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Optimal Manufacturer-Retailer Policies in a Supply chain with Defective Product and Price Dependent Demand

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Abstract

This study deals with a two-level supply chain consisting of one manufacturer and one retailer. We consider an integrated production inventory system where the manufacturer processes raw materials in order to deliver the finished product with imperfect quality to the retailer, where the number of defective product has a uniform distribution. The retailer receives product and conducts a 100% inspection. We assume that unit price charged by the retailer influences the demand of the product. Shortages are allowed and assumed to be completely backordered. The proposed model is based on the joint total profit of both the manufacturer and the retailer, and it finds the optimal ordering, shipment and pricing policies. The numerical study shows that the coordination between supply chain members is more beneficial in industries by less defective percentage at manufacturer.

Keywords: Supply chain, Pricing policy, Defective quality, Joint economic lot sizing.

1. Introduction

Today in dynamic market conditions, the supply chain coordination is becoming a key factor. In traditional inventory management, the inventory and shipment policies of supply chain members are managed separately. Therefore, the optimal lot size for one member may not result in an optimal policy for the others. To address this issue, the integrated manufacturer-retailer model has been proposed, where the joint total profit for both the retailer and the manufacturer is maximized.

The integration between supply chain members has long been debated. Goyal (1976), initiated the concept of a joint optimization problem of vendor and buyer, on the assumption that the vendor has an infinite production rate. Banerjee (1986), extended the joint economic lot size (JELS) model in which the manufacturer was obliged to order under the lot-for-lot policy. Goyal (1988), relaxed the assumption of lot-for-lot, and assumed that the production lot is shipped in a number of equal-size shipments. Goyal (1995), proposed a model where by the shipment size is raised by a factor equal to the ratio of the production rate to the demand rate. Over recent years researchers have investigated multi- vendor and buyer.

Recently, Glock and kim (2015) investigated a single-vendor–multi-retailer supply chain and considered the case where the vendor merged with one of its retailers. Sajadieh and Thorstenson (2015) studied a supply chain with a single buyer and either one or two supplier(s)/vendor(s). In addition, some researchers investigated multiple suppliers. Sajadieh et al. (2013) studied an integrated production–inventory model for a three-stage supply chain involving multiple suppliers, multiple manufacturers and multiple retailers.

In contrast, perfect-quality products are avoided in some studies; hence, the process may deteriorate and produce defective products. Porteus (1986), Lee and Rosenblatt (1986), probed the effect of defective products on the basic Economic Order Quantity (EOQ). Numerous researchers have expanded miscellaneous imperfect-quality inventory models for this critical problem involving an imperfect production process (e.g., Schwaller 1988; Zhang and Gerchak 1990; Cheng 1991; Ben-Daya and Hariga 2000; Salameh and Jaber 2000; Cardenas-Barron 2009; J.T Hsu and L.F. Hsu (2013)).

Yang and Wee (2000) investigated a joint inventory model for vendor-buyer under an imperfect production process. Another study in this area was published by Goyal et al. (2003), who introduced a simple approach to find an optimal integrated vendor–buyer inventory policy for a defective product. Hardik and Kamlesh (2014) investigated a single-vendor-single-buyer production inventory model involving defective items in both an individual and joint management system with service

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level constraint. J.T Hsu and L.F. Hsu (2012) probed an integrated inventory model for vendor-buyer under the conditions of defective products, where the vendor inspects the products. Recently, J.T Hsu and L.F. Hsu (2012) designed a model to frame an integrated vendor-buyer inventory policy for defective products, where the buyer inspects the products.

Another issue in the lot sizing area that has attracted the attention of numerous researchers is the integration of production and pricing. One of the first models of this kind was formulated by Kunreuther and Richard (1971), who incorporated pricing into the traditional EOQ model considering a linear price. Reyniers (2001) later developed a model of single-manufacturer, single-retailer distribution channel, in which the retailer faces a price-sensitive deterministic demand, on the assumption that the manufacturer has a finite production rate. A multitude of researchers have developed a joint inventory model for manufacturer-retailer, where market demand is a function of price (e.g., Viswanathan and Wang 2003; Ray et al. 2005; Bakal et al. 2008, Wang et al 2015).

Sajadieh and Jokar (2009) proposed the vendor–buyer supply chain lot sizing models where market demand is a linear function of price, assuming that the production lot is shipped in a number of equal-size shipments. Kim et al. (2011) probed an integrated inventory model for manufacturer-retailer under the conditions of price-dependent demand. The retailer places orders based on the EOQ model, and the manufacturer produces the ordered quantity on a lot-for-lot basis. Readers are referred to Sajadieh and Jokar (2008) and Glock (2012), and Ben_Daya et al. (2008), for reviews of the JELS models.

