Phase-II Monitoring of AR (1) Auto correlated Polynomial Profiles

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Abstract

In some statistical process control applications, quality of a process or product can be characterized by a relationship between a response and one or more independent variables, which is typically referred to a profile. In this paper, polynomial profiles are considered to monitor processes in which there is a first-order autoregressive relation between the error terms in each profile. A remedial measure is first proposed to eliminate the effect of autocorrelation in phase-II monitoring of auto-correlated profiles. Then, three methods are employed to monitor polynomial profiles where their performances are compared using the average run length criterion.

Keywords: Statistical process control; Polynomial profiles; Phase-II monitoring; Autocorrelation; Average run length.

1. Introduction

Sometimes, a relationship between a response and one or more independent variables, referred to a profile, can characterize the quality of a process or a product adequately. Many researchers including Stover and Brill (1998), Kang and Albin (2000), Mahmoud and Woodall (2004), Woodall et al. (2004), Wang and Tsung (2005), and Woodall (2007) discussed practical applications of profiles. Moreover, many authors, including Kang and Albin (2000), Kim et al. (2003), Mahmoud et al. (2007), Mahmoud and Woodall (2004), Mestek et al. (1994), Noorossana et al. (2010), and Stover and Brill (1998) studied Phase-I monitoring of simple linear profiles. The purpose of the Phase-I analysis is to evaluate the stability of a process and to estimate process parameters. Nonetheless, some authors including Eyvazian et al. (2011), Gupta et al. (2006), Kang and Albin (2000), Kim et al. (2003), Noorossana et al. (2004a), Zou et al. (2006), Niaki et al. (2007), and Saghaei et al. (2009) investigated Phase-II monitoring of simple linear profiles. In phase-II analysis, one is interested in detecting shifts in the process parameters as soon as possible. Sometimes more complicated models are needed to represent profiles. Kazemzadeh et al. (2008a) extended three Phase-I methods in polynomial profile monitoring. Zou et al. (2007) proposed a multivariate exponentially weighted moving average (*MEWMA*) control chart for monitoring general linear profiles in Phase II. Kazemzadeh et al. (2008b) transformed polynomial regression to an orthogonal polynomial regression model and proposed a method based on using exponentially weighted moving average (*EWMA*) control charts to monitor the parameters

of orthogonal polynomial model in Phase II. Furthermore, Amiri et al. (2012) concentrated on phase II monitoring of multiple linear regression profiles and proposed a new dimension reduction method to overcome the dimensionality problem of some of the existing multiple linear regression profile monitoring methods.

In all previous studies, it is assumed that the error terms of the model are independently and identically distributed normal random variables. However, in some cases these assumptions are violated. Noorossana et al. (2004b) investigated the effect of non-normality of the error terms on the performances of the EWMA/R method proposed by Kang and Albin (2000). Jensen et al. (2008) developed a linear mixed model (LMM) to account for the autocorrelation within a linear profile. Jensen and Birch (2009) showed that the use of mixed models has significant advantages when there is autocorrelation within nonlinear regression models. Noorossana et al. (2008) considered linear profiles and modeled autocorrelation between profiles as a first order autoregressive AR(1) process. Kazemzadeh et al. (2007 & 2009) considered polynomial profiles in the presence of between profile autocorrelation modeled by AR(1). Soleimani et al. (2009) investigated the effect of within profile autocorrelation in simple linear profiles and proposed a transformation technique to eliminate the effect of autocorrelation. Recently, the Gaussian process model has been proposed by Zhang et al. (2013) to account for the autocorrelation within linear profiles. Besides, Eghbali et al. (2013) addressed the problem of monitoring a simple linear profile that is going through a multistage process in phase II. They proposed a U statistic to eliminate the cascade effect and modified the T2

control scheme to monitor the process.

