heuristic are given below:

- **Representation of solution:** Representation of the solution is as the given GA-based heuristic.
- **Initial population:** Initially, a random solution is generated. If the solution is infeasible, then a new solution is generated; this procedure continues until the first feasible solution is achieved, then the first feasible solution is added to the initial solution. To complete the population, each new feasible solution is compared to the available solution; if it is not generated before, the solution is add it to the population. This continues by completion of the number of the population.
- **Fitness function:** The fitness function is considered equal to the value of the objective function.
- **Update process:** To update the position of a particle, the genetic operators are used. New position is indicated by x_i^{k+1},and the formula of updating is as in Eq. (44).

$$X_i^{k+1} = (X_{(pbest,i)}^k \otimes X_i^k) \vee (X_G best^k \otimes X_i^k) \vee \quad (44)$$

where $X_{Pbest,i}^k$ represents the best position of the *i*th particle, X_{Gbest}^k represents the best position among the swarm, and X_i^k represents the position of the *i*th individual solution at the *k*th iteration. Since X_i^k, X_{Pbest}^k, and X_{Gbest}^k are location–allocation arrays, the symbol ''$\otimes$'' represents the crossover operation of two individuals solutions. The symbol ''$\vee$'' indicates that the optimal solution is selected from the offsprings of $X_i^k \otimes X_{Pbest}^k$, $X_i^k \otimes X_{Gbest}^k$, $\overline{X}_i^k$, where $\overline{X}_i^k$ represents the mutation operation of X_i^k. If the generated offspring is infeasible, the operators again are used to reach a feasible solution.

- **Stop condition:** If no improvement in a number of successive iterations is obtained, then the algorithm stops.

5.4. Numerical examples

In this section, the performance of the given heuristics is evaluated. Table 1gives the values of parameters and the probability distributions functions considered when randomly generating the numerical examples. 60 ($5 \times 2 \times 2 \times 3$) examples based on the values of n, c, B, and θ have been designed.

5.5. Tuning parameters

In order to tune the parameters of the heuristics, Taguchi method (1986) is applied, and problems with 10,15, and 20 nodes for small sizes, problems with 40 nodes for medium sizes, and problems with70 nodes for large sizes are designed. Furthermore, S/N ratio is considered as in Eq (45).

$$S/N\ ratio = -10\log_{10}(RPD)^2 \quad (45)$$

In order to compute the S/N ratio, relative percentage deviation(RPD) criterion is used. The RPD values represent the difference between the best solution and the average one as in Eq. (46):

$$RPD = \frac{Alog_{sol} - Min_{sol}}{Min_{sol}} \times 100 \quad (46)$$

Alg_{sol} represents the fitness function value in each run for each problem, while Min_{sol} represents the best fitness function value for each problem. The given orthogonal arrays of Taguchi method are given for different levels in order to do the experiments. In this research, three-level experiments have been recognized as the most appropriate designs; according to Taguchi standard orthogonal arrays, L9 orthogonal is selected as the appropriate experiment design in order to tune the parameters for all heuristics. The results are represented in Tables 2-4.

Table 1
Random data of the last problems

Parameters and values															
n	c	B	θ	μ	F	a	c	r	Γ	t_1	t_2	t_3	T_1	T_2	T_3
10															
15	3	10	0.2	Poisson with mean 300	U~ (200,600)	U~(1,20)	U~(1,20)	U~(1,20)	U~(80,120)	U~(1,10)	U~(0.1,0.5)	U~(0.1,0.3)	U~(15,25)	U~(0.3,1.2)	U~(0.3,0.7)
20	4	20	0.4												
40			0.6												
70															

Table 2
Levels of tuned parameters for the GA-based heuristic

Parameters	Size of Problem	Tuned value
Size of Population	Small	100
	Medium	250
	Large	400
Number of Generation	Small	150
	Medium	250
	Large	350
Pm	Small	0.15
	Medium	0.3
	Large	0.35
Pc	Small	0.9
	Medium	0.95
	Large	0.9

Table 3
Levels of tuned parameters for the PSO-based heuristic

Parameters	Size of Problem	Tuned value
Size of Population	Small	100
	Medium	250
	Large	400
Number of Iterations	Small	150
	Medium	250
	Large	350
C1	Small	2
	Medium	2
	Large	2
C2	Small	1.5
	Medium	1.5
	Large	1.5