In this research, the four aforementioned literature branches are integrated in a model where the shipment, ordering, pricing policies, imperfect quality and backordering are optimized all together. In real world cases, the manufacturers usually accept the return products and sell them in the second market. That is our motivation to do this research. Incorporating these two features into the model increases the complexity. The study investigates an approach to adopt an optimal joint manufacturer-retailer inventory policy for a product with imperfect quality. In this model, market demand is a linear function of price. Shortages are allowed and assumed to be completely backordered. The authors have analysed how the coordination between two supply chain members is affected by the number of defective products with a uniform distribution while the end-customer demand is price sensitive in the first market. The models for the two-level supply chain were extracted from the non-joint and the joint policies.

The organization of this paper is as follows. In Section 2, the problem is introduced, and the notation and assumptions are defined.

Section 3 introduces the independent policies for the retailer and the manufacturer as well as the joint model. Moreover, an algorithm is proposed to obtain the optimal policies for the joint and independent model. Section 4 presents some numerical examples as well as sensitivity analyses of different parameters to introduce the significant aspects of the model. Finally, in Section 5 the concluding remarks are given and future research directions are provided.

2. Model Description

Consider a supply chain for an imperfect product comprising of a single manufacturer and retailer when the product is single. The retailer has an annual demand rate of $D\ (\delta_{fm})$ units for the given product and the first market demand is a linear function of price. The retailer orders a lot of size Q and the manufacturer produces the product at the production rate P in order to deliver the finished products with imperfect quality to the retailer, where the number of defective product has a uniform distribution. The production lot is shipped in a number of equal-size shipments. Retailer conducts a complete the inspection upon receiving the product and all defective products are returned back to the manufacturer upon receiving the next lot and the manufacturer sells the defective products in the second market. In other words manufacture' stock has a JIT system (Fig.1). Shortages are permitted for the retailer and assumed to be completely backordered. The suggested model is based on the joint total profit of both the manufacturer and the retailer, and finds the optimal ordering Q, number of shipments n and selling price δ_{fm}. The notations used in this paper are given next:

Decision Variables

δ_{fm}	Selling price in first market (\$/unit),
Q	Retailer's order quantity,
B	The maximum backordering quantity in units by the retailer,
n	Number of shipments.

Input Parameters

$D(\delta_{fm})$	The demand rate in first market (units/period),
D_{sm}	The demand rate in second market (units/period),
A_m	The manufacturer's setup cost,
h_m	The holding cost per dollar per year for the manufacturer,
A_r	Ordering cost for the retailer,
h_r	The holding cost per dollar per year for the retailer,
w	The price charged by the seller to the retailer (\$/unit),
δ_{sm}	Selling price in second market (\$/unit),

P	Production rate on the manufacturer,
γ	The defective percentage in Q,
$f(\gamma)$	The uniform probability density function of γ,
y	Upper bound of defective percentage in Q (γ is uniformly distributed between 0 and y),
π	The unit shortage cost per unit of time,
v	The manufacturer's unit warranty cost per defective product for the retailer,
d	The screening cost per unit,
$I(t)$	Net inventory level at time t,
$S(t)$	The shortage level at time t,

3. Mathematical Model

In this study, the profit models for the two-level supply chain were extracted from both the individual and the joint policies. In the individual model, each member in the supply chain concentrates on maximizing their own profit, whereas in the joint policy, both parties decide to cooperate and agree to obey the joint optimal policy.

3.1 Retailer's profit

The researchers assumed that the end-consumer demand in the first market, $D(\delta_{fm})$, for the product hinges on the consumer price, δ_{fm}, set by the retailer. In its simplest form, the end-consumer demand in the first market, $D(\delta_{fm})$, linearly decreases with price, i.e., $D(\delta_{fm})=a-b\delta_{fm}$, a and b are constant (a,b>0). As $D(\delta_{fm})>0$, the maximum selling price is a/b, i.e., $\delta_{fm}< a/b$. The retailer orders a lot of size Q to the manufacturer. Then the manufacturer produces n Q with a finite production rate at one set-up but ships in quantity Q to the retailer over n times and the defective products exist in lot size Q.
The anticipated profit of the retailer is calculated by the revenue from selling the finished product, minus the sum of purchasing, order processing, screening and inventory holding costs. In other words, Q, B, δ_{fm}areoptimized by maximizing the annual retailer's profit, respectively.