In this paper, the work of Soleimani et al. (2009) is first extended to include polynomial profiles. In other words, processes are considered in which the relationship between a response and a single explanatory variable is defined by a kth order polynomial regression, where it is assumed that the error terms within each profile are correlated based on a first order autoregressive model. Moreover, we assume that there is no correlation between polynomial profiles. An application of this problem is discussed by Amiri et al. (2010) in which the quality of an automobile engine is characterized by a second order polynomial profile between the torque and speed in rpm with an AR(1) autocorrelation structure between error terms within each profile. Three methods are utilized to monitor polynomial regression profiles. The performances of the methods are next evaluated through simulation studies via the average run length (ARL) criterion.

The structure of the remainder of the paper is as follows: In Section 2, the problem formulation as well as assumptions are given. The transformation technique and the three monitoring methods are presented in Section 3. In Section 4, the effect of autocorrelation on the performance of T^2 control chart is shown and the performances of the proposed methods are investigated. Concluding remarks are given in Section 5.

2. Modeling and assumptions

Having a single explanatory variable x and assuming jth sample is being collected over time, the observations are shown by $(x_i, x_i^2, ..., x_i^k, y_{ij})$; $i = 1, 2, ...n$. In other words, the subscript i shows the ith observation within each profile, and subscript j shows the jth profile collected over time. When the process is in-control, the autocorrelated polynomial profile is modeled by

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + ... + A_k x_i^k + \varepsilon_{ij}$$
$$\varepsilon_{ij} = \Phi \varepsilon_{(i-1)j} + a_{ij} \tag{1}$$

where ε_{ij}s are the correlated error terms, a_{ij}s are independent and identically distributed normal random variables with mean zero and variance σ^2, $A_0, A_1, ..., A_k$ are model parameters, and $-1 < \Phi < 1$ is the autocorrelation coefficient. Moreover, it is assumed x-values are fixed and constant from profile to profile. In this paper, we consider a Phase-II monitoring case, in which the in-control values of the parameters $A_0, A_1, ..., A_k$ and σ^2 are assumed known.

It can easily be shown that the existing autoregressive structure between the error terms, defined in Eq. (1), leads to autocorrelation between observations at different values of x in each profile. It means that, the observations in each profile can be expressed as

$$y_{ij} = A_0 + A_1 x_i + A_2 x_i^2 + ... + A_k x_i^k + \varepsilon_{ij} \tag{2}$$

And

$$y_{(i-1)j} = A_0 + A_1 x_{(i-1)} + A_2 x_{(i-1)}^2 + ... + A_k x_{(i-1)}^k + \varepsilon_{(i-1)j} \tag{3}$$

Leading to

$$y_{ij} - (A_0 + A_1 x_i + A_2 x_i^2 + ... + A_k x_i^k) =$$
$$\Phi [y_{(i-1)j} - (A_0 + A_1 x_{(i-1)} + A_2 x_{(i-1)}^2 + ... + A_k x_{(i-1)}^k)] + a_{ij} \tag{4}$$

In the next section, the transformation technique proposed by Soleimani et al. (2009) is first extended to be applied for the elimination of the autocorrelation effect. Then, three methods are utilized to monitor polynomial profiles.

3. Proposed methods

In order to eliminate the existing within-profile autocorrelation of polynomial profiles, the transformation technique proposed by Soleimani et al. (2009) is extended in the first step of the proposed methods. In this technique, all observations on the response variable are transformed via the following equation

$$Y'_{ij} = Y_{ij} - \Phi Y_{(i-1)j}. \tag{5}$$

If observations Y_{ij} and $Y_{(i-1)j}$ in Eq. (5) are replaced by their equivalents in the regression model (1), a polynomial regression model with independent error terms is obtained by

$$Y'_{ij} = A_0(1 - \Phi) + A_1(X_i - \Phi X_{i-1}) + A_2(X_i^2 - \Phi X_{i-1}^2) + ... + A_k(X_i^k - \Phi X_{i-1}^k) \tag{6}$$
$$+ (\varepsilon_{ij} - \Phi \varepsilon_{(i-1)j}) \; ; \; i = 1, 2, ..., n$$

