Journal of Optimization in Industrial Engineering,
Vol.17, Issue 2, Summer & Autumn 2024, 15-30
DOI: 10.22094/QJIE.2024.1121407

Research Article

Virtual alliance in hospital network for operating room scheduling: Benders decomposition

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Received: 01 June 2024; Revised: 10 August 2024; Accepted: 15 August 2024

Abstract

This study deals with the scheduling of operating room networks of collaborative hospitals with the arrival of emergency patients. In this study, a set of independently owned hospitals form a virtual alliance network to increase resource utilization and reduce patient waiting time. Each hospital, in collaboration with other members, is primarily responsible for providing services to its patients and may have a different objective function, which has a priority over the overall objective function of the virtual distributed scheduling collaborative hospitals. So, the objective function of the problem is divided into two categories, but the overall objective function of the network is to reduce the cost of allocating patients to hospitals and surgeons, along with the cost of operating room overtime. In this study, to make the situation more realistic, the transshipment of the patient from one hospital to another is also taken into account. For this problem, a mixed-integer mathematical programming model is presented, and the Benders decomposition algorithm is designed. The efficiency of the algorithm was compared with experiments performed with the CPLEX solver, and finally, the results were reported. The results in the one-day planning horizon show a better performance of about 5% in the Benders algorithm in all three objective functions, which is achieved in 50% of the runtime by the proposed algorithm. In the two-day planning horizon, despite the closeness of the results of the second and third objective functions in the two methods, the results of the Benders algorithm were seven times better in the makespan, which was obtained in a quarter of the runtime. Here, the more important point is the possibility of solving problems with larger dimensions with the Benders algorithm in a situation that was not possible by CPLEX due to the inherent complexity of the problem.

Keywords: Operating room scheduling; Distributed systems; Collaborative planning; Virtual alliance; Benders decomposition

1. Introduction

In recent decades, healthcare systems in different countries have faced problems such as an increase in patients' demand, service delivery, limited government support, and increased competition. Also, one of the common problems in healthcare systems in recent years is the simultaneous growth of service demand and patients' expectations of service quality and increasing healthcare costs (Chen et al. 2019). The rapid and increasing cost of healthcare systems is such that how to control these costs has become a significant problem in the healthcare systems of various countries. As a result, today, hospital managers and officials are always looking to reduce costs and improve the financial situation of hospitals in order to overcome the existing problems. Managers, on the other hand, strive to provide the highest level of patient satisfaction and seek innovative approaches that address most of these problems. These factors have led managers and decision-makers in the field of healthcare systems to try to increase productivity and the efficiency of healthcare systems. Therefore, productivity and efficiency in healthcare systems increase by creating proper planning and scheduling for all activities involved in this industry (Farughi et al., 2019).

Given that operating rooms in hospitals account for a large share of costs and are the most valuable section of the hospital, it can be said that operating room scheduling is one of the vital sections of a hospital (Abdeljaouad et al. 2020). Operating room scheduling also has a significant care unit, general wards, laboratories, and emergency department. As a result, much research has been done on the problem of operating room planning and scheduling (Noorizadegan & Seifi 2018). In the real world, increasing productivity and competitiveness has led hospital management always to try to improve performance and increase patient satisfaction. Therefore, they seek to establish parallel communication and collaborative hospitals that are from hospitals with independent ownership and separately can work together. Distributed operating rooms and sharing operating room blocks in collaborative hospitals have been proven to be a cost-effective way to improve the efficiency and productivity of operating rooms and surgeons (Roshanaei et al. 2017b).

In this study, in order to improve the competitive situation, in order to achieve the best objective function, flexibility, and cost competitiveness, an attempt has been made to address the problem of collaborative hospital operating room scheduling. In this type of network, each hospital may have a different objective function, which has a priority compared to the overall objective function of the virtual distributed scheduling network. Also, by considering two groups of patients (elective and emergency) and transferring the patient from one hospital to another according to the time of patient transportation, the conditions of the system under-study have been tried to be as close as possible to the real-world healthcare system. For this purpose, after modeling the operating room scheduling problem in collaborative hospitals in the form

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of a multi-objective model, the Benders decomposition algorithm method is proposed to solve the problem.