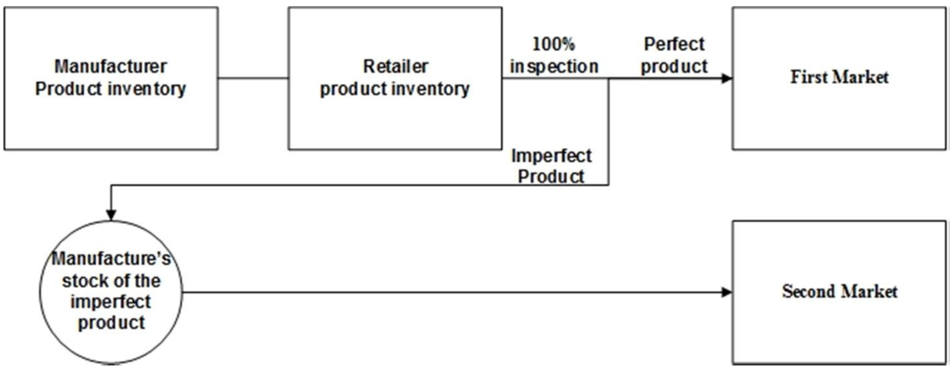

Fig. 1. The whole operation model

Note that for the perfect products, the maximum and the minimum inventory levels are equal to $I_r(0) = Q(1-\gamma)-B$ and 0 respectively. Hence, the mean inventory level for perfect and imperfect product are equal to $MI_r(t) = (Q(1-\gamma)-B)/2$ and $Q\gamma$ respectively.
As $T_1 = (Q(1-\gamma)-B)/D(\delta_{fm})$ and $T = Q(1-\gamma)/D(\delta_{fm})$, the present worth of the retailer's holding cost in the period (0,T_1] is equal to $h_r((Q(1-\gamma)-B)^2 + 2Q^2\gamma(1-\gamma))/2(a-b\delta_{fm})$.
The inventory level smoothly decreases to meet the demand. By this process, the inventory level reaches zero at time T_1 and then shortages are allowed to happen. Similarly, it can also be proven that the present worth of the shortage cost during the period [T_1, T] is $\pi B^2/2(a-b\delta_{fm})$.

The expected retailer's annual profit function is obtained by the use of the following formula:

$$\text{Maximize } E\Pi_r(Q,\delta_{fm},B) = (a-b\delta_{fm})(\delta_{fm}-w) - \frac{1}{2Q(1-E[\gamma])}\big(2(a-b\delta_{fm})(dQ+A_r) + (h_r+\pi)B^2 + h_rQ(Q(E[(1-\gamma)^2] + 2(E[\gamma]-E[\gamma^2]) - 2B(1-E[\gamma])))\big), \quad (1)$$

S.t.: $\delta_{fm}<\frac{a}{b}$; $Q>0$

Manufacturer's profit

After computing the retailer's order quantity, unit backordered and the selling price, the orders are received by the manufacturer at a known interval. The mean inventory level for the manufacturer is equal to:

$$MI_m = \frac{nQ^{*2}}{P} - \frac{n^2Q^{*2}}{2P} + \frac{n(n-1)Q^{*2}(1-\gamma)}{2(a-b\delta_{fm})}. \quad (2)$$

Furthermore, the present worth of the manufacturer's holding cost is calculated by:

$$h_m I_m^{-} = h_m \left(\frac{nQ^{*2}}{P} - \frac{n^2Q^{*2}}{2P} + \frac{n(n-1)Q^{*2}(1-\gamma)}{2(a-b\delta_{fm})}\right). \quad (3)$$

Therefore, by adding the setup and warranty costs for imperfect product, the expected manufacturer's annual profit function is computed by the use of the following formula:

$$\text{Maximize} E\Pi_m(n) = (a - b\delta_{fm})w + D_{sm}\delta_{sm} - \frac{(a-b\delta_{fm})(nQ^*vE[\gamma]+A_m)}{nQ^*(1-E[\gamma])} - h_m(\frac{Q^*(a-b\delta_{fm})(2-n)}{2p(1-E[\gamma])} + \frac{(n-1)Q^*}{2}) \quad (4)$$

S.t.: *n* is integer.

The joint manufacturer-retailer inventory model

In this section, that the expected total profit of the system, $E\Pi_{js}(Q, \delta_{fm}, B, n)$, is introduced. The optimal policy of the joint system is attained by the use of the following formula:

$$\text{Maximize } E\Pi_{js}(Q, \delta_{fm}, B, n) = (a - b\delta_{sm})\delta_{fm} + D_{sm}\delta_{sm} - \frac{1}{2Q(1-E[\gamma])}(\frac{2(a-b\delta_{fm})(nQd+nQvE[\gamma]+nA_r+A_m)}{n} + (h_r+\pi)B^2 + h_r Q\left(E[(1-\gamma)^2]Q - 2B(1-E[\gamma]) + 2Q(E[\gamma]-E[\gamma^2])\right) - h_m(\frac{Q^2((a-b\delta_{fm})(2-n)+(n-1)p(1-E[\gamma]))}{p})). \quad (5)$$

If the defective percentage has a uniform distribution, then we have:

$$\text{Maximize } \Pi_{js}(Q, \delta_{fm}, B, n) = (a - b\delta_{sm})\delta_{fm} + D_{sm}\delta_{sm} - \frac{1}{Q(2-y)}(\frac{3(a-b\delta_{fm})(2nQd+nQvy+2nA_r+2A_m)}{n} + (h_r+\pi)B^2 + \frac{h_rQ\left((3-3y+y^2)Q-3B(2-y)+Q(3y-2y^2)\right)}{3} - h_m(\frac{Q^2(2(a-b\delta_{fm})(2-n)+(n-1)p(2-y))}{2p})). \quad (6)$$