that results in

$$Y'_{ij} = A'_0 + A'_1 X'_i + A'_2 X'^2_i + ... + A'_k X'^k_i + a_{ij} \tag{7}$$

where $Y'_{ij} = Y_{ij} - \Phi Y_{(i-1)j}$, $\tag{8}$

$$X'_i = X_i - \Phi X_{i-1}, \; X'^2_i = X_i^2 - \Phi X_{i-1}^2, ..., X'^k_i = X_i^k - \Phi X_{i-1}^k, \tag{9}$$

and

$$A'_0 = A_0(1 - \Phi), A'_1 = A_1, A'_2 = A_2, ..., A'_k = A_k \tag{10}$$

In Eq. (7), a_{ij}s are independent random variables with mean zero and variance σ^2. In this paper, we consider Phase-II monitoring of polynomial profiles where Φ is assumed a known parameter. Now, three control charts are developed to monitor the parameters of the polynomial profile in Eq. (7).

3.1. T^2 method

The first method is a modified version of the T^2 control chart proposed by Kang and Albin (2000).To reduce the effect of autocorrelation that exists between error terms within profiles, all the parameters, $(A_0, A_1, A_2, ..., A_k)$, of the original model are replaced by their transformed ones. This method is used when the number of parameters (k) is not very large. The modified T^2 statistic is obtained by

$$T_j^2 = \begin{bmatrix} \left(\hat{A}_{oj}', \hat{A}_{1j}', \hat{A}_{2j}', ..., \hat{A}_{kj}'\right) - \\ \left(A_{oj}', A_{1j}', A_{2j}', ..., A_{kj}'\right) \end{bmatrix}^T \Sigma^{-1}$$
$$\begin{bmatrix} \left(\hat{A}_{oj}', \hat{A}_{1j}', \hat{A}_{2j}', ..., \hat{A}_{kj}'\right) - \\ \left(A_{oj}', A_{1j}', A_{2j}', ..., A_{kj}'\right) \end{bmatrix} \quad (11)$$

where

$$\Sigma = [\sigma^2(X^T X)^{-1}] \quad (12)$$

3.2. Residual-based T^2

In the second method, the residuals of the transformed model is used, where the residual are obtained as

$$e_{ij} = y_{ij}' - (A_0' + A_1' X_i' + A_2' X_i'^2$$
$$+ ... + A_k' X_i'^k) \quad ; \quad i = 1, 2, ..., n \quad (13)$$

The T^2 statistics and the upper control limit for the residual-based T2 chart, $T_{residual}^2$ thereafter, are determined using the following equations, respectively.

$$T_j^2 = (\underline{e}_j - \underline{0}) \Sigma_{\underline{e}_j}^{-1} (\underline{e}_j - \underline{0})^T \quad (14)$$

$$UCL = \chi_{n-1,\alpha}^2 \quad (15)$$

where $\underline{e}_j = (e_{2j}, e_{3j}, ..., e_{nj})^T$, $\Sigma_{\underline{e}_j} = \sigma^2 I$, I is the

identity matrix , $\underline{0}$ is a zero vector, n is the number of x values, and $\chi_{n-1,\alpha}^2$ is the upper $100(1-\alpha)$ percentile of the chi-square distribution with $n-1$ degrees of freedom.