Table 4
Levels of tuned parameters for the improved hybrid

Parameters	Size of Problem	Tuned value
Size of Population	Small	100
	Medium	250
	Large	400
Number of Iterations	Small	150
	Medium	250
	Large	350

Table 5
Results of metahuristic algorithms' solutions

Node	b	C	θ	LINGO		GA		hybrid PSO		Pso	
				CPU(S)	Obj	CPU(S)	Obj	CPU(S)	Obj	CPU(S)	Obj
10	10	3	0.2	3224	3348	5.1	3348	13.08	3348	11.41	3354
			0.4	3242	3351	4	3351	16.37	3595	11.31	3353
			0.6	3237	3369	4.97	3372	18.09	3485	12.09	3379
		4	0.2	3341	3190	4.68	3190	20.09	3199	12.71	3332
			0.4	3376	3180	7.19	3182	19.8	3185	12.39	3225
			0.6	3370	3194	4.27	3207	13.26	3210	11.79	3282
	20	3	0.2	3392	3355	4.51	3360	12.36	3355	11.63	3366
			0.4	3420	3370	4.87	3372	15.91	3563	11.57	3380
			0.6	3435	3387	7.79	3397	16.53	3401	11.53	3383
		4	0.2	3654	3192	4.32	3202	19.03	3192	12.33	3219
			0.4	3688	3197	5.01	3198	19.51	3197	12.06	3267
			0.6	3671	3202	8.24	3202	15.83	3202	11.67	3302
15	10	3	0.2	-	-	15.19	5845	26.23	6306	12.82	6382
			0.4	-	-	15.83	5821	25.12	5952	12.84	5964
			0.6	-	-	12.13	6389	33.38	6351	18.32	6488
		4	0.2	-	-	15.49	5721	24.65	5967	14.86	6205
			0.4	-	-	15.38	5644	38.75	6037	16.05	6560
			0.6	-	-	17.33	5365	34.59	5702	18.5	6537
	20	3	0.2	-	-	15.23	5616	28.95	5956	19.07	6571
			0.4	-	-	14.43	5715	27.52	6055	19.32	6324
			0.6	-	-	13.65	5823	29.61	5812	22.12	6206
		4	0.2	-	-	13.11	5373	27.21	5760	22.44	6499
			0.4	-	-	13.33	5547	26.32	5811	23.01	6344
			0.6	-	-	12.12	5412	35.8	5507	22.76	6420
20	10	3	0.2	-	-	27.62	8713	44.21	9767	31.2	10016
			0.4	-	-	19.93	8877	45.33	9586	28.62	10102
			0.6	-	-	28.07	8928	40.2	9337	30.48	10009
		4	0.2	-	-	18.56	8877	44.45	9375	27.94	10001
			0.4	-	-	15.01	8995	37.26	9461	29.38	9978
			0.6	-	-	18.62	8689	43.3	9295	31.16	10033
	20	3	0.2	-	-	34.58	8689	87.37	9581	33.54	10055
			0.4	-	-	27.88	8711	47.63	9566	29.86	10070
			0.6	-	-	21.78	8877	51.1	9234	30.5	9989
		4	0.2	-	-	36.08	9088	41.79	9210	29.4	9995
			0.4	-	-	28.35	8585	66.75	9426	30.58	10005
			0.6	-	-	20.46	8676	46.6	9179	29.34	10062
40	10	3	0.2	-	-	275.37	30031	268.54	30124	241.22	30768
			0.4	-	-	190.75	29261	303.57	30232	241.52	30788
			0.6	-	-	141.09	29649	331.34	30132	239.29	30575
		4	0.2	-	-	241.41	28950	291.54	30086	239.45	30844
			0.4	-	-	173.35	29142	299.43	30133	238.78	30355
			0.6	-	-	252.19	28836	298.54	30112	238.33	30888
	20	3	0.2	-	-	231.21	29315	314.43	30001	244.58	31374
			0.4	-	-	232.08	29140	310.23	30276	243.45	30744
			0.6	-	-	257.62	29024	302.46	29999	239.58	30768
		4	0.2	-	-	243.35	29076	351.21	30069	244.22	30646
			0.4	-	-	307.15	28766	377.53	30094	269.45	30744
			0.6	-	-	232.92	29137	325.37	30012	260.29	30974
70	10	3	0.2	-	-	599.21	76890	762.51	76999	677.43	77112
			0.4	-	-	6503.91	76895	614.93	77043	711.21	77234
			0.6	-	-	599.25	76898	823.65	77153	850.06	77333
		4	0.2	-	-	935.31	76702	995.21	76884	906.54	77189
			0.4	-	-	1129.1	76623	1070.02	76602	943.54	77003
			0.6	-	-	707.34	76601	925.67	76998	870.56	77068
	20	3	0.2	-	-	786.9	76693	1034.44	76875	843.24	77234
			0.4	-	-	526.72	76880	1045.65	77125	854.45	77496
			0.6	-	-	953.12	77001	975.55	77122	967.67	77833
		4	0.2	-	-	705.97	76873	1136.66	76985	902.78	77676
			0.4	-	-	869.21	76854	1234.28	77133	956.32	77765
			0.6	-	-	1037.54	76407	912.12	76452	995.76	77558

