A Hybrid Intuitionistic Fuzzy Multi-criteria Group Decision Making Approach for Supplier Selection

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Abstract

Due to the increasing competition of globalization, selection of the most appropriate supplier is one of the key factors for the success of a supply chain management. Due to conflicting evaluations and insufficient information about the criteria, Intuitionistic fuzzy sets (IFSs) are considered as an impressive tool and utilized to specify the relative importance of the criteria. The aim of this paper is to develop a new approach to solving the decision making processes. Thus, an intuitionistic fuzzy multi-criteria group decision making approach is proposed. Interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) is utilized to aggregate individual opinions of decision makers into a group opinion. A linear programming model is used to obtain the weights of the criteria. Then, a combined approach based on GRAand TOPSIS method is introduced and applied to the ranking and selection of the alternatives. Finally a numerical example for supplier selection is given to illustrate the feasibility and effectiveness of the proposed method. A combined method based on GRA and TOPSIS associated with intuitionistic fuzzy set has a high chance of success for multi-criteria decision-making problems since it contains vague perception of decision makers' opinions. Therefore, intuitionistic fuzzy sets can be used for dealing with uncertainty in multi-criteria decision-making problems such as project selection, manufacturing systems, pattern recognition, medical diagnosis as well as many other areas of management decision problems.
Keywords:Multi-Criteria Group Decision Making,Supplier Selection, Interval-Valued Intuitionistic Fuzzy Set, TOPSIS Method, GRA Method.

1. Introduction

Supply Chain Management (SCM) has received recently considerable attention in both academia and industry. The major aims of SCM are to reduce supply chain risk and production costs, maximize revenue, improve customer service, and optimize inventory levels. These, in turn, result in increased competitiveness and profitability. Effective purchasing function plays an important role in successful SCM. The most important activity of the purchasing function is the selection of appropriate supplier as it brings about significant savings for the organization.

One of the well-known studies on supplier selection was conducted by Chai, Liu, & Ngai, (2013)who reviewed and classified 123 articles regarding the supplier selection problem. Chou, Chang, & Shen (2008) identified three key stages for supplier selection including rating stage, aggregation stage, and selection stage, respectively. Several methodologies have been proposed to address the supplier selection problem. The systematic analysis for supplier selection includes categorical method, analytic hierarchy process (AHP)(X. Deng, Hu, Deng, & Mahadevan, 2014; Levary, 2008), analytic network process (ANP)(Eshtehardian, Ghodousi, & Bejanpour, 2013; Y. Lin, Lin, Yu, & Tzeng, 2010), mathematical programming(Hsu, Chiang, & Shu, 2010; Kull & Talluri, 2008; C. Lin, Chen, & Ting, 2011; Wu & Blackhurst, 2009), and artificial intelligence (AI) techniques(Guneri, Ertay, & Yucel, 2011; Lee & Ouyang, 2009; Tseng, 2011; J. Xu & Yan, 2011).

Most of these methods do not seem to address the complex and unstructured nature and context of many today's purchasing decisions. In many existing decision models in the literature, only quantitative criteria have been considered for supplier selection. Several influence factors are often not taken into account in the decision-making process, such as incomplete information,

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additional qualitative criteria and imprecision preferences. Therefore, fuzzy set theory (FST)has been applied to supplier selection recently. C. Li, Fun, & Hung, (1997)discussed the application of FST to supplier selection. Arabzad, Ghorbani, Razmi, & Shirouyehzad, (2014); C.-T. Chen, Lin, & Huang, 2006; Kannan, Jabbour, & Jabbour, (2014)extended the concept of TOPSIS (technique for order preference by similarity to an ideal solution) method to develop a methodology for solving supplier selection problems in fuzzy environment. Bayrak, Celebi, & Taşkin, (2007) presented a fuzzy multi-criteria group decision-making approach to supplier selection based on fuzzy arithmetic operation. Önüt, Kara, & Işik, (2009)developed a supplier evaluation approach based on ANP and TOPSIS methods for the supplier selection. In the type of fuzzy multi-criteria model, grey relational analysis (GRA) is suggested as a tool for implementing a multiple criteria performance scheme, which is used to identify solutions from a finite set of alternatives (Kuo, Yang, & Huang, 2008) and it has been proven to be useful for dealing with poor, incomplete, and uncertain information (Szmidt & Kacprzyk, 2000). GRA is a part of grey system theory, which is suitable for solving problems with complicated interrelationships between multiple factors and variables.GRA has been successfully applied in solving a variety of MADM problems(Bali, Kose, & Gumus, 2013; W.-H. Chen, Tsai, & Kuo, 2005; Olson & Wu, 2006).

This paper proposes an intuitionistic fuzzy multi-criteria group decision making approach for supplier selection problem which employs a combined method based on GRA and TOPSIS for ranking the suppliers. The importance of the criteria and the impact of alternatives on criteria provided by decision makers are difficult to precisely express by crisp data in the selection of supplier problem. Interval-valued intuitionistic fuzzy sets (IVIFSs) introduced by Atanassov & Gargov, (1989)are a suitable way to deal with this challenge and applied in many decision-making problems in uncertain environment. In group decision-making problems, aggregation of expert opinions is very important to appropriately perform evaluation process. Therefore, IIFOWA operator is utilized to aggregate all individual decision makers' opinions for rating the importance of criteria and the alternatives. The TOPSIS method was presented by Hwang & Yoon, (1981)considering both positive-ideal and negative-ideal solution. It is one of the popular methods in multi-attribute decision-making problem. Also GRA method was originally developed by J.-L. Deng, (1989) which is an impact evaluation model that can measure the degree of similarity or difference between two sequences based on the relation.GRA method only considers the shape similarity of data sequence curve of alternative's attribute to that of ideal solution(J.-L. Deng, 1989). However, TOPSIS method only considers the position approximation (Opricovic & Tzeng, 2004). By combining GRA and TOPSIS method, we present a combined approach that can accurately reflect the relationship between alternative's data and ideal solutions. Therefore, GRA method is combined with TOPSIS in intuitionistic fuzzy environment, which has not been studied yet.

The remainder of this paper is organized as follows: Section 2 gives some basic concepts and related knowledge of IVIFSs. In Section 3, we present a brief introduction of interval-valued intuitionistic fuzzy GRA (IVIFGRA) method. In Section 4, interval-valued intuitionistic fuzzy TOPSIS (IVIFTOPSIS) method is introduced. In Section5, based on IVIFGRA and IVIFTOPSIS, we present a combined decision making approach. In Section 6, for multi-criteria group decision making, an approach is given. In Section7, a numerical example is presented to illustrate the proposed approach and to demonstrate its feasibility and practicality. Finally, a short conclusion is given.

2. IVIFSs and Related Knowledge

To introduce our new approach, some relevant concepts are illustrated in this section.

Definition**:**

(Atanassov & Gargov, 1989). Let $D[0,1]$ be the set of all closed subintervals of the interval$[0,1]$. Let($X \neq \Phi$)be a given set. An interval-valued intuitionistic fuzzy set A in X is given by$A = \{(x, \mu_A(x), v_A(x)) : x \in X\}$, where$\mu_A$: $X \to D[0,1]$, v_A: $X \to D[0,1]$and $0 \leq sup_x \mu_A(x) + sup_x v_A(x) \leq 1$.

