Failure Mode and Effects Analysis Using Generalized Mixture Operators

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Abstract

Failure mode and effects analysis (FMEA) is a method based on teamwork to identify potential failures and problems in a system, design, process and service in order to remove them. The important part of this method is determining the risk priorities of failure modes using the risk priority number (RPN). However, this traditional RPN method has several shortcomings. Therefore, in this paper we propose a FMEA which uses generalized mixture operators to determine and aggregate the risk priorities of failure modes. In a numerical example, a FMEA of the LGS gas type circuit breaker product in Zanjan Switch Industries in Iran is presented to further illustrate the proposed method. The results show that the suggested approach is simple and provides more accurate risk assessments than the traditional RPN.
Keywords: Failure mode and effects analysis (FMEA); Generalized mixture operators; Fuzzy set; Risk priority number (RPN).

1. Introduction

Failure mode and effects analysis (FMEA) in design is a systematic method used to define, identify and eliminate known and/or potential failures, problems, and errors from the design of a product before the first product comes out of the production line. The FMEA is a proactive action, that is, the FMEA team predict potential problems and their causes and effects. They also define appropriate actions to remove or lessen the measure of occurrence. In other words, the main purpose of this method is to do a proactive action toward what will be happened in the future. In contrast with corrective reactions, proactive corrective actions have lower costs and take shorter time in the preliminary stages of product design.

The FMEA was first proposed by NASA in the1960s for their obvious reliability requirements. Very soon it was used to improve safety in the processes involved in chemical industries. Then in 1977 it was similarly used and promoted by Ford Motor Company (Sharma et al., 2005; Chang and Wen, 2010). The FMEA improves the design process through the following techniques and strategies:

1. Helping the designer team to assess the requirements of the design.

2. Increasing the possibility of considering potential failure modes and their effects on the costumer.

3. Providing a framework for examining and evaluating decision makers' (DMs) suggestions and required actions to reduce the risk of failure.

4. Providing a ranked list of potential failure modes to establish a scheme for improving the design and ratification of design control methods.

Modifying the failure modes in a FMEA is based on the ranking that they do. In this regard, prioritization is done by a RPN which is determined as follows:

$\mathrm{RPN} = \mathrm{S} \times \mathrm{O} \times \mathrm{D}$

where (O) is the probability of failure occurrence, (S) is the severity of the failure, and (D) is the probability of the failure being detected. These three factors are measured using the scores from one to ten according to Tables 1-3 where numbers one and ten show the least and the most important risk factor, respectively. A failure mode with a higher RPN has a higher priority and is assumed to be more important.

However, the traditional RPN has some shortcomings and has been criticized on several grounds (Bowles and Peláez, 1995; Chang et al., 2001; Sankar and Prabhu, 2001; Chin et al., 2009). Some of these disadvantages are as follows:

• Different combinations of O, S and D may produce exactly the same value of RPN, but their hidden risk implications may be totally different. For example, two different events with the values of 2, 3, 2 and 4, 1, 3 for O, S and D, respectively, have the same RPN value of 12. However, the hidden risk implications of the two events may not be necessarily the same. This may cause a waste of

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resources and time, and in some cases the high risk event may go unnoticed.

• Differences in the relative importance between O, S, and D are not considered and thus it is assumed that the three risk factors have equal importance. This may not be the case in a practical application of FMEA.

• The mathematical formula for calculating the RPN is not incorrect but it is questionable and debatable. There is no rationale behind multiplying O, S, and D to produce the RPN.

• The three risk factors are difficult to be precisely evaluated. Much information in the FMEA can be expressed in a linguistic or fuzzy way.

• The RPN is not continuous with many holes and is widely distributed at the bottom of the scale from 1 to 1000. This causes problems in interpreting the meaning of differences between different RPNs. For example, is the difference between 1 and 2 the same as or less than the difference between 900 and 1000?

• Small variations in one rating may lead to many different effects on the RPN, depending on the values of other factors (Liu et al., 2011).

A number of approaches have been proposed to solve the problems of FMEA. For example, Bowles and Peláez (1995) presented a fuzzy logic-based approach in order to prioritize failures in a system FMECA, which used linguistic terms to describe O, S, D and the riskiness of failure. The relationships between the riskiness and O, S, D were characterized by a fuzzy if–then rule base which was developed from expert knowledge and expertise. Then crisp ratings for O, S and D were fuzzified to match the premise of each possible if–then rule. All the rules that had any truth in their premises were fired to contribute to the fuzzy conclusion. The fuzzy conclusion was finally defuzzified by the weighted mean of the maximum method (WMoM) as the ranking value of the risk priority.

In another study, Chang et al. (1999) applied fuzzy theory to eliminate the conversion debate by directly evaluating the linguistic assessment of factors. They used some fuzzy linguistic terms such as very low, low, moderate, high, and very high to evaluate the degrees of the risk factors O, S and D. They also used the grey theory to obtain the risk priority numbers by assigning the relative weighting coefficient without any utility function.