The retailer's independent optimal solution

Taking the second partial derivative of $E\Pi_r(Q, \delta_{fm}, B)$ with respect to *B*, we achieve,

$\partial^2\Pi_r(Q, \delta_{fm}, B)/\partial B^2 = -2(h_r + \pi)/Q(2-y) < 0.$

Thus, $E\Pi_r$ (Q,δ_{fm},B) is concave in *B* for given values of the retailer's selling price δ and the order quantity *Q*. The optimal backordering size can subsequently be obtained as,

$B^* = h_rQ(1 - \text{E}[\gamma])/(h_r + \pi)$. By replacing the optimal backordering size into Eq. (1) and simplifying, we have:

$$\text{Maximize } E\Pi_r(Q, \delta_{fm}, B) = (a - b\delta_{fm})(\delta_{fm} - w) - \frac{1}{2Q(1-\text{E}[\gamma])(h_r+\pi)}(2(h_r+\pi)(a-b\delta_{fm})(dQ + A_r) + h_rQ^2(\text{E}[(1-\gamma)^2] + 2(E[\gamma]-E[\gamma^2]) - h_r(1-E[\gamma])^2) \quad (7)$$

The retailer's profit is concave in *Q* for given values of the retailer's selling price δ_{fm}. The optimal order quantity can be calculated by solving the first order differential equation of the profit model for the retailer, which yields, $Q^* = \sqrt{(a - b\delta_{fm})A_r/K}$, where, $K = h_r((h_r + \pi)(E[(1-\gamma)^2] + 2(E[\gamma] - E[\gamma^2]) - h_r(1 - E[\gamma])^2)/2(h_r + \pi)$.

By replacing the optimal order size into Eq. (2) and simplifying it, we have:

$$E\Pi_r(\delta_{fm}) = (a - b\delta_{fm})(\delta_{fm} - w) - \frac{d(a-b\delta_{fm})}{1-E[\gamma]} - \sqrt{\frac{4(a-b\delta_{fm})A_rh_r((h_{br}+\pi)(E[(1-\gamma)^2]+2(E[\gamma]-E[\gamma^2]))-h_r(1-E[\gamma])^2)}{2(1-E[\gamma])^2(h_r+\pi)}} \quad (8)$$

To maximize $E\Pi_r(\delta_{fm})$, can be approximated by $d_0{\delta_{fm}}^2 + d_1\delta_{fm} + d_2$ as follow: (see, Scheid 1988);

$E\Pi_r(\delta) = (a\delta_{fm} - b{\delta_{fm}}^2) + db\delta_{fm}/(1 - \text{E}[\gamma]) - ((4aA_rh_r((h_r + \pi)(\text{E}[(1-\gamma)^2] + 2(\text{E}[\gamma] - \text{E}[\gamma^2])) - h_r(1 - \text{E}[\gamma])^2)/2(1 - \text{E}[\gamma])^2(h_r + \pi))\text{^}(\frac{1}{2}))(d_0{\delta_{fm}}^2 + d_1\delta_{fm} + d_2)$,

where d_0= (-8+4√2) $(b/a)^2$,d_1=(12-7√2) (b/a) and d_2=3√2-4.

The second derivative of $E\Pi_r(\delta_{fm})$ with respect to δ_{fm} can be calculated as: $-2b - 4Jd_0 \leq 0$.

The expression above is concave in δ if $a^3 > (2^{10}b^2A_r/(1 - \text{E}[\gamma]))/(h_r(\text{E}[(1-\gamma)^2]/2(1 - \text{E}[\gamma]) - h_r(1 - \text{E}[\gamma])/2(h_r + \pi) + (\text{E}[\gamma] - \text{E}[\gamma^2])/(1 - \text{E}[\gamma])))$, differentiating $E\Pi_r(\delta_{fm})$ with respect to δ_{fm}, equating it to zero and solving it for δ_{fm}, yields

$${\delta_{fm}}^* = \frac{(a+bw-2Jd_1)(1-E[\gamma])+db}{(2b+4Jd_0)(1-E[\gamma])}, \quad (9)$$

Where, $J = \sqrt{\frac{aA_bh_r}{(1-E[\gamma])}\left(\frac{\text{E}[(1-\gamma)^2]+2(\text{E}[\gamma]-\text{E}[\gamma^2])}{2(1-\text{E}[\gamma])} - \frac{h_r(1-E[\gamma])}{2(h_r+\pi)}\right)}$.