The intervals$\mu_A(x)$ and $v_A(x)$ denote the degree of belongingness and the degree of non-belongingness of the element x to the set$A$, respectively. Thus, for each$x \in X$,$\mu_A(x)$ and $v_A(x)$ are closed intervals whose lower and upper end points are denoted by$\mu_{AL}(x)$, $\mu_{AU}(x)$ and$v_{AL}(x)$, $v_{AU}(x)$, respectively. A can be denoted by,

$$A = \{(x, [\mu_{AL}(x), \mu_{AU}(x)], [v_{AL}(x), v_{AU}(x)]) : x \in X\}, \quad (1)$$

where$0 \leq \mu_{AU}(x) + v_{AU}(x) \leq 1$, $\mu_{AL}(x) \geq 0$ and$v_{AL}(x) \geq 0$. In addition the set of all the IVIFS in X is shown by IVIFS(X).For each element x, the unknown degree (uncertainty degree) of an intuitionistic fuzzy interval of $x \in X$ in A can be defined as follows:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) = [1 - \mu_{AU}(x) - v_{AU}(x), 1 - \mu_{AL}(x) - v_{AL}(x)] \quad (2)$$

An IVIFS value is denoted by $A = ([a, b], [c, d])$ for convenience.

Definition**:**

(Ze-Shui, 2007a). Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$ be any two IVIFNs, then their operational laws can be defined as follows:

$$\bar{\tilde{\alpha}}_1 = ([c_1, d_1], [a_1, b_1]) \tag{3}$$

$$\tilde{\alpha}_1 + \tilde{\alpha}_2 = \left(\begin{bmatrix} a_1 + a_2 - a_1 a_2 \\ , b_1 + b_2 - b_1 b_2 \end{bmatrix}, [c_1 c_2, d_1 d_2] \right) \tag{4}$$

$$\tilde{\alpha}_1 . \tilde{\alpha}_2 = \left(\begin{matrix} [a_1 a_2, b_1 b_2], \\ [c_1 + c_2 - c_1 c_2, d_1 + d_2 - d_1 d_2] \end{matrix} \right) \tag{5}$$

$$\lambda \tilde{\alpha}_1 = \left(\begin{matrix} \begin{bmatrix} 1 - (1 - a_1)^{\lambda}, 1 - \\ (1 - b_1)^{\lambda} \end{bmatrix}, \\ [c_1{}^{\lambda}, d_1{}^{\lambda}] \end{matrix} \right), \lambda \geq 0 \tag{6}$$

Definition:
(Ze-Shui, 2007a). Let $\tilde{\alpha} = ([a, b], [c, d])$ be an IVIFN. Then the score function (S) is defined by:

$$S(\tilde{\alpha}) = 1/2(a - c + b - d) \tag{7}$$

Where $S(\tilde{\alpha}) \in [-1,1]$. The greater the value of $(\tilde{\alpha})$, the greater IVIFN$\tilde{\alpha}$.

Definition:
Let $\tilde{\alpha}_1 = ([a_1, b_1], [c_1, d_1])$,$\tilde{\alpha}_2 = ([a_2, b_2], [c_2, d_2])$,...,$\tilde{\alpha}_n = ([a_n, b_n], [c_n, d_n])$ be n IVIFNs, and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of these IVIFNs. The weighted score function is defined as:

$$W_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \omega_1 S(\tilde{\alpha}_1) + \omega_2 S(\tilde{\alpha}_2) + \dots + \omega_n S(\tilde{\alpha}_n) \tag{8}$$

(Ze-Shui, 2007a) developed the interval-valued intuitionistic weighted arithmetic aggregation operator(IIFWA) and the interval-valued intuitionistic fuzzy ordered weighted aggregation operator (IIFOWA) to aggregate interval-valued intuitionistic fuzzy information.

Definition:
(Ze-Shui, 2007a). Let$\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$, $j = (1,2, \dots, n)$be a collection of IVIFNs.The IIFWA operator is further defined by:

$$IIFWA_{\omega}(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left(\begin{bmatrix} 1 - \prod_{j=1}^{n} (1 - a_j)^{\omega_j}, \\ 1 - \prod_{j=1}^{n} (1 - b_j)^{\omega_j} \end{bmatrix}, \left[\prod_{j=1}^{n} c_j{}^{\omega_j}, \prod_{j=1}^{n} d_j{}^{\omega_j} \right] \right) \tag{9}$$

Where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of$\tilde{\alpha}_j (j = 1,2, \dots, n)$;$\omega_j \in [0,1]$, and $\sum_{j=1}^{n} \omega_j = 1$.

(Z.-S. Xu & Chen, 2007a) proposed the interval-valued intuitionistic fuzzy ordered weighted aggregation (IIFOWA) operator to aggregate IVIFNs. The operator is characterized by reordering the IVIFNs in descending order. A weight,w_j, is associated with a particular ordered position. The arguments are endowed with new weights w_j rather than the initial weightsω_j.

Definition:
(Z.-S. Xu & Chen, 2007a). Let $\tilde{\alpha}_j = ([a_j, b_j], [c_j, d_j])$, $j = (1,2, \dots, n)$be a collection of IVIFNs, and$(\tilde{\alpha}_{\sigma(1)}, \tilde{\alpha}_{\sigma(2)}, \dots, \tilde{\alpha}_{\sigma(n)})$be a permutation of$(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n)$, such that $\tilde{\alpha}_{\sigma(j-1)} \geq \tilde{\alpha}_{\sigma(j)}$ for all j, and let $\tilde{\alpha}_{\sigma(j)} = ([a_{\sigma(j)}, b_{\sigma(j)}], [c_{\sigma(j)}, d_{\sigma(j)}])$, then the IIFOWA operator can be defined by:

$$IIFOWA_w(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left(\left[1 - \prod_{j=1}^{n} (1 - a_{\sigma(j)})^{w_j}, 1 - \prod_{j=1}^{n} (1 - b_{\sigma(j)})^{w_j} \right], \left[\prod_{j=1}^{n} c_{\sigma(j)}{}^{w_j}, \prod_{j=1}^{n} d_{\sigma(j)}{}^{w_j} \right] \right) \tag{10}$$

Where $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of the IIFOWA operator, $w_j \in [0,1]$,and $\sum_{j=1}^{n} w_j = 1$. The weight vector of the IIFOWA operator can be determined by the method of (Z. Xu, 2005) , which uses the perspective of normal distribution to gain weights. In this way, it can reduce the influence of unfair arguments in the final results by assigning low weights to the "optimistic" or "pessimistic" discretions.

Definition:
(Szmidt & Kacprzyk, 2000). For two intuitionistic fuzzy sets A and$B$ in $X = \{x_1, x_2, \dots, x_n\}$, the normalized Hamming distance is defined as follows:

$$d_h(A, B) = \frac{1}{2n} \sum_{i=1}^{n} (|\mu_A(x_i) - \mu_B(x_i)| + |v_A(x_i) - v_B(x_i)| + |\pi_A(x_i) - \pi_B(x_i)|) \tag{11}$$

Definition:
(Szmidt & Kacprzyk, 2000).For two intuitionistic fuzzy sets A and B in $X = \{x_1, x_2, \dots, x_n\}$, the normalized Euclidean distance is defined as follows:

$$d_e(A, B) = \sqrt{\frac{1}{2n} \sum_{i=1}^{n} \begin{matrix} (\mu_A(x_i) - \mu_B(x_i))^2 + \\ (v_A(x_i) - v_B(x_i))^2 + \\ (\pi_A(x_i) - \pi_B(x_i))^2 \end{matrix}} \tag{12}$$

Clearly these distances satisfy the conditions of the metric.

