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Research Article

Measuring the Performance of Medical Diagnostic Laboratories Based On Interval Efficiencies

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Abstract

The classic data envelopment analysis (DEA) models have overlooked the intermediate products, internal interactions and the absence of data certainty, and dealt with analyzing the network within the "Black Box" mode. This results in the loss of important information and at times, a considerable modification occurs in efficiency results. In this paper, a three-stage network model is considered with additional inputs and undesirable outputs and the efficiency of the network is obtained as interval efficiency in the presence of the imprecise datum. The proposed model simulates the internal structure of a diagnostic lab (the pre-test, the test and the post-test). In this study, the criteria for evaluation are obtained by using the Fuzzy Delphi method. Due to the social, economic and environmental problems of health care organizations, the importance of sustainability criteria is evident in the case study indicators. We utilized the multiplicative DEA approach to measure the efficiency of a general system and a heuristic technique was used to convert non-linear models into linear models. Ultimately, this paper concentrates on the interval efficiency to rank the units.

***Keywords*:** Network DEA; Medical Diagnostic Laboratories; Sustainability, Imprecise data; Additional inputs; Undesirable outputs

1. Introduction

Performance appraisal is a key part in the formulation and implementation of organizational policies. Today, all organizations have depicted the importance of having a system for measuring performance. As a principle, every organization should measure its performance capacities as far as possible. The absence of an effective assessment or evaluation system is directly related to the disintegration of an organization and this shortcoming is considered an organizational disease; for without measuring, there shall be no basis for judgments, opinions and evaluations. As whatever cannot be evaluated, cannot be even fittingly managed. So as to ensure a correct management, every organization must use scientific models for the evaluation of performance, so that its efforts and the results achieved from its performance can be appraised. Enhancing the performance and effectiveness of organizations is essential for survival in global markets today. The laboratory services industry is not an exception and moves them towards new approaches to make high quality health value for people. Therefore, laboratory diagnostic services in the health system are the leading provider of diagnostic services due to the nature of the services in this sphere. In this regard, the strengths and weaknesses of the units are known. Then, the units should be compared, in order to eliminate the defects and fortify their strong points. Due to the lack of resources and the significance and sensibility of treatment and community health, the optimal utilization of available resources seems essential. Improving the performance of the precise. The goal of this approach is to provide high efficiency services. Even a negligible upgrade in lab improvement programs will facilitate in paying attention to serving the people. Due to the large number of laboratory units and the establishment of new units, an extremely new condition has developed that has made the scope of the competition extremely broad and considerable; so, experienced laboratories cannot contend in a field with the present structures and inefficient bodies. In this regard, every laboratory must be aware of the efficiency of its service activities and identify the reasons of its unit efficiency and inefficiency. Since laboratories contain three main processes (the pre-test, the test and the post-test) and have a multi-stage nature, in this study we apply Network Data Envelopment Analysis (NDEA) models to measure performance. This paper, considers a three-stage network, which, in addition to middle variables, has additional undesirable inputs and additional undesirable outputs. In fact, DEA has proposed as a theoretical framework for performance analysis, but its application in the field of health care is not wide. This network is proposed to evaluate the performance and ranking of laboratory units with consideration of sustainability criteria (economic, social and environmental). Therefore, we considered a three-stage network of three main laboratory processes. In this regard, the pre-test process consists of the reception unit and the sample unit. The process of test consists of a test unit. Finally, the post-test process consists of the test results unit. The paper unfolds as follows: Section 2 reviews the literature on the DEA approach. Section 3 discusses the research methodology. Then, the mathematical modeling of

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the problem is described. In Section 4, a heuristic approach is introduced to solve the nonlinear program. Section 5, comprises of the result of a case study of the paper. The case study includes 25 private medical diagnostic tests in Tehran and subsequently, we shall analyze the results. Finally, Section 6 concludes the paper.

2. Literature Review

Data Envelopment Analysis (DEA) is a non-parametric method to measure the relative efficiency of a set of analogous decision-making units (DMUs), with multiple inputs and outputs (Khalili-Damghani et al., 2015). Farrell (1957) considered a model for performance evaluation with an input and an output for the first time. Charnes, Cooper and Rhodes developed Farrell's model for multiple inputs and outputs and dubbed it as "CCR" in honor of its makers (Charnes et al., 1978). Charnes, Cooper and Banker extended the DEA models and registered this model as their own, known as "BCC" (Banker et al., 1984). A failure to consider the internal correlations and the intermediate variables of complex systems (*i.e.* two or multiple stage processes), was a major flaw or defect, with which the classic DEA models, such as, the CCR and BCC were labeled. In fact, these models considered the systems as black boxes that would only be satisfied with the initial inputs and final outputs (Tone and Tsutsui, 2009). In order to overcome this problem, Fare and Grosskopf (2000) introduced the Network Data Envelopment Analysis (NDEA) models. With the assistance of the sub-series and parallels, including the intermediate variables, complex systems were simulated and a more genuine evaluation was computed (Chen and Yan, 2011). From Kao's viewpoint, NDEA models can be categorized into three groups, namely, series, parallel and hybrid models. When the activities of a system are prolonged with respect to each other, the network has a serial structure, and at times when the activities are parallel to each other, the network has a parallel structure. Similarly, it maintains a hybrid mode, when there is a combination between the series and parallel set-up (Kao, 2009). Generally, the multiplicative and additive approaches are used to compute the network performance in the parallel and serial mode, respectively, (kao, 2006). After introducing the NDEA models, numerous studies have been conducted in this field, and a combination of this field with branches of the game theory, brought about NDEA studies in cooperative and non-cooperative modes, which can be mentioned hereunder. Li et al. (2012) presented a model for a two-stage structure, a phase that holds a more important standpoint for managers; they have named this phase as "leader" and the other is known as "follower". In order to calculate the efficiency, initially, the efficiency of the leader phase was maximized to the optimum and thus, the efficiency of the follower phase was secured by maintaining a constant efficiency in the phase of the leader. This exemplary, was designated as a decentralized control or a Stackelberg Game, which has been widely utilized by researchers in recent years. Du et al. (2015) analyzed a parallel network in the cooperative and non-cooperative modes and stated that, the efficiency of the former was more than that of the latter mode. An et al. (2017) procured a network, comprising of two stages, which interacted with each other; they computed the efficiency of this network in a cooperative and non-cooperative mode or condition. In another research, Zhou et al. (2018), evaluated a multi-stage network in the non-cooperative and Black Box mode and compared the results.

The role of undesirable factors has been extremely crucial in NDEA, in the recent years; for instance, Liu et al. (2016) utilized the clustering methods and described this sphere as one of the four critical spheres or domains of DEA, from the researchers' viewpoint. Undesirable factors are one of the critical arenas that are accounted for DEA. For the first time, Fare et al. (1989) considered the undesirable factors to evaluate the efficiency in DEA models. Lu and Lo (2007) classified the undesirable outputs within a framework of three modes; the first method was to overlook all the undesirable outputs. The second method was to restrict the expansion of the undesirable outputs, or these undesirable outputs were to be considered as a nonlinear DEA model; whereas, the third technique, which was taken under contemplation for the undesirable outputs, was as an input, signified with a negative sign, as an output and/or was handled by imposing a single downward conversion. In the past few years, the role of the undesirable factors in DEA models has made considerable progress and the tasks of Wang et al. (2013) and Wu et al. (2016) are significant in this respect.

Classic DEA models, such as, CCR and BCC models were proposed for certainty in data, as they do not deal with datum uncertainty. In fact, the fundamental assumption in these models is that the amount of data in relevance with the inputs and outputs is an accurate numerical value. However, in most cases, in the business environment, determining values for inputs and outputs is not feasible in reality (Khalili-Damghani et al., 2015). Ben-Tal and Nemirovski (2000) have demonstrated that an extremely slight deviation in the data leads to an unjustified response or a considerable change in the efficiency results. Hence, they proclaimed that the results of the classical DEA methods with determined parameters are not reliable. Kao (2006) stated that the reason for the absence of the presence of reliability, in terms of human judgment and concept, DEA models with imprecise data could play a more important role in evaluating efficiency in factual issues. Wang et al. (2009) expressed that in the presence of imprecise data, DEA models are capable of drawing insights for companies in variable and ambiguous conditions, in order to have a more realistic assessment of their own. Thereby, it is extremely essential to consider the uncertainty in the available data and the manner of dealing with it during the evaluation of efficiency by utilizing DEA methods. In most of the initial

DEA tasks, uncertainty was ignored, but in recent years, several models have been under discussion for imprecise or inaccurate data (Azizi, 2013). Cooper et al. (1999) utilized a technique for DEA to confront imprecise data. Cooper et al. (2001) developed a method for converting a nonlinear planning problem into linear programming, by considering the alternative variables for determining the efficiency of the DMUs in the presence of imprecise data. Imprecise data have several criterions and varied models have been designed to oppose this aspect (Amirteimoori and Kordrostami, 2014). One of the most widespread approaches in the context with data uncertainty conditions is utilizing an interval DEA model (Farzipoor Saen, 2009). Despotis and Smirlis (2002) developed the CCR model and rendered a model, in which the efficiency evaluation is calculated by considering the interval data. Entani et al. (2002) used DEA with a pessimistic approach in the presence of interval data. Despotis et al. (2006) rendered a method in which the unspecified and imprecise values were replaced with a series of intervals and they utilized the DEA intervals for evaluating the efficiencies of units. Aghayi et al. (2013) modeled a two-stage network to consider the uncertainty in input and output data. In this research, uncertainty was modeled intermittently and the efficiency results were expressed in terms of the intervals with the upper and lower bounds. Khalili-Damghani (2015) piloted a model that calculated the efficiency evaluation in the presence of interval data and undesirable outputs. Since the evaluation of a system involves a wide range of economic, social and environmental indicators, this leads to complex multi-criteria decision-making problems. A possible way to simplify the assessment is to define the concept of sustainability and to determine the importance of economic, social and environmental indicators (Gerdessen and Pascucci, 2013). Extensive research has been conducted on methods and applications of DEA, but these efforts have focused mainly on assessing DMUs in area of engineering. More recently, researchers have used the DEA to evaluate system performance by considering sustainability factors. However, many of these studies cover only environmental and economic aspects, but social dimension has been neglected as a dimension of sustainability (Galán-Martín et al., 2016). Table (1) reviews the studies that have applied the game theory methods in DEA. The last row of table (1) presents characteristics of the current paper.