In the following, in Section 2, the related papers are reviewed. Section 3 is dedicated to problem definition, mathematical model, and problem complexity. In Section 4, the Benders decomposition algorithm and how to implement it in the proposed model are introduced. In Section 5, numerical results will be reported. Finally, after concluding, future researches are suggested for future studies in Section 6.

2. Literature Review

The operating room scheduling was first studied in 1953 by Adair (Adair 1953). In most of the optimization research based on operating room scheduling, simple hypotheses have been used, which often include: deterministic in the duration of surgery, lack of emergency patient arrival, sufficient resources, and centralized operating room scheduling problems. In the following, some papers in this field are discussed.

Tyler et al. (2003) used a simulation method to increase efficiency by about 85% to 90% in operating room scheduling. He has considered the reduction of patients' waiting time in the hospital. He has also identified the factors affecting the optimal and efficient use of resources in the operating room scheduling. Jebali et al. (2006) introduced a two-stage programming approach to operating room scheduling. The first stage involves assigning surgeries to operating rooms. The second stage involves determining the order and sequence of operations assigned to operating rooms for optimal use, taking into account the various resource constraints.

Arnaout and Kulbashian (2008) considered the operating room scheduling problem as a parallel machine scheduling problem with sequence-dependent setup times in order to minimize completion time. Then they presented a new heuristic method to solve the model. In their study, Zonderland et al. (2010) considered the single operating room scheduling hypothesis and developed the planning and scheduling process of elective and semi-urgent surgeries based on Markov's decision theory. Their objective function was to treat semi-urgent patients and canceled elective patients to treat urgent patients due to the capacity of the operating room for a long time. In Van Essen et al. (2012), for emergency surgeries, operating room scheduling for elective surgeries was introduced by breaking the elective surgery planning. Addis et al. (2016) presented a model with two approaches combining off-line and online decisions and scheduling with the objective function of minimizing patient tardiness penalties. In the model presented by them, off-line scheduling is implemented and modified due to the existence of canceled surgeries or their rescheduling, along with the arrival of the emergency patients by the results obtained during in-line scheduling. Ceschia et al. (2016) developed a dynamic scheduling model for operating room scheduling, taking into account elective patients with new patient arrivals. They also sought to reduce overtime and unauthorized operating room tardiness. They designed a solution based on local search and explored the search space using a composite neighbourhood method. Hamid et al. (2018) proposed a multi-objective model for planning and scheduling the surgery of elective patients in operating rooms with three objective functions: minimization of the total waiting time of patients based on patient preference, minimization of the costs related to the use of operating rooms, and minimization of the total completion time of patients' surgeries. They used the Epsilon-constraint method to solve the presented model.