3. GRA method for Multiple Attribute Decision Making Problems with Interval-valued Intuitionistic Fuzzy Information

GRA method was originally developed by (J.-L. Deng, 1989)and has been successfully applied in solving a variety of MADM problems (Lahby & Adib, 2013; G. Li, Yamaguchi, & Nagai, 2008; Mehregan, Jafarnejad, & Dabbaghi, 2014). The main procedure of GRA is firstly

translating the performance of all alternatives into a comparability sequence. This step is called gray relational generating. According to these sequences, an ideal target sequence is defined. Then, the gray relational coefficient between all comparability sequences and ideal target sequence is calculated. Finally, based on these gray relational coefficients, the gray relational degree between ideal target sequence and every comparability sequences is calculated. If a comparability sequence translated from an alternative has the highest gray relational degree between the ideal target sequence and itself, that alternative will be the best choice.
Let $Y = \{Y_1, Y_2, \ldots, Y_n\}$ be a discrete set of alternatives, and $G = \{G_1, G_2, \ldots, G_p\}$ be the set of attributes, $w = (w_1, w_2, \ldots, w_p)^T$ is the weighting vector of the attribute $G_j (j = 1,2, \ldots, p)$, where $w_j \in [0,1]; \sum_{j=1}^{p} w_j = 1$.
Suppose that $\tilde{R} = (\tilde{r}_{ij})_{n\times p} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])_{n\times p}$ is the interval-valued intuitionistic fuzzy decision matrix, where $[a_{ij}, b_{ij}]$ indicates the degree that the alternative Y_i satisfies the attribute G_j given by the decision-maker, $[c_{ij}, d_{ij}]$ indicates the degree that the alternative Y_i does not satisfy the attribute G_j given by the decision maker, $[a_{ij}, b_{ij}] \subset [0,1]$, $[c_{ij}, d_{ij}] \subset [0,1]$, $b_{ij} + d_{ij} \le 1$, $i = (1,2, \ldots, n)$, $j = (1,2, \ldots, p)$.

In the following, we illustrate GRA method to solve interval-valued intuitionistic fuzzy MADM. The method involves the following steps:

Step1: Determine the ideal with interval-valued intuitionistic fuzzy information.

$$\tilde{r}^{+} = \begin{pmatrix} [a_1{}^{+}, b_1{}^{+}], [c_1{}^{+}, d_1{}^{+}], [a_2{}^{+}, b_2{}^{+}], \\ [c_2{}^{+}, d_2{}^{+}], \ldots, [a_p{}^{+}, b_p{}^{+}], [c_p{}^{+}, d_p{}^{+}] \end{pmatrix} \quad (13)$$

Where

$$\tilde{r}_j{}^{+} = ([a_j{}^{+}, b_j{}^{+}], [c_j{}^{+}, d_j{}^{+}]) = \left(\left[\max_i a_{ij}, \max_i b_{ij}\right], \left[\min_i c_{ij}, \min_i d_{ij}\right]\right), j \in 1,2, \ldots, p$$

Step2: Calculate the gray relational coefficient of each alternative from ideal using the following equation,

$$\xi_{ij}^{+} = \frac{\min\limits_{1\le i\le n}\min\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^{+}) + \rho \max\limits_{1\le i\le n}\max\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^{+})}{d(\tilde{r}_{ij}, \tilde{r}_{ij}^{+}) + \rho \max\limits_{1\le i\le n}\max\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^{+})}, i = (1,2, \ldots, n), j = (1,2, \ldots, p) \quad (14)$$

Where the identification coefficient = 0.5 . and it uses the normalized Hamming distance.
Step3: Calculating the degree of gray relational coefficient of each alternative from ideal using the following equation,

$$\xi_i^{+} = \sum_{j=1}^{p} w_j \xi_{ij}^{+}, \qquad i = (1,2, \ldots, n) \quad (15)$$

Step5: Rank all the alternatives $Y_i (i = 1,2, \ldots, n)$ and select the best one(s) in accordance with $\xi_i^{+} (i = 1,2, \ldots, n)$. If any alternative has the highest ξ_i^{+} value, then, it is the most important alternative.

4. TOPSIS method for multiple attribute decision making problems with interval-valued intuitionistic fuzzy information

TOPSIS method was presented by Hwang & Yoon, (1981)and has been successfully applied to solving a variety of MADM problems(Du, Gao, Hu, Mahadevan, & Deng, 2014; Z. Xu & Zhang, 2013).TOPSIS is based on the concept that the chosen alternative should have the shortest distance from the positive ideal solution (PIS) and the longest distance from the negative ideal solution (NIS).
Let Y, G, w and $\tilde{R}$ be the same as presented in section3. The procedure for interval-valued intuitionistic fuzzy TOPSIS method has been given as follows:
Step1: Determine the positive ideal and the negative ideal using interval-valued intuitionistic fuzzy information.
The ideal alternative is a hypothetical alternative in which all attribute values correspond to the best level. On the contrary, the anti-ideal alternative is also a hypothetical alternative in which all attribute values correspond to the worst level. Denote the positive ideal alternative with interval-valued intuitionistic fuzzy information, A^{+}, and the anti-ideal alternative with interval-valued intuitionistic fuzzy information, A^{-}, as follow:

$$A^{+} = \left\{ \left(\begin{pmatrix} \max\limits_i \tilde{r}_{ij} \,\middle|\, j \in J \end{pmatrix}, \\ \begin{pmatrix} \min\limits_i \tilde{r}_{ij} \,\middle|\, j \in J' \end{pmatrix} \right| i = 1,2, \ldots, n \right\} = \{\tilde{r}_1{}^{+}, \tilde{r}_2{}^{+}, \ldots, \tilde{r}_p{}^{+}\} \quad (16)$$

$$A^{-} = \left\{ \left(\begin{pmatrix} \min\limits_i \tilde{r}_{ij} \,\middle|\, j \in J \end{pmatrix}, \\ \begin{pmatrix} \max\limits_i \tilde{r}_{ij} \,\middle|\, j \in J' \end{pmatrix} \right| i = 1,2, \ldots, n \right\} = \{\tilde{r}_1{}^{-}, \tilde{r}_2{}^{-}, \ldots, \tilde{r}_p{}^{-}\} \quad (17)$$

Where $\tilde{r}_j{}^{+} = ([a_j{}^{+}, b_j{}^{+}], [c_j{}^{+}, d_j{}^{+}])$ and

$\tilde{r}_j{}^{-} = ([a_j{}^{-}, b_j{}^{-}], [c_j{}^{-}, d_j{}^{-}]), j = 1,2, \ldots, p$.

Where J and J' are the attribute sets of the larger-the-better type (such as benefit) and the smaller-the-better type (such as cost), respectively.

Step2: Calculate the separation measures, using the normalized Euclidean distance.

The separation measures, d_i^{+} and d_i^{-}, of each alternative from interval-valued intuitionistic fuzzy positive-ideal and negative-ideal solutions are calculated as follow:

$$d_e(Y_i, A^+) = d_i^{+} = \sqrt{\frac{1}{2p}\sum_{j=1}^{p}\left(\begin{array}{c}\left(\mu_{Y_i}(x_j) - \mu_{A^+}(x_j)\right)^2 + \\ \left(v_{Y_i}(x_j) - v_{A^+}(x_j)\right)^2 + \\ \left(\pi_{Y_i}(x_j) - \pi_{A^+}(x_j)\right)^2\end{array}\right)} \quad (18)$$

$$d_e(Y_i, A^-) = d_i^{-} = \sqrt{\frac{1}{2p}\sum_{j=1}^{p}\left(\begin{array}{c}\left(\mu_{Y_i}(x_j) - \mu_{A^-}(x_j)\right)^2 + \\ \left(v_{Y_i}(x_j) - v_{A^-}(x_j)\right)^2 + \\ \left(\pi_{Y_i}(x_j) - \pi_{A^-}(x_j)\right)^2\end{array}\right)} \quad (19)$$

Step3: Calculate the relative closeness coefficient to the interval-valued intuitionistic fuzzy ideal solution.