3.3. $EWMA/R$

An *EWMA* control chart in combination with the *R*-chart is employed as the third method to monitor not only the average value of the residuals, but also to detect shifts in the process variance. These charts are the same as the ones proposed by Kang Albin (2000), where the residuals are obtained using Eq. (12) and the average value of the

residuals for the *j*th profile are calculated by

$$\bar{e}_j = \sum_{i=2}^{n} e_{ij} / (n-1) \quad (16)$$

The *EWMA* control chart statistic, denoted by z_j for $j = 1, 2, ...,$ is given by

$$z_j = \theta \bar{e}_j + (1-\theta) z_{j-1} \quad (17)$$

where $\theta, (0 < \theta \leq 1),$ is a smoothing constant and $z_0 = 0$. The lower and the upper control limits for the *EWMA* control chart are

$$LCL = -L\sigma\sqrt{\theta/(2-\theta)(n-1)} \quad (18)$$

$$UCL = L\sigma\sqrt{\theta/(2-\theta)(n-1)} \quad (19)$$

respectively, where $L(>0)$ is a constant selected to give a specified in-control *ARL*.

The *R* control chart statistic denoted by R_j is calculated by $R_j = max(e_{ij}) - min(e_{ij})$ with the lower and upper control limits as

$$LCL = \sigma(d_2 - Ld_3) \quad \text{and} \quad UCL = \sigma(d_2 + Ld_3) \quad (20)$$

respectively, where $L(>0)$ is a constant chosen to give a specified in-control *ARL*. The values of d_2 and d_3 are constants that depend on the sample size *n*.

In the next section, the performances of the above control charts are evaluated based on out-of-control average run lengths using simulation experiments.

4. Simulation Experiments

In this section, we first evaluate the performance of the $T_{residual}^2$ control chart for monitoring polynomial profiles when within-profile autocorrelation is present and the proposed transformation method is not utilized. The following example is used to study the performance:

$$y_{ij} = 3 + 2x_i + x_i^2 + \varepsilon_{ij} \quad (21)$$

$$\varepsilon_{ij} = \Phi\varepsilon_{(i-1)j} + a_{ij} \quad (22)$$

where a_{ij} follows a normal distribution with mean zero and variance one and *x*-values are 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. In the simulation experiments the effect of different autocorrelation coefficients Φ on the performance of $T_{residual}^2$ control chart under different shifts in the intercept λ, the second parameter β, the third parameter δ, and error standard deviation γ using the in-control average run length

criterion is studied in 10,000 simulation runs. The results are summarized in Table 1. In this table, λ, β, δ and γ are measured in multiples of σ and the in-control average run length is considered 200.

As shown in Table 1, when the transformation technique is not used, the in-control *ARLs* of $T^2_{residual}$ control chart

decrease in the presence of autocorrelation within profiles, leading to its poor performance. Moreover, this effect is more considerable when the autocorrelation coefficient gets larger.

Table 1

The effect of autocorrelation on in-control ARL performance of $T^2_{residual}$ control chart under different shifts in intercept, second parameter, third parameter, and error standard deviation without utilizing the proposed transformation method