Braglia (2000) developed a multi-attribute failure mode analysis (MAFMA) based on the analytic hierarchy process (AHP) technique, which considered the four different factors of O, S, D, and expected cost as decision attributes, possible causes of failure as decision alternatives, and the selection of failure cause as the decision goal. The goal, attributes and alternatives formed a three-level hierarchy, in which the pairwise comparison matrix was used to estimate the attribute weights and the local priorities of the causes with respect to the expected cost attribute. Moreover, the conventional scores of O, S and D were normalized as the local priorities of the causes with respect to O, S, and D, respectively, and the weight composition technique in the AHP was utilized to synthesize the local priorities into the global priority, based on which the possible causes of failure were ranked (Chin et al., 2009).

Braglia et al. (2003) also used the fuzzy TOPSIS (a technique for determining order preference based on similarity to the ideal solution) for ranking failure modes. Through this method, the three risk factors O, S and D and their relative importance could be assessed by triangular fuzzy numbers instead of precise crisp numbers.

Seyed-Hosseini et al. (2006) proposed a method called decision making trial and evaluation laboratory (DEMATEL) for reprioritization of failure modes in the FMEA, which prioritizes alternatives based on the severity of effect or influence and direct and indirect relationships between them. However, this approach could not address the shortcomings of the conventional RPN. In fact, when each cause of failure is assigned to only one potential failure mode, the risk ranking orders obtained by the DEMATEL approach corresponds with the ones obtained by the conventional RPN method.

More recently, Chin et al. (2009) proposed a FMEA which uses data envelopment analysis (DEA) to determine the risk priorities of failure modes. Their proposed method measures the overall risks of failure modes. Then the risk priorities are determined in terms of the overall risks rather than maximum or minimum ones.

Last but not least, Liu et al. (2011) suggested a FMEA using the fuzzy evidential reasoning (FER) approach for improving assessment grades obtained from team members and the grey theory in order to increase the accuracy of the prioritization of failure modes in the FMEA. A review and comparison of many of these methods can be found in Chin et al. (2009).

In this article, we use generalized mixture operators to rank failure modes. Using generalized mixture operators, we can select the alternative with the best scores of the most important criteria instead of selecting the alternative with the best scores of most criteria. In the FMEA, (S) is usually the most important factor in the prioritization process and other criteria like (O) and (D) are less important. With applying generalized mixture operators to the FMEA, we can prioritize failure modes with respect to their scores for the most important attributes. Thus, the prioritization process is done respectively according to (S), (O) and (D). It should be pointed out that because of the sensitivity of this method; the prioritization process is very accurate.

Table 1
Traditional rating for the occurrence of a failure (Sankar and Prabhu, 2001; Xu et al., 2002; Chin et al., 2009)

Rating	Probability of occurrence	Failure probability
10	Very high: failure is almost inevitable	>1 in 2
9		1 in 3
8	High: repeated failures	1 in 8
7		1 in 20
6	Moderate: occasional failures	1 in 80
5		1 in 400
4		1 in 2000
3	Low: relatively few failures	1 in 15,000
2		1 in 150,000
1	Remote: failure is unlikely	<1 in 1,500,000

Table 2
Traditional rating for the severity of a failure (Sankar and Prabhu, 2001; Xu et al., 2002; Chin et al., 2009)

Rating	Effect	Severity of effect
10	Hazardous without warning	Very high ranking when a potential failure mode affects the safe system operation without warning
9	Hazardous with warning	Very high severity ranking when a potential failure mode affects the safe system operation with warning
8	Very high	System inoperable with destructive failure without compromising safety
7	High	System inoperable with equipment damage
6	Moderate	System inoperable with minor damage
5	Low	System inoperable without damage
4	Very low	System operable with significant degradation of performance
3	Minor	System operable with some degradation of performance
2	Very minor	System operable with minimal interference
1	None	No effect

Table 3
Traditional rating for detection (Sankar and Prabhu, 2001; Xu et al., 2002; Chin et al., 2009)

Rating	Detection	Likelihood of detection by design control
10	Absolute uncertainty	The design control cannot detect the potential cause/mechanism and subsequent failure mode
9	Very remote	Very remote chance the design control will detect the potential cause/mechanism and subsequent failure mode
8	Remote	Remote change the design control will detect the potential cause/mechanism and subsequent failure mode
7	Very low	Very low chance the design control will detect the potential cause/mechanism and subsequent failure mode
6	Low	Low chance the design control will detect the potential cause/mechanism and subsequent failure mode
5	Moderate	Moderate chance the design control will detect the potential cause/mechanism and subsequent failure mode
4	Moderately high	Moderately high chance the design control will detect the potential cause/mechanism and subsequent failure mode
3	High	High chance the design control will detect the potential cause/mechanism and subsequent failure mode
2	Very high	Very high chance the design control will detect the potential cause/mechanism and subsequent failure mode
1	Almost certain	Design control will detect the potential cause/mechanism and subsequent failure mode

The rest of this paper is organized as follows: section 2 reviews the related literature especially some studies on fuzzy sets and generalized mixture operators. Section 3 presents the proposed approach. Then in section 4 a numerical example is given to illustrate the potential applications of the new approach in the FMEA, and the final section provides conclusions.

2. Preliminaries

In this section, in addition to some background, we introduce the mathematical tools of fuzzy sets theory and generalized mixture operators used in the proposed method.

2.1. Fuzzy sets

Fuzzy sets which are the generalizations of crisp sets were first introduced by Zadeh (1965) as a way of solving problems involving imprecise or vague data.