The relative closeness coefficient of an alternative Y_i with respect to the interval-valued intuitionistic fuzzy positive-ideal solution A^+ is defined as follows:

$$f_i = \frac{d_i^{-}}{d_i^{+} + d_i^{-}} \ , i = 1,2,\dots,n. \quad (20)$$

Step4: Rank the alternatives.

When the relative closeness coefficient of each alternative is determined, alternatives are ranked according to descending order of f_i.

5. A Combined Approach Based on GRA and TOPSIS Method for Multiple Attribute Decision Making Problems with Interval-valued Intuitionistic Fuzzy Information

GRA method only considers the shape similarity of data sequence curve of alternative's attribute to that of ideal solution's. However, TOPSIS method only considers the position approximation. By combining GRA and TOPSIS method, we present a combined approach that can accurately reflect the relationship between alternative's data and ideal solutions. The introduced method involves the following steps:

Step1: Determine the positive ideal and negative ideal solution with interval-valued intuitionistic fuzzy information as given in (13) and (21).

$$\tilde{r}^- = \left(\begin{array}{c}[a_1^-, b_1^-],[c_1^-, d_1^-], \\ [a_2^-, b_2^-],[c_2^-, d_2^-], \dots, \\ [a_p^-, b_p^-],[c_p^-, d_p^-]\end{array}\right) \quad (21)$$

Where

$$\tilde{r}_j^{+} = \left([a_j^+, b_j^+],[c_j^+, d_j^+]\right) = \left(\left[\max_i a_{ij}, \max_i b_{ij}\right], \left[\min_i c_{ij}, \min_i d_{ij}\right]\right), j \in 1,2,\dots,p$$

$$\tilde{r}_j^{-} = \left([a_j^-, b_j^-],[c_j^-, d_j^-]\right) = \left(\left[\min_i a_{ij}, \min_i b_{ij}\right], \left[\max_i c_{ij}, \max_i d_{ij}\right]\right), j \in 1,2,\dots,p$$

Step2: Calculate the gray relational coefficients of each alternative from PIS and NIS using the following equations, respectively Equations (14) and (22).

$$\xi_{ij}^- = \frac{\min\limits_{1\le i\le n}\min\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^-) + \rho \max\limits_{1\le i\le n}\max\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^-)}{d(\tilde{r}_{ij}, \tilde{r}_{ij}^-) + \rho \max\limits_{1\le i\le n}\max\limits_{1\le j\le p} d(\tilde{r}_{ij}, \tilde{r}_{ij}^-)}, i = (1,2,\dots,n), j = (1,2,\dots,p) \quad (22)$$

Where the identification coefficient $\rho = 0.5$. And using the normalized Hamming distance.

Step3: Calculating the degree of gray relational coefficients of each alternative from PIS and NIS using the following equations, respectively Equation (15) and (23).

$$\xi_i^- = \sum_{j=1}^{p} w_j \xi_{ij}^- \ , \qquad i = (1,2,\dots,n) \quad (23)$$

The basic principle of the GRA method is that the chosen alternative should have the "largest degree of grey relation" from the positive ideal solution and the "smallest degree of grey relation" from the negative ideal solution.

Step4: Calculate the relative grey relational degree of each alternative from the positive ideal solution using the following equation:

$$\xi_i = \frac{\xi_i^+}{\xi_i^+ + \xi_i^-} \ , i = 1,2,\dots,n. \quad (24)$$

Step5: Rank all the alternatives $Y_i (i = 1,2,\dots,n)$ and select the best one(s) in accordance with $\xi_i (i = 1,2,\dots,n)$. The alternative with the highest ξ_i value is the most important alternative.

6. A Hybrid Intuitionistic Fuzzy Multi-criteria Group Decision Making Approach for Supplier Selection

Today's decision making is a major challenge faced by companies. Especially in this case, supplier selection is a complicated decision-making problem involving multi-criteria, alternatives and decision makers. However, there are a few studies on multi-criteria decision-making

(MCDM) involving multiple decision-makers in an interval-valued intuitionistic fuzzy environment. In order to avoid partiality caused by an individual subject's judgment, the group decision-making method is used to integrate different opinions and reach the best decision with a common solution. Comparing with individual decision making, group decision making can elicit more complete information about the problem and provide more selective alternatives(T.-Y. Chen, Wang, & Lu, 2011). Therefore, we proposed an interval-valued intuitionistic fuzzy multi-criteria group decision-making (MCGDM) with combined method based on GRA and TOPSIS for supplier selection problem.

The preference relation on criteria based on IVIFSs can be concisely expressed in a pair-wise comparison matrix. Suppose that there is a set of criteria $G = \{g_1, g_2, \ldots, g_p\}$, and a set of decision makers (experts) $E = \{e_1, e_2, \ldots, e_m\}$. Each expert has to compare the relative importance of each pair of criteria with IVIFSs.

Definition**:**

(T.-Y. Chen et al., 2011). If an interval-value intuitionistic fuzzy preference relation matrix $\tilde{G}_k$ on the set X is defined as $\tilde{G}_k = \left(\tilde{g}_{ij}^{(k)}\right)_{p\times p} \subset X \times X$, then:

$$\tilde{G}_k = \begin{bmatrix} \tilde{g}_{11}^{(k)} & \cdots & \tilde{g}_{1p}^{(k)} \\ \vdots & \ddots & \vdots \\ \tilde{g}_{p1}^{(k)} & \cdots & \tilde{g}_{pp}^{(k)} \end{bmatrix} \tag{25}$$

Where $\tilde{g}_{ij}^{(k)} = \left([a_{ij}, b_{ij}]^{(k)}, [c_{ij}, d_{ij}]^{(k)}\right)$, $i, j = (1,2, \ldots, p)$ is an IVIFS. $[a_{ij}, b_{ij}]^{(k)}$ indicates the expert e_k's interval-valued intuitionistic fuzzy preference degree for the criterion g_i when the criteria g_i and g_j are compared and criterion g_i is preferred over the other one ; also $[c_{ij}, d_{ij}]^{(k)}$ indicates the expert e_k's interval-valued intuitionistic fuzzy preference degree for the criterion g_i when the criteria g_i and g_j are compared and criterion g_j is preferred over the other one ; $[a_{ij}, b_{ij}]^{(k)} \subset [0,1]$, $[c_{ij}, d_{ij}]^{(k)} \subset [0,1]$, $[a_{ji}, b_{ji}]^{(k)} = [c_{ij}, d_{ij}]^{(k)}$, $[c_{ji}, d_{ji}]^{(k)} = [a_{ij}, b_{ij}]^{(k)}$, $[a_{ii}, b_{ii}]^{(k)} = [c_{ii}, d_{ii}]^{(k)} = [0.5, 0.5]$, and $b_{ij}^{(k)} + d_{ij}^{(k)} \le 1$, $i, j = (1,2, \ldots, p)$ and $k = (1,2, \ldots, m)$.

Definition**:**

(Z. Xu, 2007b). Let $\tilde{G}_k = \left(\tilde{g}_{ij}^{(k)}\right)_{p\times p}$ be an interval-valued intuitionistic fuzzy preference relation matrix. If $\tilde{g}_{ij}^{(k)} = \tilde{g}_{ik}^{(k)} . \tilde{g}_{kj}^{(k)}$ for all i, j, k, then $\tilde{G}_k$ is called the consistent interval-valued intuitionistic fuzzy preference relation matrix.

