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Research Article

The multiple attribute group decision-making problems with interval-valued intuitionistic fuzzy numbers: A linear programming approach

Maryam Arshi [1], **Abdollah Hadi-Vencheh** [2], **Adel Aazami** [3,*], **Tara Hamlehvar** [4]

1. Department of Mathematics, Najafabad Branch, Islamic Azad University, Isfahan, Iran
2. Department of Mathematics, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran
3. Department of Global Business and Trade, Institute for Transport and Logistics Management, Vienna University of Economics and Business, Vienna, Austria
4. Department of Industrial & System Engineering, Auburn University, Alabama State, United States

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Abstract

The objective of this manuscript is to introduce an innovative methodology for addressing multiple attribute group decision-making (MAGDM) problems utilizing interval-valued intuitionistic fuzzy sets (IVIFS). The proposed approach solves the problem using a mathematical programming methodology. In the present investigation, a group decision-making problem characterized by IVIF multiple attributes is conceptualized as a linear programming model and resolved expeditiously. The models that are being proposed have been reformulated into two analogous linear programming (LP) models through the application of a variable transformation and the concept of aggregation operators. The obtained LP models are solvable by common approaches. The principal benefit of the suggested methodology is its facilitation of decision-makers (DM) in identifying an alternative that exhibits optimal performance, and the decision-making process does not rely on DM knowledge. Application of the proposed method is represented in a decision-making problem, and the results are compared with similar methods, proving the compatibility of the proposed method with previous ones. The solid and understandable logic with computational easiness are the main advantages of the proposed method.

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* **Corresponding Author:**
Adel Aazami
Department of Global Business and Trade, Institute for Transport and Logistics Management, Vienna University of Economics and Business, Vienna, Austria
E-Mail: **adel.aazami@wu.ac.at**

1. Introduction

Decision-making primarily examines the framework within which an individual decision-maker (DM) or a collective decision-making group deliberates upon a course of action in an environment characterized by uncertainty. The field of decision theory facilitates the identification of the alternative that possesses the highest expected value, which can also be interpreted as the likelihood of attaining a prospective outcome. The overarching objective of formulating decision-making theory is to aid individuals in selecting from a predetermined array of alternatives. In the domain of operations research, the principal domains of inquiry frequently encompass multi-criteria decision-making problems. From a managerial perspective, decision-making problems are generally classified into two predominant categories: selection and planning problems (Simon, 1977). In the context of MCDM (multi-criteria decision-making), challenges arise when multiple criteria must be fulfilled to reach an effective decision. Furthermore, Multi-Criteria Decision Making (MCDM) encompasses both MODM (Multi-Objective Decision Making) and MADM (Multi-Attribute Decision Making) (Climaco, 2012). Generally, in MODM, trouble with making plans is considered. However, by MADM, selections are issued. A MADM problem can be described as follows: Let $B=\{ b_1, b_2, \dots, b_m\}$ denote a non-empty, finite collection of decision alternatives, and $D= \{ d_1, d_2, \dots, d_n \}$ is a finite set of goals, criteria, or attributes, consistent with the desirability of a judged alternative. MADM aims to decide the most desirable alternative with the very best diploma of desirability, admired for all related objectives (Zimmermann, 1987).

Human information is regularly inexact and incomplete. (Yovits, 1988) stated that uncertainty could also arise because of approximate or partial records. The maximum of our facts regarding our adjacent phenomena is decided approximately or partially. Consequently, observing a few frameworks to deal with these uncertainties appears essential.

The fuzzy set theory is among the broadly common frameworks about uncertainty (Zadeh, 1965). Fuzzy sets represent an extension of classic set theory in which each element within a universal set is allocated a degree of membership. Unlike classical sets, which clearly distinguish non-membership and membership, fuzzy sets allow for varying degrees of membership. This theory is extensively used in addressing decision-making problems.

(Grattan Guinness, 1976) and (Gau & Buehrer, 1993) suggested that simply presenting a linguistic expression within a fuzzy set (FS) might not always be adequate. To address this, (Atanassov, 2016) introduced the concept of intuitionistic fuzzy sets (IFS), expanding on Zadeh's original fuzzy sets by incorporating the aspect of hesitancy.

The Interval-valued Intuitionistic Fuzzy Set (IVIFS) demonstrates significant utility in the context of MAGDM or MADM problems, wherein the components of vector W or matrix D are characterized as IVIFNs. It is worth noting that the famous MADM approaches were developed in IVIF forms. The technique for Order of Preference by Similarity to the Ideal Solution (TOPSIS) problem was extended by (Tan, 2011), (Ye, 2010), and (Park, Park, et al., 2011) under IVIF frameworks with specific distances within IVIFNs and anti-ideal and ideal descriptions. (Park, Cho, et al., 2011) gift an IVIF extension of the VIKOR (Vise Kriterijumska Optimizacija I Kompromisno Resenje) approach. A closeness coefficient was proposed by (Li, 2011) based on a nonlinear programming technique to solve IVIF MADM problems. (Arshi et al., 2023, 2024) introduced an innovative methodology aimed at addressing multiple attribute group decision-making (MAGDM) problems through the application of interval-valued intuitionistic fuzzy sets (IVIFS) and neutrosophic numbers. MADM problems were solved by (Chen et al., 2012), who introduced innovative fuzzy ranking techniques for Intuitionistic Fuzzy Values (IFVs), utilizing Interval-Valued Intuitionistic Fuzzy (IVIF) weighted average operators. The IVIF continuous weighted entropy was proposed by (Jin et al., 2014), and a method was developed for MAGDM problems conditional over the IVIF attitudinal expected score function and weighted relative closeness. (Garg & Kumar, 2020) introduce an innovative MADM approach using an IVIF (interval-valued intuitionistic fuzzy) set framework. This methodology integrates the Technique for Order Preference by Similarity to the Ideal Solution (TOPSIS). (Hajiagha et al., 2015) introduced a linear programming framework for multi-attribute group decision-making (MAGDM) challenges. This framework addressed a sequence of issues in an iterative manner, culminating in a definitive score computed for each alternative.