(-) means that lingo could not solve the problems in a reasonable amount of time, CPU represents the time, and obj represents the objective function value.

6. Experimental Results

In order to assess the performance of the heuristics, the quality of solutions for the small-sized problems is compared with those obtained from lingo solver. The results based on applications of the three algorithms from the view point of run time and solution are represented in Table 5. Samples of convergence diagrams of the heuristics are indicated in Fig. 3-5. In order to assess the efficiency of the heuristics, a special criterion is used. The criterion is a relative percentage ratio (RPI) which is used for the objective function values and CPU time assessment. Each heuristic is run four times per an example, and the best and worst objective functions (values) are founded; the values of this index are between 0 and 100. Smaller values of this index represent better performance. RPI values are computed as in Eq. (47)-(48).

$$RPI_{sol}=\left|\frac{best_{sol}-Alg_{sol}}{best_{sol}-Worst_{sol}}\right| \quad (47)$$

$$RPI_{time}=\left|\frac{best_{time}-Alg_{time}}{best_{time}-Worst_{time}}\right| \quad (48)$$

The results of RPI as given by Eqs (47)-(48) are given in Table 6. As it is clear from Figs.6-11, the RPI index for both objective functions and an CPU time is of better performance for GA for all sizes.

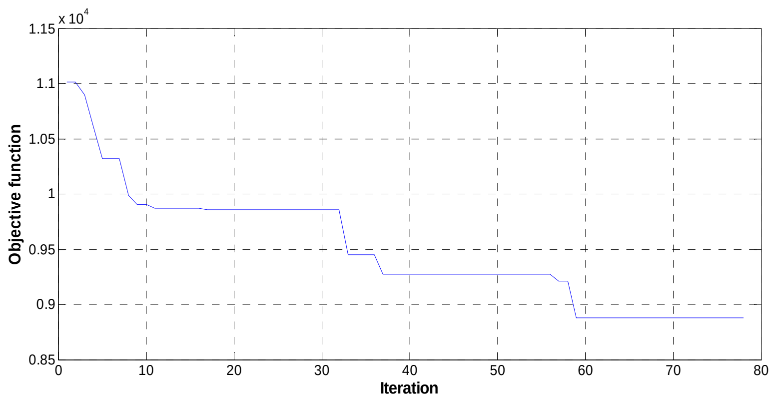

Fig. 3. Convergence diagram of Genetic algorithm for problem with 20 nodes

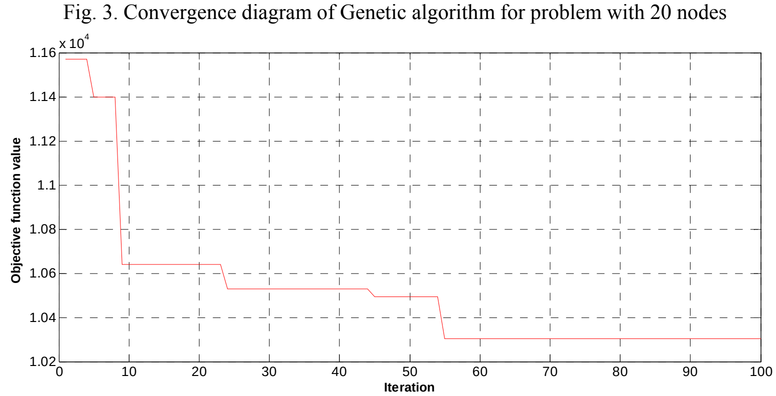

Fig. 4. Convergence diagram of PSO algorithm in for problem with 20 nodes

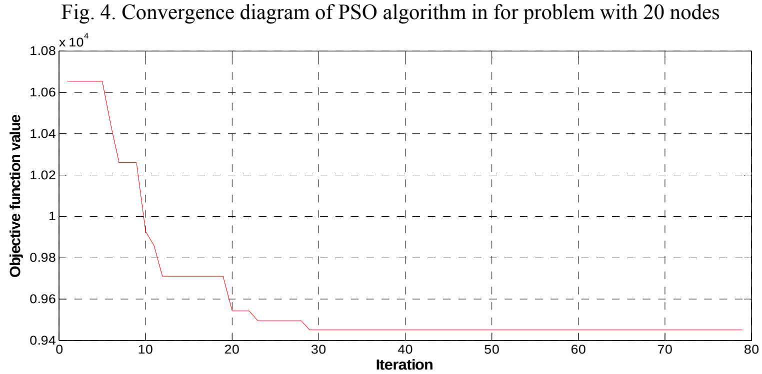

Fig. 5.Convergence diagram of improved hybrid PSO algorithm for problems with 20 node

Table 6
Results of RPI indexes

Node	b	c	θ	GA RPI Time	GA RPI ObjFun	Hybrid PSO RPI Time	Hybrid PSO RPI ObjFun	PSO RPI Time	PSO ObjFun
10	10	3	0.2	2.999	0	58.38	0	54.07	1.063
			0.4	1.536	11.18	67.607	20.186	59.094	0.31
			0.6	2.276	28.251	75.829	51.868	55.947	1.046
		4	0.2	22.58	39.321	62.766	30.123	57.32	23.706
			0.4	0	22.185	53.6	41.887	46.04	7.45
			0.6	2.298	12.647	87.642	51.983	46.723	8.3865
	20	3	0.2	19.319	0	77.574	35.654	62.723	2.614
			0.4	0.012	0	84.573	43	55.193	1.246
			0.6	11.918	16.498	74.784	37.373	57.734	0
		4	0.2	16.268	1.654	89.336	36	55.511	4.477