Definition**:**

The criterion values can also be expressed in a decision matrix based on IVIFSs to discern the performance of each alternative with respect to criteria. Now, suppose there exists a set of alternatives $Y = \{y_1, y_2, \ldots, y_n\}$ which consist of n non-inferior decision-making alternatives, a set of criteria $G = \{g_1, g_2, \ldots, g_p\}$, and a set of experts $E = \{e_1, e_2, \ldots, e_m\}$.

Definition**:**

(T.-Y. Chen et al., 2011).If an interval-value intuitionistic fuzzy decision matrix $\tilde{D}_k$ on the set X is defined as $\tilde{D}_k = \left(\tilde{d}_{ij}^{(k)}\right)_{n\times p} \subset X \times X$, then:

$$\tilde{D}_k = \begin{array}{c} \\ y_1 \\ \vdots \\ y_n \end{array} \begin{array}{c} g_1 \quad \cdots\cdots\cdots \quad g_p \\ \begin{bmatrix} \tilde{d}_{11}^{(k)} & \cdots & \tilde{d}_{1p}^{(k)} \\ \vdots & \ddots & \vdots \\ \tilde{d}_{n1}^{(k)} & \cdots & \tilde{d}_{np}^{(k)} \end{bmatrix} \end{array} \tag{26}$$

Where $\tilde{d}_{ij}^{(k)} = \left([a_{ij}, b_{ij}]^{(k)}, [c_{ij}, d_{ij}]^{(k)}\right)$, $i = (1,2, \ldots, n)$, $j = (1,2, \ldots, p)$, is an IVIFS. $[a_{ij}, b_{ij}]^{(k)}$ indicates the extent to which the expert e_k considers the alternative y_i to satisfy the criterion g_j of the fuzzy concept ''excellence.'' Also $[c_{ij}, d_{ij}]^{(k)}$ indicates the extent to which the expert e_k considers the alternative y_i does not satisfy the criterion g_j of the fuzzy concept ''excellence.'' In addition, $[a_{ij}, b_{ij}]^{(k)} \subset [0,1]$, $[c_{ij}, d_{ij}]^{(k)} \subset [0,1]$, $0 \le b_{ij}^{(k)} + d_{ij}^{(k)} \le 1$, $i = (1,2, \ldots, n)$, $j = (1,2, \ldots, p)$, $k = (1,2, \ldots, m)$.

Therefore, our proposed multi-criteria group decision making approach based on interval-valued intuitionistic fuzzy environment is as follows:

In the first stage, the preference relation matrices for criterion weights are required. Experts have used the IVIFNs to express their performances. In the condition where the criterion weights are unknown, experts delivered the preference relations on criteria by pairwise comparison.

Step I-1: Use (25) to set up the interval-valued intuitionistic fuzzy preference relation on criteria as Equation (27).

$$\tilde{G}_k = \left(\tilde{g}_{ij}^{(k)}\right)_{p\times p} \tag{27}$$

Step I-2: Apply the operation in (9) to aggregate each row of preference relations as Equation (28).

$$\tilde{g}_i^{(k)} = IIFA_\omega\left(\tilde{g}_{i1}^{(k)}, \tilde{g}_{i2}^{(k)}, \ldots, \tilde{g}_{ip}^{(k)}\right), \ i = 1,2, \ldots, n\ ; \ k = 1,2, \ldots, m. \tag{28}$$

Step I-3: Use (8) to calculate weighted score function for each aggregate's IVIFNs as Equation (29).

$$R_i^{(k)} = W_\omega\left(\tilde{g}_i^{(k)}\right) \tag{29}$$

Step I-4: Reordering the weighted score function results in descending order based on previous step, such that

$\tilde{g}_{i_{\sigma(j-1)}} \geq \tilde{g}_{i_{\sigma(j)}}$ for all, and let $\tilde{g}_{i_{\sigma(j)}} = ([a_{\sigma(j)}, b_{\sigma(j)}], [c_{\sigma(j)}, d_{\sigma(j)}])$.

Step I-5: Apply the operation in (10) to integrate experts' opinions on criteria, and express the criterion weights in the interval-valued intuitionistic fuzzy format as follows:

$$\tilde{g}_i = IIFOWA_w(\tilde{g}_i^{(1)}, \tilde{g}_i^{(2)}, \dots, \tilde{g}_i^{(m)})\,.$$

Step I-6: Convert the criterion weights from interval-valued intuitionistic fuzzy formats $\tilde{g}_i$, $i = 1,2,\dots,p$ into interval-valued fuzzy formats $\tilde{\tilde{g}}_i$; $i = 1,2,\dots,p$.

Step I-7: Determine the scope of attribute weights, and achieve the criterion weights g_i within the lower and upper boundaries.

In the second stage, the decision matrixes of criterion values are another required input for our proposed approach. Again experts have used the IVIFNs to express their opinions.

Step II-1: Use (26) to set up the decision matrixes of criterion values as Equation (30).

$$\tilde{D}_k = \left(\tilde{d}_{ij}^{(k)}\right)_{n\times p} \tag{30}$$

Step II-2: Use (8) to calculate weighted score function for each element in the decision matrixes as Equation (31).

$$R_{ij}^{(k)} = W_\omega\left(\tilde{d}_{ij}^{(k)}\right) \tag{31}$$

Step II-3: Reordering the weighted score function results in descending order based on previous step, such that $\tilde{d}_{ij_{\sigma(j-1)}} \geq \tilde{d}_{ij_{\sigma(j)}}$ for all, and let $\tilde{d}_{ij_{\sigma(j)}} = ([a_{\sigma(j)}, b_{\sigma(j)}], [c_{\sigma(j)}, d_{\sigma(j)}])$.

Step II-4: Apply the operation in (10) to integrate experts' opinions on criterion values, and establish the aggregated decision matrix of criterion values as Equation (32).

$$\dot{\tilde{D}} = \left(\dot{\tilde{d}}_{ij}\right)_{n\times p} \tag{32}$$

Where $\dot{\tilde{d}}_{ij} = IIFOWA_w\left(\tilde{d}_{ij}^{(1)}, \tilde{d}_{ij}^{(2)}, \dots, \tilde{d}_{ij}^{(m)}\right); i = 1,2,\dots,n\,; j = 1,2,\dots,p\,.$

Step II-5: Calculate the weighted score function for each alternative in the condition of unknown criterion weights.

Step II-6: Use a linear programming model to calculate the exact criterion weights. The optimization is defined by a sum of weighted score function grades of each alternative as given in (33), and subject to the weight assumption $H(x)$.

$$Max \sum_{i=1}^{n} \sum_{j=1}^{p} W_j S(\dot{\tilde{d}}_{ij}) \tag{33}$$

St. $H(x)$

Step II-7: Using the combined method based on GRA and TOPSIS to rank the alternatives.

When the relative grey relational degree of each alternative is determined, alternatives are ranked according to descending order of ξ_i .

7. Numerical Example

A manufacturing and engineering company has decided to select the most appropriate supplier for one of the key elements in its manufacturing process. After pre-evaluation, four suppliers have remained as alternatives for further evaluation. In order to evaluate alternative suppliers, a committee composed of five decision makers has been established. The criteria considered in the selection process were producing ability (g_1), financial issues (g_2), delivery time (g_3) and services (g_4).

The procedure for selection of the most appropriate supplier contains the following steps:

Step 1: Determine the weights of the criteria.