(Sadabadi et al., 2021) converted a fuzzy multiple criteria decision-making (FMCDM) issue into two linear programming models using a simple additive weighting method (SAW). (Garg & Arora, 2018) proposed a nonlinear programming framework that employs the TOPSIS methodology to address the complexities inherent in multi-attribute decision-making problems. (Wan et al., 2020) developed a method for addressing MAGDM problems using IVIF data. They defined the weights of every DM with admiration for the attribute. Subsequently, an interval-programming multi-objective model was solved to determine the weights of the attributes. Lastly, the comprehensive interval values of the alternatives were used to rank them. (Saffarzadeh et al., 2020) proposed a method for determining the weights of Decision Makers (DMs) in Group Multiple Criteria Decision-Making (GMCDM) problems based on interval data. (Mohammadghasemi et al., 2019) proposed the TOPSIS method with the incorporation of Gaussian Interval Type-2 Fuzzy Sets (GIT2FSs) as a more flexible alternative to conventional triangular Membership Functions (MFs). Their work emphasized the effectiveness of GIT2FSs in accurately modeling curved MFs. (Tang & Meng, 2018) proposed the Induced Generalized Symmetrical Interval-Valued Intuitionistic Fuzzy Choquet-Shapley (IG-SIVIFCS) operator, which takes into account the global importance of elements as well as their global interactions. (Elsayed, 2024) proposed a multiple decision-making model (MCDM) to evaluate and schedule environmentally friendly fuel alternatives with the aim of reducing greenhouse gas emissions. This method includes the effect of removing the value (MEREC) to obtain the weight of the value and using the TODIM method to give the options using triangular neutrosophic numbers. (Ishizaka & Siraj, 2018) assessed public transport satisfaction using a combined MCDM technique. This approach integrates Delphi, Group AHP (GAHP), and the Preference Ranking Organization Method for Enrichment of Assessments.

(Abdollahi & Pour-Moallem, 2020) proposed a method combining TOPSIS and entropy to rank various Demand Response Resources (DRRs). (Abootalebi et al., 2018) developed an approach to address Group Multiple Criteria Decision-Making (GMCDM) problems, specifically for situations where data is precise with crisp values. (Meng & Tan, 2017) explored intuitive, hesitant, fuzzy linguistic distance measures (Pamu ar et al., 2018) presented a hybrid Multi-Criteria Decision-Making (MCDM) framework that amalgamates Interval Rough Analytic Hierarchy Process (IRAHP) with Multi-Attributive Border Approximation Area Comparison (MABAC). (Hajek & Froelich, 2019) presented how TOPSIS can be integrated with intuitionistic fuzzy cognitive maps valued by intervals to enhance group decision-making effectiveness. (Isen & Boran, 2018) created a hybrid model utilizing a genetic algorithm, an adaptive neuro-fuzzy inference system, and fuzzy c-means for inventory classification. (Perez-Canedo & Verdegay, 2023) employed a lexicographic approach to ascertain distinct optimal fuzzy objective values, juxtaposing these findings with outcomes derived from linear ranking function methodologies. Furthermore, they elucidated the applicability of the lexicographic method in the domains of dietary planning and the analysis of time-cost trade-off problems within fuzzy contexts. Table 1 shows a brief description of the literature review.

Given that the attributes of interval-valued intuitionistic fuzzy sets can substantially improve the articulation of uncertainty and the administration of ambiguous data in decision-making contexts, this research endeavors to introduce an innovative framework for addressing Multi-Attribute Group Decision-Making (MAGDM) problems utilizing IVIF information. For this purpose, we employ linear programming (LP) techniques to solve the problem. As we know, solving the LP models is easy and simple. Indeed, the salient contributions of this research encompass: firstly, the presentation of an innovative methodology for tackling Multiple Attribute Group Decision-Making (MAGDM) problems when confronted with Interval-Valued Intuitionistic Fuzzy (IVIF) data; second, employing linear programming to solve an MCDM problem with IVIF numbers; third, the proposed methodology is simple and can be extended to an MCDM problem with any number of alternative and criteria.

The present investigation is organized in the following manner: Section 2 offers a comprehensive overview of IVIFSs along with essential concepts, and Section 2 addresses the specific problem and its formulations. Section 3 elucidates the proposed method for addressing the problem. A numerical example demonstrating the applicability of the proposed technique is presented in Section 4. Finally, Section 5 draws several conclusions.

Table 1
Brief description of the literature

Author	Year	Method or subject
Jin et al.	2014	The IVIF continuous weighted entropy
Ishizaka & Siraj	2018	A public transport satisfaction using a combined MCDM problem
Abootalebi et al.	2018	Determining expert weights in the GMADM problem
Isen & Boran	2018	Genetic algorithm, adaptive neuro-fuzzy inference system, and fuzzy c-means
Hajek & Froelich	2019	Integration of TOPSIS with interval-valued intuitionistic fuzzy cognitive maps
Mohammadghasemi et al.	2019	Using TOPSIS method with the incorporation of Gaussian Interval Type-2 Fuzzy Sets
Saffarzadeh et al.	2020	Presenting a method to determine decision makers' weight (DMs) in GMCDM
Abdollahi & Pour-Moallem	2020	A technique based on TOPSIS and entropy
Garg & Kumar	2020	Integration of IVIF and TOPSIS
Wan et al.	2020	Solving MAGDM problems with Atanassov's interval-valued intuitionistic fuzzy values
Sadabadi et al.	2021	Transforming MAGDM into linear programming model
Khan et al.	2021	Using linguistic interval-valued Q-rung Orthopair fuzzy TOPSIS method
Azam et al.	2022	A Decision-Making Approach for the Evaluation of Information Security Management
Perez-Canedo & Verdagay	2023	Using lexicographic method
Arshi et al.	2023	Transforming MAGDM into a linear programming model
Elsayed	2024	Using MCDM to evaluate and schedule environmentally friendly fuel alternatives
Arshi et al.	2024	Solving MAGDM problems with interval-valued neutrosophic numbers

2. Mathematical Foundation

(Atanassov & Gargov, 1989) generalized the IFS idea to IVIFSs. Consider E[0,1] as the complete collection of closed subintervals within the interval [0,1]. Let Y denote a particular non-empty set in which an IVIFS (Interval-Valued Intuitionistic Fuzzy Set) is articulated through the expression $\hat{B}=\{y, \mu_{\hat{B}}(y), \nu_{\hat{B}}(y) \,|\, y \in Y\}$, where $\mu_{\hat{B}}: Y \to E[0,1]$, $\nu_{\hat{B}}: Y \to E[0,1]$ under the condition $0 < sup_y \mu_{\hat{B}}(y) + sup_y \nu_{\hat{B}}(y) \le 1$.