Unlike crisp sets, in fuzzy sets the degree of membership of each element is between zero to one. In other words, each fuzzy set is defined by a membership function which assigns a value within the unit interval [0, 1] to each element in the universe of discourse

Some basic definitions of fuzzy sets used throughout this paper are as follows:

Definition 1. (Fuzzy number). A fuzzy number $\widetilde{A}$ is a normal and convex fuzzy subset of X. Here, the 'normality' implies that (Deng et al., 2011):

$$\exists x \in R, \quad \vee_x \mu_{\widetilde{A}}(x) = 1$$

and 'convex' means that:

$$\forall x_1 \in X, \quad x_2 \in X, \qquad \forall \alpha \in [0,1],$$
$$\mu_{\tilde{A}}(\alpha x_1 + (1-\alpha)x_2) \geq \min\big(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\big). \tag{1}$$

Definition 2.The trapezoidal $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x) | x \in X\}$ fuzzy number can be denoted as $\widetilde{A} = (a, b, c, d)$ where b and c are the central values $(\mu_{\widetilde{A}}(b \leq x \leq c) = 1), a$ is the left spread, and d is the right spread (Bashiri and Bardi, 2011).

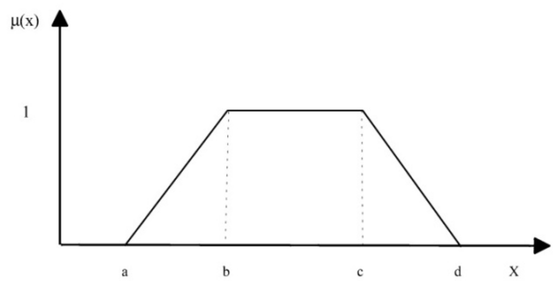

Fig. 1. Trapezoidal fuzzy number A

Note that if $b = c$, then $\tilde{A}$ is called a normal triangular fuzzy number.

Definition 3.A linguistic variable is a variable whose values are linguistic terms (Palaneeswaran and Kumaraswamy, 2001). The concept of linguistic variable is very useful for dealing with situations which are too complex or too ill-defined to be reasonably described by the conventional quantitative expressions (Zadeh, 1975).

Linguistic values can also be represented by fuzzy numbers (Bashiri and Bardi, 2011).

2.2. Generalized mixture operators

Generalized mixture operators are extended forms of weighted averaging in which weighting functions ($f_i(x_i)$) are replaced with constant weights (w_i). Weighted averaging or simple additive weighting (SAW) method is a scoring technique in the multiple attribute decision making (MADM) used for ranking alternatives and is defined as:

$$D(A_i) = \frac{\sum_{j=1}^{n} x_{ij} \times w_j}{\sum_{j=1}^{n} w_k} \tag{2}$$

Recently, within the generalized mixture approach, Marques-Pereira and Riberio (2003a, 2003b) proposed the linear and quadratic weight generating functions. They also studied the monotonicity of generalized mixture operators which are made by the weight generating functions.

In this regard, the generalized mixture operator is defined as:

$$Mn_f(x_1, x_2, \dots, x_n) = \frac{\sum_{i=1}^{n} f_i(x_i) x_i}{\sum_{i=1}^{n} f_i(x_i)} \tag{3}$$

where $\boldsymbol{f} = (f_1, f_2, \dots, f_n)$ is a vector of weighting functions which are supposed to be continuous and $\boldsymbol{X} = (x_1, x_2, \dots, x_n)$ is a vector of satisfaction values. And we have:

$$w_i(\boldsymbol{X}) = \frac{f_i(x_i)}{\sum_{i=1}^{n} f_i(x_i)} \tag{4}$$

$w_i(\boldsymbol{X})$ are positive weighting functions with the normalization condition $\sum_{i=1}^{n} w_i(\boldsymbol{X}) = 1$.

The characteristics of weighting functions and generalized mixture operators are discussed by Marques-Pereira and Riberio (2003a, 2003b).

In this study, we use the quadratic weighting function because this kind of weighting function is more sensitive to the attribute satisfaction levels than the linear weighting function. The quadratic weighting function is defined as:

$$q(x) = 1 + (\beta - \gamma)x + \gamma x^2 \tag{5}$$

and the effective weight generating function is:

$$Q(x) = \alpha \frac{q(x)}{q(1)} = \alpha \frac{1 + (\beta - \gamma)x + \gamma x^2}{1 + \beta} \tag{6}$$

where $0 \le \gamma \le 1$ and $\gamma \le \beta \le \beta_c(\gamma)$ and $0 < \alpha \le 1$. In addition, the critical beta function $\beta_c(\gamma)$ is defined as: $\beta_c(\gamma) = 1 + \gamma$ for $0 \le \gamma \le .5$ and $\beta_c(\gamma) = \gamma + 2\sqrt{\gamma(1-\gamma)}$ for $0.5 \le \gamma \le 1$ (Marques-Pereira and Riberio, 2003a, 2003b).

It is obvious that $0 < Q(0) = \alpha/(1+\beta) \le Q(1) = \alpha \le 1$, so the parameter αcontrols the value $Q(1)$ when the criteria satisfaction value is one. The parametric condition $0 \le \gamma \le 1$ also controls the measure of curvature in the effective weight generating function, and the parameter β controls the ratio between the largest and the smallest values of the effective function (11) when the attribute satisfaction values are zero and one.