Construct the interval-valued intuitionistic fuzzy preference relation matrices on the criteria based on pairwise comparison.

$$\tilde{G}_1 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.4,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.3,0.5],[0.2,0.4]) \\ ([0.1,0.2],[0.4,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.1,0.2]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.2,0.3],[0.5,0.6]) & ([0.1,0.2],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) \\ ([0.2,0.4],[0.3,0.5]) & ([0.1,0.3],[0.6,0.7]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_2 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.3,0.4]) & ([0.5,0.7],[0.2,0.3]) & ([0.5,0.7],[0.2,0.3]) \\ ([0.3,0.4],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.4,0.6],[0.1,0.3]) & ([0.4,0.5],[0.1,0.2]) \\ ([0.2,0.3],[0.5,0.7]) & ([0.1,0.3],[0.4,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.7],[0.1,0.2]) \\ ([0.2,0.3],[0.5,0.7]) & ([0.1,0.2],[0.4,0.5]) & ([0.1,0.2],[0.5,0.7]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_3 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.7,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) \\ ([0.1,0.2],[0.7,0.8]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.7],[0.2,0.3]) & ([0.4,0.6],[0.2,0.3]) \\ ([0.1,0.2],[0.6,0.7]) & ([0.2,0.3],[0.5,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) \\ ([0.2,0.3],[0.6,0.7]) & ([0.2,0.3],[0.4,0.6]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_4 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.3,0.4]) & ([0.3,0.4],[0.5,0.6]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.3,0.4],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.5,0.6]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.5,0.6],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) & ([0.5,0.6],[0.3,0.4]) \\ ([0.1,0.2],[0.7,0.8]) & ([0.3,0.4],[0.5,0.6]) & ([0.3,0.4],[0.5,0.6]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

$$\tilde{G}_5 = \begin{bmatrix} ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.4,0.6]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) \\ ([0.4,0.6],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.3]) \\ ([0.3,0.4],[0.5,0.6]) & ([0.2,0.3],[0.6,0.7]) & ([0.5,0.5],[0.5,0.5]) & ([0.3,0.4],[0.2,0.3]) \\ ([0.3,0.4],[0.4,0.5]) & ([0.1,0.3],[0.6,0.7]) & ([0.2,0.3],[0.3,0.4]) & ([0.5,0.5],[0.5,0.5]) \end{bmatrix}$$

Opinions of decision makers on criteria were aggregated. The scope of attribute weights was determined, using the steps of I-2 till I-7 .

$$0.2556 \le g_1 \le 0.8601, \qquad 0.1721 \le g_3 \le 0.7734,$$
$$0.2201 \le g_2 \le 0.8546, \qquad 0.1572 \le g_4 \le 0.7488.$$

Step 2: Construct the aggregated interval-valued intuitionistic fuzzy decision matrix based on the opinions of decision makers.

IVIFNs are used to rate each alternative supplier with respect to each criterion by five decision makers.

$$\tilde{D}_1 = \begin{bmatrix} ([0.3,0.5],[0.4,0.5]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.2,0.3]) & ([0.4,0.7],[0.0,0.1]) \\ ([0.6,0.8],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.8],[0.1,0.2]) & ([0.5,0.7],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.7,0.8],[0.0,0.1]) & ([0.5,0.7],[0.2,0.3]) & ([0.6,0.8],[0.1,0.2]) \\ ([0.2,0.3],[0.4,0.5]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.6],[0.3,0.4]) & ([0.4,0.5],[0.1,0.3]) \end{bmatrix}$$

$$\tilde{D}_2 = \begin{bmatrix} ([0.5,0.6],[0.3,0.4]) & ([0.4,0.6],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.6],[0.1,0.2]) \\ ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.5],[0.3,0.4]) & ([0.5,0.7],[0.1,0.2]) \\ ([0.6,0.8],[0.1,0.2]) & ([0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.7,0.9],[0.0,0.1]) \\ ([0.4,0.6],[0.3,0.4]) & ([0.4,0.5],[0.0,0.1]) & ([0.4,0.5],[0.2,0.4]) & ([0.4,0.6],[0.1,0.2]) \end{bmatrix}$$

$$\tilde{D}_3 = \begin{bmatrix} ([0.5,0.7],[0.2,0.3]) & ([0.5,0.6],[0.1,0.2]) & ([0.5,0.6],[0.2,0.4]) & ([0.4,0.6],[0.1,0.3]) \\ ([0.5,0.6],[0.1,0.2]) & ([0.5,0.7],[0.2,0.3]) & ([0.3,0.6],[0.2,0.4]) & ([0.6,0.8],[0.0,0.1]) \\ ([0.5,0.8],[0.1,0.2]) & ([0.5,0.8],[0.1,0.2]) & ([0.4,0.7],[0.2,0.3]) & ([0.5,0.8],[0.0,0.2]) \\ ([0.4,0.6],[0.1,0.3]) & ([0.4,0.6],[0.0,0.1]) & ([0.3,0.5],[0.2,0.4]) & ([0.4,0.6],[0.2,0.3]) \end{bmatrix}$$

$$\tilde{D}_4 = \begin{bmatrix} ([0.3,0.4],[0.4,0.6]) & ([0.6,0.7],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.0,0.1]) \\ ([0.5,0.8],[0.1,0.2]) & ([0.6,0.7],[0.2,0.3]) & ([0.4,0.6],[0.1,0.4]) & ([0.5,0.6],[0.1,0.3]) \\ ([0.7,0.8],[0.1,0.2]) & ([0.7,0.8],[0.0,0.1]) & ([0.5,0.7],[0.2,0.3]) & ([0.6,0.7],[0.1,0.2]) \\ ([0.2,0.3],[0.4,0.5]) & ([0.5,0.7],[0.1,0.3]) & ([0.5,0.6],[0.1,0.3]) & ([0.4,0.5],[0.1,0.3]) \end{bmatrix}$$

$$\tilde{D}_5 = \begin{bmatrix} ([0.3,0.4],[0.4,0.6]) & ([0.6,0.7],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.7],[0.0,0.1]) \\ ([0.4,0.6],[0.2,0.4]) & ([0.6,0.8],[0.1,0.2]) & ([0.5,0.6],[0.3,0.4]) & ([0.7,0.8],[0.1,0.2]) \\ ([0.5,0.7],[0.1,0.3]) & ([0.7,0.9],[0.0,0.1]) & ([0.5,0.6],[0.2,0.4]) & ([0.6,0.9],[0.0,0.1]) \\ ([0.2,0.3],[0.5,0.6]) & ([0.5,0.7],[0.1,0.3]) & ([0.4,0.6],[0.1,0.2]) & ([0.4,0.6],[0.1,0.3]) \end{bmatrix}$$

Opinions of decision makers on criterion values were aggregated. The aggregated decision matrix of criterion values was established, using the steps of II-2 tillII-4.

$$\dot{\tilde{D}} = \begin{bmatrix} ([0.3774,0.5226],[0.3458,0.4774]) & ([0.5359,0.6563],[0.1081,0.2093]) & ([0.5900,0.6928],[0.1698,0.2815]) & ([0.4792,0.6450],[0.0000,0.1332]) \\ ([0.5279,0.7198],[0.1081,0.2161]) & ([0.5677,0.7185],[0.1776,0.2815]) & ([0.4480,0.6097],[0.1956,0.3702]) & ([0.5859,0.7511],[0.0000,0.1774]) \\ ([0.6164,0.7907],[0.1000,0.2093]) & ([0.6600,0.8216],[0.0000,0.1273]) & ([0.4780,0.6619],[0.2262,0.3381]) & ([0.5917,0.8431],[0.0000,0.1500]) \\ ([0.2763,0.4239],[0.3374,0.4677]) & ([0.4483,0.6306],[0.0000,0.1657]) & ([0.4019,0.5677],[0.1777,0.3288]) & ([0.4000,0.5677],[0.1178,0.2867]) \end{bmatrix}$$