The intervals $\mu_{\hat{B}}(y)$, $\nu_{\hat{B}}(y)$ represent the degrees of non-membership and membership associated with the element y to the set B. Therefore, for every $y \in Y, \mu_{\hat{B}}(y)$and $\nu_{\hat{B}}(y)$

denote closed intervals with upper and lower endpoints represented by $\mu_{BL}(y), \mu_{BU}(y), \nu_{BL}(y)$, and $\nu_{BU}(y)$.

The IVIFS B is denoted by

$$B = \{< y, [\mu_{BL}(y), \mu_{BU}(y)], [\nu_{BL}(y), \nu_{BU}(y)] > | y \in Y\} \quad (1)$$

Where $0 < \mu_{BU}(y) + \nu_{BU}(y) \leq 1$, $\mu_{BL}(y), \nu_{BL}(y) \geq 0$. An *IVIF* set value is represented by $\hat{B}$= *([e,f], [g,h])* for convenience, and termed as an IVIFN. If $\hat{B}_1$ = *([e₁,f₁], [g₁,h₁])* and $\hat{B}_2$=*([e₂,f₂], [g₂,h₂])* be any two IVIFNs; their operational laws are determined as follows (Wan et al., 2020):

$$\hat{B}_1 + \hat{B}_2 = ([e_1 + e_2 - e_1e_2, f_1 + f_2 - f_1f_2], [g_1g_2, h_1h_2]) \quad (2)$$

$$\hat{B}_1 . \hat{B}_2 = ([e_1e_2, f_1f_2], [g_1 + g_2 - g_1g_2, h_1 + h_2 - h_1h_2]) \quad (3)$$

$$\lambda\hat{B}_1 = ([1 - (1 - e_1)^\lambda, 1 - (1 - f_1)^\lambda], [g_1^\lambda, h_1^\lambda]) \ , \ \lambda \geq 0 \quad (4)$$

The score function was defined by (Xu, 2007) for an IVIFN $\hat{B}$=*([e, f], [g, h])* as

$$s(\hat{B}) = \frac{1}{2}(e\text{-}g\text{+}f\text{-}h) \quad (5)$$

And its accuracy functions as

$$h(\hat{B}) = \frac{1}{2}(e+g+f+h) \quad (6)$$

for two IVIFNs $\hat{B}_1$ and $\hat{B}_2$,

1. If $(\hat{B}_1)$ <s$(\hat{B}_2)$, then $\hat{B}_1$ is smaller compared to $\hat{B}_2, \hat{B}_1 < \hat{B}_2$.

2. If s$(\hat{B}_1)$ = s$(\hat{B}_2)$, then

(2.1) When h$(\hat{B}_1)$ = h$(\hat{B}_2)$ thus, $\hat{B}_1 = \hat{B}_2$.

(2.2) When h$(\hat{B}_1)$ < h$(\hat{B}_2)$ thus, $\hat{B}_1$ is smaller than $\hat{B}_2, \hat{B}_1 < \hat{B}_2$ (Xu [3 ·])

Let $\hat{B}_j$ =*([eⱼ, fⱼ], [gⱼ, hⱼ])* ,*j= 1,2, … , n* as a set of IVIFNs. Then, the generalized interval intuitionistic fuzzy weighted average GIFWB$_w(\hat{B}_1, \hat{B}_2, \dots, \hat{B}_n)$ GIIFWB$_w(\hat{B}_1, \hat{B}_2, \dots, \hat{B}_n)$ is defined as follows:

$$GIIFWB_w(B_1, B_2, \dots, B_n) = (w_1\hat{B}_1^\lambda + w_2\hat{B}_2^\lambda + \dots + w_n\hat{B}_n^\lambda)^{\frac{1}{\lambda}} \quad (7)$$

in which $\lambda > 0$, and W= $[w_1, w_2, \dots, w_n]^T$ represents the weight vector with $w_j \geq 0$, *j= 1, 2, …, n*, and $\sum_{j=1}^n w_j = 1$. It is found that GIIFWB is also an IVIFN, which is determined as (Zhao et al., 2010):

$$GIIFWB_w(\hat{B}_1, \hat{B}_2, \dots, \hat{B}_n) = ([(1 - \prod_{j=1}^n (1 - e_j^\lambda)^{w_j})^{\frac{1}{\lambda}}, (1 - \prod_{j=1}^n (1 - f_j^\lambda)^{w_j})^{\frac{1}{\lambda}}], [1\text{-}(1 - \prod_{j=1}^n (1 - (1 - g_j)^\lambda)^{w_j})^{\frac{1}{\lambda}}, 1 - (1 - \prod_{j=1}^n (1 - (1 - h_j)^\lambda)^{w_j})^{\frac{1}{\lambda}}]) \quad (8)$$

If λ =0, thus, Eq. (6) is changed into IIFWB (interval intuitionistic fuzzy weighted average).

Assuming a collective of K decision-makers evaluating the set of alternatives *B={B₁, B₂, …, Bₙ}* regard to criteria set *D={D₁, D₂, …, Dₙ}*, each decision maker individually makes the evaluations along with an individual decision matrix A^k=$[\hat{x}_{ij}^k]$. The $\hat{x}_{ij}^k$ are expressed in the form of an IVIFN $\hat{x}_{ij}^k = [(\underline{\mu}_{ij}^k, \overline{\mu}_{ij}^k), (\underline{\nu}_{ij}^k, \overline{\nu}_{ij}^k)]$, $1 \leq i \leq m ; 1 \leq j \leq n$. The issue at hand pertains to the determination of hierarchies or evaluations of alternatives, thereby facilitating decision-makers in selecting their preferred alternative(s) or hierarchical arrangement.