Table 1
Classification of Studies on DEA-Game Theory method

Reference	Type of game	Structure of network	Additional inputs	Undesirable output	Type of modelling	Type of data	Sustainability
Hwang et al. (2013)	Cooperative	One-stage	-	✓	Linear programming	Precise data	-
Azizi (2013)	Cooperative	One-stage	-	-	Linear programming	Imprecise data	-
Despotis and Smirlis (2002)	Cooperative	One-stage	-	-	Linear programming	Imprecise data	-
Shabanpour et al. (2017)	Cooperative	one-stage	-	-	Linear programming	Precise data	-
Kao and Hwang (2008)	Cooperative	Two-stage	-	-	Linear programming	Precise data	-
Aghayi et al. (2013)	Cooperative	Two-stage	-	-	Linear programming	Imprecise data	-
Wang et al. (2014)	Cooperative	Two-stage	-	-	Linear programming	Imprecise data	-
Kou et al. (2016)	Cooperative	Two-stage	✓	-	Linear programming	Precise data	-
Li et al. (2012)	Non-cooperative	Two-stage	✓	-	Linear programming	Precise data	-
Liang et al. (2008)	Cooperative and Non-cooperative	Two-stage	-	-	Linear programming	Precise data	-
Wu et al. (2015)	Cooperative	Two-stage	✓	✓	Linear programming	Precise data	-
Zhou et al. (2018)	Non-cooperative	Two -stage	-	-	Non- linear programming	Precise data	-
An et al. (2017)	Cooperative and Non-cooperative	Two-stage	✓	-	Non- linear programming	Precise data	-
Wu et al. (2016)	Cooperative and Non-cooperative	Two -stage	✓	✓	Non- linear programming	Precise data	-
Du et al. (2015)	Cooperative and Non-cooperative	Three -stage	-	-	Linear programming	Precise data	-
Yousefi et al.	Cooperative	Three -stage	✓	-	Non- linear	Imprecise	-

(2017)					programming	data	
Badiezadeh et al. (2018)	Cooperative	Three -stage	✓	✓	Linear programming	Precise data	-
current paper	Cooperative	Three -stage	✓	✓	Non- linear programming	Imprecise data	✓

According to the above literature, the following shortcomings in terms of the problems of the Measuring and Efficiency Decomposition of Medical Diagnostic Laboratories can be highlighted: (1) A network is proposed to evaluate the performance and ranking of laboratory units with consideration of sustainability criteria (economic, social and environmental); (2) The researches performed in the network, focused on two stages, however, the current research considers the three-stage processes, which in addition to intermediate measures, also has additional inputs and undesirable outputs; (3) Interval data is utilized to evaluate efficiency to make results more realistic; (4) a heuristic approach is suggested to convert the nonlinear models into linear models.

3. Methodology

In this study, the methodology is designed in three steps; in the first step, since variables are not identified and there is no structural and theoretical guidance, the factors affecting each dimension of the model by analyzing organizational documents, library studies, observation and the interview will be obtained. In the second step, for evaluating and screening the findings of this stage, experts' opinions and the Fuzzy Delphi method are applied to achieve consensus about the influential factors. In the Fuzzy Delphi method, the information is received from the experts in the form of a written language analyzed by a Fuzzy method. The implementation of the Fuzzy Delphi method in an overview is shown in Fig.1. In the third step, a NDEA approach is designed to measure the performance of laboratories.

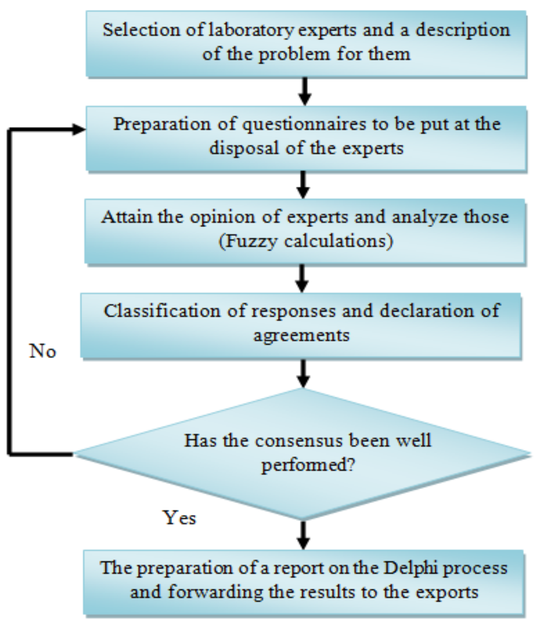

Fig. 1 Fuzzy Delphi Method Implementation Algorithm

3.1. Identification of effective indicators

The questionnaire was designed with the aim of obtaining experts' opinions about the extent to which they agreed with the model criteria. Thus, the experts have expressed their consent through verbal variables such as Very Low, Low, Medium, High and Very High. Since different characteristics of individuals affect their mental interpretation of the qualitative variables, by defining the range of qualitative variables, the experts with the same mindset responded to the questions. These variables are defined in the form of triangular fuzzy numbers according to table (2).

Table 2
Triangular Fuzzy Numbers of Linguistic variables

Linguistic variables	Triangular fuzzy number	Defuzzification value
Very low effect	(0.75, 1, 1)	0.75
Low effect	(0.5, 0.75, 1)	0.5625
Medium effect	(0.25, 0.5, 0.75)	0.3125
High effect	(0, 0.25, 0.5)	0.0625
Very high effect	(0, 0, 0.25)	0.0625

It should be noted that in table (2), the defuzzification value were calculated by Formula (1) as follows.

$$x = m + \frac{\beta - \alpha}{4} \tag{1}$$

Initially, we used two methods of documentation and observation to obtain the most important indicators in the laboratory field and to compile indicators. Some of the indicators we required were in the form of documents and reports and partly were attained by means of internal and external articles in the hospital and medical diagnostic laboratories. Then, by attending laboratories, we obtained the overall effective factors in laboratory processes through the observation of the organization. The appropriate indicators are showed in Table (3) following a library study approach and observation in laboratories.

Table 3
Indicators effective in evaluating the performance of medical diagnostic laboratories

Row	Indicator	Documentation						observation
		Checklist of quality assessment of labs	Leleu et al. (2014)	Asandului et al. (2014)	Abu Bakar et al. (2009)	Yousefi et al. (2017)	Patra and Ray (2018)	
1	Sum of the scores of the laboratory standards	✓						
2	Garbage weight	✓				✓		
3	Average sample transfer time	✓			✓			
4	Number of patients' admitted		✓	✓			✓	
5	Number of active experiments							✓
6	Correct number of tests	✓						✓
7	Test response time							✓
8	Number of false tests	✓						✓
9	Available space for service	✓						
10	Average waiting time for sampling	✓						✓
11	Cost of consumables					✓		
12	Staff wage	✓						
13	Number of responses of the prepared tests	✓						
14	Safety cost of test unit	✓				✓		
15	Number of kits							✓
16	Safety cost of sampling unit	✓				✓		
17	Lab profit							✓
18	Income from admission	✓						
19	Cost of laboratory space and land value	✓						
20	Number of samples	✓						✓
21	Cost of staff welfare							✓

To extract the factors, in addition to benefiting from the indices obtained in table (3), the effective criteria of the three laboratory processes (the pre-test, the test and the post-test) were obtained from the Fuzzy Delphi method. The conceptual model, along with the descriptions of the criteria, has been sent to the experts and the number of their agreement with each of the criteria has been taken. Given the proposed options and the linguistic variables defined in the questionnaire, the results of the review of the responses are presented in table (4). According to the results of table (4), the Fuzzy average of each of the criteria is calculated conforming to Formulas (2) and (3). In formula (2), A_i represents the expert opinion i and n the number of experts. In formula (3), A_{ave} is the average of expert opinion.

$$A_i = \left(a_1^{(i)}, a_2^{(i)}, a_3^{(i)}\right), i = 1,2, \dots, n \tag{2}$$

$$A_{ave} = (m_1, m_2,\ m_3) = \left(\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_3^{(i)}\right) \tag{3}$$