The special characteristic of this method is that the weights of attributes depend continuously on the attributes satisfaction values. In some cases an important attribute with a low satisfaction value should necessarily have less effect on the overall evaluation of the alternative while a less important attribute with higher satisfaction values should have more effect on the overall evaluation of the alternative.

In other words, the quadratic and linear weight generating functions introduced in Marques-Pereira and Riberio (2003a, 2003b) can penalize (or reward) alternatives that have lower (or higher) satisfaction values for the attributes, particularly when an attribute is of high or very high importance (Marques-Pereira and Riberio, 2003b). Moreover, the generalized mixture operator is more sensitive to the variations of satisfaction values criteria than the ordered weighted average (OWA) operator and weighted averaging. Several examples of the mentioned quality are provided in Marques-Pereira and Riberio (2003a, 2003b).

3. The Proposed Method

3.1. Assessment of the risk factors using linguistic variables

Linguistic variables are frequently used in the FMEA since there are superabundant uncertainties in the FMEA procedure and decision makers tend to do assessment through linguistic variables. Likewise, in this paper we use linguistic variables for assessing the risk factors and their relative weights. The evaluation grade set is defined as a fuzzy set as follows:

$N = \{N_{11}, N_{22}, \dots, N_{77}\}$ = {very low (VL), low (L), slightly low (SL), medium (M), slightly high (SH), high (H), very high (VH)}.

In the present paper, individual assessment grades are approximated by the trapezoidal fuzzy numbers (Table 4 and Figure 2), and preliminary weights are approximated by the triangular fuzzy numbers (Table 5 and Figure 3) that are special forms of the trapezoidal fuzzy numbers.

Table 4
Linguistic variables for the ratings

Linguistic terms	Fuzzy numbers
Very low	(0, 0, 0.077, 0.154)
Low	(0.077, 0.154,0.231, 0.308)
Slightly low	(0.231, 0.308, 0.385, 0.462)
Medium	(0.385, 0.462, 0.3538, 0.615)
Slightly high	(0.538, 0.615, 0.692, 0.769)
High	(0.692, 0769, 0846, 0923)
Very high	(0.846,0923, 1, 1)

In addition, membership function values are determined through results of a questionnaire administered to the FMEA team members.

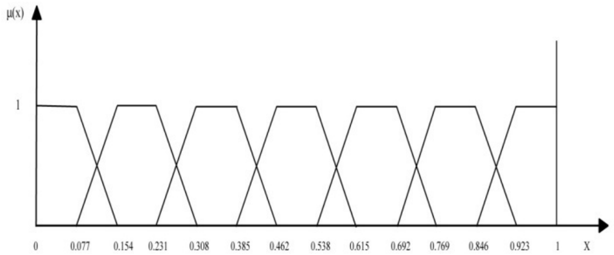

Fig. 2. The linguistic term set N with seven terms about rating

In order to make generalizations, we assume that fuzzy individual assessment grades $\{N_{ij}\}, i = 1, 2, \dots, 7; j = 1, 2, \dots, 7$ are not independent on each other and only two adjacent fuzzy individual assessment grades may intersect.

Table 5
Linguistic variables for the preliminary relative importance weights

Linguistic terms	Fuzzy numbers
Very low	(0, 0.1, 0.25)
Low	(0.15, 0.3, 0.45)
Moderate	(0.35, 0.5, 0.65)
High	(0.55, 0.7, 0.85)
Very high	(0.75, 0.9, 1)

If $\{N_{ii}\},\ i = 1, 2, \dots, 7$ and $\{N_{jj}\},\ j = i + 1\ to\ 7; i = 1, 2, \dots, 7$ are two adjacent fuzzy numbers, then $\{N_{ij}\}, i = 1, 2, \dots, 7; j = 1 + i$ will be the interval fuzzy number. Since $\{N_{ii}\},\ i = 1, 2, \dots, 7$ and $\{N_{jj}\},\ j = i + 1 \text{ to } 7; i = 1, 2, \dots, 7$ are trapezoidal fuzzy numbers, we can assume that the interval fuzzy number $\{N_{ij}\}, i = 1, 2, \dots, 7; j = 1 + i$ is a trapezoidal fuzzy number, as shown in Figure 4.

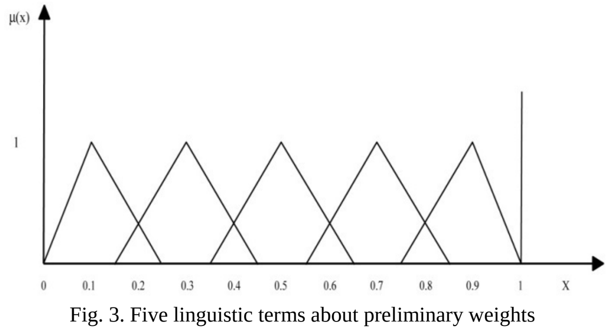

Fig. 3. Five linguistic terms about preliminary weights

When team members' assessment grade is not exactly $\{N_{ii}\}, i = 1, 2, \dots, 7$ or $\{N_{jj}\},\ j = i + 1 \text{ to } 7; i = 1, 2, \dots, 7$ and their assessment is something between $\{N_{ii}\},\ i = 1, 2, \dots, 7$ and $\{N_{jj}\},\ j = i + 1 \text{ to } 7; i = 1, 2, \dots, 7$, or they are not confident that their assessment grade is $\{N_{ii}\}, i = 1, 2, \dots, 7$ or $\{N_{jj}\},\ j = i + 1 \text{ to } 7; i = 1, 2, \dots, 7$, evaluation will be more flexible through the aforesaid method.