An aggregated decision matrix A is initially formulated via the GIIFWB methodology (by assigning a predetermined weight to diverse experts) or through the IIFWB approach (taking into account the perspectives of all experts as equivalent, i.e. $w_k = \frac{1}{K}$, $k = 1,2, \dots, K$). The aggregated decision matrix will be obtained in the form of *A*=$[\hat{x}_{ij}]$ where,

$$\hat{x}_{ij} = GIIFWB_w(\hat{x}_{ij}^1, \hat{x}_{ij}^2, \dots, \hat{x}_{ij}^k) \quad (9)$$

The extended variant of the aggregated matrix A is denoted as:

$$A = \begin{bmatrix} \hat{x}_{11} & \cdots & \hat{x}_{1n} \\ \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \cdots & \hat{x}_{mn} \end{bmatrix} \quad (10)$$

When addressing the issue of ranking alternatives within set B, the subsequent formulation is taken into account for the Multi-Attribute Group Decision-Making (MAGDM) problem:

$$\begin{aligned} gI_i = Max \sum_{j=1}^n w_j^g \hat{x}_{ij} \\ s.t \\ \sum_{j=1}^n w_j^g \hat{x}_{ij} \leq \hat{1} \quad 1 \leq i \leq m \quad (i) \\ w_j^g \geq 0 \quad 1 \leq j \leq n \quad (ii) \\ \sum_{j=1}^n w_j^g = 1 \quad (iii) \\ w_j^g \geq w_k^g \forall (j,k) \in \{1, \dots, n\} \ j \neq k \quad (iv) \\ w_j^g \geq \quad (v) \end{aligned} \quad (11)$$

in which $\hat{1}$ represents an IVIFN, like [(0.9, 0.95), (0.01, 0.05)], and w_j , $1 \leq j \leq n$ denotes the criterion j importance weight. This framework, originating from data envelopment analysis and proposed by (Ramanathan, 2006) as the R-model, operates as a weighted linear optimization instrument for the resolution of multi-criteria inventory classification problems. It aims to maximize the overall score of each alternative i through an objective function that is a linear representation of the criteria. In accordance with constraint (i), the cumulative score associated with each alternative is required to remain below the value of 1, employing analogous weights for the respective alternative. Concurrently, constraint (ii) delineates that all weights attributed to the criteria must possess a positive value. The summation of weights is restricted by the arbitrary constraints (iii)-(v) as normalized, and some predetermined preferences are imposed over criteria weights. A lower bound is determined by constraint (v) to prevent no criteria weight to be 0. This model is solved repeatedly for each alternative and ranked in descending order based on their scores. To enhance its functionality, we extended the R-model and proposed a corresponding weighted linear optimization model, organized in the following structure:

$$
\begin{aligned}
& bI_i = Min \sum_{j=1}^{n} w_j^b \, \hat{x}_{ij} \\
& \quad s.t. \\
& \sum_{j=1}^{n} w_j^b \hat{x}_{ij} \geq \hat{1} \quad 1 \leq i \leq m \\
& w_j^b \geq 0 \qquad 1 \leq j \leq n \\
& \sum_{j=1}^{n} w_j^b = 1 \\
& w_j^b \geq w_k^b \forall (j,k) \in \{1, \dots, n\}, j \neq k \\
& w_j^b \geq \varepsilon
\end{aligned} \tag{12}
$$

To calculate the weights in model (12) for the ith alternative, a weighted linear optimization is employed. This process includes constraints ensuring that the weighted sum of all alternatives, using the same set of weights, must be greater than or equal to $\hat{1}$.

3. Development of the proposed method

The MAGDM issues delineated in Equations (11) and (12) can be conceptualized as dual linear programming problems under the context of Interval-Valued Intuitionistic Fuzzy (IVIF) information.

An optimization problem is formally defined as a mathematical construct intended to ascertain the supremum or infimum of a particular real-valued function f across a designated subset G of a universal set X, that is to sa

$$\alpha = inf\{\, f(x) : x \in G \,\}, \quad G \subseteq X \tag{13}$$

In the context of optimization problems, acquiring the value of α or correspondingly, an $x_0 \in G$ that $f(x_0) = \alpha$ (Ponstein, 2004) is incorporated. Two matrix representations of linear programming are delineated in Equations (14) and (15).

$$
\begin{aligned}
& Max\ CX \\
& s.t \\
& AX \leq b\,, \quad X \geq 0
\end{aligned} \tag{14}
$$

And

$$
\begin{aligned}
& Min\ CX \\
& s.t \\
& AX \geq b\,, \quad X \geq 0
\end{aligned} \tag{15}
$$

In this context, X denotes the column vector of decision variables, A signifies the technological matrix, C represents the row vector of cost (or profit) coefficients, and b indicates the right-hand side vector corresponding to resources. Under the presumption of the certainty axiom, every element of matrix A, in addition to the vectors b and C, is ascertained to be deterministic in nature.