Table 4
The average expert opinions after the first round polls

Row	Indicator	Triangular Fuzzy average	Defuzzification value average (x)
1	Sum of the scores of the laboratory standards	(0.63 0.88 0.98)	0.65
2	Cost of laboratory space and land value	(0.05 0.30 0.55)	0.11
3	Average sample transfer time	(0.53 0.78 0.98)	0.58
4	Garbage weight	(0.60 0.85 0.95)	0.63
5	Number of patients' admitted	(0.40 0.65 0.83)	0.44
6	Number of active experiments	(0.45 0.70 0.90)	0.50
7	Number of false tests	(0.50 0.75 0.90)	0.54
8	Number of samples	(0.68 0.85 0.98)	0.71
9	Available space for service	(0.53 0.78 0.95)	0.57
10	Correct number of tests	(0.48 0.73 0.88)	0.51
11	Number of kits	(0.58 0.83 0.95)	0.61
12	Average waiting time for sampling	(0.55 0.80 0.98)	0.59
13	Staff wage	(0.63 0.93 0.98)	0.69
14	Cost of staff welfare	(0.08 0.25 0.50)	0.14
15	Test response time	(0.63 0.88 0.98)	0.65
16	responses of the prepared tests	(0.30 0.55 0.78)	0.36
17	Income from admission	(0.58 0.83 0.95)	0.61
18	Cost of consumables	(0.58 0.83 0.95)	0.61
19	Safety cost of test unit	(0.45 0.40 0.65)	0.51
20	Lab profit	(0.38 0.63 0.88)	0.44
21	Safety cost of sampling unit	(0.68 0.85 0.98)	0.71

In table (4), the triangular fuzzy average and the defuzzification operations are calculated by Formula (3) and Formula (1), respectively. According to Cheng et al. (2002), if the difference between the two rounds were lower than the threshold of very small (0.1), consensus has been made and the process will stop. Therefore, in the polls of the second round, by making the necessary changes in the criteria, the second questionnaire was prepared and, along with the previous point of view of each expert and their degree of difference with the viewpoint of other experts, was again given to the members of the expert group. In the second round, the members of the expert group responded to the design questions again according to the views of other members of the group, and regarding the changes made to the criteria, the results of which are presented in table (5). The results of the second round, as in the first round, were analyzed by means of Formulas (1) and (3), the results of which are shown in table (5). Also in table (5), the difference between the mean of first round and second round is also indicated.

Table 5
The average expert opinions after The Polls of the second Round

Row	Indicator	Triangular Fuzzy average	Defuzzification value average (x)	Difference of the first and second Rounds
1	Sum of the scores of the laboratory standards	(0.7 0.78 0.93)	0.74	0.09
2	Cost of laboratory space and land value	(0.10 0.18 0.43)	0.16	0.05
3	Average sample transfer time	(0.58 0.73 0.95)	0.63	0.05
4	Garbage weight	(0.78 0.83 0.95)	0.81	0.18
5	Number of patients' admitted	(0.48 0.63 0.85)	0.53	0.09
6	Number of active experiments	(0.53 0.70 0.95)	0.59	0.09
7	Number of false tests	(0.70 0.78 0.93)	0.74	0.20
8	Number of samples	(0.75 0.80 0.93)	0.78	0.07
9	Available space for service	(0.65 0.73 0.95)	0.71	0.14
10	Correct number of tests	(0.50 0.60 0.78)	0.54	0.03
11	Number of kits	(0.65 0.75 0.93)	0.69	0.08
12	Average waiting time for sampling	(0.63 0.73 0.90)	0.67	0.08
13	Staff wage	(0.75 0.78 0.88)	0.78	0.09
14	Cost of staff welfare	(0.13 0.18 0.43)	0.19	0.05
15	Test response time	(0.68 0.75 0.90)	0.71	0.06
16	responses of the prepared tests	(0.38 0.53 0.75)	0.43	0.07
17	Income from admission	(0.65 0.73 0.88)	0.69	0.08
18	Cost of consumables	(0.65 0.73 0.88)	0.69	0.08
19	Safety cost of test unit	(0.53 0.65 0.85)	0.58	0.07
20	Lab profit	(0.43 0.60 0.85)	0.49	0.05
21	Safety cost of sampling unit	(0.75 0.80 0.93)	0.78	0.07

As can be seen in table (5), expert members have reached consensus in all criteria, except "the waste weight, the number of false tests, and the available pace for service", because the difference between the first round and the second round is lower than the threshold of very small (0.1). Therefore, we poll to continue for only three criteria: "the waste weight, the number of false tests, and the available pace for service". At this round, a third questionnaire containing three criteria "the waste weight, the number of false tests, and the available pace for service" was designed. According the previous point of view of each expert and their degree of difference with the viewpoint of other experts, the questionnaire was again given to the members of the expert group. The fuzzy analysis of the results obtained from this step is included in table (6).

Table 6
The average expert opinions after The Polls of the third Round

Row	Indicator	Triangular Fuzzy average	Defuzzification value average (x)	Difference of the second and third Rounds
1	Garbage weight	(0.70 0.95 1.10)	0.74	0.07
2	Number of false tests	(0.63 0.88 1.08)	0.68	0.06
3	Available space for service	(0.55 0.80 1.08)	0.62	0.09

As table (6) shows, the difference between the second round and the third round is smaller than the threshold of very low (0.1). According to the results by Cheng et al. (2002), the consensus stops at this round. Finally, according to table (5), the two criteria "the cost of staff welfare and the cost of laboratory space and land value" that are in the range of Very low effect to Medium effect are eliminated. The criteria of the sum of the scores of the laboratory standards, the average sample transfer time, the garbage weight, the number of active experiments, the number of false tests, the available space for service, the number of kits, the average waiting time for sampling, the staff wage, the test response time, the income from admission, the cost of consumables, the safety cost of sampling unit and the safety cost of test unit are in the range of Very high effect to High effect. Other criteria including the number of patients' admitted, the correct number of tests, the number of replies to the prepared tests and the lab profit are in the range of Medium effect. Out of twenty-one effective criteria of diagnostic labs, in two rounds of the polls, by eliminating three criteria: "the cost of laboratory space and land value, and the cost of staff welfare", finally, eighteen executable criteria were identified in the area of diagnostic laboratories. Table (7) shows effective final indicators for evaluating the performance of diagnostic laboratories by Fuzzy Delphi method.

Table 7
Effective Indicators for the performance evaluation of medical Diagnostic Laboratories

Row	Indicator	Row	Indicator
1	Sum of the scores of the laboratory standards	11	Income from admission
2	Garbage weight	12	Cost of consumables
3	Average sample transfer time	13	Safety cost of test unit
4	Number of patients' admitted	14	Safety cost of sampling unit
5	Number of active experiments	15	Average waiting time for sampling
6	Correct number of tests	16	Test response time
7	Number of false tests	17	responses of the prepared tests
8	Available space for service	18	Lab profit
9	Staff wage	19	Number of samples
10	Number of kits		

3.2. Model description

A network of three stages is considered as shown in Fig. 2. It actually simulates a medical diagnostic lab in the real world. This network of three stages consists of three main processes (pre-testing, testing and post-testing). Suppose a set of n homogeneous DMUs, which is denoted by DMU_j (j=1,..., n). The first stage is the reception unit, the second stage involves the sampling and test unit and the third stage covers the test results unit. The pre-test process has a stage called the reception unit. In the reception unit, the desired input and the undesirable input are denoted by $x_{i_1j}(i_1 = 1,2,\dots,I_1)$ and $x_{i_2j}(i_2 = 1,2,\dots,I_2)$, respectively; the undesired output and the desirable output are denoted by $y_{r_1j}(r_1 = 1,2,\dots,R_1)$ and $y_{r_2j}(r_2 = 1,2,\dots,R_2)$, respectively. The testing process consists of a stage called the sampling & testing unit. The intermediate measures are specified between the reception unit and the sampling & test unit by $z_{d_1j}(d_1 = 1,2,\dots,D_1)$. The desired additional inputs and undesirable additional outputs of the sampling and testing unit are displayed by $x_{i_3j}(i_3 = 1,2,\dots,I_3)$ and $y_{r_3j}(r_3 = 1,2,\dots,R_3)$, respectively. The post-test process has a stage called the test results unit. The intermediate measures between the sampling and testing unit and the test results unit are illustrated by $z_{d_2j}(d_2 = 1,2,\dots,D_2)$. The desired additional inputs to the test results unit are shown by $x_{i_4j}(i_4 = 1,2,\dots,I_4)$. Finally, the desired outputs of the test results unit are represented by $y_{r_4j}(r_4 = 1,2,\dots,R_4)$.