Afterwards, the optimal order quantity and the optimal backordering quantity can be obtained as follows, respectively:

$$Q^* = \sqrt{A_r\left(a - b\left(\frac{(a+bw-2Jd_1)(1-E[\gamma])+db}{(2b+4Jd_0)(1-E[\gamma])}\right)\right)/k}, \quad (10)$$

$$B^* = \frac{h_r(1-E[\gamma])}{h_r+\pi}\sqrt{A_r\left(a - b\left(\frac{(a+bw-2Jd_1)(1-E[\gamma])+db}{(2b+4Jd_0)(1-E[\gamma])}\right)\right)/k}. \quad (11)$$

As mentioned, earlier, the defective percentage,γ, has a uniform distribution:

$$f(\gamma) = \begin{cases} \frac{1}{y} & 0 \leq x < y \\ 0 & Otherwise \end{cases}.$$

The retailer's annual profit function is then given as follow:

$$\text{Maximize } \Pi_r(Q, \delta_{fm}, B) = (a - b\delta_{fm})(\delta_{fm} - w) - \frac{1}{3Q(2-y)}(3(2(a - b\delta_{fm})(dQ + A_r) + B^2(h_r + \pi)) + h_r Q(Q(3 - y^2) + 3BQ(2 - y))). \quad (12)$$

Therefore, the selling price, optimal order quantity and the optimal backordering quantity can be computed as

$$\delta_{fm}{}^* = \frac{(a+bw-2J'd_1)(2-y)+2db}{(2-y)(2b+4J'd_0)},$$

$$Q^* = \sqrt{\frac{2A_r(a - b(((a + bc - 2J'd_1)(2 - y) + 2db)/(2 - y)(2b + 4J'd_0)))}{K'}},$$

and $B^* = (2 - y)\, h_r \sqrt{\frac{2A_r\left(a - b\left(\frac{(a+bc-2J'd_1)(2-y)+2db}{(2-y)(2b+4J'd_0)}\right)\right)}{K'}} / (h_r + \pi)$, respectively,

Where, $J' = \sqrt{2aA_r h_r(3 - y^2)/3(2 - y^2)}, K' = h_r(h_r(12 - 4y^2 - 3(2 - y)^2) + \pi(12 - 4y^2))/h_r + \pi$.

The manufacture's independent optimal solution

The manufacturer's expected profit is concave in n and the value of n can simply be computed as,

$$n = \sqrt{\frac{2A_m p(a - b\delta_{fm})/h_m(1 - E[\gamma])Q^2(p(1 - E[\gamma])}{-a + b\delta_{fm})}}.$$

As the value of n is a positive integer, the optimal value of n, indicated by n*, 'is given by: n*= [n], when $E\Pi_m{}^*([n]) \leq E\Pi_m{}^*([n] + 1)$;

Otherwise, n*= [n] +1, where [n] is the greatest integer smaller than n.

The defective percentage has a uniform distribution; therefore, the manufacturer's annual profit function is defined by the use of the following formula:

$$\text{Maximize } \Pi_m(n) = (a - b\delta_{fm})w + D_{sm}\delta_{sm} - \frac{1}{nQ(2-y)p}(2p(a - b\delta)(vynQ^* + A_m) + h_m(2Q^*(a - b\delta_{fm})(2 - n) + pQ^*(2 - y)(n - 1)) \quad (13)$$

The value of n is$(8(A_m p(a - b\delta_{fm})/h_m(2 - y)Q^{*2}(p(2 - y) - a + b\delta_{fm}))^{1/2}$, therefore total system profit per unit time for the individual supply chain is shown by:

$$\Pi_i(Q, \delta_{fm}, B, n) = \Pi_r(Q, \delta_{fm}, B) + \Pi_m(n).$$

Solution procedure for the joint model

The total system profit $\Pi_{js}(Q, \delta_{fm}, B, n)$ is concave in B, Q for the given values of the retailer's selling price δ_{fm} and the number of shipments n.

The optimal order quantity and backorder quantity when the defective percentage,γ, has a uniform distribution, can be computed as follows:

$$Q^* = \sqrt{\frac{2(a-b\delta_{fm})(nA_r+A_m)}{n(2-y)X}}, \quad (14)$$

$$B^* = h_r\sqrt{\frac{2(a-b\delta_{fm})(nA_r+A_m)}{n(2-y)X}}/h_r + \pi, \quad (15)$$

Where, $X = \frac{1}{(2-y)}\left(h_r\left(\frac{(3-y^2)}{3} - \frac{h_r(2-y)^2}{4(h_r+\pi)}\right) + h_m\left(\frac{2(a-b\delta_{fm})(n-2)+(n-1)p(2-y)}{2p}\right)\right)$.