LP is prolonged below specific uncertainty frameworks. As an example, refer to the study of (Charnes & Cooper, 1959), (Madansky, 1960) for stochastic LP, (Chen et al., 2004), (Ishibuchi & Tanaka, 1990), (Dang & Forrest, 2009), (Jimenez et al., 2007), and (Kaur & Kumar, 2013) for fuzzy LP. Nevertheless, the wide variety of works on LP with IVIF information is restricted. In this context, an innovative methodology has been developed to address the linear programming (LP) challenges, wherein the parameters are articulated as Interval-Valued Intuitionistic Fuzzy Numbers (IVIFNs). In this investigation, two linear programming models incorporating IVIF data, with parameters denoted as A, b, and C, are established as IVIFNs.

$$
\begin{aligned}
& Max\ \hat{C}X \\
& s.t. \\
& \hat{A}X \leq \hat{b} \\
& X \geq 0
\end{aligned} \tag{16}
$$

And

$$
\begin{aligned}
& Min\ \hat{C}X \\
& s.t. \\
& \hat{A}X \geq \hat{b} \\
& X \geq 0
\end{aligned} \tag{17}
$$

For a long-form, IVIF-LP in Eqs. (16) and (17) stated as:

$$
\begin{aligned}
& Max \sum_{j=1}^{n} \hat{c}_j x_j \\
& s.t. \\
& \sum_{j=1}^{n} \hat{a}_{ij} x_j \leq \hat{b}_i \qquad 1 \leq i \leq m \\
& x_j \geq 0 \qquad 1 \leq j \leq n
\end{aligned} \tag{18}
$$

And

$$
\begin{aligned}
& Min \sum_{j=1}^{n} \hat{c}_j x_j \\
& s.t. \\
& \sum_{j=1}^{n} \hat{a}_{ij} x_j \geq \hat{b}_i \qquad 1 \leq i \leq m \\
& x_j \geq 0 \qquad 1 \leq j \leq n
\end{aligned} \tag{19}
$$

The definite parameters in Eq. (18), (19) are IVIFNs set as:

$\hat{c}_j = [(c_{1j}, c_{2j}), (c_{3j}, c_{4j})]$, $1 \leq j \leq n$ where (c_{1j}, c_{2j}) and (c_{3j}, c_{4j}) are membership and non-membership intervals, respectively.

$\hat{a}_{ij} = [(a_{1ij}, a_{2ij}), (a_{3ij}, a_{4ij})]$, $1 \leq i \leq m; 1 \leq j \leq n$ where(a_{1ij}, a_{2ij}) and (a_{3ij}, a_{4ij}) are membership and non-membership intervals, respectively.

$\hat{b}_i = [(b_{1i}, b_{2i}), (b_{3i}, b_{4i})]$, $1 \leq\ i \leq m$ where (b_{1i}, b_{2i}) is membership and (b_{3i}, b_{4i}) is a non-membership interval.

Now, take into account the objective function $\sum_{j=1}^{n} \hat{c}\, x_j$. Since objective function coefficients $\hat{c}_j,\ 1 \le j \le n$ are IVIFNs; thus, the objective function is represented as a linear combination of these Interval Valued Intuitionistic Fuzzy Numbers (IVIFNs) by means of non-negative coefficients $x_j \ge 0, 1 \le j \le n$. The outcomes of this linear combination may be obtained through an interactive process utilizing both the multiplication and summation operators, which are delineated in relations (2) and (4), respectively. This inductive procedure comprises scalar multiplication in conjunction with the IVIF summation operation, and it comprehensively includes all relevant operations. A simple transformation of variables may be employed to diminish the total number of operations. The variable t is defined as follows:

$$t = \frac{1}{x_1 + x_2 + \cdots + x_n} \tag{20}$$

Now, the objective function $\sum_{j=1}^{n} \hat{c}_j x_j$ is multiplied by t. Determining the variable $tx_j = y_j, j = 1, \dots, n,$ the objective function is converted into:

$$\sum_{j=1}^{n} \hat{c}_j y_j \tag{21}$$

Since $\sum_{j=1}^{n} y_j = 1$ and $y_j \ge 0, 1 \le j \le n$, relation (21) can be inferred as IIFWA of a set of IVIFNs $\hat{c}_j, 1 \le j \le n$. Using relation (8), relation (21) is converted as:

$$(\left[\left(1-\prod_{j=1}^{n}(1-c_{1j})^{y_j}\right),\left(1-\prod_{j=1}^{n}(1-c_{2j})^{y_j}\right)\right],[\prod_{j=1}^{n}c_{3j}^{y_j},\prod_{j=1}^{n}c_{4j}^{y_j}]) \tag{22}$$

Using t in Equation (20) helps in deriving a closed form for the objective functions. As for the explanation of the score functions in Equation (5), an IVIFN is optimized by increasing its membership degree while its non-membership degree is decreased. Also, reducing the membership degree will minimize an IVIFN while increasing its non-membership degree. Assume two interval numbers $A = [\underline{a}, \overline{a}]$ and $B = [\underline{b}, \overline{b}]$. Then, $A \ge B$ if $\underline{a} \ge \underline{b}$ and $\overline{a} \ge \overline{b}$ (Wang & Li, 2012), thus, Eq. (22) will be maximized if $(1-\prod_{j=1}^{n}(1-c_{1j})^{y_j})$ and $(1-\prod_{j=1}^{n}(1-c_{2j})^{y_j})$ are maximized while $\prod_{j=1}^{n} c_{3j}^{y_j}$ and $\prod_{j=1}^{n} c_{4j}^{y_j}$ are minimized. These conditions are met when:

$\prod_{j=1}^{n}(1-c_{1j})^{y_j}$ and $\prod_{j=1}^{n}(1-c_{2j})^{y_j}$ are minimized, and simultaneously;

$\prod_{j=1}^{n} c_{3j}^{y_j}$ and $\prod_{j=1}^{n} c_{4j}^{y_j}$ are minimized.

Consequently, the singular objective function associated with the IVIF-LP problem is reformulated as:

$$Min\ (\prod_{j=1}^{n}(1-c_{1j})^{y_j},\prod_{j=1}^{n}(1-c_{2j})^{y_j},\prod_{j=1}^{n}c_{3j}^{y_j},\prod_{j=1}^{n}c_{4j}^{y_j}) \tag{23}$$

Given that the natural logarithm function exhibits an increasing characteristic, the process of minimizing the aforementioned IVIF components is tantamount to the "minimization" of the natural logarithm of these components, as:

$$Min\ (\sum_{j=1}^{n} y_j.Ln\,(1-c_{1j}),\sum_{j=1}^{n} y_j.Ln\,(1-c_{2j}),\sum_{j=1}^{n} y_j.Ln\,(c_{3j}),\sum_{j=1}^{n} y_j.Ln\,(c_{4j})) \tag{24}$$