In recent years, due to the importance of the problem of distributed scheduling in multi-factory networks, researchers have tried to expand this topic in the field of healthcare systems. Additionally, in most of the optimization research based on operating room scheduling, simple hypotheses have been used, most of which include: operating room scheduling in a single hospital, the deterministic surgery time, lack of dynamic patient emergency arrival, and availability of resources. For this reason, Wang et al. (2016) have applied operating room scheduling in Toronto, Canada, to hospital networks with the hypothesis of stochastic surgical duration, emergency arrival, and limited and shared resources between hospitals. In this study, the objective function of minimizing the cost of surgery according to the type of surgery, the cost of operating room openness, the cost of allocating patients to hospitals, and the cost of tardiness of patients' surgery to the next scheduling horizon were considered. Roshanaei et al. (2017b) believe that operating room scheduling plays an essential role in the profitability of hospitals and their optimal use leads to reducing the cost of surgical services to patients, reducing the waiting time for surgery, and increasing patient satisfaction. Therefore, in their paper, they addressed the problem of operating room planning and scheduling in an independent hospital that has been extended to a strategic multi-hospital network. In the proposed model, a set of patients, surgeons, and operating rooms work together to achieve different objective functions. These objective functions include minimizing operating room costs, surgeons' costs, and the additional cost of the operating room, as well as maximizing the allocation of elective patients to the planning horizon. In their research, Roshanaei et al. (2017a) planned and scheduled the operating room, from an independent hospital to several hospitals, in collaboration with a distributed strategic network; So that the proposed model assigns each patient to the hospital and the daily planning horizon according to the waiting time and health status. Then, the new approach of propagating logic-based Benders' decomposition approach to solve the scheduling problem was presented. They also used a mixed-integer programming model to schedule distributed operating rooms. Roshanaei et al. (2020) studied the collaborative scheduling of the distributed operating room as a location-allocation model. Here, two levels of balancing decisions are studied: (*i*) daily macro imbalance among collaborating hospitals, and (*ii*) daily micro imbalance among open operating rooms in each hospital according to the number of patients assigned to each operating room. This model is formulated as a nonlinear mixed-integer programming mathematical model and consists of two exact solution techniques based on the new single-stage and two-stage branch-and-check method.

Rabbani et al. (2022) studied the patient appointment scheduling problem considering clustered patients in outpatient chemotherapy clinics. To minimize the completion time of all treatments, and maximize the use of nurses' skills, the authors proposed a bi-objective mathematical programming model. They also utilized a hybrid approach based on Torabi-Hassini to solve this problem in large sizes. Lotfi and Behnamian (2022) studied the operating room scheduling of hospital networks with virtual alliance. In that research, by considering the elective patients and non-elective patients, first, a mixed-integer mathematical programming model was proposed. Then, the authors proposed an NSGA-II and memetic-based algorithm with the learning mechanism. Yang et al. (2023) studied multi-stage resource-constrained operating room scheduling problem. First, they modeled this problem as a mixed integrated programming. Second, the authors proposed a slack speed-up-based discrete artificial bee colony algorithm. Bargetto et al. (2023) considered an integrated operating room planning and scheduling problem that includes sequence, capacity and due date constraints and human resources. In that research, a model of the sequence-dependent operating room cleaning times was proposed. To solve this model, they designed a branch-and-price-and-cut algorithm based on the time-indexed formulation where a column generation scheme relies on a label-correcting algorithm. Wang et al. (2023) studied the impact of emergency arrival uncertainty on operating room planning under a non-operative anesthesia mechanism. For different operating room settings, they showed that the non-operative anesthesia mechanism can significantly improve the operating room utilization in comparison with the traditional one. Gür et al. (2024) considered stochastic operating room scheduling under the uncertainty of operation times. They determined separate coefficients of variability for each operation, taking into account the variability factors. To evaluate the variability factors, based on the PROMETHEE method, in this study, the analytical network process method was used. Fallahpour et al. (2024) studied the complexities of integrated operating room planning and scheduling with a focus on elective and emergency patients in an uncertain environment. They developed a mixed integer programming model to minimize inactivity and patient wait times while optimizing high-priority resource allocation. Here, also, an enhanced epsilon constraint method was used for the proposed model. Rahmani Manshadi (2024) proposed a robust mixed-integer binary programming model considering different preferences for hospitals, surgeons, and patients. To simultaneously maximize the efficiency of available resources, minimizing the patients waiting time, and minimizing surgery costs, the author utilized the augmented epsilon constraint approach. Based on the stochastic programming method proposed by Bertsimas and Sim, in this model, a rolling horizon method was applied to reschedule the program after cancellation.

Table (1) summarizes the reviewed papers on operating room scheduling. As it is clear, no research has discussed operating room scheduling concerning virtual alliances. In this research, the proposed solution method, along with modeling the problem under-study, is also quite innovative.