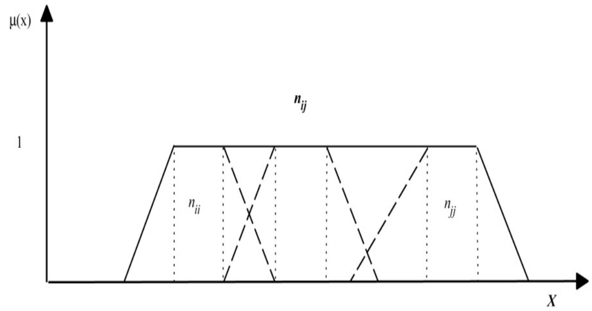

Fig. 4. The interval fuzzy grade set (Liu et al, 2011)

3.2. Aggregating team members' judgments

Suppose there are L decision makers $(D_1, D_2, \dots, D_L)$ in a FMEA team, who assess $\boldsymbol{M}$ failure modes $(F_1, F_2, \dots, F_M)$ with respect to $\boldsymbol{G}$ risk factors $(R_1, R_2, \dots, R_G)$. The weights defined by the decision makers for the risk factors is shown by $\widetilde{W}_l = (w_{1l}, w_{2l}, w_{3l})\ ; l = 1, 2, \dots, L$, too. The weight of each decision maker is denoted by $\delta_l\ ; l = 1, 2, \dots, L$, in which $\sum_{l=1}^{L} \delta_l = 1$.

In order to simplify calculations of the aggregating procedure, team members' assessments should be defuzzified through one of the defuzzification methods. The centroid defuzzification method defines the centroid coordinate of $\tilde{A}$ just in the horizontal axis as its defuzzified value and can be expressed as follows (Uehara and Hirota, 1998; Wang, 2009):

$$\tilde{x}_0(\tilde{A}) = \frac{\int_a^d x\mu_{\tilde{A}}(x)dx}{\int_a^d \mu_{\tilde{A}}(x)dx} \tag{7}$$

Compared with other defuzzification methods, the centroid defuzzification is more advantageous as it considers the degree of membership of each element in the fuzzy set. Hence, we use this method for the defuzzification of team members' opinions. Yet for simplification purposes, in this article the centroid formula for assessment grades and preliminary weights is:

$$n_{ii} = \frac{1}{3}\left[a + b + c + d - \frac{dc - ab}{(d + c) - (a + b)}\right] \tag{8}$$

$$W_l = \frac{a + b + c}{3} \tag{9}$$

The overall assessment grades and the preliminary weights of the risk factors can be obtained by the arithmetic mean. For each assessment grade, the arithmetic mean can be shown as:

$$\acute{n}_{ıȷ} = \frac{\sum_{l=1}^{L} n_{ijl}\delta_l}{\sum_{l=1}^{L}\delta_l} \quad (10)$$

$i = 1, 2, \dots, 7\ ;\ \ j = 1, 2, \dots, 7$

$l = 1, 2, \dots, L$

and the overall preliminary weight of the risk factor R_g of L decision makers is denoted as:

$$\acute{w}_g = \frac{\sum_{l=1}^{L} W_{gl}\delta_l}{\sum_{l=1}^{L}\delta_l} \quad (11)$$

$l = 1, 2, \dots, L$

$g = 1, 2, \dots, G$

3.3. Prioritizing failure modes by generalized mixture operators

In this phase, the overall assessments obtained from equation (10) are used as different values of x in equation (6), and the overall preliminary weights resulted from equation (11) are used as the parameter α in equation (6). Values of the parameters β and γ are arbitrary and depend on team members' preferences and problems conditions. To facilitate the calculations, β and γ are assumed to be equal for each criterion. If we suppose $\acute{n}_{ıȷg}\ ; g = 1, 2, \dots, G$ are the overall assessment grades (that were obtained from equation (10)) with respect to the risk factor R_g, then for each failure mode equation (6) turns out to be:

$$Q_g(\acute{n}_{ıȷg}) = \acute{w}_g \frac{q(\acute{n}_{ıȷg})}{q(1)} = \acute{w}_g \frac{1 + (\beta - \gamma)\acute{n}_{ıȷg} + \gamma {\acute{n}_{ıȷg}}^2}{1 + \beta}$$

$i = 1, 2, \dots, 7\ ;\ \ j = 1, 2, \dots, 7$

$g = 1, 2, \dots, G$ (12)

As shown in equation (12), the effective weight generating function is composed of two different parts. One portion is made of the parameter α and the other portion is made of the fractional expression. In this function, the parameter α can be replaced by the pre-weights given by experts and x_i variables can be replaced by the assessment grades of the decision making matrix. Thus, the weights of the risk factors depend on both the pre-weights given by experts and the values of the decision making matrix. Therefore, in this method we can use experience of experts with considering the particular circumstances of the issue according to the decision making matrix. Effective weight generating functions make a balance between the weights provided by experts and the weights resulted from the decision making matrix. In this method, if satisfaction values of the decision making matrix are high, we can conclude that the pre-weight of that particular risk factor provided by the expert in the particular alternative is correct; otherwise, the pre-weight must be corrected.