Replacing expression Q^* and B^*into Eq. (6) and simplifying it, we can have $\Pi_{js}(\delta_{fm}, n)$;

$$\Pi_{js}(\delta_{fm}, n) = (a - b\delta_{fm})(\delta - \frac{2d+vy}{(2-y)}) + D_{sm}\delta_{sm} - \sqrt{8\frac{(a-b\delta_{fm})}{(2-y)}\left(A_r + \frac{A_m}{n}\right)X}, \quad (16)$$

For a given value of $D(\delta_{fm})$, maximizing Π_{js} is the same as minimizing the following expression:

$$\Pi_{js}{}' = 8\frac{D(\delta_{fm})}{(2 - y)}\left(A_r + \frac{A_m}{n}\right)X. \quad (17)$$

$\Pi_{js}{}'(\delta_{fm}, n)$ is convex in δ_{fm} for known values of n. Modifying the algorithm introduced by sajadieh and Jokar (2009), the optimal values of four decision variables for the joint model are obtained as follow:

Step 1.Initialize by setting $\Pi_{js}{}^* = 0$, set n =1.

Step 2.DetermineLO, using a one dimensional search algorithm such as Newton. If LO<a, then set $D(\delta_{fm})$=LO; otherwise, $D(\delta_{fm})$ = a.

Step 3.Set δ_{fm} = (a-D (δ_{fm}))/b.

Step 4.Compute the value of backorder quantity

$$B^* = h_r\sqrt{\frac{2(a-b\delta_{fm})(nA_r+A_m)}{n(2-y)X}}/h_r + \pi$$

Step 5.Compute the value of order quantity

$$Q^* = \sqrt{\frac{2(a-b\delta_{fm})(nA_r+A_m)}{n(2-y)X}}$$

Step 6. Calculate Π_{js}

$$\text{Maximize } \Pi_{js}(Q, \delta_{fm}, B, n) = (a - b\delta_{fm})\delta_{fm} + D_{sm}\delta_{sm} - \frac{1}{Q(2 - y)}\left(\frac{3(a - b\delta_{fm})(2nQd + nQvy + 2nA_r + 2A_m)}{n} + (h_r + \pi)B^2 + \right.$$

$$\frac{h_r Q\big((3-3y+y^2)Q-3B(2-y)+Q(3y-2y^2)\big)}{3} - h_m\left(\frac{Q^2(2(a-b\delta_{fm})(2-n)+(n-1)p(2-y))}{2p}\right)).$$

Step 7.If $\Pi_{js}>\Pi_{js}^*$, therefore set $\Pi_{js}^* = \Pi_{js}(\delta_{fm}, Q, B, n)$; $n^* = n$, $\delta^* = \delta_{fm}$, $Q^* = Q$ and $B^* = B$.

Step 8.Set $n=n+1$. If $n \le n_{up}$, then go to Step 2, otherwise stop. The value of n_{up} can simply be calculated as following procedures;
It is first presumed that n is a continuous variable. Taking the second partial derivative of Π_{js}' with respect to n, the second derivative is positive. Subsequently, Π_{js}' is strictly convex in n, and the value of n can simply be obtained as,

$$n = \sqrt{\frac{h_r p(12(\pi+h_r y)-y^2 p(7h_r+4\pi))+6h_m(h_r+\pi)(p(2-y)+4D\ (\delta_{fm}))}{6A_r h_m(h_r+\pi)(p(2-y)-2D\ (\delta_{fm}))}}.$$

The upper bounds of shipments witnessed when $\delta_{fm} = 0$, then we have:

$$n_{up} = \left\lceil \sqrt{\frac{h_r p\big(12(\pi+h_r y)-y^2 p(7h_r+4\pi)\big)+6h_m(h_r+\pi)(p(2-y)+4a)}{6A_r h_m(h_r+\pi)(p(2-y)-2a)}} \right\rceil \quad (18)$$

4. Numerical Examples and Sensitivity Analysis

Consider a two-level supply chain system that replenishes the retailer's orders instantly. This system is not perfect, i.e. it produces some defective products. The inspection process that screens out the defective products is perfect. Moreover in this system the market's demand depends to the unit price charged by the retailer.

The parameters needed for analyzing the above system situation are given below:

h_r=\$6/unit/year,

h_m=\$5/unit/year,

π = \$7/unit/year,

A_r=\$25/order,

A_m=\$150/set up,

p=5500 unit/year,

v=\$10/unit,

d=\$0.7/unit,

a=3000,

w=\$10/unit.

D_{sm}=1000 units/year

δ_{sm}=\$20/unit

If the demand function is:

$D\ (\delta_{fm})$ =3000-10δ_{fm},

Another assumes that the defective percentage random variable, γ, follows uniformly distributed with

$$f(\gamma) = \begin{cases} 5 & 0 \le \gamma < 0.2 \\ 0 & \text{otherwise} \end{cases}$$

Thus, the maximum profit and the optimal values of Q, B, δ_{fm}, n under joint optimization and individual optimization are given in Table 1.