Table 1
Literature review summary

Reference	Single-Objective	Multi-Objective	Network	Deterministic	Uncertainty	Elective patients	Emergency patients	Exact Method	(Meta) Heuristic	Simulation	Virtual Alliance
Zonderland et al. (2010)	✓				✓	✓	✓	✓			
van Essen et al. (2012)	✓				✓	✓	✓	✓	✓	✓	
Meskens et al. (2013)		✓		✓		✓		✓			
Wang et al. (2014)	✓				✓	✓	✓	✓	✓		
Xiang et al. (2015)	✓			✓		✓			✓		
Wang et al. (2016)	✓		✓		✓	✓	✓	✓		✓	
Roshanaei et al. (2017a)		✓	✓	✓		✓		✓			
Roshanaei, et al. (2017b)	✓		✓	✓		✓		✓			
Silva and de Souza (2020)	✓				✓	✓	✓				
Behmanesh and Zandieh (2019)		✓			✓	✓					
Roshanaei et al. (2020)	✓		✓	✓		✓		✓			

Rabbani et al. (2022)		✓		✓		✓		✓	✓		
Lotfi and Behnamian (2022)		✓	✓	✓		✓	✓		✓		✓
Yang et al. (2023)	✓			✓		✓		✓	✓		
Bargetto et al. (2023)	✓			✓		✓		✓			
Wang et al. (2023)	✓				✓		✓				
Gür et al. (2024)	✓				✓		✓	✓			
Fallahpour et al. (2024)		✓			✓	✓	✓	✓			
Rahmani Manshadi (2024)		✓			✓	✓		✓			
Present study		✓	✓	✓		✓	✓	✓			✓

According to Table 1, it can be said that although a limited number of research have tried to solve multi-objective problems, but most of the research carried out in the field of single-objective scheduling. As expected, the number of studies conducted in hospital networks is small, and among these studies, Virtual Alliance has not been investigated in almost any study. Considering that uncertain conditions are closer to the real world, although the number of research conducted in this situation is complex, solving the problem with uncertainty is almost equal to solving the problem with certainty. According to the review conducted here, in most studies, elective patients have been considered in scheduling. Also, despite the Np-hard nature of the studied scheduling problem and the need to use (Meta)heuristic algorithms to solve it, most researchers have been interested in presenting exact methods in the articles and in the meantime, very few papers have used simulation to solve the problem.

3. Problem Definition and Modeling

Operating rooms play a key role in healthcare systems and are one of the most valuable resources available in any hospital for two reasons; (*i*) deal with the health of the patient, and (*ii*) spend a lot of money and resources from hospitals on operating rooms (Heydari & Soudi, 2016). Therefore, high flexibility and reduction of operating room costs can play an important role in healthcare systems. As a result, the involvement of several hospitals and the creation of a collaborative hospital is felt more than ever today. Hospital managers are also interested in creating collaborative hospitals so that they can work in such networks to improve the competitive situation and increase their patient satisfaction. Sharing distributed resources (operating rooms, surgeons, nurses, staff, equipment, and transportation) creates collaborative hospitals, one of which is the formation of a virtual alliance network. In virtual collaborative hospitals, each hospital needs to offer services to patients. However, this can be affected by things like the technology available in each hospital, how many operating rooms they have, and the speed and skills of their surgeons. So, in these networks, the main issue is how to manage the network's limited resources based on the needs of the hospitals. Therefore, this type of collaboration is part of a short-term collaboration and lasts as long as hospitals achieve more benefits than when they work individually (Behnamian 2014). Also, in this type of network, each hospital may have different objective functions to each other, which are prioritized compared to the overall objective function of the virtual alliance distributed scheduling network. Following Lotfi and Behnamian (2022), the main assumptions of the model of collaborative hospital scheduling of elective and non-elective patients are as follows:

- There are H heterogeneous parallel hospitals that have parallel operating rooms.
- Patients are divided into elective patients and non-elective patients.
- Non-elective patients are emergency patients.
- Elective patients should be scheduled that are independent of each other.
- The number of elective and non-elective patients and their arrival time is deterministic.
- Non-elective patients can be postponed to the future for a short period in the absence of facilities.
- Each patient must be assigned to precisely one of the hospitals.
- First, the elective patients and then the emergency patients are assigned to the operating rooms.
- The arrival of the last emergency patient is before the end of the surgery of the last elective patient.
- Surgeons are considered to be the shared resources of hospitals.
- By considering the transportation of patients between hospitals, the patient can go to another hospital to reduce the waiting time and the completion time.
- There are always enough ambulances to transport patients between hospitals.

Assuming that each hospital is responsible for providing health services to its patients, in order to balance the improvement of the overall objective function of the network and increase patient satisfaction to reduce waiting time, it is possible to transfer patients to a hospital other than the hospital of origin for surgery. This can only happen, if it is possible to move patients from one hospital to another that has an available operating room, or if the total time for the patient's treatment is shorter than the time at the hospital they are currently in. Therefore, considering the time of transportation between hospitals has made the problem more practical in the real world.

3.1 Mathematical model

In this section, based on Lotfi and Behnamian (2022), a mixed-integer linear programming model is explained. The used indices, parameters, and decision variables are introduced below.

Indices

p, p'	Set of elective patients	$p, p' = 1, \dots, P$
e, e'	Set of emergency patients	$e, e' = 1, \dots, E$
i, j, k	Set of patients for surgery	$i = P \cup E$, $i = \lvert P \rvert + \lvert E \rvert$
h, q, q'	Set of hospitals	$h, q, q' = 1, \dots, n-1, n, n+1, \dots, H$
d	Days of the week, scheduling horizon	$d = 1, \dots, D$
s	Set of surgeons	$s = 1, \dots, S$

Parameters

t_{hq}	The time interval between hospitals h and q
a_j	Patient j surgery time
O_h	Number of operating rooms in each hospital
w_{jh}	If patient j is first in hospital h for surgery, he will get a value of one; otherwise, its value will be zero
D_e	Deadline for emergency patient e
V_s	Skill and speed of the surgeon s
pre_j	The setup time of the operating room for the patient's surgery j
$clean_j$	Time to clean the operating room after the patient's surgery j
$Tt(j,s)$	Total surgery time of patient j includes (setup time, surgery time of patient j by surgeon s and cleaning time of the operating room), $Tt_{js} = pre_j + (a_j/V_s) + clean_j$
B_{hd}	Servicing time of operating rooms in hospital h in day d
$cost_{jhs}$	The cost of assigning patient j to hospital h by surgeon s
cos_{hd}	Cost of overtime opening of the operating room in hospital h in day d
M	A positive big number
n	Number of patients

Decision variables

y_{jhds}	If patient j is assigned to hospital h on day d to surgeon s is equal to 1, otherwise 0
x_{ijhds}	If patient j is assigned immediately after patient i to hospital h on day d to surgeon s on surgery d is equal to 1, otherwise 0
z_{shd}	If surgeon s in hospital h on day d is equal to 1, otherwise 0
F_{jhd}	A continuous variable of the completion time of surgery of patient j in hospital h on day d
c_{jh}	Continuous variable to determine the completion time of surgery of patient j in the hospital of origin h
$cmax_h$	The longest completion time for surgery at the hospital of origin h
$CMmax$	The longest completion time of surgery
$over_{hd}$	Maximum operating room overtime in hospital h on day d

In this study, two dummy patients, numbered 0 and $n+1$, are created that take no time to process in any hospital ($c_{0h} = 0$). Also, in order to reduce the number of problem decision variables, the indices of the number of operating rooms in each hospital have been omitted. Additionally, zero jobs represent the operating rooms in each hospital. The objective function for the first group of hospitals (from 1 to n) is the total completion times of surgery. The second group (from $n+1$ to H) focuses on the time it takes to complete the last surgery.