By the aforesaid formula, generalized weights of each risk factor with respect to each failure mode can be calculated. The above-mentioned values are briefly shown in the following matrix:

$$\begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_M \end{bmatrix} = \begin{bmatrix} Q_1(\acute{n}_{ıȷ1}) & Q_1(\acute{n}_{ıȷ2}) & \cdots & Q_1(\acute{n}_{ıȷG}) \\ Q_2(\acute{n}_{ıȷ1}) & Q_2(\acute{n}_{ıȷ2}) & \cdots & Q_2(\acute{n}_{ıȷG}) \\ \vdots & \vdots & \dots & \vdots \\ Q_M(\acute{n}_{ıȷ1}) & Q_M(\acute{n}_{ıȷ2}) & \cdots & Q_M(\acute{n}_{ıȷG}) \end{bmatrix}$$

Finally, aggregated assessment grades of each failure mode with respect to each risk factor are gained from equation (3) as follows:

$$Mn_Q(\acute{n}_{ıȷg}) = \frac{\sum_{g=1}^{G} Q_g(\acute{n}_{ıȷg})\acute{n}_{ıȷg}}{\sum_{g=1}^{G} Q_g(\acute{n}_{ıȷg})} \quad (13)$$

$i = 1, 2, \dots, 7\ ;\ \ j = 1, 2, \dots, 7$

$g = 1, 2, \dots, G$

Priorities of failure modes are based on their scores obtained from equation (13). In this regard, the highest score shows the highest priority of the identified failure modes.

Another advantage of generalized mixture operators is the dominance effect of criteria. It means that if the satisfaction value of a criterion approaches a complete number (which in this paper is 1), the dominance effect of this particular criterion increases. For example, the dominance effect of criterion C_i with the satisfaction value of 0.9 is very bigger than that of another criterion with the satisfaction value of 0.8, because in generalized mixture operators generating functions depends constantly on the satisfaction values of criteria and they increase or decrease the role of each criterion in equation 13 according to the parameters β and γ and pre-weights provided by experts.

This characteristic is very useful in the FMEA since, as we will show in section 4, it makes it easy to recognize the failure modes with higher satisfaction values, especially the risk factor S.

3.4. The procedure of the proposed method

The procedure of the proposed method can be condensed into nine steps and explained as follows:

Step 1. Listing potential failure modes of the component or product by the FMEA team.

Step 2. Listing all possible causes of each failure mode.

Step 3. Defining a suitable scale and preliminary weight for each risk factor by the FMEA team.

Step 4. Assessing each failure mode with respect to the risk factors.

Step 5. Converting linguistic assessments into fuzzy numbers through equations (8) and (9).

Step 7. Aggregating the team members' opinion into the overall opinion through equations (10) and (11).

Step 8. Ranking failure modes through equation (13).

4. Case Study

In this section, this study uses a real world case in order to demonstrate the proposed approach. The proposed method is used to improve the quality of a product of Zanjan Switch Company which is one of the largest manufacturers of medium and high voltage circuit breakers and disconnectors in Iran.

The product is LGS which is a live tank SF6 auto puffer circuit-breaker designed for 72.5 kV and a rated breaking current of 25-31.5 kA. In order to improve it, the Corporation plans to minimize its failures. Due to the high volume sale of the LGS, the company initially intends to rank the failures in this product. To do so, four decision makers are selected based on the extent of their familiarity with the product and their experience. More specifically, the FMEA team members are a system engineer, a design engineer, a manufacturing engineer, and a representative for the services purchased. These team members are assigned these relative weights, respectively: 0.2, 0.4, 0.25 and 0.15. According to the experience of the decision makers, the assessment grades and the preliminary weights of the risk factors *S*, *O*, and *D* are defined and organized in Tables 6 and 7.

Now, the assessment grades of the team are defuzzified through equation (8). The defuzzified preliminary weights are presented in Table 8.

Then, different opinions of the four members of the FMEA team are synthesized using equations (10) and (11). The aggregated assessments are reported in Table 9. The aggregated preliminary weights for S, O and D are 0.853, 0.46 and 0.314, respectively. Having obtained the overall assessment grades and preliminary weights, now we can use equation (12) to determine the effective weights of each risk factor.

Table 6

Assessment information on the fifteen failure modes by the four members of the FMEA team