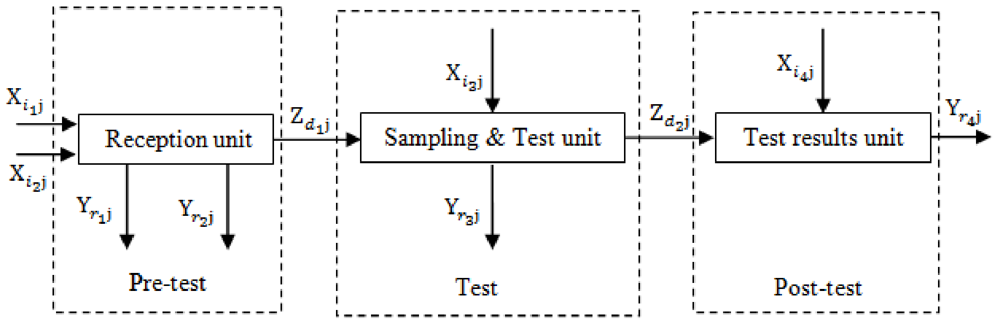

Fig. 2 The structure of three-stage network system with additional inputs and undesirable outputs

It is mainly for three reasons that researchers are most likely to use input-oriented models for efficiency analysis. Primarily, demand is in a state of growth and the estimation of demand is a complex issue. Secondly, managers have more control over inputs than outputs; and thirdly, this model reflects the initial goals or objectives of policy-makers for being responsible for the demands of the people and the units should reduce costs and/or limit the use of resources. Thus, we utilize the input-axis model in this research. In accordance with Korhonen and Luptacik (2004), we signify the undesirable outputs in the models with a negative mark. We assign v_{i_1} and v_{i_2} as the weights of the inputs to the reception unit $x_{i_1 j}(i_1 = 1,2,\dots,I_1)$ and $x_{i_2 j}(i_2 = 1,2,\dots,I_2)$, respectively. In addition, we denote $z_{d_1 j}(d_1 = 1,2,\dots,D_1)$ as the weights of the intermediate measures between the reception unit and sampling & testing unit. Finally, we introduce u_{r_1} and u_{r_2} as the weights on the outputs flowing from the reception unit $y_{r_1 j}(r_1 = 1,\dots,R_1)$ and $y_{r_2 j}(r_2 = 1,\dots,R_2)$, respectively. It should be noted that the intermediate measures $z_{d_1 j}(d_1 = 1,2,\dots,D_1)$ play a dual role in the reception unit and in the sampling and testing unit. In the reception unit, η_{d_1} acts as the weight of the output. According to the three-stage network, shown in Fig.2, the efficiency of the reception unit is determined by $\theta_0^{Reception\ unit}$. Typically, the efficiency of the reception unit can be measured by the following model (4).

$$\theta_0^{Reception\ unit} = max\ \frac{\sum_{d_1=1}^{D_1}\eta_{d_1}z_{d_1 o}+\sum_{r_2=1}^{R_2}u_{r_2}y_{r_2 o}-\sum_{r_1=1}^{R_1}u_{r_1}y_{r_1 o}}{\sum_{i_1=1}^{I_1}v_{i_1}x_{i_1 o}-\sum_{i_2=1}^{I_2}v_{i_2}x_{i_2 o}}$$
$$s.t.\quad \frac{\sum_{d_1=1}^{D_1}\eta_{d_1}z_{d_1 j}+\sum_{r_2=1}^{R_2}u_{r_2}y_{r_2 j}-\sum_{r_1=1}^{R_1}u_{r_1}y_{r_1 j}}{\sum_{i_1=1}^{I_1}v_{i_1}x_{i_1 j}-\sum_{i_2=1}^{I_2}v_{i_2}x_{i_2 j}} \le 1 \quad , \quad j = 1,\dots,n \tag{4}$$
$$\eta_{d_1}, u_{r_1}, u_{r_2}, v_{i_1}, v_{i_2} \ \ge\ \varepsilon\ ;\ d_1 = 1,\dots,D_1;\ r_1 = 1,\dots,R_1; r_2 = 1,\dots,R_2;\ i_1 = 1,\dots,I_1;\ i_2 = 1,\dots,I_2.$$

In the sampling and testing unit, we assume v_{i_3} and u_{r_3} are the weights on inputs $x_{i_3 j}(i_3 = 1,\dots,I_3)$ and outputs $y_{r_3 j}(r_3 = 1,\dots,R_3)$, respectively. Let η_{d_2} denote the weights on the intermediate measures to the sampling and testing unit $z_{d_2 j}(d_2 = 1,2,\dots,D_2)$. We show the efficiency of sampling and testing unit by $\theta_0^{Sampling\ \&\ Test\ unit}$. The efficiency of sampling and testing unit can be calculated by solving model (5).

$$\theta_0^{Sampling\ \&\ Test\ unit} = \max\ \frac{\sum_{d_2=1}^{D_2}\eta_{d_2}z_{d_2 o}-\sum_{r_3=1}^{R_3}u_{r_3}y_{r_3 o}}{\sum_{i_3=1}^{I_3}v_{i_3}x_{i_3 o}+\sum_{d_1=1}^{D_1}\eta_{d_1}z_{d_1 o}}$$
$$\text{s.t.}\quad \frac{\sum_{d_2=1}^{D_2}\eta_{d_2}z_{d_2 j}-\sum_{r_3=1}^{R_3}u_{r_3}y_{r_3 j}}{\sum_{i_3=1}^{I_3}v_{i_3}x_{i_3 j}+\sum_{d_1=1}^{D_1}\eta_{d_1}z_{d_1 j}} \le 1\ ,\quad j = 1,\dots,\text{n} \tag{5}$$
$$\eta_{d_1}, \eta_{d_2}, u_{r_3}, v_{i_3} \ge\ \varepsilon;\ d_1 = 1,\dots,D_1;\ d_2 = 1,\dots,D_2;\ r_3 = 1,\dots,R_3;\ i_3 = 1,\dots,I_3.$$

In the test results unit, we assume v_{i_4} and u_{r_4} are the weights on inputs $x_{r_4 j}(i_4 = 1,\dots,I_4)$ and outputs $y_{r_4 j}(r_4 = 1,2,\dots,R_4)$, respectively. In addition, we adopt η_{d_2} as the weights on the intermediate measures to the test results unit. The efficiency of the reception unit is shown by $\theta_0^{Test\ results\ unit}$. The efficiency of the test reception unit is defined by applying model (6).

$$\theta_0^{Test\ results\ unit} = \max \ \frac{\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 o}}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o} + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}}$$

$$\text{s.t.} \quad \frac{\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 j}}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 j} + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j}} \le 1 \quad , \quad j = 1, \dots, n \tag{6}$$

$$\eta_{d_2}, u_{r_4}, v_{i_4} \ \ge \ \varepsilon; \ d_2 = 1, \dots, D_2; \ r_4 = 1, \dots, R_4; \ i_4 = 1, \dots, I_4.$$

In this study, the intermediate measures are evaluated and re-modeled, regardless of the dual role (as inputs of one stage or as output of the next stage). Therefore, we used the same weights for the intermediate measures. This is a common assumption in DEA studies (Kao and Hwang, 2008). For the network system shown in Fig. 2, the reception unit, the sampling & testing unit and the test results unit are connected by intermediate measures $z_{d_1 j} (d_1 = 1,2, \dots, D_1)$ and $z_{d_2 j} (d_2 = 1,2, \dots, D_2)$ in series. The overall efficiency of the system can be calculated by means of formula (7) conforming to the tandem system of Kao and Hwang (2008):

$$\theta_o^{overall} = max \ \theta_0^{Reception\ unit} \, . \, \theta_0^{Sampling\ \&\ Test\ unit} . \, \theta_0^{Test\ results\ unit} \tag{7}$$

Where $\theta_o^{overall}$ is the efficiency of whole system. The overall efficiency DMU_0 can be achieved by solving fractional program (8).

$$\theta_o^{\text{overall}} = \max \ \frac{\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 o} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 o}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o} - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}} . \frac{\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o} - \sum_{r_3=1}^{R_3} u_{r_3} y_{r_3 o}}{\sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}} . \frac{\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 o}}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o} + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}}$$

$$\text{s.t.} \quad \frac{\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 j} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j}}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j} - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}} \le 1 , \ j = 1, \dots, n$$

$$\frac{\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j} - \sum_{r_3=1}^{R_3} u_{r_3} y_{r_3 j}}{\sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 j} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j}} \le 1, \ j = 1, \dots, n \tag{8}$$

$$\frac{\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 j}}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 j} + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j}} \le 1 \quad , \quad j = 1, \dots, n$$

$$\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, u_{r_4}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \ge \varepsilon; d_1 = 1, \dots, D_1; d_2 = 1, \dots, D_2; r_1 = 1, \dots, R_1; r_2 = 1, \dots, R_2;$$
$$r_3 = 1, \dots, R_3; \ r_4 = 1, \dots, R_4; \ i_1 = 1, \dots, I_1; \ i_2 = 1, \dots, I_2; i_3 = 1, \dots, I_3; i_4 = 1, \dots, I_4.$$