$$\min z1 = \sum_{j=1}^{n} \sum_{h=1}^{n} c_{jh} \tag{1}$$

$$\min z2 = CM_{max} \tag{2}$$

$$\min z3 = \sum_{j=1}^{n} \sum_{h=1}^{H} cost_{jh} . y_{jhds} + \sum_{h=1}^{H} \sum_{d=1}^{D} cos_{hd} . over_{hd} \tag{3}$$

$$s.t.: \sum_{h=1}^{H} \sum_{d=1}^{D} \sum_{s=1}^{S} y_{phds} = 1 \qquad \forall\, p \in \{1, \dots, n\} \tag{4}$$

$$\sum_{h=1}^{H}\sum_{d=d_e}\sum_{s=1}^{S} y_{ehds} = 1 \qquad \forall\, e \in \{1,\dots,E\} \tag{5}$$

$$\sum_{\substack{p=0\\p\neq p'}}^{p}\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} x_{pp'hds} = 1 \qquad \forall\, p' \in \{1,\dots,P\} \tag{6}$$

$$\sum_{p=1}^{p}\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} x_{pehds} \leq 1 \qquad \forall\, e \in \{1,\dots,E\} \tag{7}$$

$$\sum_{\substack{e=1\\e\neq e'}}^{E}\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} x_{ee'hds} \leq 1 \qquad \forall\, e' \in \{1,\dots,E\} \tag{8}$$

$$\sum_{j=1}^{n}\sum_{s=1}^{S} x_{0jhds} = O_h \qquad \forall\, h \in \{1,\dots,H\}, d \in \{1,\dots,D\} \tag{9}$$

$$\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} x_{0jhds} \leq 1 \qquad \forall\, j \in \{1,\dots,n\} \tag{10}$$

$$\sum_{\substack{i=0\\i\neq j}}^{n} x_{ijhds} = y_{jhds} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{11}$$

$$\sum_{\substack{k=1\\k\neq j}}^{n+1} x_{jkhds} = y_{jhds} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{12}$$

$$\sum_{\substack{j=1\\j\neq i}}^{n}\sum_{h=1}^{H}\sum_{s=1}^{S} x_{ijhds} \leq 1 \qquad \forall\, i \in \{1,\dots,n\}, d \in \{1,\dots,D\} \tag{13}$$

$$\sum_{h=1}^{H}(x_{ijhds} + x_{jihds}) \leq 1 \qquad \forall i \in \{1,\dots,n-1\}\ , j > i\ , \forall d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{14}$$

$$\sum_{h=1}^{H} z_{hds} \leq 1 \qquad \forall\ h \in \{1,\dots,H\}, s \in \{1,\dots,S\} \tag{15}$$

$$y_{jhds} \leq z_{hds} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{16}$$

$$\sum_{\substack{j=1\\j\neq i}}^{n} x_{ijhds} \leq 1 \qquad \forall i \in \{0,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{17}$$

$$\sum_{\substack{i=0\\i\neq j}}^{n} Tt_{js}.x_{ijhds}\ \leq B_{hd} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{18}$$

$$f_{jhd} \geq Tt_{js}\sum_{\substack{i=0\\i\neq j}}^{n} x_{ijhds} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{19}$$

$$f_{jhd} - f_{ihd} \geq Tt_{js} - M\left(1 - x_{ijhds}\right) \qquad \forall i,j \in \{1,\dots,n\}, i \neq j, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{20}$$

$$c_{jh} \geq w_{jh}\left(f_{jqd} + 2t(h,q).y_{jqds}\right) \qquad \forall\, j \in \{1,\dots,n\}, h,q \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{21}$$

$$cmax_{\mathrm{h}} \geq w_{jh}\left(f_{jqd} + 2t(h,q).y_{jqds}\right) \qquad \forall\, j \in \{1,\dots,n\}, h,q \in \{n+1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{22}$$

$$CM_{max} \geq cmax_h \qquad \forall\, h \in \{n+1,\dots,H\} \tag{23}$$

$$over_{hd} \geq F_{jhd} - B_{hd} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\} \tag{24}$$

$$x_{ijhds}, y_{jhds}, z_{shd} \in \{0,1\} \qquad C_{max} \geq 0, CM_{max} \geq 0, C_{jh} \geq 0, f_{jhd} \geq 0, over_{hd} \geq 0 \tag{25}$$