All the elements of the above vector are in [0,1]. Thus, minimizing it corresponds to "minimization" of the summation of its elements. That is,

$$Min \sum_{j=1}^{n} y_j.Ln\,((1-c_{1j})(1-c_{2j})c_{3j}c_{4j} \tag{25}$$

Now, consider the ith constraint $\sum_{j=1}^{n} \hat{a}_{ij} x_j (\le = \ge)\, \hat{b}_i$, for a specific value $i, 1 \le i \le m$ in relations (18) and (19). To address this constraint, both sides are multiplied by t, relation (20). Hence, the initial constraint is transformed into $\sum_{j=1}^{n} \hat{a}_{ij} y_j (\le = \ge) t\hat{b}_i$. Taking into account the constraint's left side, The IIFWA operator of a set of IVIFNs, denoted as $\hat{a}_{ij}, 1 \le j \le n$, can be expressed as follows:

$$([(1-\prod_{j=1}^{n}(1-a_{1ij})^{y_j}),(1-\prod_{j=1}^{n}(1-a_{2ij})^{y_j})],[\prod_{j=1}^{n}a_{3ij}^{y_j},\prod_{j=1}^{n}a_{4ij}^{y_j}]) \tag{26}$$

The product $t\hat{b}$ can be handled on the right side, in terms of the scalar multiplication in Eq. (4), as:

$$([1-(1-b_1)^t, 1-(1-b_2)^t],[b_3^t, b_4^t]) \tag{27}$$

In relation to $\le$ kind constraints, the membership on the right side must be inferior to that of the left constraints, while its non-membership should be superior. This comparative interval principle is:

$$\begin{cases} 1-\prod_{j=1}^{n}(1-a_{1ij})^{y_j} \le 1-(1-b_1)^t \\ 1-\prod_{j=1}^{n}(1-a_{2ij}) \le 1-(1-b_2)^t \\ \prod_{j=1}^{n} a_{3ij}^{y_j} \ge b_3^t \\ \prod_{j=1}^{n} a_{4ij}^{y_j} \ge b_4^t \end{cases} \tag{28}$$

The constraints set in Eq. (28) are converted in the linear structure using the logarithm Neperien function:

$$\begin{cases} \sum_{j=1}^{n} y_j Ln(1-a_{1ij}) \geq tLn(1-b_1) \\ \sum_{j=1}^{n} y_j Ln(1-a_{2ij}) \geq tLn(1-b_2) \\ \sum_{j=1}^{n} y_j Ln(a_{3ij}) \geq tLn(b_3) \\ \sum_{j=1}^{n} y_j Ln(a_{4ij}) \geq tLn(b_4) \end{cases} \quad (29)$$

Utilizing a similar theoretical framework, an analogous linear set of constraints is formulated for each constraint that is of equal or greater significance:

$$\begin{cases} \sum_{j=1}^{n} y_j Ln(1-a_{1ij}) \leq tLn(1-b_1) \\ \sum_{j=1}^{n} y_j Ln(1-a_{2ij}) \leq tLn(1-b_2) \\ \sum_{j=1}^{n} y_j Ln(a_{3ij}) \leq tLn(b_3) \\ \sum_{j=1}^{n} y_j Ln(a_{4ij}) \leq tLn(b_4) \end{cases} \quad (30)$$

All the above $\leq$ or $\geq$ signs in relations (29) and (30) will be altered into equality for equality type constraints.
Ultimately, the IVIF-LP problems in relations (18) and (19) are transformed into equivalent LP problems, as illustrated in relations (31) and (32). Upon resolution of this issue, the optimal parameters t* and y_j^*, $1 \leq j \leq n$ are established. By employing a reverse transformation in accordance with Equation (20), the optimal values of the initial variablesx_j^*, $1 \leq j \leq n$, are determined.

$$gI_i = Min \sum_{j=1}^{n} y_j.\ Ln((1-c_{1j})(1 - c_{2j})c_{3j}c_{4j})$$

s.t.

$$\begin{cases} \sum_{j=1}^{n} y_j Ln(1-a_{1ij}) \geq tLn(1-b_1) \\ \sum_{j=1}^{n} y_j Ln(1-a_{2ij}) \geq tLn(1-b_2) \\ \sum_{j=1}^{n} y_j Ln(a_{3ij}) \geq tLn(b_3) \\ \sum_{j=1}^{n} y_j Ln(a_{4ij}) \geq tLn(b_4) \end{cases} \quad (31)$$

$$\sum_{j=1}^{n} y_j = 1$$

$$t, y_j \geq 0; \quad j = 1,2,\dots,n$$

And

$$bI_i = Max \sum_{j=1}^{n} y_j Ln((1-c_{1j})(1-c_{2j})c_{3j}c_{4j})$$

s.t.

$$\begin{cases} 1-\prod_{j=1}^{n}(1-a_{1ij})^{y_j} \leq 1-(1-b_1)^t \\ 1-\prod_{j=1}^{n}(1-a_{2ij}) \leq 1-(1-b_2)^t \\ \prod_{j=1}^{n} a_{3ij}^{y_j} \geq b_3^t \\ \prod_{j=1}^{n} a_{4ij}^{y_j} \geq b_4^t \end{cases}$$

$$\begin{cases} \sum_{j=1}^{n} y_j Ln(1-a_{1ij}) \leq tLn(1-b_1) \\ \sum_{j=1}^{n} y_j Ln(1-a_{2ij}) \leq tLn(1-b_2) \\ \sum_{j=1}^{n} y_j Ln(a_{3ij}) \leq tLn(b_3) \\ \sum_{j=1}^{n} y_j Ln(a_{4ij}) \leq tLn(b_4) \end{cases} \quad (32)$$

$$\sum_{j=1}^{n} y_j = 1$$

$$t, y_j \geq 0, \quad 1 \leq j \leq n$$

The problems in relations (31) and (32) are two linear programming problems that are solved using ordinal techniques. Such a process can be carried out to resolve the MAGDM problems in relations (11) and (12).
Solving relations (11) and (12), we reap the values gI_i and bI_i. Since gI_i and bI_i are oriented by respectively the weights with the highest and lowest favorability for the ith alternative. We can refer to these as the "good" and "bad" indexes for a multi-attribute decision-making problem. By integrating these two extremes, we are able to construct a composite index as follows:

$$nI_i(\lambda) = \lambda.\frac{gI_i - gI^-}{gI^* - gI^-} + (1-\lambda).\frac{bI_i - bI^-}{bI^* - bI^-} \quad (33)$$