Table 1
Maximum profit and the optimal values under joint optimization and individual optimization

joint optimization					individual optimization				
Q	B	δ_{fm}	n	profit	Q	B	δ_{fm}	n	profit
63.541	26.39	150.455	3	221136	45.859	19.049	154.654	18	122494

The profit increases obtained by joint optimization as compared to individual optimization is:
$\Delta = \Pi_j - \Pi_I$ therefore, Δ is: 98642.

From equations above and the results of the sensitivity analysis for some parameters that are presented in table 2-6, in Table 2, 5 and 6, assumed that the b is set to 50 and other parameters are fixed. In Table 4, the defective percentage is set to 0.2 and the other parameters are not changed.

In Table 2, under joint optimization the size of a production batch of product increases when the defective percentage rises. However, a clear trend can not be found under individual scenario. Also as Table 2 indicates, coordination compared to individual optimization grows when the defective percentage decreases. In addition, in Table 2, when the defective percentage rises, the unit price charged by the retailer, increases under both joint and individual optimization. However it has a higher effect on the unit price charged by the retailer under joint optimization.

Table 2
Optimal solutions for different defective percentage γ is uniformly distributed between 0 and Y

Y	0.001	0.01	0.1	0.2	0.3	0.4
Q_j	60.08	60.21	61.52	63.16	51.19	31.08
B_j	27.71	27.65	26.97	26.23	20.08	16.35
δ_{fm_j}	29.93	29.95	30.18	30.4	30.74	31.08
nj	3	3	3	3	4	5
Πj	41564	40895	36232	33654	32009	31248
Q_i	10.68	10.6	9.79	9.02	8.35	7.77
B_i	23.17	23.08	22.32	21.72	21.3	21.05
δ_{fm_i}	34.72	34.72	34.7	34.69	34.67	34.65
n_i	15	15	16	18	19	21
Π_i	23248	23236	23226	23210	23195	23147
Δ	18316	17659	13006	10444	8814	8101

Fig. 2 shows this behavior of Δ decreases by Y. For instance, increasing the defective percentage from 0.001 to 0.4 results in 96308 decrease in Δ when b = 10. This value is 17982 when b = 30, and is 7827 when b= 50.

In Table3, assumed that the b is set to 50 and other parameters are as follow: h_r=\$20/unit/year,h_m=\$5/unit/year, π = \$25/unit/year,A_r=\$31/order,A_m=\$151/set up,p=5500 unit/year,d=\$0.7/unit,a=3000,w=\$10/unit.D_{sm}=1000 units/year, δ_{sm}=\$20/unit.

In Table 3, the decrease in maximum total profit seems to be faster under individual scenario respect to joint optimization.

In Table 4, by increasing b, Δ decreases .The decrease in Δ seems to be faster for small value of b (10-20). Thus, coordination between supply chain members increases when b is low.

In Table 5, when h_r increases, the number of shipments per lot will rise, therefore, the buyer can reduce the holding cost.

In Table 6 that is due to the fact that the purchasing price does not affect the joint total profit. However, under individual optimization, higher purchasing prices make the retailer increase the selling prices. As a result, the demand as well as the total profit decreases.

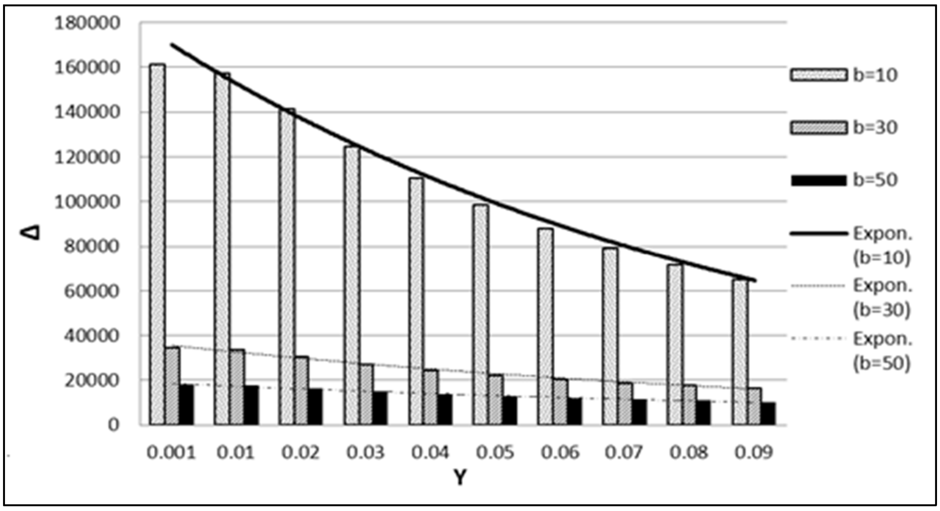

Fig. 2. Relationship between the Δ and the defective percentage

Table 3
Optimal solutions for different warranty cost v

v	10	12	14	16	18	20
Q_j	16.28	16.29	16.30	16.31	16.32	16.33
Bj	5.79	5.793	5.795	5.798	5.801	5.803
δ_{fm_j}	31.14	31.12	31.1	31.08	31.06	31.04
nj	8	8	8	8	8	8
Π_j	37166	37165	37165	37164	37163	37163
Q_i	11.45	11.45	11.45	11.45	11.45	11.45
B_i	4.07	4.07	4.07	4.07	4.07	4.07
δ_{fm_i}	34.74	34.74	34.74	34.74	34.74	34.74
n_i	36	36	36	36	36	36
Π_i	31371	30739	30108	29476	28845	28214
Δ	5795	6426	7057	7688	8318	8949