Objective functions (1) and (2) show the goals of the network. Objective function (3) calculates the cost of assigning a patient to a hospital and a surgeon, as well as the cost of overtime for operating rooms in the entire network. Constraint (4) means that each surgery for selected patients can only be performed in one hospital, takes one day, and requires one surgeon during the scheduled time period. Constraint (5) means that each emergency patient is associated with one hospital and one surgeon on the day of arrival. Constraint (6) states that each elective patient in each hospital must be operated on by one surgeon only after another in only one hospital. Constraints (7) and (8) say that each emergency patient in each hospital can only have surgery in one operating room, and this will happen only after the elective patients have their surgeries. Constraint (9) says that each operating room in every hospital must have one dummy patient to start its process. Constraint (10) states that after the dummy patient, to each operating room, at most one patient can be assigned. Constraints (11) and (12) say that if a patient is assigned to hospital h on day d, there is exactly one patient before and after her/him. Constraint (13) says that in each operating room, only one patient can be operated at a time. Constraint (14) prevents situations in which a patient cannot be both the next patient and the previous patient for another patient. Constraint (15) says that each surgeon can work at only one hospital each day. Constraint (16) makes sure that if a patient is at hospital h on day d, then surgeon s is also in that hospital. Constraint (17) says that at most one surgery can be done after each surgery. Constraint (18) states that the total completion time taken for the surgery must be smaller equal to the time the operating room is available. Constraint (19) figures out how long each patient's surgery will take at the hospital they are assigned to. Constraint (20) shows the relationship between the completion time of patient surgery i, j, which are immediately followed in one operating room. Constraint (21) represents the completion time of each patient's surgery at her/his origin hospital, including the time required to transfer the patient to hospital h. Constraints (22) and (23) refer to the makespan. Constraint (24) shows the maximum overtime that any hospital can has in its operating room for each day. In the end, the model variables are explained by Constraint (25).

3.2 The complexity of the problem

Given that the parallel machine scheduling problem with the objective function of makespan is NP-hard (Behnamian, 2014), considering parallel operating room scheduling in a hospital is at least as difficult as this. Therefore, the scheduling of operating rooms in a collaborative hospital has at least the difficulty of a single hospital scheduling problem, and thus the distributed scheduling of operating rooms will be an NP-hard problem.

4. Benders' Decomposition Algorithm

Benders' decomposition model was proposed in 1962 by Benders with the objective that instead of solving a large and time-consuming problem, smaller problems can be solved iteratively, leading to more efficient problem-solving and reduced solution time (Benders, 1962). The flowchart of the Benders' decomposition algorithm is shown in Figure (1).

The general idea of this algorithm is to divide the problem into two master problems (MP) and sub-problems (ZahediAnaraki & Esmaeilian 2021). The sub-problem usually consists of continuous variables, and the master problem consists of discrete and mixed variables. So this method is widely used in solving mixed-integer programming problems (Benders, 2005). After decomposing the model into two master and sub-problems, the discrete and mixed variables in the master problem (MP) are solved, and the solution obtained from the mixed variables is fixed and given to the sub-problem (SP). Through the following problem, according to the results of mixed-fixed variables, other variables of the model are solved, and the constraints of the model can be checked (Ghezavati 2015). If the mentioned problem solutions become infeasible, it leads the model towards convergence by producing Bender's cuts (based on repetition).