			0.4	4.758	1.29	85.8621	37	48.62	12.419
			0.6	19.454	0	65.3869	38	44.731	12.755
15	10	3	0.2	3.271	31.768	23.9308	65.59	7.677	71.166
			0.4	23.299	27.895	39.097	36.052	14.995	36.799
			0.6	5.539	46.574	34.515	43.804	16.892	53.79
		4	0.2	8.333	0	14.039	18.317	7.253	36.038
			0.4	18.061	9.905	55.487	40.546	19.994	68.014
			0.6	16.114	0	44.529	17.124	18.718	59.552
	20	3	0.2	22.299	22.285	48.984	33.873	32.566	72.92
			0.4	16.028	4.524	42.9014	12	29.802	44.459
			0.6	4.956	14.419	19.296	13.626	17.874	53.064
		4	0.2	15.04	0	37.028	10.821	32.206	65.162
			0.4	12.92	17.483	36.828	33.85	38.188	66.893
			0.6	14.208	0	51.367	5.986	44.384	63.516
20	10	3	0.2	15.609	0	61.362	47.621	25.482	65.247
			0.4	0.034	10.651	77.842	48.224	26.631	75.569
			0.6	13.422	13.537	38.278	35.166	18.36	70.703
		4	0.2	7.111	11.247	59.663	19.5	26.181	71.162
			0.4	0.81	15.974	39.172	16.374	25.586	65.197
			0.6	0.035	0.65	43.735	12.906	22.222	67.88
	20	3	0.2	22.606	0	99.706	46.506	21.081	71.22
			0.4	20.199	1.978	47.508	16.28	22.944	78.801
			0.6	0.001	10.941	66.879	17.474	19.89	71.475
		4	0.2	27.849	21.661	39.576	8.78	14.13	69.348
			0.4	11.528	0	75.136	15.205	15.223	68.765
			0.6	10.721	0	57.508	24.206	26.615	66.698
40	10	3	0.2	33.554	46.03	5.311	50.065	11.103	78.004
			0.4	25.396	19.034	79.942	56.223	47.891	77.518
			0.6	2.656	14.359	38.588	37.099	46.204	43.832
		4	0.2	54.56	0.82	65.553	47.415	53.474	78.506
			0.4	14.24	6.395	76.475	50.715	53.317	56.171
			0.6	68.88	3.039	77.643	48.144	59.577	75.574
	20	3	0.2	0.016	10.999	24.243	34.728	11.116	82.22
			0.4	0.018	18.148	65.992	59.882	10.001	77.075
			0.6	41.193	5.915	61.58	42.187	31.532	70.796
		4	0.2	43.413	0	87.291	35.363	43.767	55.911
			0.4	67.128	0	95.211	41.383	52.084	61.639
			0.6	0.075	13.848	99.675	41.451	29.551	71.798
70	10	3	0.2	5.043	36.319	56.904	54.423	29.884	62.825
			0.4	12.87	0.207	66.117	30.641	32.282	70.186
			0.6	0.029	0.133	56.432	34.136	62.725	58.232
		4	0.2	45.94	0.155	75.829	28.304	31.585	75.738
			0.4	44.9	2.196	46.446	0	26.935	41.945
			0.6	0.014	0	53.178	41.225	47.037	48.494
	20	3	0.2	6.889	28.08	79.98	39.462	28.077	61.913
			0.4	0.011	4.206	80	33.653	63.154	78.245
			0.6	14.202	0	38.846	12.359	27.214	84.984
		4	0.2	0.202	31.125	79.98	39.1	45.696	88.319
			0.4	0.024	0	50.71	25.881	23.861	84.508
			0.6	39.928	6.919	33.858	9.856	31.244	82.049
average				16.55	13.856	62.887	32.793	26.186	57.455