Failure mode	Risk factor											
	Severity (S)				Occurrence (O)				Detection (D)			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
Gas leak	n_4	n_{44}	n_{67}	n_{44}	n_{55}	n_{45}	n_{44}	n_{45}	n_{44}	n_{12}	n_{22}	n_{22}
Engine failure	n_6	n_{67}	n_{77}	n_{77}	n_{11}	n_{11}	n_{11}	n_{11}	n_{12}	n_{11}	n_{12}	n_{11}
High ohmic resistance	n_3	n_{23}	n_{23}	n_{22}	n_{22}	n_{22}	n_{12}	n_{12}	n_{44}	n_{34}	n_{44}	n_{34}
Damper oil leaks	n_1	n_{12}	n_{23}	n_{23}	n_{12}	n_{22}	n_{12}	n_{11}	n_{11}	n_{22}	n_{11}	n_{12}
Numerator failure	n_3	n_{34}	n_{44}	n_{34}	n_{11}	n_{12}	n_{11}	n_{11}	n_{34}	n_{44}	n_{34}	n_{33}
Coil failure	n_6	n_{67}	n_{66}	n_{66}	n_{66}	n_{56}	n_{55}	n_{55}	n_{66}	n_{56}	n_{56}	n_{66}
Failure density manometer	n_4	n_{45}	n_{34}	n_{44}	n_{33}	n_{33}	n_{34}	n_{34}	n_{34}	n_{34}	n_{34}	n_{44}
Limit switches do not charging the spring	n_5	n_{55}	n_{44}	n_{55}	n_{11}	n_{11}	n_{11}	n_{12}	n_{11}	n_{22}	n_{22}	n_{12}
Failure to stay connected	n_5	n_{44}	n_{56}	n_{44}	n_{12}	n_{11}	n_{22}	n_{12}	n_{12}	n_{12}	n_{11}	n_{12}
Reset command	n_4	n_{45}	n_{44}	n_{44}	n_{12}	n_{11}	n_{11}	n_{11}	n_{11}	n_{33}	n_{23}	n_{22}
Performance indicator	n_1	n_{12}	n_{12}	n_{11}	n_{11}	n_{11}	n_{12}	n_{12}	n_{55}	n_{55}	n_{44}	n_{56}
Disconnecting and connecting simultaneously	n_4	n_{34}	n_{56}	n_{55}	n_{33}	n_{23}	n_{12}	n_{12}	n_{44}	n_{34}	n_{33}	n_{33}
Permanent alarm	n_1	n_{12}	n_{12}	n_{11}	n_{22}	n_{11}	n_{12}	n_{22}	n_{23}	n_{33}	n_{22}	n_{23}
Going over the handle	n_1	n_{22}	n_{12}	n_{11}	n_{45}	n_{44}	n_{45}	n_{45}	n_{12}	n_{22}	n_{12}	n_{11}
Impossibility of closure density manometer	n_4	n_{44}	n_{45}	n_{45}	n_{34}	n_{33}	n_{33}	n_{23}	n_{44}	n_{33}	n_{34}	n_{44}

Table 7

Preliminary weights considered by the FMEA team members

DFMEA team members	Severity (S)	Occurrence (O)	Detection (D)
$\boldsymbol{D_1}$	0.88	0.5	0.30
$\boldsymbol{D_2}$	0.88	0.3	0.12
$\boldsymbol{D_3}$	0.70	0.5	0.30
$\boldsymbol{D_4}$	0.88	0.5	0.50

Table 8
Defuzzied crisp numbers for fuzzy assessment grades of the FMEA team members

Failure mode	Risk factor											
	Severity (O)				Occurrence (O)				Detection (D)			
	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4	D_1	D_2	D_3	D_4
Gas leak	0.577	0. 500	0.839	0.500	0.623	0.577	0.500	0.577	0.500	0.136	0.192	0.192
Engine failure	0.807	0.839	0.940	0.940	0.060	0.060	0.060	0.060	0.136	0.060	0.136	0.060
High ohmic resistance	0.346	0.269	0.269	0.192	0.192	0.192	0.136	0.136	0.500	0.423	0.500	0.423
Damper oil leaks	0.060	0.136	0.269	0.269	0.136	0.192	0.136	0.060	0.060	0.192	0.060	0.136
Numerator failure	0.423	0.423	0.500	0.423	0.060	0.136	0.060	0.060	0.423	0.500	0.423	0.346
Coil failure	0.807	0.839	0.807	0.807	0.807	0.730	0.623	0.623	0.807	0.730	0.730	0.807
Failure density manometer	0.500	0.577	0.423	0.500	0.346	0.346	0.423	0.423	0.423	0.423	0.423	0.500
Limit switches are not charging the spring	0.730	0.623	0.500	0.623	0.060	0.060	0.060	0.136	0.060	0.192	0.192	0.136
Failure to stay connected	0.623	0.500	0.730	0.500	0.136	0.060	0.192	0.136	0.136	0.136	0.060	0.136
Reset command	0.500	0.577	0.500	0.500	0.136	0.060	0.060	0.060	0.060	0.346	0.269	0.192
Performance indicator	0.060	0.136	0.136	0.060	0.060	0.060	0.136	0.136	0.623	0.623	0.500	0.730
Disconnect and connect simultaneously	0.500	0.423	0.730	0.623	0.346	0.269	0.136	0.136	0.500	0.423	0.346	0.346
Permanent alarm	0.060	0.136	0.136	0.060	0.192	0.060	0.136	0.192	0.269	0.346	0.192	0.269
Going over the handle	0.060	0.192	0.136	0.060	0.577	0.500	0.577	0.577	0.136	0.192	0.136	0.060
Impossibility of closure density manometer	0.500	0.500	0.577	0.577	0.423	0.346	0.346	0.269	0.500	0.346	0.423	0.500