It shall be assumed that some of the data in model (8) are unreliable and incapable of being accurately determined and we only know that they are within their upper and lower bounds. Despotis and Smirlis (2002) have calculated the efficiency of DMUs in the presence of intervals and have proposed models for the upper and lower bound efficiencies. Therefore, by developing the task of Despotis and Smirlis we shall modify model (8), with the assumption that the variables are bounded in the presence of undesirable outputs as in the figure below:

$$\theta_o^{overall(U)} = max \ \frac{\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^u + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 o}^u - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 o}^L}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o}^L - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}^U} . \frac{\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^u - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3 0}^L}{\sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o}^L + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^L} . \frac{\sum_{r_4=1}^{R_4} u_{r_3} y_{r_4 o}^u}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o}^L + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^L}$$

$$s.t. \quad \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j}^L + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 j}^L + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}^L - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 j}^u - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}^u \le 0 \quad , \ \forall j \neq o$$

$$\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j}^L - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3 j}^u - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 j}^u - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 j}^u \le 0, \ \forall j \neq o \tag{9}$$

$$\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 j}^L - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 j}^u - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 j}^u \le 0 \quad , \quad \forall j \neq o$$

$$\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^u + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 o}^u + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}^u - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 o}^L - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o}^L \le 0$$

$$\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^u - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3 o}^L - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o}^L - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^L \le 0,$$

$$\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4 o}^u - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o}^L - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^L \le 0$$

$$\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, u_{r_4}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \ge \varepsilon; d_1 = 1, \dots, D_1; d_2 = 1, \dots, D_2; r_1 = 1, \dots, R_1; r_2 = 1, \dots, R_2;$$
$$r_3 = 1, \dots, R_3; \ r_4 = 1, \dots, R_4; \ i_1 = 1, \dots, I_1; \ i_2 = 1, \dots, I_2; i_3 = 1, \dots, I_3; i_4 = 1, \dots, I_4.$$

$$\theta_o^{overall(L)} = max \ \frac{\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^L + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2 o}^L - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1 o}^U}{\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o}^U - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}^L} . \frac{\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^L - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3 0}^U}{\sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o}^U + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1 o}^U} . \frac{\sum_{r_4=1}^{R_4} u_{r_3} y_{r_4 o}^L}{\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o}^U + \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2 o}^U}$$

$$s.t.\quad \sum_{d_1=1}^{D_1}\eta_{d_1} z_{d_1j}^U + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2j}^U + \sum_{i_2=1}^{I_2} v_{i_2}x_{i_2j}^U - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1j}^L - \sum_{i_1=1}^{I_1} v_{i_1}x_{i_1j}^L \le 0 \quad, \forall j\neq o$$
$$\sum_{d_2=1}^{D_2}\eta_{d_2} z_{d_2j}^U - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3j}^L - \sum_{i_3=1}^{I_3} v_{i_3}x_{i_3j}^L - \sum_{d_1=1}^{D_1}\eta_{d_1} z_{d_1j}^L \le 0,\ \forall j\neq o \tag{10}$$
$$\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4j}^U - \sum_{i_4=1}^{I_4} v_{i_4}x_{i_4j}^L - \sum_{d_2=1}^{D_2}\eta_{d_2} z_{d_2j}^L \le 0 \quad , \quad \forall j\neq o$$
$$\sum_{d_1=1}^{D_1}\eta_{d_1} z_{d_1o}^L + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2o}^L + \sum_{i_2=1}^{I_2} v_{i_2}x_{i_2o}^L - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1o}^U - \sum_{i_1=1}^{I_1} v_{i_1}x_{i_1o}^U \le 0$$
$$\sum_{d_2=1}^{D_2}\eta_{d_2} z_{d_2o}^L - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3o}^U - \sum_{i_3=1}^{I_3} v_{i_3}x_{i_3o}^U - \sum_{d_1=1}^{D_1}\eta_{d_1} z_{d_1o}^U \le 0,$$
$$\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4o}^L - \sum_{i_4=1}^{I_4} v_{i_4}x_{i_4o}^U - \sum_{d_2=1}^{D_2}\eta_{d_2} z_{d_2o}^U \le 0$$
$$\eta_{d_1},\eta_{d_2},u_{r_1},u_{r_2},u_{r_3},u_{r_4},v_{i_1},v_{i_2},v_{i_3},v_{i_4} \ge \varepsilon; d_1=1,\dots,D_1; d_2=1,\dots,D_2; r_1=1,\dots,R_1; r_2=1,\dots,R_2;$$
$$r_3=1,\dots,R_3;\ r_4=1,\dots,R_4;\ i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3; i_4=1,\dots,I_4.$$

Model (9) measures the efficiency under the most desirable conditions that is known as the 'upper bound efficiency'; whereas, model (10) measures the efficiency under the most undesirable conditions and is known as the 'lower bound efficiency'. It should be noted that the undesirable outputs have similar characteristics to that of the inputs, thereby, their boundaries are also considered like the inputs. Models (9) and (10) are nonlinear and are obtained by multiplying their objective function. In the fourth section of this paper, an innovative approach is used to solve them. Let us assume that the models (9) and (10) are resolved, the interval efficiency of the network illustrated in Fig. 2 for DMU_0 in the $\left[\theta_o^{overall(L)},\theta_o^{overall(U)}\right]$ manner. In order to compare the interval efficiency and the ranking of DMUs, a minimax regret-based approach was proposed by Wang et al. (2005) and we shall use this approach to rank the units. Wang et al. has also stated that, the interval efficiency with the slightest waste of efficiency would be the optimal efficiency interval. They defined the criteria for the minimax regret-based approach for the efficiency interval as $= A_i[a_i^L,a_i^U],(i = 1 \dots n)$ which has been described in formula (14) as below:

$$R(A_i) = max\left(\max_{j\neq i}\left(a_j^U\right) - a_i^L, 0\right), (i = 1 \dots n) \tag{11}$$

We first calculate the maximum loss for each interval and consider the minimum loss, as the most desirable or favorable interval. In the next stage, we eliminate the desirable interval and repeatedly, in the same manner, from the remaining *n-1* select the optimal interval and this is iteratively performed until only one interval efficiency remains; and the lowest position is assigned to this.

4. Model Solution

Models (9) and (10) cannot be turned into linear models because of the additional inputs and outputs in the stages. Thus, we propose the heuristic approach given hereunder, for solving models (9) and (10). This approach shall be founded on model (9). We know that the objective function of model (9) is the product of the efficiency of the three phases *i.e.*

$$\theta_o^{overall(U)} = max\ \theta_0^{Reception\ unit(U)}.\theta_0^{Sampling\ \&\ Test\ unit(U)}.\theta_0^{Test\ resı}$$

. We consider $\theta_0^{Reception\ unit(U)}$ and $\theta_0^{Sampling\ \&\ Test\ unit(U)}$ as variables in the objective function, which are between the $\left[0, \theta_o^{Reception\ unit(U)\text{-max}}\right]$ and $\left[0, \theta_o^{Sampling\ \&\ Test\ unit(U)\text{- max}}\right]$ intervals and change respectively. We describe $\theta_0^{Reception\ unit(U)}$ and $\theta_0^{Sampling\ \&\ Test\ unit(U)}$ in the figure below, so that we can move them within the intervals.

$$\theta_o^{Reception\ unit(U)} = \theta_o^{Reception\ unit(U)\text{-max}} - k_1\Delta\varepsilon,\ k_1=0,1,\dots,\left[\frac{\theta_o^{Reception\ unit(U)\text{- max}}}{\Delta\varepsilon}\right]+1 \tag{12}$$

$$\theta_o^{Sampling\ \&\ Test\ unit(U)} = \theta_o^{Sampling\ \&\ Test\ unit(U)\text{-max}} - k_2\Delta\varepsilon,\quad k_2=0,1,\dots,\left[\frac{\theta_o^{Sampling\ \&\ Test\ unit(U)\text{- max}}}{\Delta\varepsilon}\right]+1$$

In the formula (12), we consider $\Delta\varepsilon$ as a step size and of a very small value and define $\theta_o^{Reception\ unit(U)-max}$ and $\theta_o^{Sampling\ \&\ Test\ unit(U)-max}$ respectively, as the maximum efficiency of stages (1 and 2) in Fig. 2. From the models rendered hereunder, they can be calculated.

$$\theta_o^{Reception\ unit(U)\text{-max}} = \max\left\{\theta_o^{1U} \mid \theta_j^{1L}\le 1, \theta_j^{2L}\le 1, \theta_j^{3L}\le 1, \forall j\neq o, \theta_o^{1U}\le 1, \theta_o^{2U}\le 1, \theta_o^{3U}\le 1\right\} \tag{13}$$

$$\theta_o^{Sampling\ \&\ Test\ unit(U)\text{-max}} = \max\left\{\theta_o^{2U} \mid \theta_j^{1L}\le 1, \theta_j^{2L}\le 1, \theta_j^{3L}\le 1, \forall j\neq o, \theta_o^{1U}\le 1, \theta_o^{2U}\le 1, \theta_o^{3U}\le 1\right\}$$

All the variables are non-negative in model (13). The aforementioned models have attained a maximum efficiency in the first and second stages, on the condition that, the efficiency of the stages is less than (1). These models are fractions and by utilizing the Charnes-Cooper conversion (1962), like the one given below, they are modified into linear models.