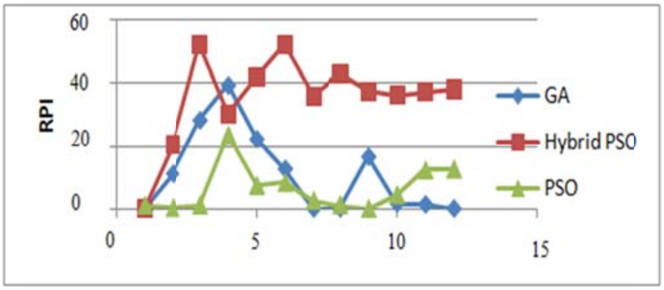

Fig. 6. **RPI$_{sol}$** index for network with 10 nodes

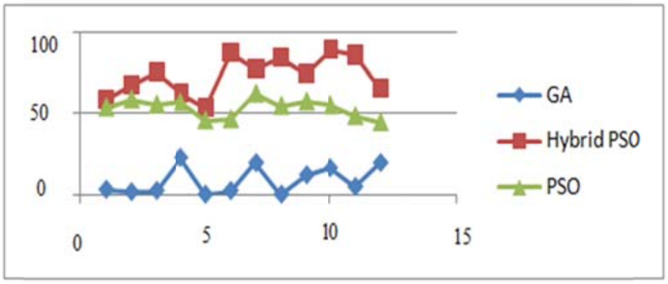

Fig .7. **RPI$_{time}$**index for network with 10 nodes

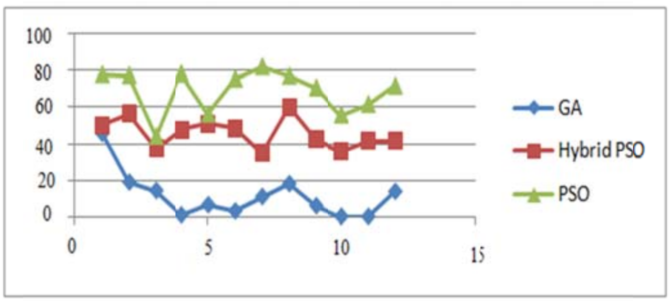

Fig. 8. **RPI$_{sol}$** index for network with 40 nodes

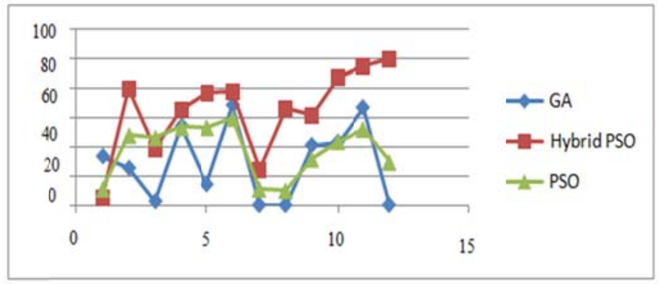

Fig. 9. **RPI$_{time}$** index for network with 40 nodes

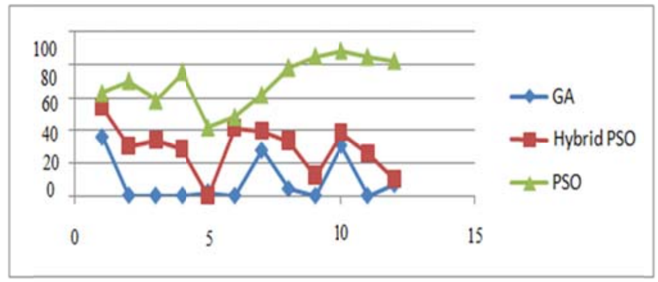

Fig. 10. **RPI$_{sol}$** index for network with 70 nodes

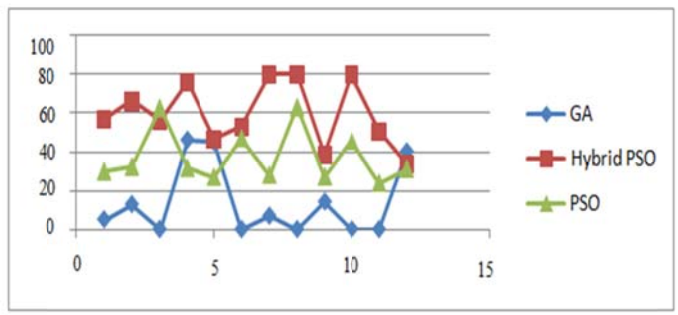

Fig. 11. **RPI$_{time}$** index for network with 70 nodes

Analysis of variance (ANOVA) is applied utilizing Minitab16. According to the results, in confidence level of 0.95, the mean equality hypothesis for time index is rejected for all problem sizes, and the mean equality hypothesis for objective function index is rejected for medium and large-sized problems. The results are given in Tables 7-8.

Table 7
Results of ANOVA test for time index($\alpha = 0.05$)

Size	level	Mean	Std.dev	Pooled St Dev	F	P
Small	GA	14.86	8.92	11.60	20.74	0.00
	PSO	20.18	7.99			
	Hybrid PSO	32.06	16.12			
Medium	GA	231.54	44.94	31.32	24.25	0.00
	PSO	245.01	9.71			
	Hybrid PSO	314.52	28.80			
Large	GA	1279	1656	975	24482.83	0.00
	PSO	77375	283			
	Hybrid PSO	961	169			

Table 8
Results of ANOVA test for objective function index($\alpha = 0.05$)

Size	level	Mean	Std.dev	Pooled StDev	F	P
Small	GA	5927	2302	2548	0.58	0.561
	PSO	6575	2782			
	Hybrid PSO	6227	2537			
Medium	GA	29194	350	251	122.05	0.00
	PSO	30789	243			
	Hybrid PSO	30106	85			
Large	GA	76776	171	229	21.78	0.00
	PSO	77375	283			
	Hybrid PSO	76948	219			

7. Conclusions and Future Researches

In this paper, a hub covering location model is proposed in which the hubs behave as M/M/c queuing systems. A nonlinear model considering constraints for entrance flow and transportation time is presented. The model was linearized. Since the problem is NP-hard, three GA, PSO, and Hybrid PSO-based heuristics were proposed to solve the problem. Then, a number of numerical examples with three different sizes of small, medium, and large was designed, and the performance of the heuristics was evaluated. The results indicated that the GA-based heuristic dominates others for all types of the problems. According to the results, in confidence level of 0.95, the mean equality hypothesis for time index is rejected for all problem sizes, and the mean equality hypothesis for objective function index is rejected for medium and large-sized problems. The proposed model can be used in establishing airports, post offices, passenger terminal, etc. Also, other queuing systems, such as G/G/1 and G/G/M, can be used to develop a more realistic model. On the other hand, the problem can be developed to a multi-period one, in which the effects of time value of money are considered. The entrance flow as fuzzy number can be a new idea in order to extend the model.

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