Where $gI^* = \max\{gI_i, 1 \leq i \leq m\}$, $gI^- = \min\{gI_i, 1 \leq i \leq m\}$, $bI^* = \max\{bI_i, 1 \leq i \leq m\}$, $bI^- = \min\{bI_i, 1 \leq i \leq m\}$. The parameter $0 \leq \lambda \leq 1$ acts as a control variable that represents the decision maker's inclination towards the bad and good indexes. If $\lambda = 1, nI_i$ becomes a normalized version of the good index I_i. Conversely, when λ equals $0, nI_i$ transforms into a normalized version of the bad index I_i. In various cases, (33) includes both indices. If decision-makers do not have a strong preference, choosing $\lambda = 0.5$ would be a fairly reasonable and neutral option.

4. Using in MAGDM

Here, a MAGDM problem is used to recommend that undergraduate students illustrate using the presented method. (Wang & Li, 2012) proposed this problem for the first time.
Without losing the generality, it is assumed that three committee members (DMs) exist as d_1, d_2, and d_3 and four students (x_1, x_2, x_3, and x_4) because the finalists are followed using initial screening. Using all DMs, it is agreed to assess such candidates in opposition to 4 capabilities of educational information, a_1; college English take a look at Band rating, a_2; teamwork abilities, a_3; and studies potentials, a_4. Here, it is assumed that there is an agreement in the group to evaluate qualitative features on five linguistic terms. A conversion scale that correlates IVIFNs with linguistic terms is reported in Table 2.

Table 2
IVIF scale utilized to evaluate alternative

Linguistic terms	IVIFNs
'Very nice' (VN)	([0.9,0.95],[0.02,0.05])
'Nice' (N)	([0.7,0.75],[0.2,0.25])
'Fair' (F)	([0.5,0.55],[0.4,0.45])
'Poor' (P)	([0.2,0.25],[0.7,0.75])
'Very poor' (VP)	([0.02,0.05],[0.9,0.95])

It is important to note that the technique presented is not affected by the IVIF scale used to convey the judgments of decision-makers. Therefore, any arbitrary scale can be performed. Nevertheless, here, the presented scale of (Wang & Li, 2012) is utilized. The decision-makers may use the suggested scale in Table 2 to represent their different ideas.
For the assessment of subjective characteristics, the aforementioned scale or an alternative measurement is employed. In order to address objective (quantitative) attributes, it is imperative to implement a methodology that transforms specific quantitative data into Table 2.
IVIFNs. It is supposed that an objective attribute is assessed as (hundred, a hundred and fifty, a hundred and twenty, ninety) for a hard and fast, which consists of 4 alternatives. The values of attribute vectors are then normalized. The obtained normalized vector was 0.7, 1, 0.8, and 0.6. Subsequently, the desirability of these values can be ascertained by the Decision Maker (DM) utilizing a linguistic scale. For example, assuming that the DM articulates its perspective in proximity to the aforementioned normalized vector as (VN, F, N, F). Consequently, the Interval Valued Intuitionistic Fuzzy (IVIF) scale functions to convert these linguistic values into their respective IVIF representations.
Table 2 also represents the DMs' IVIF evaluations on alternative performances regarding various criteria.

(Wang & Li, 2012) estimated the criteria's weight vector as [0.268,0.394,0.151,0.185]. The aggregated decision matrix considers decision matrices in Table 3.
Taking into account the alternative x_1, in terms of Eq. (11), the formulation of the MAGDM model is:

$$Max\ [(0,0),(0.5,0.749)]w_1 + [(0.548,0.571),(0.177,0.296)]w_2 + ([0.578,0.630],[0.317,0.370])w_3 + ([0.856,0.915],[0.043,0.085])w_4$$

s.t.

$$[(0,0),(0.5,0.749)]w_1 + [(0.548,0.571),(0.177,0.296)]w_2 + ([0.578,0.630],[0.317,0.370])w_3 + ([0.856,0.915],[0.043,0.085])w_4 \le [(0.9,0.95),(0.01,0.05)]$$

$$([0.872,0.926],[0.037,0.055])w_1 + ([0.219,0.228],[0.319,0.532])w_2 + ([0.200,0.250],[0.700,0.750])w_3 + ([0.500,0.550],[0.400,0.450])w_4 \le [(0.9,0.95),(0.01,0.05)]$$

$$([0.435,0.462],[0.269,0.403])w_1 + ([0.878,0.914],[0.036,0.059])w_2 + ([0.578,0.630],[0.317,0.370])w_3 + ([0.753,0.822],[0.117,0.178])w_4 \le [(0.9,0.95),(0.01,0.05)]$$

$$([0.218,0.231],[0.385,0.577])w_1 + ([0,0],[0.413,0.690])w_2 + ([0.578,0.630],[0.317,0.370])w_3 + ([0.500,0.550],[0.400,0.450])w_4 \le [(0.9,0.95),(0.01,0.05)]$$

$$\sum_{j=1}^{4} w_j = 1$$

$$w_j \ge 0, \quad 1 \le j \le 4$$

This IVIF linear programming problem can be solved via converting it into a corresponding Eq. (31) model.

$$Min - 0.98y_1 - 4.59y_2 - 4y_3 - 10.01y_4$$

s.t.