Table 4
Optimal solutions for different price sensitivity *b*

b	10	30	50	70	90
Q_j	63.54	63.31	63.09	62.86	62.64
Bj	26.39	26.3	26.2	26.11	26.02
δ_{fm_j}	150.45	50.45	30.45	21.88	17.12
nj	3	3	3	3	3
Π_j	221136	71146	41157	28310	21178
Q_i	45.85	27.26	21.72	18.85	17.06
B_i	19.04	11.32	9.02	7.83	7.08
δ_{fm_i}	154.65	54.67	34.69	26.14	21.4
n_i	9	15	18	20	21
Π_i	122494	48644	28293	18668	13012
Δ	98642	22502	12864	9642	8166

Table 5
Optimal solutions for different buyer's holding cost hr

h_r	2	6	10	14	18	20
Q_j	113.36	50.08	31.64	22.98	19.68	18.47
Bj	7.55	8.72	8.13	7.42	7.41	7.39
δ_{fm_j}	30.39	30.44	30.47	30.13	30.52	30.53
n_j	3	4	5	6	6	6
Π_j	41951	41033	40187	39364	38775	38482
Q_i	30.58	18.66	15.16	13.35	12.21	11.77
B_i	2.03	3.25	3.89	4.31	4.60	4.71
δ_{fm_i}	34.67	34.70	34.73	34.74	34.76	34.76
n_i	12	19	23	26	29	30
Π_i	31371	30739	30108	29476	28845	28214
Δ	10580	10295	10079	9888	9931	10269

Table 6
Optimal solutions for different purchasing price w

W	1	4	8	12	16	20
Q_j	61.52	61.52	61.52	61.52	61.52	61.52
B_j	26.97	26.97	26.97	26.97	26.97	26.97
δ_{fm_j}	30.18	30.18	30.18	30.18	30.18	30.18
n_j	3	3	3	3	3	3
Π_j	41376	41376	41376	41376	41376	41376
Q_i	20.81	21.33	22	22.65	23.28	23.89
B_i	9.12	9.35	9.64	9.93	10.20	10.47
δ_{fm_i}	30.16	31.67	33.69	35.72	37.74	39.76
n_i	17.9	16.87	15.54	14.31	13.17	12
Π_i	29231	28532	27203	25425	23198	20527
Δ	12145	12844	14173	15952	18178	20850

5. Conclusions

In the present study, a joint manufacturer-retailer production-inventory-marketing model is developed for a defective product by a uniform distribution. The contribution of the research to the joint economic lot-sizing literature is to incorporate the pricing policy as well as the imperfect quality into the previous joint manufacturer-retailer models. Moreover, the manufacturers accept the return products and sell them in the second market. Incorporating these two features into the model increases the complexity. In other words, a more comprehensive model is introduced in which the unit price charged by the retailer influences the demand of the products being sold in the first market and the production process is not perfect. Thereby, this study establishes a link between the literature on shipment, ordering, pricing policies, imperfect quality and the joint economic lot-sizing literature. The objective is to maximize the total joint annual profits incurred by the manufacturer and the retailer. The retailer inspects the product and delivers perfect-quality product to the first market and the rejected products sent back to the manufacture to be sold in the second market. Shortages are permitted for the retailer and are completely backordered. The researchers assume the policy in which the shipment quantity is delivered to the retailer is identical at each shipment. The anticipated annual individual and joint total profits are derived and appropriate procedures to find the optimal solution are presented. The sensitivity analysis shows that coordination between supply chain members increases when the defective percentage is reduced, and the warranty cost increases. The warranty cost has more effect on decreasing maximum total profit under individual optimization. Moreover, the profit increase rises by purchasing price. However, under individual optimization, higher purchasing prices force the retailer to increase the selling prices. As a result, the demand as well as the total profit decreases. When the buyer's holding cost increases, the number of shipments per lot will rise; therefore, the buyer can reduce the holding cost.

There are more potentials to extend the present work. For example, other parameters of the system that ware not included in this paper, such as perishability of product could be added to the model. The impact of marketing effort can be considered in the model. Future research may investigate the application of other shipment policies. Developing a model to the multi-manufacturer case is also suggested for future research. Another extension of a model is to consider the possibility of incorrect classification for a perfect product as a defective one and vice versa.

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