λ (Shift in the intercept)												
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
	0	200	183.1	144.9	101.4	66.4	41.5	26.1	16.3	10.6	7.0	4.8
Φ	0.1	168.4	153.7	119.8	81.6	53.3	33.4	21.5	13.9	9.2	6.3	4.5
Autocorrelation	0.3	58.5	54.2	43.2	32.7	23.6	16.4	11.9	8.7	6.3	4.8	3.7
coefficients	0.5	14.9	14.4	13.3	11.2	9.3	7.7	6.2	5.1	4.2	3.5	3.0
	0.7	4.8	4.7	4.5	4.4	4.1	3.7	3.5	3.1	2.8	2.6	2.4
	0.9	2.2	2.1	2.1	2.1	2.1	2.0	2.0	2.0	1.9	1.9	1.8
β (Shift in the second parameter)												
		0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
	0	200	160.6	92.4	44.0	20.2	9.8	5.2	3.1	2.1	1.5	1.3
Φ	0.1	168.9	132.4	73.2	35.3	17.3	8.7	4.8	3.1	2.1	1.6	1.2
Autocorrelation	0.3	57.7	47.5	30.9	17.7	10.1	6.2	3.9	2.6	1.9	1.6	1.3
coefficients	0.5	15.1	13.5	10.7	7.7	5.7	4.1	3.1	2.3	1.9	1.6	1.4
	0.7	4.8	4.6	4.3	3.7	3.3	2.8	2.4	2.1	1.8	1.6	1.4
	0.9	2.1	2.1	2.1	2.0	2.0	1.9	1.8	1.7	1.7	1.6	1.5
δ (shift in the third parameter)												
		0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
	0	200	36.3	3.97	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Φ	0.1	169.5	30.1	3.7	1.3	1.1	1.0	1.0	1.0	1.0	1.0	1.0
Autocorrelation	0.3	58.3	15.7	3.2	1.4	1.1	1.0	1.0	1.0	1.0	1.0	1.0
coefficients	0.5	15.1	7.3	2.8	1.5	1.1	1.1	1.0	1.0	1.0	1.0	1.0
	0.7	4.9	3.7	2.3	1.5	1.2	1.1	1.0	1.0	1.0	1.0	1.0
	0.9	2.2	2.1	1.8	1.5	1.2	1.1	1.1	1.0	1.0	1.0	1.0
γ (shift in the standard deviation)												
		1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
	0	200	46.9	17.1	8.2	4.7	3.2	2.5	1.9	1.6	1.5	1.3
Φ	0.1	168.2	41.5	15.6	7.7	4.6	3.2	2.3	1.9	1.6	1.4	1.3
Autocorrelation	0.3	57.8	20.5	9.6	5.5	3.6	2.7	2.1	1.8	1.6	1.4	1.3
coefficients	0.5	15.1	7.9	4.8	3.3	2.5	2.1	1.8	1.6	1.4	1.3	1.2
	0.7	4.9	3.4	2.7	2.2	1.8	1.6	1.5	1.4	1.3	1.2	1.2
	0.9	2.3	1.9	1.7	1.6	1.4	1.4	1.3	1.3	1.2	1.2	1.1

When the proposed transformation method is used however, the performances of $T^2_{residual}$, T^2, and *EWMA/R* are then compared employing the same example introduced earlier in (21-22). Two autocorrelation coefficients $\Phi = 0.1$ (weak autocorrelation) and $\Phi = 0.9$ (strong autocorrelation) are considered where all control-charting methods are designed to have an overall in-control *ARL* of 200. To achieve this, the smoothing constant θ in the *EWMA* control chart is set 0.2. Furthermore, in the

EWMA/R control chart, we set the value of L equal to 2.973 for both $\Phi = 0.1$ and $\Phi = 0.9$ autocorrelation coefficients. For T^2 and $T^2_{residual}$ charts *UCLs* are set 12.84 and 23.59, respectively. We used 10,000 simulation runs to study out-of-control *ARL* under different shifts in the intercept, the second parameter, the third parameter, and the error standard deviation. The results are summarized in Tables 2 through 5.

Table 2

Out-of-control ARL comparisons under shifts from A_0 to $A_0 + \lambda\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	λ										
Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$T^2_{residual}$	198.5	188.5	152.9	112.8	78.5	53.2	34.7	23.1	14.9	10.3	6.9
T^2	200	173.7	122.3	76.2	44.6	26.9	16.6	10.4	7.8	4.7	3.5
EWMA/R	200	11.8	38.7	17.4	10.1	6.9	5.2	4.2	3.6	3.1	2.7
$\Phi = 0.9$	λ										
Methods	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
$T^2_{residual}$	199.8	197.6	199.3	197.6	198.6	196.6	195.1	190.2	188.4	186.5	184.1
T^2	200.1	198.4	196.7	196.2	195.2	193.0	187.3	183.0	180.6	172.2	169.8
EWMA/R	200	200	192.1	187.1	179.2	164.5	152.0	137.6	125.2	112.5	102.1