$$\begin{aligned}
&\theta_o^{Reception\ unit(\mathrm{u})\text{-max}} = \max \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{u} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2o}^{u} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1o}^{L}\\
&\text{s.t.} \quad \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1o}^{L} - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2o}^{u} = 1\\
&\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1j}^{L} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2j}^{L} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2j}^{L} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1j}^{u} - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1j}^{u} \le 0 \quad , \forall j\neq o\\
&\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2j}^{L} - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3j}^{u} - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3j}^{u} - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1j}^{u} \le 0, \ \forall j\neq o \qquad (14)\\
&\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4j}^{L} - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4j}^{u} - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2j}^{u} \le 0 \quad , \quad \forall j\neq o\\
&\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{u} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2o}^{u} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2o}^{u} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1o}^{L} - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1o}^{L} \le 0\\
&\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2o}^{u} - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3o}^{L} - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3o}^{L} - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{L} \le 0,\\
&\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4o}^{u} - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4o}^{L} - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2o}^{L} \le 0\\
&\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, u_{r_4}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \ge \varepsilon; d_1 = 1, \dots, D_1; d_2 = 1, \dots, D_2; r_1 = 1, \dots, R_1; r_2 = 1, \dots, R_2;\\
&r_3 = 1, \dots, R_3;\ r_4 = 1, \dots, R_4;\ i_1 = 1, \dots, I_1;\ i_2 = 1, \dots, I_2; i_3 = 1, \dots, I_3; i_4 = 1, \dots, I_4.
\end{aligned}$$

$$\begin{aligned}
&\theta_o^{Sampling\ \&\ Test\ unit(u)_\text{-max}} = \max \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2o}^{u} - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3o}^{L}\\
&\text{s.t.} \quad \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3o}^{L} + \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{L} = 1\\
&\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1j}^{L} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2j}^{L} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2j}^{L} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1j}^{u} - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1j}^{u} \le 0 \quad , \forall j\neq o\\
&\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2j}^{L} - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3j}^{u} - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3j}^{u} - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1j}^{u} \le 0, \ \forall j\neq o \qquad (15)\\
&\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4j}^{L} - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4j}^{u} - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2j}^{u} \le 0 \quad , \quad \forall j\neq o\\
&\sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{u} + \sum_{r_2=1}^{R_2} u_{r_2} y_{r_2o}^{u} + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2o}^{u} - \sum_{r_1=1}^{R_1} u_{r_1} y_{r_1o}^{L} - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1o}^{L} \le 0\\
&\sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2o}^{u} - \sum_{r_2=1}^{R_2} u_{r_3} y_{r_3o}^{L} - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3o}^{L} - \sum_{d_1=1}^{D_1} \eta_{d_1} z_{d_1o}^{L} \le 0,\\
&\sum_{r_4=1}^{R_4} u_{r_4} y_{r_4o}^{u} - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4o}^{L} - \sum_{d_2=1}^{D_2} \eta_{d_2} z_{d_2o}^{L} \le 0\\
&\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, u_{r_4}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \ge \varepsilon; d_1 = 1, \dots, D_1; d_2 = 1, \dots, D_2; r_1 = 1, \dots, R_1; r_2 = 1, \dots, R_2;\\
&r_3 = 1, \dots, R_3;\ r_4 = 1, \dots, R_4;\ i_1 = 1, \dots, I_1;\ i_2 = 1, \dots, I_2; i_3 = 1, \dots, I_3; i_4 = 1, \dots, I_4.
\end{aligned}$$

For determining the value of $\theta_o^{Reception\ unit(\mathrm{U})\text{-max}}$ and $\theta_o^{Sampling\ \&\ Test\ unit(\mathrm{U})\text{-max}}$ with the assistance of models (14) and (15), we alter model (9) and convert it to the following model.

$$\begin{aligned}
&\theta_o^{\mathrm{overall(U)}} = \max \Big\{ \theta_0^{Reception\ unit(U)} . \theta_0^{Sampling\ \&\ Test\ unit(U)} . \theta_0^{Test\ results\ unit(U)} \Big| \ \theta_j^{Reception\ unit(L)} \le 1,\\
&\theta_j^{Sampling\ \&\ Test\ unit(L)} \le 1, \theta_j^{Test\ results\ unit(L)} \le 1,\ \forall j\neq o, \theta_o^{Reception\ unit(u)} \le 1, \theta_o^{Sampling\ \&\ Test\ unit(u)} \le 1, \theta_o^{Test\ results\ unit(U)}\\
&\le 1, \theta_o^{Reception\ unit(U)} = \frac{O_o^{Reception\ unit(U)}}{I_o^{Reception\ unit(L)}}, \theta_o^{Sampling\ \&\ Test\ unit(U)} = \frac{O_o^{Sampling\ \&\ Test\ unit(U)}}{I_o^{Sampling\ \&\ Test\ unit(L)}},\\
&\theta_o^{Reception\ unit(\mathrm{U})} \in \Big[0, \theta_o^{Reception\ unit(U)\text{-max}}\Big], \theta_o^{Sampling\ \&\ Test\ unit(U)} \in \Big[0, \theta_o^{Sampling\ \&\ Test\ unit(\mathrm{U})\text{-max}}\Big] \Big\} \qquad (16)
\end{aligned}$$

It should be observed that in the model (16), we consider $\theta_o^{Reception\ unit(\mathrm{u})}$ and $\theta_o^{Sampling\ \&\ Test\ unit(\mathrm{u})}$ in the objective function as two variables and two constraints which specify these two variables and together with its interval modifications, it was supplemented to the model. In models (4) and (5), we have described the efficiencies of stages (1) and (2) and in the model (16), we have briefly illustrated it in the form of outputs and inputs of each stage or $\theta_o^{Reception\ unit(u)} = \frac{O_o^{Reception\ unit(u)}}{I_o^{Reception\ unit(l)}}$ and $\theta_o^{Sampling\ \&\ Test\ unit(u)} = \frac{O_o^{Sampling\ \&\ Test\ unit(u)}}{I_o^{Sampling\ \&\ Test\ unit(l)}}$. The model (16) is a fractional model and by utilizing the Charnes-Cooper conversion (1962), like the one given below, it is modified into linear models.

$$
\begin{aligned}
&\theta_o^{overall(U)} = \max \theta_o^{Reception\ unit(u)} . \theta_o^{Sampling\ \&\ Test\ unit(u)} . \left(\textstyle\sum_{r_4=1}^{R_4} u_{r_4}\, y_{r_4 o}^u\right)\\
&s.t. \quad \textstyle\sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o}^L + \sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 o}^L = 1\\
&\textstyle\sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 j}^L + \sum_{r_2=1}^{R_2} u_{r_2}\, y_{r_2 j}^L + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 j}^L - \sum_{r_1=1}^{R_1} u_{r_1}\, y_{r_1 j}^u - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 j}^u \le 0 \quad , \ \forall j \neq o\\
&\textstyle\sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 j}^L - \sum_{r_2=1}^{R_2} u_{r_3}\, y_{r_3 j}^u - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 j}^u - \sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 j}^u \le 0, \ \forall j \neq o \qquad (17)\\
&\textstyle\sum_{r_4=1}^{R_4} u_{r_4}\, y_{r_4 j}^L - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 j}^u - \sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 j}^u \le 0 \quad , \quad \forall j \neq o\\
&\textstyle\sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 o}^u + \sum_{r_2=1}^{R_2} u_{r_2}\, y_{r_2 o}^u + \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}^u - \sum_{r_1=1}^{R_1} u_{r_1}\, y_{r_1 o}^L - \sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o}^L \le 0\\
&\textstyle\sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 o}^u - \sum_{r_2=1}^{R_2} u_{r_3}\, y_{r_3 o}^L - \sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o}^L - \sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 o}^L \le 0,\\
&\textstyle\sum_{r_4=1}^{R_4} u_{r_4}\, y_{r_4 o}^u - \sum_{i_4=1}^{I_4} v_{i_4} x_{i_4 o}^L - \sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 o}^L \le 0\\
&\textstyle\sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 o}^u + \sum_{r_2=1}^{R_2} u_{r_2}\, y_{r_2 o}^u - \sum_{r_1=1}^{R_1} u_{r_1}\, y_{r_1 o}^L - \theta_o^{Reception\ unit(u)} \left(\sum_{i_1=1}^{I_1} v_{i_1} x_{i_1 o}^L - \sum_{i_2=1}^{I_2} v_{i_2} x_{i_2 o}^u\right) = 0\\
&\textstyle\sum_{d_2=1}^{D_2} \eta_{d_2}\, z_{d_2 o}^u - \sum_{r_3=1}^{R_3} u_{r_3}\, y_{r_3 o}^L - \ \theta_o^{Sampling\ \&\ Test\ unit(u)} \left(\sum_{i_3=1}^{I_3} v_{i_3} x_{i_3 o}^L + \sum_{d_1=1}^{D_1} \eta_{d_1}\, z_{d_1 o}^L\right) = 0\\
&\theta_o^{Reception\ unit(u)} \in \left[0, \theta_o^{Reception\ unit(u)\text{-}max}\right]\\
&\theta_o^{Sampling\ \&\ Test\ unit(u)} \in \left[0, \theta_o^{Sampling\ \&\ Test\ unit(u)\text{-}max}\right]\\
&\eta_{d_1}, \eta_{d_2}, u_{r_1}, u_{r_2}, u_{r_3}, u_{r_4}, v_{i_1}, v_{i_2}, v_{i_3}, v_{i_4} \ge \varepsilon; d_1 = 1, \dots, D_1; d_2 = 1, \dots, D_2; r_1 = 1, \dots, R_1; r_2 = 1, \dots, R_2;\\
&r_3 = 1, \dots, R_3;\ r_4 = 1, \dots, R_4;\ i_1 = 1, \dots, I_1;\ i_2 = 1, \dots, I_2;\ i_3 = 1, \dots, I_3;\ i_4 = 1, \dots, I_4.
\end{aligned}
$$

In model (17), by utilizing formula (12), we increase the values of k_1 and k_2 independently, from (0) to a high level for each one, so that each time the model can be solved with the new $\theta_o^{Reception\ unit(u)}$ and $\theta_o^{ampling\ \&\ Test\ unit(u)}$. We resolve all the returns of the conditions of the k_1 and k_2 models and illustrate the responses with $\theta_o^{overall(u)}(k_1, k_2)$. By comparing the overall values of $\theta_o^{overall(u)}(k_1, k_2)$, we describe the maximal efficiency of the network (Fig. 2) as $\theta_o^{overall*} = \max\ \theta_o^{overall(u)}(k_1, k_2)$. It should be noted that, we have tested our proposed approach in three modes and each time we have considered two stages as variables. Since the efficiency of Fig. 2 is unique, the results of these three methods are remarkably in approximation to each other and in order to explain we have raised one of these three conditions to describe our above approach.