As mentioned in section 3, values of the parameters β and γ are arbitrary, so we assume the values of β and γ are 1.5 and 0.7, respectively. Thus, for example, the effective weight of the risk factor (S) with respect to the failure mode of engine failure can be calculated as follows:

$$Q_1(0.867) = 0.853\frac{q(0.867)}{q(1)}$$
$$= 0.853\frac{1 + (1.5 - 0.7)0.867 + 0.7 \times 0.867^2}{1 + 1.5} \quad = 0.757$$

Now, we should normalize the pre-weights to obtain the normalized weights as shown below:

$$Q_1(.\text{Engine failure}) = 0.757/(0.757 + 0.193 + 0.137)$$
$$= 0.696$$

Table 9
Aggregated assessment grades of the FMEA team members

Failure modes	Severity (S)	Occurrence (O)	Detection (D)
1	0.477	0.584	0.307
2	0.867	0.060	0.102
3	0.281	0.170	0.465
4	0.159	0.128	0.105
5	0.434	0.075	0.419
6	0.813	0.718	0.780
7	0.504	0.377	0.442
8	0.647	0.079	0.125
9	0.584	0.129	0.125
10	0.515	0.090	0.182
11	0.087	0.090	0.630
12	0.550	0.247	0.423
13	0.087	0.157	0.273
14	0.134	0.562	0.128
15	0.531	0.356	0.458

The pre-weights and normalized weights are reported in Table 10.

Table 10
Pre-weights and normalized weights of each criterion with respect to each failure mode

Failure mode	Pre- weight Normalized weight					
	Severity (S)	Occurrence (O)	Detection (D)	Severity (S)	Occurrence (O)	Detection (D)
1	0.526	0.314	0.165	0.5234	0.3124	0.1642
2	0.757	0.193	0.137	0.696	0.178	0.126
3	0.436	0.213	0.192	0.519	0.253	0.228
4	0.390	0.205	0.137	0.533	0.280	0.187
5	0.504	0.196	0.184	0.570	0.222	0.208
6	0.721	0.356	0.258	0.540	0.267	0.193
7	0.539	0.258	0.188	0.547	0.262	0.191
8	0.618	0.196	0.139	0.648	0.206	0.146
9	0.582	0.205	0.139	0.629	0.221	0.150
10	0.545	0.198	0.147	0.612	0.223	0.165
11	0.367	0.198	0.225	0.465	0.250	0.285
12	0.563	0.228	0.184	0.577	0.234	0.189
13	0.367	0.210	0.160	0.498	0.285	0.217
14	0.380	0.307	0.140	0.460	0.371	0.169
15	0.553	0.253	0.191	0.555	0.254	0.191

The ranks of failure modes are acquired by equation (13), and are presented in Table11.

Table 11
Aggregated assessment grades of the FMEA team members, results of the proposed method and the risk priority ranking

Failure mode	Generalized mixture operator	Priority ranking	Traditional RPN	Priority ranking
1	0.482	3	0.085	3
2	0.626	2	0.005	12
3	0.295	11	0.022	6
4	0.141	15	0.002	14
5	0.351	10	0.014	7
6	0.781	1	0.455	1
7	0.459	5	0.084	4
8	0.453	7	0.006	11
9	0.415	8	0.009	9
10	0.365	9	0.008	10
11	0.242	13	0.005	12
12	0.455	6	0.057	5
13	0.147	14	0.004	13
14	0.292	12	0.010	8
15	0.472	4	0.086	2

As shown in Table 11, the failure mode numbered 6 has the top priority for correction. The failure mode numbered 2 is the second main priority as identified by the generalized mixture operators but is the twelfth priority in the traditional RPN. But as shown in Table 9, the satisfaction value of the risk factor S in the failure mode numbered 2 is 0.867 and it has the highest assessment grade for the risk factor S among the other failure modes.

Practically, the risk factor S has the most important role among the three risk factors in the FMEA. Thus, this satisfaction value for the failure mode numbered 2 shows the hazard of error. But, the traditional RPN is not able to consider a priority higher than 12 for the failure mode numbered 2 as it does not take account of the weights of risk factors. Besides, if we consider the SAW method as an aggregation method, it can consider priority 3 for this failure mode, because weights of values in this method are constant. Therefore, it fails to distinguish between high and low satisfaction values. This shows that the generalized mixture operator which is made of the quadratic weighting function is more complete and realistic than the traditional RPN.

As mentioned in section 3.3, using weight generating functions, we can also make decisions by considering experts' experiences and circumstances of the problem. Furthermore, we can make weight generating function for each criterion and towards each alternative without any interference caused by other criteria and alternatives. As a result, through this method we counteract any virtual increases and decreases of criteria in the aggregating process. In short, generalized mixture operators have special characteristics which cannot be found in other MADM methods.

5. Conclusion

This paper proposed a novel method for assessing the risk of failures in the FMEA. The suggested method helps with the two main controversial parts of the FMEA that are the aggregation of team members' opinions and ranking the failure modes. Because of difficulty in acquiring precise assessment information on failure modes, we proposed a method which allows the risk factors and their important weights to be evaluated in a linguistic manner and fuzzy rule. The main part of this paper used the generalized mixture operators through which the weights of the risk factors can be dependent on the satisfaction values of the risk factors and experts' experiences. This method allowed us to penalize the risk factors with low satisfaction values and reward criteria with well- satisfaction values. Finally the results showed that the proposed method provides more accurate and responsible data for decision makers to identify the most critical failure modes and assign limited resources to the most serious risk items.

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