Table 3

Out-of-control ARL comparisons under shifts from A_1 to $A_1 + \beta\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	β										
Methods	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$T^2_{residual}$	200.3	164.1	103.1	53.4	26.7	13.2	7.1	4.2	2.6	1.8	1.4
T^2	200.1	190.6	158.9	122.2	90.4	64.1	45.5	32.2	22.8	16.7	12.4
EWMA/R	200.7	63.4	16.7	7.9	5.1	3.8	3.0	2.5	2.5	2.0	1.8
$\Phi = 0.9$	β										
Methods	0	0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
$T^2_{residual}$	197.8	197.1	186.7	179.6	165.2	149.3	131.8	114.3	99.4	86.3	73.1
T^2	198.4	196.5	192.2	180.4	169.5	152.9	141.1	126.8	112.8	98.5	85.1
EWMA/R	197.3	181.90	133.8	85.3	55.1	35.7	25.1	18.5	14.1	11.3	9.4

Table 4

Out-of-control ARL comparisons under shifts from A_2 to $A_2 + \delta\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	δ										
Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
$T^2_{residual}$	200.7	43.6	5.2	1.5	1.1	1.1	1.0	1.0	1.0	1.0	1.0
T^2	199.2	174.6	127.5	80.1	49.7	29.4	18.5	11.9	7.7	5.4	3.9
EWMA/R	200.3	8.4	3.1	2.1	1.5	1.1	1.1	1.0	1.0	1.0	1.0
$\Phi = 0.9$	δ										
Methods	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1
$T^2_{residual}$	197.1	159.1	92.7	45.2	20.8	10.1	5.3	3.1	2.1	1.5	1.2
T^2	200.4	191.6	167.8	138.4	109.7	84.2	63.6	47.6	35.7	27.4	20.3
EWMA/R	200.2	60.6	15.7	7.5	4.9	3.6	2.9	2.4	2.1	1.9	1.8

Table 5

Out-of-control ARL comparisons under standard deviation shifts from σ to $\gamma\sigma$ with $\Phi = 0.1$ and $\Phi = 0.9$

$\Phi = 0.1$	γ										
Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
$T^2_{residual}$	200.5	47.2	16.6	8.1	4.7	3.2	2.4	1.9	1.6	1.4	1.3
T^2	197.4	72.1	32.4	17.9	11.5	8.0	5.9	4.6	3.7	3.2	2.7
EWMA/R	199.3	61.7	24.7	12.4	7.2	4.7	3.4	2.6	2.1	1.8	1.6
$\Phi = 0.9$	γ										
Methods	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
$T^2_{residual}$	199.3	47.1	17.2	7.9	4.6	3.1	2.3	1.9	1.6	1.4	1.3
T^2	200.6	70.4	32.8	18.1	11.3	7.9	5.8	4.6	3.7	3.2	2.7
EWMA/R	199.4	61.2	24.8	7.1	4.7	3.4	2.6	2.2	2.2	1.8	1.6

The results in Table 2 show that under the intercept shift from A_0 to $A_0 + \lambda\sigma$ in both weak and strong autocorrelations ($\Phi = 0.1$ and $\Phi = 0.9$), while the *EWMA/R* chart uniformly performs better than the other two, the $T^2_{residual}$ chart has the worst performance. Further, it can be seen that the out-of-control *ARL*s for the strong autocorrelation case are larger than the ones in the weak autocorrelation situation.

Under the shifts in the second parameter from A_1 to $A_1 + \beta\sigma$, the results in Table 3 show that while the *EWMA/R* chart performs uniformly better than the other two charts in both the weak and the strong autocorrelation cases, the T^2 chart performs better than $T^2_{residual}$.

As shown in Table 4, for the third parameter shifts under the strong autocorrelation coefficient, the results are similar to those obtained for the intercept and the second parameter shifts. Moreover, under the weak autocorrelation coefficient when very small shifts are present, the *EWMA/R* chart performs better than the other two schemes, as expected. However, the $T^2_{residual}$ chart is the best in medium shifts. Meanwhile, as the magnitude of the shift increases the performance of the *EWMA/R* and T^2 become similar.