5. Case Study Description

Our case study is related to the private laboratories of Tehran. In this regard, the sample size in this study includes 25 medical diagnostic laboratories selected by cluster sampling from private laboratories in Tehran. In this section, we will examine the performance of private medical diagnostic laboratories in Tehran. Medical diagnostic laboratories cover three main processes: the pre-test process, the testing process, and the post-test process, as shown in Fig. 3.

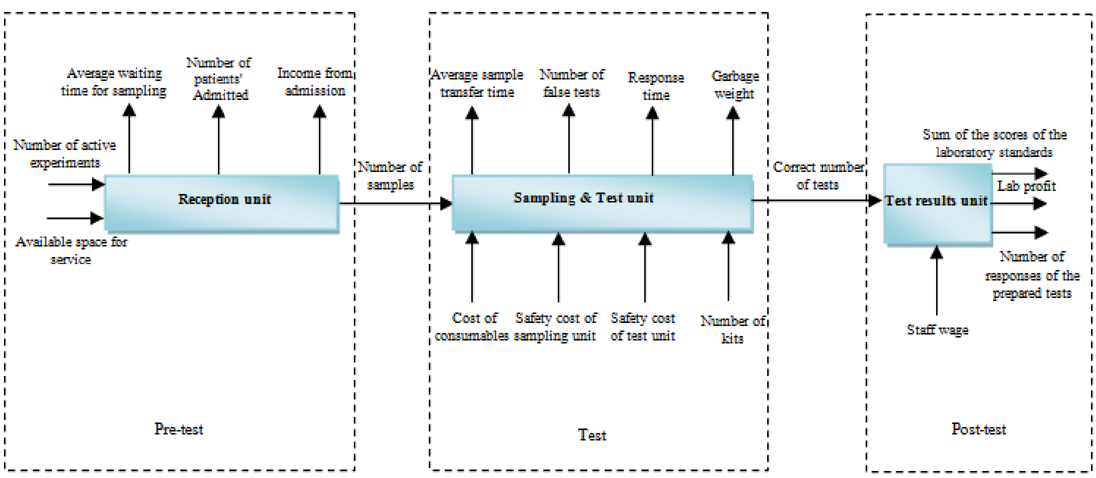

Fig. 3 Three-stage network of a medical diagnostic laboratory

In this study, to evaluate the performance of the medical diagnostic laboratory, economic, social and environmental criteria are considered simultaneously. The environmental criterion is related to the waste weight. This criterion is intended to protect the health of the staff, patients, other clients, and the environment. Staff safety as social criteria is the costs that each service center payments for dangers that exist and accidents which happen in workplace. Cost of consumables, staff wage, income from admission and lab profit are introduced as economic criteria. The input, intermediary, and output variables are according to table (8):

Table 8
The notation of input variables, Intermediary variables and output variables

Input variables		Intermediary variables		Output variables	
Number of active experiments	x_1	Number of samples	z_1	Average waiting time for sampling	y_1
				Number of patients' admission	y_2
Available pace for service	x_2	Correct number of tests	z_2	Income from admission (economic criterion)	y_3
Cost of consumables (economic criterion)	x_3			Average sample transfer time	y_4
Safety cost of sampling unit (social criterion)	x_4			Number of false tests	y_5
Safety cost of test unit (social criterion)	x_5			Test response time	y_6
Number of kits	x_6			Garbage weight (environmental criterion)	y_7
Staff wage (economic criterion)	x_7			Number of responses of the prepared tests	y_8
				Sum of the scores of the laboratory standards	y_9
				Lab profit (economic criterion)	y_{10}

Table (9) and (10) show the data set. Note that data is collected for 2017. The inputs and the intermediate measures of network in table (9) and the outputs of network are shown in table (10).

Table 9
Set of inputs and intermediate measures for the 25 diagnostic laboratories in 2017

DMU	x_1	x_2	x_3	x_4	x_5	x_6	x_7	z_1	z_2
1	360	150	33548847	6894265	208000	17235663	100000000	15075	60358
2	600	170	34474338	7314342	300000	18285854	107200000	15906	63864
3	350	189	25158959	5342857	199000	13357143	81000000	11665	46163
4	355	177	20012808	4250000	200000	10625000	136500000	9279	36928
5	510	200	37166644	7892857	280000	19732143	133300000	17232	69021
6	490	164	32592288	6921429	251000	17303571	45000000	15111	60163
7	180	153	24587164	5221429	180000	13053571	85800000	11400	45572
8	210	166	22871781	4857143	220000	3122759	145000000	10604	42543
9	520	199	31448699	6592689	300000	16481722	52500000	14581	57565
10	450	184	25730753	5448044	231000	13620111	2500000	11930	47569
11	300	179	28589726	6038030	202000	15095075	135000000	13255	52719
12	430	130	17725630	3744819	210000	9362048	95000000	8218	32692
13	140	230	11500000	2453885	100000	6134711	14000000	1979	19671
14	510	185	34178450	6888259	288000	17220648	90720000	5938	59029
15	190	181	11513435	2324744	200000	5811860	14300000	1432	20245
16	150	190	19216893	3959367	222000	9898418	35100000	3008	33405
17	330	165	27889449	5584030	270000	13960074	49400000	5171	48265
18	170	180	12180596	2426467	204000	6066167	24700000	5298	21118
19	200	202	16620398	3375873	198000	8439682	50700000	5577	28794
20	400	159	11544005	2303674	100000	5759185	61600000	5029	20044
21	390	200	3025208	600315	100000	1500788	5500000	524	5201
22	310	205	25896678	5103203	100000	12758008	39600000	11266	44396
23	155	184	12135684	2386994	210000	5967485	18000000	4772	20805
24	550	192	39873519	7868954	230000	19672385	112500000	18557	68505
25	220	186	12938332	2613335	203000	6533339	40800000	3977	22767

Table 10
Set of outputs for the 25 diagnostic laboratories in 2017

DMU	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}
1	[11, 12.1]	3154	689617349	[30, 36]	16	[360, 684]	183	3154	196	470332740
2	[12, 16,8}	8069	720408345	[60, 54]	11	[120, 132]	212	8069	164	352401311
3	[11,13.2]	4533	529872381	[20, 28]	13	[300, 420]	111	4533	128	447306672
4	[14, 17.6]	4448	420982833	[40, 68]	10	[1440, 2448]	130	4448	110	235913267
5	[13, 16.9]	7441	769246533	[90, 99]	7	[600, 900]	215	7441	88	418550629
6	[16, 24]	7082	680334166	[40, 60]	10	[2880, 3744]	106	7082	145	405896651
7	[16, 22.4]	3840	509072663	[180, 194]	9	[10080, 19152]	108	3840	135	389141484
8	[18, 19.8]	3731	464686047	[15, 24]	10	[360, 648]	157	3731	98	119343357
9	[18, 32.4]	7763	659268893	[60, 78]	8	[2880, 3456]	220	7763	170	248745783
10	[18, 21.6]	4203	544804438	[65,104]	8	[480, 768]	229	4203	85	352605529
11	[16, 20.8]	4680	603802982	[75, 90]	10	[180, 306]	108	4680	105	453500152
12	[17, 27.2]	3830	374481901	[25, 47.5]	11	[1440, 2016]	132	3830	115	134636905
13	[7, 9.7]	960	245388458	[30, 42]	37	[1440, 1872]	113	960	100	195620696
14	[13, 22.1]	2895	688825933	[60, 90]	54	[4320, 6480]	247	2895	100	440068576
15	[13, 18.2]	1202	232474385	[30, 33]	56	[4320, 4752]	236	1033	195	167714823
16	[14, 16.8]	2501	395936735	[40, 68]	80	[1440, 1728]	244	2490	200	232762056
17	[18, 32.4]	4174	558402974	[15, 24]	63	[360, 648]	263	4174	188	325069421
18	[16, 20.8]	4390	242646690	[50,65]	72	[540, 1026]	240	4390	138	117273460
19	[15, 22.5]	5088	337587290	[90, 108]	68	[300, 450]	270	5088	145	163651337
20	[16, 17.6]	4280	230367401	[180, 324]	73	[10080, 13104]	246	4280	138	93160537
21	[10, 14]	345	60031530	[240, 312]	42	[1440, 2448]	141	345	168	33905219
22	[22, 26.4]	8973	510320333	[15, 17.5]	170	[300, 420]	286	8973	200	279479110
23	[11, 18.7]	875	238699416	[35, 66.5]	40	[2880, 3456]	193	875	110	126109253
24	[26, 33.8]	5348	786895417	[60, 90]	214	[420, 462]	406	5348	188	429980558
25	[12, 15.8]	937	261333549	[30, 54]	55	[180, 288]	204	937	200	136948542

Note that the environmental criterion is (the waste weight) in kilograms, economic criteria and social criteria are (safety cost of sampling unit and safety cost of test unit) in ten thousand Tomans, and time-based criteria are in minutes. According the opinions of the experts, we consider the step size in the models $\Delta\varepsilon=0.01$ and $\varepsilon=0.001$. We implemented our heuristic approach, for the two models (9 and 10). Table (11), illustrates the maximum efficiency of the stages, in the condition of the upper and lower bounds.