Step 3: Calculate the weighted score function for each alternative in the condition of unknown criterion weights by utilizing Eq. (8)as follows:

$$\begin{aligned} W_\omega\left(\dot{\tilde{d}}_1\right) &= g_1((1/2(0.3774-0.3458+0.5226-0.4774))) \\ &+ g_2((1/2(0.5359-0.1081+0.6563-0.2093))) \\ &+ g_3((1/2(0.5900-0.1698+0.6928-0.2815))) \\ &+ g_4((1/2(0.4792-0.0000+0.6450-0.1332))) \\ W_\omega\left(\dot{\tilde{d}}_2\right) &= g_1((1/2(0.5279-0.1081+0.7198-0.2161))) \\ &+ g_2((1/2(0.5677-0.1776+0.7185-0.2815))) \\ &+ g_3((1/2(0.4480-0.1956+0.6097-0.3702))) \\ &+ g_4((1/2(0.5859-0.0000+0.7511-0.1774))) \\ W_\omega\left(\dot{\tilde{d}}_3\right) &= g_1((1/2(0.6164-0.1000+0.7907-0.2093))) \\ &+ g_2((1/2(0.6600-0.0000+0.8216-0.1273))) \\ &+ g_3((1/2(0.4780-0.2262+0.6619-0.3381))) \\ &+ g_4((1/2(0.5917-0.0000+0.8431-0.1500))) \\ W_\omega\left(\dot{\tilde{d}}_4\right) &= g_1((1/2(0.2763-0.3374+0.4239-0.4677))) \\ &+ g_2((1/2(0.4483-0.0000+0.6306-0.1657))) \\ &+ g_3((1/2(0.4019-0.1777+0.5677-0.3288))) \\ &+ g_4((1/2(0.4000-0.1178+0.5677-0.2867))) \end{aligned}$$

Then use a linear programming model to calculate the exact criterion weights.

$$Max\ W_\omega\left(\dot{\tilde{d}}_1\right)+W_\omega\left(\dot{\tilde{d}}_2\right)+W_\omega\left(\dot{\tilde{d}}_3\right)+W_\omega\left(\dot{\tilde{d}}_4\right)$$

$$s.t.\ \begin{cases} 0.2556 \le g_1 \le 0.8601, \\ 0.2201 \le g_2 \le 0.8546, \\ 0.1721 \le g_3 \le 0.7734, \\ 0.1572 \le g_4 \le 0.7488, \\ g_1+g_2+g_3+g_4 = 1. \end{cases}$$

$$g_1 = 0.2556, \quad g_3 = 0.1721,$$
$$g_2 = 0.2201, \quad g_4 = 0.3522.$$

Step 4: Construct the aggregated weighted interval-valued intuitionistic fuzzy decision matrix. The aggregated weighted intuitionistic fuzzy decision matrix was constructed following the determination of the weights of the criteria and the rating of the alternatives by utilizing Eq. (6)as follows:

$$\dot{\tilde{D}}_w = \begin{bmatrix} ([0.1537,0.2293],[0.6880,0.7707]) & ([0.2369,0.3135],[0.4567,0.5764]) \\ ([0.2323,0.3611],[0.4567,0.5830]) & ([0.2557,0.3601],[0.5441,0.6399]) \\ ([0.2864,0.4236],[0.4444,0.5764]) & ([0.3161,0.4551],[0.0000,0.4839]) \\ ([0.1076,0.1765],[0.6821,0.7652]) & ([0.1890,0.2958],[0.0000,0.5310]) \end{bmatrix}$$

$$\begin{matrix} ([0.2695,0.3401],[0.5355,0.6399]) & ([0.2053,0.3057],[0.0000,0.4916]) \\ ([0.1888,0.2821],[0.5629,0.7047]) & ([0.2669,0.3873],[0.0000,0.5439]) \\ ([0.2046,0.3175],[0.5925,0.6825]) & ([0.2705,0.4791],[0.0000,0.5126]) \\ ([0.1656,0.2557],[0.5442,0.6759]) & ([0.1647,0.2557],[0.4708,0.6440]) \end{bmatrix}$$

With regard to the influence of third parameter in calculation of distance between the alternatives in interval-valued intuitionistic fuzzy environment, the degrees of uncertainty for each of the above alternatives have been added to the aggregated weighted interval-valued intuitionistic fuzzy decision matrix using Eq. (2) as follows:

$$\dot{\tilde{D}}_w = \begin{bmatrix} ([0.1537,0.2293],[0.6880,0.7707],[0.0000,0.1583]) & ([0.2369,0.3135],[0.4567,0.5764],[0.1101,0.3064]) \\ ([0.2323,0.3611],[0.4567,0.5830],[0.0559,0.3110]) & ([0.2557,0.3601],[0.5441,0.6399],[0.0000,0.2002]) \\ ([0.2864,0.4236],[0.4444,0.5764],[0.0000,0.2692]) & ([0.3161,0.4551],[0.0000,0.4839],[0.0610,0.6839]) \\ ([0.1076,0.1765],[0.6821,0.7652],[0.0583,0.2103]) & ([0.1890,0.2958],[0.0000,0.5310],[0.1732,0.8110]) \end{matrix}$$

$$\begin{matrix} ([0.2695,0.3401],[0.5355,0.6399],[0.0200,0.1950]) & ([0.2053,0.3057],[0.0000,0.4916],[0.2027,0.7947]) \\ ([0.1888,0.2821],[0.5629,0.7047],[0.0132,0.2483]) & ([0.2669,0.3873],[0.0000,0.5439],[0.0689,0.7331]) \\ ([0.2046,0.3175],[0.5925,0.6825],[0.0000,0.2029]) & ([0.2705,0.4791],[0.0000,0.5126],[0.0082,0.7295]) \\ ([0.1656,0.2557],[0.5442,0.6759],[0.0684,0.2902]) & ([0.1647,0.2557],[0.4708,0.6440],[0.1002,0.3645]) \end{bmatrix}$$

Step 5: Obtain the positive-ideal and the negative-ideal solution with interval-valued intuitionistic fuzzy information, using Eq. (13) and Eq. (21) respectively.

$$A^+ = [([0.2864,0.4236],[0.4444,0.5764],[0.0000,0.2692]) \quad ([0.3161,0.4551],[0.0000,0.4839],[0.0610,0.6839])$$

$$([0.2695,0.3401],[0.5355,0.6399],[0.0200,0.1950]) \quad ([0.2705,0.4791],[0.0000,0.4916],[0.0292,0.7295])]$$

$$A^- = [([0.1076,0.1765],[0.6880,0.7707],[0.0527,0.2044]) \quad ([0.1890,0.2958],[0.5441,0.6399],[0.0643,0.2669])$$

$$([0.1656,0.2557],[0.5925,0.7047],[0.0396,0.2420]) \quad ([0.1647,0.2557],[0.4708,0.6440],[0.1002,0.3645])]$$

Step 6: Calculate the separation measures, using the normalized Hamming distance.