Finally, the results in Table 5 show that under the standard deviation shift from σ to $\gamma\sigma$ in both weak and strong autocorrelation situations, the $T^2_{residual}$ control chart performs uniformly better than other two charts. In addition, similar performances are obtained for both weak and strong autocorrelations. This means that the autocorrelation coefficient does not affect the out-of-control *ARL* under the standard deviation shift.

5. Conclusions

In this paper, first, the effect of within-profile autocorrelation on the performance of a $T^2_{residual}$ chart designed to monitor polynomial profiles under independency of the error terms was investigated. As shown in Table 1, when the transformation technique was not used, the in-control *ARL*s of $T^2_{residual}$ scheme would decrease in the presence of autocorrelation within profiles, leading to its poor performance. Moreover, this effect was more considerable when the autocorrelation coefficient was larger. Then, the transformation technique of Soleimani et al. (2009) that was originally proposed for simple linear profile was extended and employed for the polynomial profile. Finally, the performances of T^2, $T^2_{residual}$, and *EWMA/R* control charts in terms of out-of-control average run lengths using the transformation technique in 10,000 simulation runs showed that the *EWMA/R* scheme performs better than the other charts under the step shifts in the regression parameters. However, the $T^2_{residual}$ method had better performance in comparison with the other two methods under the shifts in the standard deviation. We also showed that autocorrelation would not affect the out-of-control *ARL* of the chart under the standard deviation shift.

References

[1] Amiri, A., Jensen, W.A., Kazemzadeh, R.B. (2010). A case study on monitoring polynomial profiles in the automotive industry. Quality and Reliability Engineering International, 26(5), 509-520.

[2] Amiri, A., Eyvazian, M., Zou, C., Noorossana R. (2012). A parameters reduction method for monitoring multiple

linear regression profiles. International Journal of Advanced Technology, 58(5-8), 621-629.

[3]   Eghbali Ghahyazi, M., Niaki, S.T.A., Soleimani, P. (2013). On the monitoring of linear profiles in multi-stage processes. To appear in Quality and Reliability Engineering International. http://dx.doi.org/10.1002/qre.1531.

[4]   Eyvazian, M., Noorossana, R., Saghaei, A., Amiri, A. (2011). Phase II monitoring of multivariate multiple linear regression profiles. Quality & Reliability Engineering International, 27(2), 281-296.

[5]   Gupta, S., Montgomery, D.C., Woodall, W.H. (2006). Performance evaluation of two methods for online monitoring of linear calibration profiles. International Journal of Production Research, 44(10), 1927–1942.

[6]   Jensen, W.A., Birch, J.B. (2009). Profile monitoring via nonlinear mixed model. Journal of Quality Technology, 41(1), 18–34.

[7]   Jensen, W.A., Birch, J.B., Woodall, W.H. (2008). Monitoring correlation within linear profiles using mixed models. Journal of Quality Technology, 40(2), 167–183.

[8]   Kang, L., Albin, S.L. (2000). On-line monitoring when the process yields a linear profile. Journal of Quality Technology, 32(4), 418–426.

[9]   Kazemzadeh, R.B., Noorossana, R., Amiri, A. (2007). Statistical monitoring of autocorrelated polynomial profiles. In Proceedings of the 9th Islamic Countries Conference on Statistical Sciences. Shah Alam, Malaysia, December 12–14.

[10] Kazemzadeh, R.B., Noorossana, R., Amiri, A. (2008a). Phase-I monitoring of polynomial profiles. Communications in Statistics-Theory and Methods, 37(10), 1671–1686.

[11] Kazemzadeh, R.B., Noorossana, R., Amiri, A. (2008b). Monitoring polynomial profiles in quality control applications. The International Journal of Advanced Manufacturing Technology, 42(7), 703-712.