$$0y_1 - 0.79y_2 - 0.86y_3 - 1.94y_4 \ge -2.30t$$
$$0y_1 - 0.85y_2 - 0.99y_3 - 2.47y_4 \ge -3t$$
$$-0.69y_1 - 1.73y_2 - 1.15y_3 - 3.15y_4 \ge -4.61t$$
$$-0.29y_1 - 1.22y_2 - 0.99y_3 - 2.47y_4 \ge -3t$$
$$-2.06y_1 - 0.25y_2 - 0.22y_3 - 0.69y_4 \ge -2.30t$$
$$-2.60y_1 - 0.26y_2 - 0.29y_3 - 0.80y_4 \ge -3t$$
$$-3.30y_1 - 1.14y_2 - 0.36y_3 - 0.92y_4 \ge -4.61t$$
$$-2.90y_1 - 0.63y_2 - 0.29y_3 - 0.80y_4 \ge -3t$$
$$-0.57y_1 - 2.10y_2 - 0.86y_3 - 1.40y_4 \ge -2.30t$$
$$-0.62y_1 - 2.45y_2 - 0.99y_3 - 1.73y_4 \ge -3t$$
$$-1.31y_1 - 3.32y_2 - 1.15y_3 - 2.15y_4 \ge -4.61t$$
$$-0.91y_1 - 2.83y_2 - 0.99y_3 - 1.73y_4 \ge -3t$$
$$-0.25y_1 - 0y_2 - 0.86y_3 - 0.69y_4 \ge -2.30t$$
$$-0.26y_1 - 0y_2 - 0.99y_3 - 0.80y_4 \ge -3t$$
$$-0.95y_1 - 0.88y_2 - 1.15y_3 - 0.92y_4 \ge -4.61t$$
$$-0.55y_1 - 0.37y_2 - 0.99y_3 - 0.80y_4 \ge -3t$$

$$\sum_{j=1}^{4} y_j = 1$$

$$y_j \geq 0 , 1 \leq j \leq 4$$

$$t \geq 0$$

The model (12) and the transformer to model (32) are written similarly.

Table 3
Aggregated decision matrix

	A_1	A_2	A_3	A_4
x_1	([0,0],[0.5,0.749])	([0.548,0.571],[0.177,0.296])	([0.578,0.630],[0.317,0.370])	([0.856,0.915[,[0.043,0.085])
x_2	([0.872,0.926],[0.037,0.055])	([0.219,0.228],[0.319,0.532])	([0.200,0.250],[0.700,0.750])	([0.500,0.550],[0.400,0.450])
x_3	([0.435,0.462],[0.269,0.403])	([0.878,0.914],[0.036,0.059])	([0.578,0.630],[0.317,0.370])	([0.753,0.822],[0.117,0.178])
x_4	([0.218,0.231],[0.385,0.577])	([0,0],[0.413,0.690])	([0.578,0.630],[0.317,0.370])	([0.500,0.550],[0.400,0.450])

Table 4
Objective values for alternative

	gI_i	bIi
x_1	-10.01	-0.98
x_2	10.86	-1.15
x_3	-10.72	-3.44
x_4	-4.60	-1.27

We applied the suggested model with $\lambda = 0.5$.

$bI^* = -0.98,\quad bI^- = -3.44,\qquad gI^* = -4.60,\ gI^- = -10.86$

$nI_1 = 0.5675,\quad nI_2 = 0.465,\quad nI_3 = 0.011,\quad nI_4 = 0.94$

Table 5
Ranking of alternatives with different methods

	The proposed method	TOPSIS-IVF	(Wan & Dong, 2020)	(Wan et al., 2020)
x_1	2	3	2	2
x_2	3	2	3	3
x_3	4	1	1	1
x_4	1	4	4	4

5. Using in MAGDM

Numerous practical problems can be formulated within exceptional managerial, engineering, social, and economic fields as the MCDM problems. According to (Vincke, 1992), the significant difficulties of MCDM problems are caused by not being precisely determined and the lack of definite solutions for them. Such challenges are strengthened when the uncertainty notion is considered as an inevitable feature of such problems. Numerous attempts exist to provide analysts with instruments to deal with MCDM problems within indefinite situations. Regarding the capabilities of IVIFSs in explaining the ill-defined and uncertain data in DM problems, two new linear programming-based models were presented in the present work for solving MAGDM problems with IVIF information. Driven by Data Envelopment Analysis (DEA), a rigorous logical framework is delineated by the suggested formulation. Several models were resolved iteratively within this methodology, culminating in an ultimate score for each alternative. These scores are employed to rank and compare alternatives.

The proposed technique involves solving multiple models, with each model corresponding to a different alternative. After setting up the problem for the first alternative. Upon the establishment of the problem for the initial alternative, the objective functions were modified in accordance with multiple alternatives, while the region continued to be invariant and feasible. Therefore, it is crucial to develop multiple objective functions that share similar constraints. Considering the suggested model as an LP problem with IVIF parameters, a method is designed for solving such a problem, using an applied IVIFNs aggregation operator and a variable transformation to solve the crisp problem. Then, we proposed a combinational model. This model is a standardized index in the interval [0,1] through two weight sets as the most and least favourable for each alternative. At last, a numerical example was provided to explain the proposed methodology.

The primary benefits of the proposed method can be outlined as follows: firstly, it enhances the motivation to use mathematical optimization models when addressing MAGDM problems. Recognizing this, the method offers a framework for decision-making through the resolution of two sets of optimization models. Secondly, the rationale behind developing this method is clear and well-founded. Maximizing or minimizing the weighted average of each alternative, with scores restricted to less than one, closely resembles the principles of Data Envelopment Analysis (DEA), a well-established method. Importantly, the model requires less information than other methods since attribute weights do not need to be predetermined; the model determines them internally. Despite the wide-ranging applications of IVIF in MAGDM (multi-attribute group decision-making) problems, mathematical programming with IVIF information has received limited attention. We plan to develop more generalized interactive aggregation operators utilizing various fuzzy environments (Khan et al., 2019; Azam et al., 2022; Khan et al., 2020, 2021). For future research, one can extend the proposed methodology to other types of neutrosophic numbers. For instance, an extension of the proposed method in the presence of a single-valued trapezoidal neutrosophic number (SVTrNN). Further, in the context of this paper, future research can formulate the linear programming model with IVNNs. The application of IVNNs can also be examined to find the utility function of decision-makers in multi-attribute utility theory.

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