The separation of each alternative from the positive-ideal solution, A^+, is given as:

$$d_{ij}^{\ +} = \begin{bmatrix} 0.0547 & 0.0748 & 0.0000 & 0.0298 \\ 0.0146 & 0.0875 & 0.0182 & 0.0119 \\ 0.0000 & 0.0000 & 0.0134 & 0.0026 \\ 0.0606 & 0.0358 & 0.0235 & 0.0868 \end{bmatrix}$$

Similarly, the separation of each alternative from the negative-ideal solution, A^-, is given as:

$$d_{ij}^{\ -} = \begin{bmatrix} 0.0124 & 0.0189 & 0.0235 & 0.0779 \\ 0.0524 & 0.0164 & 0.0070 & 0.0753 \\ 0.0613 & 0.0879 & 0.0126 & 0.0868 \\ 0.0014 & 0.0816 & 0.0096 & 0.0000 \end{bmatrix}$$

Step 7: Calculate the gray relational coefficients of each alternative from PIS and NIS.

$$\xi_{ij}^+ = \begin{bmatrix} 0.4443 & 0.3691 & 1.0000 & 0.5945 \\ 0.7502 & 0.3333 & 0.7064 & 0.7857 \\ 1.0000 & 1.0000 & 0.7650 & 0.9435 \\ 0.4194 & 0.5500 & 0.6502 & 0.3352 \end{bmatrix}$$

$$\xi_{ij}^- = \begin{bmatrix} 0.7806 & 0.6999 & 0.6512 & 0.3607 \\ 0.4563 & 0.7285 & 0.8628 & 0.3686 \\ 0.4175 & 0.3333 & 0.7773 & 0.3362 \\ 0.9685 & 0.3501 & 0.8202 & 1.0000 \end{bmatrix}$$

Step 8: Calculate the degree of gray relational coefficients of each alternative from PIS and NIS.

$\xi_1^+ = 0.5763$, $\xi_2^+ = 0.6634$, $\xi_3^+ = 0.9396$, $\xi_4^+ = 0.4582$.
$\xi_1^- = 0.5927$, $\xi_2^- = 0.5553$, $\xi_3^- = 0.4323$, $\xi_4^- = 0.8180$.

Step 9: Calculate the relative grey relational degree of each alternative from the positive ideal solution.

$\xi_1 = 0.4930, \xi_2 = 0.5444, \xi_3 = 0.6849, \xi_4 = 0.3590$.

Step 10: Rank all the alternatives $Y_i (i = 1,2, \dots, n)$ and select the best one(s) in accordance with $\xi_i (i = 1,2, \dots, n)$. The alternative with the highest ξ_i value is the most important alternative.

$$\xi(\dot{\tilde{d}}_3) > \xi(\dot{\tilde{d}}_2) > \xi(\dot{\tilde{d}}_1) > \xi(\dot{\tilde{d}}_4)$$

$$y_3 \succ y_2 \succ y_1 \succ y_4$$

Thus, according to calculations made in previous stages, the relative grey relational degree of each alternative is obtained. The third alternative has the greatest degree of

acceptance while the fourth alternative has the least degree of acceptance.
The following figures (fig.1 and fig.2) show that although there is an overall ranking of the alternatives, the decision maker(s) can observe ranking of the alternatives in every single criteria and ca select each of them according to different situations.

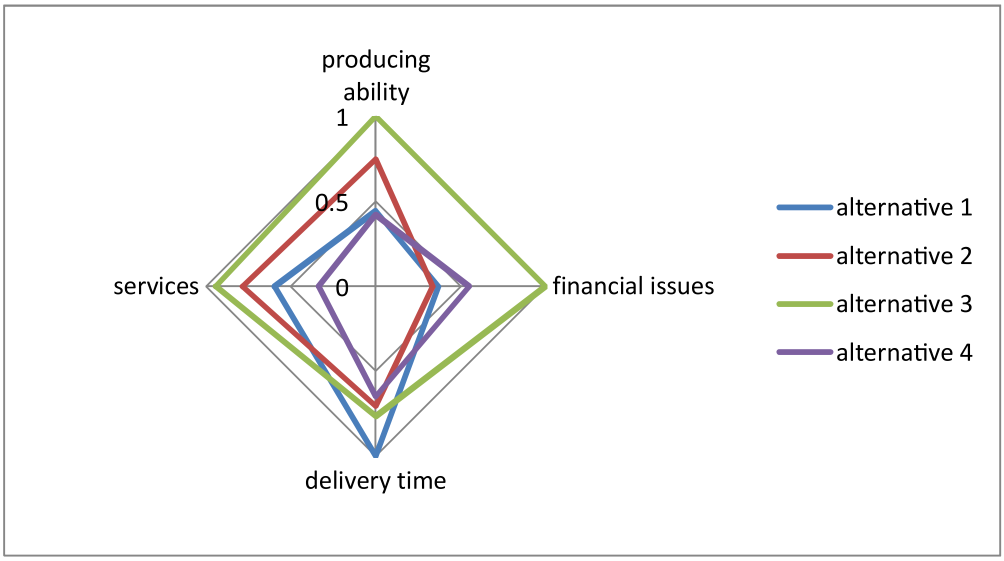

Fig.1.The shape similarity between positive ideal solution and the alternatives.

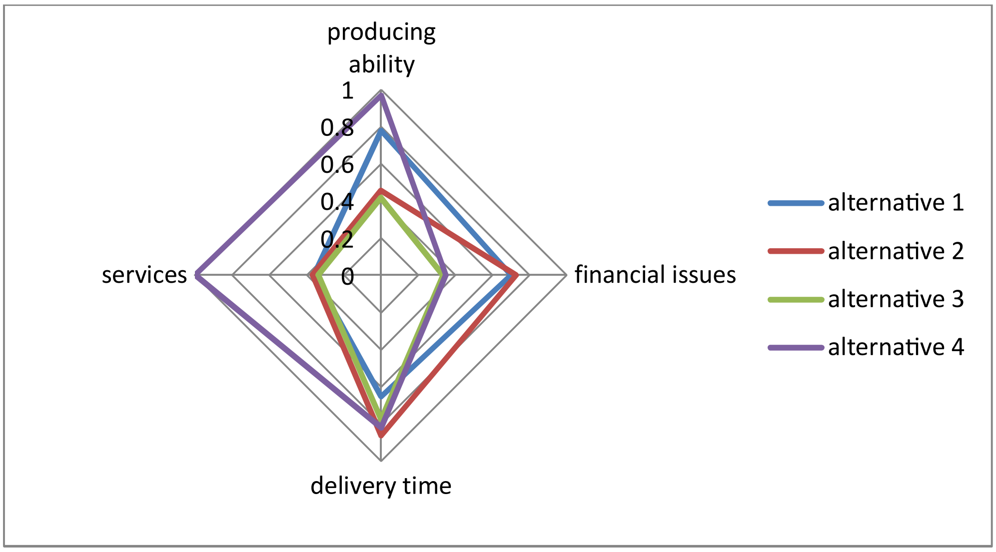

Fig.2.The shape similarity between negative ideal solution and the alternatives.

8. Conclusions

This study presents an interval-valued intuitionistic fuzzy multi-criteria group decision-making (MCGDM) with combined method based on GRA and TOPSIS for supplier selection problem. IFSs are a suitable way to deal with uncertainty. In the evaluation process, the ratings of each alternative with respect to each criterion and the weights of each criterion were given by IVIFNs. Also IIFOWA was utilized to aggregate opinions of decision makers. After interval-valued intuitionistic fuzzy positive-ideal solution and interval-valued intuitionistic fuzzy negative-ideal solution were calculated based on the normalized Hamming distance, the relative grey relational degree of alternatives were obtained and alternatives were ranked. A combined method based on GRA and TOPSIS associated with intuitionistic fuzzy set has a high chance of success for multi-criteria decision-making problems as it contains vague perception of decision makers' opinions. Therefore, in future, intuitionistic fuzzy set can be used for dealing with uncertainty in multi-criteria decision-making problems such as project selection, manufacturing systems, pattern recognition, medical diagnosis and many other areas of management decision problems.

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