[12] Kazemzadeh, R.B., Noorossana, R., Amiri, A. (2009). Phase-II monitoring of autocorrelated polynomial profiles in AR(1) processes. Scientia Iranica, 17(1), 12-24.

[13] Kim, K., Mahmoud, M.A., Woodall, W.H. (2003). On the monitoring of linear profiles. Journal of Quality Technology, 35(3), 317–328.

[14] Mahmoud, M.A., Parker, P.A., Woodall, W.H., Hawkins, D.M. (2007). A change point method for linear profile data. Quality and Reliability Engineering International, 23(2), 247–268.

[15] Mahmoud, M.A., Woodall, W.H. (2004). Phase-I analysis of linear profiles with calibration applications. Technometrics, 46(4), 380–391.

[16] Mestek, O., Pavlik, J., Suchánek, M. (1994). Multivariate control charts: Control charts for calibration curves. Fresenius' Journal of Analytical Chemistry, 350(6), 344–351.

[17] Niaki, S.T.A., Abbasi, B. Arkat, J. (2007). A generalized linear statistical model approach to monitor profiles. International Journal of Engineering Transactions A: Basics, 20(3), 233-242.

[18] Noorossana, R., Amiri, A., Vaghefi, S.A., Roghanian, E. (2004a). Monitoring quality characteristics using linear profile. In Proceedings of the 3rd International Industrial Engineering Conference. Tehran, Iran.

[19] Noorossana, R., Vaghefi, S.A., Amiri, A. (2004b). The effect of non-normality on monitoring linear profiles. In Proceedings of the 2nd International Industrial Engineering Conference. Riyadh, Saudi Arabia.

[20] Noorossana, R., Amiri, A., Soleimani, P. (2008). On the monitoring of autocorrelated linear profiles. Communications in Statistics-Theory and Methods, 37(3), 425–442.

[21] Noorossana, R., Eyvazian, M., Amiri, A., Mahmoud, M.A. (2010). Statistical monitoring of multivariate multiple linear regression profiles in phase I with calibration application. Quality & Reliability Engineering International, 26(3), 291-303.

[22] Saghaei, A., Mehrjoo, M. Amiri, A. (2009). A CUSUM-based method for monitoring simple linear profiles. The International Journal of Advanced Manufacturing Technology, 45(11), 1252-1260.

[23] Soleimani, P., Noorossana, R., Amiri, A. (2009). Simple linear profiles monitoring in the presence of within profile autocorrelation. Computers and Industrial Engineering, 57(3), 1015-1021.

[24] Stover, F.S., Brill, R.V. (1998). Statistical quality control applied to ion chromatography calibrations. Journal of Chromatography A, 804(1–2), 37–43.

[25] Wang, K., Tsung, F. (2005). Using profile monitoring techniques for a data-rich environment with huge sample size. Quality and Reliability Engineering International, 21(7), 677–688.

[26] Woodall, W.H. (2007). Current research on profile monitoring. Revista Producão, 17(3), 420–425.

[27] Woodall, W.H., Spitzner, D.J., Montgomery, D.C., Gupta, S. (2004). Using control charts to monitor process and product quality profiles. Journal of Quality Technology, 36(3), 309–320.

[28] Zhang, Y., He, Z., Zhang, C., Woodall, W.H. (2013). Control charts for monitoring linear profiles with within-profile correlation using Gaussian process models. To Appear in Quality and Reliability Engineering International. DOI: 10.1002/qre.1502.

[29] Zou, C., Zhang, Y., & Wang, Z. (2006). Control chart based on change-point model for monitoring linear profiles. IIE Transactions, 38(12), 1093–1103.

[30] Zou, C., Tsung, F., Wang, Z. (2007). Monitoring general linear profiles using multivariate exponentially weighted moving average schemes. Technometrics, 49(4), 395–408.