Table 11
Results of the maximum efficiencies of the first and second stages in Upper and Lower bound

DMU	Lower bound		Upper bound	
	$\theta_o^{Reception\ unit(L)-max}$	$\theta_o^{Sampling\ \&\ Test\ unit(L)-max}$	$\theta_o^{Reception\ unit(U)-max}$	$\theta_o^{Sampling\ \&\ Test\ unit(U)-max}$
1	1.00000	1.00000	1.00000	1.00000
2	0.53009	1.00000	1.00000	1.00000
3	0.64259	1.00000	1.00000	1.00000
4	0.41894	0.99202	0.50034	0.99210
5	0.70478	1.00000	1.00000	1.00000
6	0.49031	1.00000	0.68122	1.00000
7	1.00000	1.00000	1.00000	1.00000
8	0.79709	1.00000	1.00000	1.00000
9	0.44745	0.99754	0.58512	1.00000
10	0.42756	0.99735	0.51235	0.99826
11	0.71135	1.00000	1.00000	1.00000

12	0.30715	0.99611	0.30759	0.99616
13	1.00000	0.99053	1.00000	1.00000
14	0.47652	1.00000	0.69306	1.00000
15	0.43938	1.00000	0.43961	1.00000
16	1.00000	1.00000	1.00000	1.00000
17	0.59718	1.00000	0.72458	1.00000
18	0.61824	0.99303	0.61848	0.99306
19	0.61217	0.97818	0.61249	0.97917
20	0.20316	0.99253	0.20329	0.99270
21	0.05399	0.99364	0.05413	0.99370
22	0.58160	1.00000	0.59127	1.00000
23	0.72549	0.99510	1.00000	0.99515
24	0.53182	0.99482	0.66471	0.99895

By studying the values of k, we observed that, in this case study, the overall efficiency is optimized when the values of k are zero, which means that the optimal efficiency values of the first and second stages are equal to the maximum limit and their minimum limit value. In order to compare and perform the ranking of the interval efficiencies, we shall utilize Wang's minimax regret-based approach explained in Section (3) of this paper. Table (12), displays the range of overall efficiency changes and the ratings obtained for the DMUs.

Table 12
Results based on interval efficiency

DMU	$[\theta_o^{overall(L)}, \theta_o^{overall(U)}]$	RANK
1	[0.78012, 0.79044]	4
2	[0.39627, 0.74755]	11
3	[0.64259, 1]	5
4	[0.33272, 0.3974]	20
5	[0.52334, 0.74256]	10
6	[0.42698, 0.59323]	14
7	[0.87265, 0.88457]	2
8	[0.35309, 0.44297]	19
9	[0.30207, 0.39598]	21
10	[0.42643, 0.51146]	15
11	[0.63411, 0.89142]	6
12	[0.19124, 0.19152]	24
13	[0.99053, 1]	1
14	[0.35994, 0.52351]	16
15	[0.40758, 0.40779]	17
16	[0.78536, 0.79536]	3
17	[0.46667, 0.56623]	13
18	[0.61393, 0.61419]	7
19	[0.53874, 0.53956]	9
20	[0.20164, 0.20181]	23
21	[0.05365, 0.05379]	25
22	[0.5816, 0.59127]	8
23	[0.47067, 0.6488]	12
24	[0.36586, 0.45917]	18
25	[0.28429, 0.28444]	22

Therefore, the performance of 25 DMUs is rated as follows:

$$\mathrm{DMU}_{13} > \mathrm{DMU}_{7} > \mathrm{DMU}_{16} > \mathrm{DMU}_{1} > \mathrm{DMU}_{3} > \mathrm{DMU}_{11} > \mathrm{DMU}_{18} > \mathrm{DMU}_{22} > \mathrm{DMU}_{19} > \mathrm{DMU}_{5} > \mathrm{DMU}_{2} > \mathrm{DMU}_{23} > \mathrm{DMU}_{17} > \mathrm{DMU}_{6} > \mathrm{DMU}_{10} > \mathrm{DMU}_{14} > \mathrm{DMU}_{15} > \mathrm{DMU}_{24} > \mathrm{DMU}_{8} > \mathrm{DMU}_{4} > \mathrm{DMU}_{9} > \mathrm{DMU}_{25} > \mathrm{DMU}_{20} > \mathrm{DMU}_{12} > \mathrm{DMU}_{21},$$

Where symbol " $>$ " means that the interval performance is better. The results of this study indicate that most private labs in Tehran are not efficient. The reasons of inefficiency of laboratories can be identified as follow: 1) one of the most important sources of municipal wastes production is hospitals, health centers, physicians, clinics and medical diagnostic laboratories. Among them, the laboratories produce large amount of infectious waste that threaten health and the environment. Releasing this waste into the environment can cause and transmit a variety of diseases, including hepatitis B, C, and AIDS. Proper management of waste plays an important role in the performance of laboratories. 2) Analyzing the standards and criteria that any laboratory system needs to be upgraded. Therefore, the quality management achievements in the lab has some advantages such as enhancing the assurance of the accuracy of the results provided by the labs, ensuring the continuous calibration of lab equipment, standardizing the procedures for the management of laboratories, and improving the level of customer-oriented of laboratories. 3) The ability to differentiate and excel a laboratory is the extensive coverage of services. Therefore, increasing the geographic coverage of services of each laboratory is important. 4) Factors such as currency fluctuations, price increases of kits, and the cost of implementing quality standards indicate that the laboratories need to control and manage costs. On average, 45% of the total required cost is related to the consumables in each laboratory. Therefore, the costs management has a significant role in increasing efficiency. Considering the reasons mentioned for increasing the efficiency of laboratories, the following solutions are suggested: 1) Separation of laboratory wastes at the place of production, collection and labeling, transportation to the location of safe place, packing, temporary storage, transportation from the place of production and loading, and also the final disposal stage. All steps should be designed according to the performance and capacity of each laboratory. All staff members should be educated and notified of the procedures in writing. 2) Investigating the factors and determining the status of the laboratory and recording the results in the form of weaknesses and strengths, and determining the gap between the existing and the desired situation may provide appropriate and effective strategies for standardizing the laboratories. 3) Provision of services to smaller laboratories in view of the increasing diversity and capacity of the experiments is one of the ways in which successful labs operate. The use of sampling units and the use of information and communication technology is also one of the requirements for the coverage of services. 4) The operation management approach by identifying and eliminating unnecessary points reduces additional laboratory costs and increases productivity.

6. Conclusion

In this paper, a model for evaluating efficiency and ranking complex networks, in the presence of imprecise datum, from the interval criteria has been presented. This is capable of bringing the network efficiency interval and the efficiency of the stages into hand; and then rates the units, on the basis of these intervals achieved. This system allows us to consider the intermediate variables to open the structure of the Black Box and gather crucial information, from the various efficient and inefficient points of the system, to be put at the disposal of managers. In this study, we measured the efficiency of selected private medical diagnostic laboratories in Tehran through DEA approach. In this regard, the efficiency evaluation of 25 private medical diagnostic laboratories in Tehran was examined. To evaluate the efficiency of medical diagnostic laboratories, there are several indicators that may be used for different approaches, but in most studies, researchers, regardless of the approach they use to evaluate, try to find better or more appropriate indicators. Therefore, it is important to investigate and identify the most effective factors for evaluating the performance of medical diagnostic laboratories. Therefore, in order to facilitate the correct and well-informed decision-making in this area, given the lack of literature, the identification of effective factors was done using the Fuzzy Delphi method.

We simulated a three-stage network with series structure of a medical diagnostic lab in a real world. This network model of three main laboratory processes (the pre-test process, the test process and the post-test process). The pre-testing process contains the reception unit. The testing process consists of the sampling and testing unit. Finally, the post-test process includes the test results unit. The model presented in this article is an innovative model, and similar research was not found in the field of medical diagnostic laboratories as a network analysis.

According to managers, we used a CCR multiplicative model to measure the performance of the lab units and then to convert nonlinear programs into linear programs by a heuristic method. In addition, we utilize the Wang (2005) minimax regret-based approach to compare and rank the interval efficiencies that come to hand. The results of the ranking illustrated that, the DMUs namely, (13) and (21) were the best and the poorest units, respectively, in relative to interval efficiencies between the 25 units. We found out that variant performance indicators, such as, monitoring and control of wastes, geographic coverage, review of the pre-test process (the reception unit), providing appropriate and effective strategies for standardization of laboratories and cost of consumables (kits) due to currency fluctuations are significant elements to determine efficiency. Other results of this research are providing application indicators that can be used in future research by other researchers. The heuristic approach proposed in this research can be used to solve a hybrid three-stage system. The model becomes complex for higher-stage systems, due to the presence of additional inputs and outputs; thereby, it increases the solution time significantly. To overcome the problem we can change the movement step ("$\Delta\varepsilon$"). We provided the results of this research to the managers that might lead to improved laboratory services by adopting appropriate approaches. Also, since the activities of an enterprise such as medical diagnostic laboratories is not sectional over a period of time, and it is a continuous activity, the cross-sectional efficiency assessment cannot provide a realistic answer to the performance of laboratories. Therefore, network analysis in dynamic mode is recommended.

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