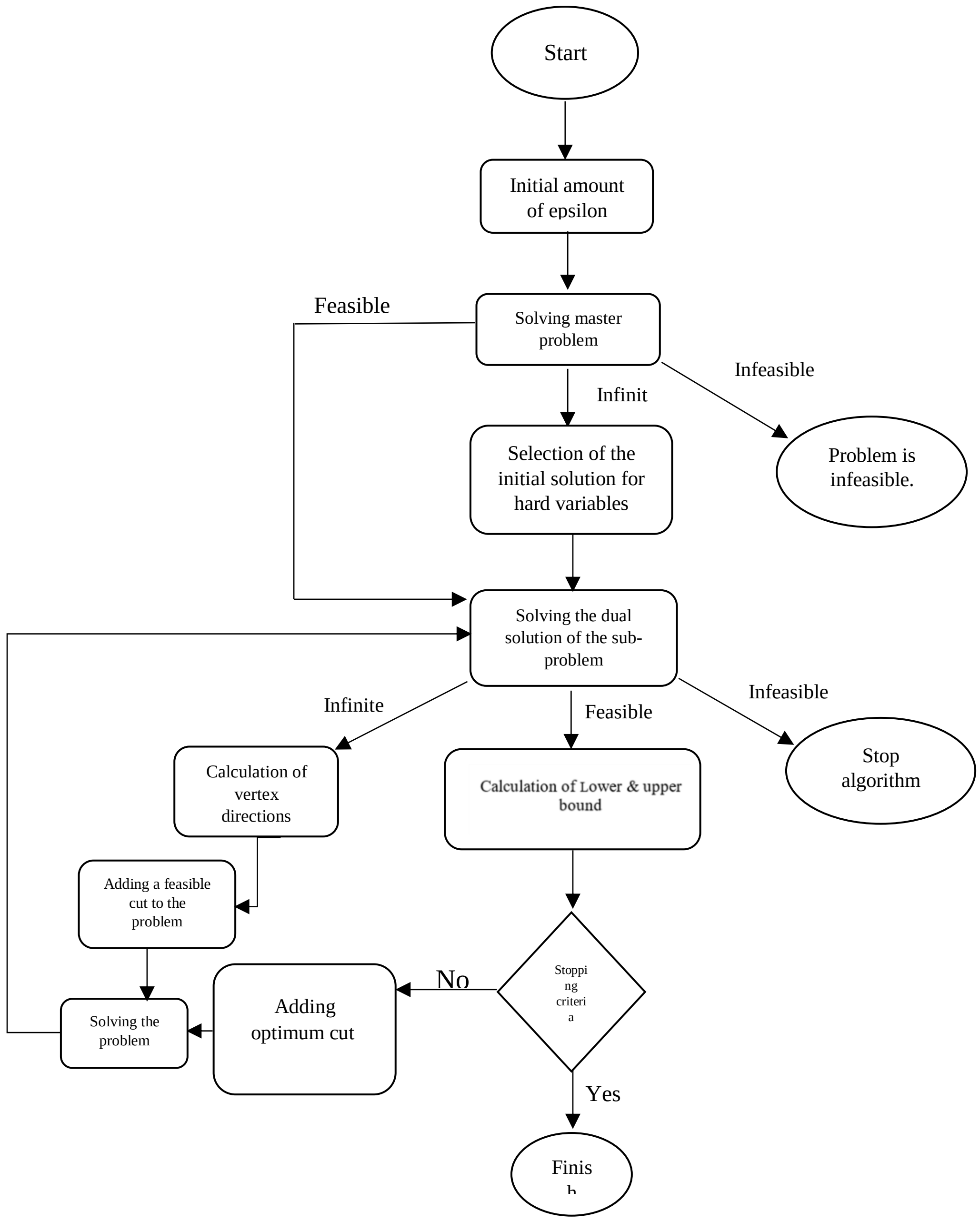


Fig. 1. Benders' decomposition algorithm

4.1 Multi-objective problem

In this section, the multi-objective problem must first be turned into a single-objective problem. In this regard, the epsilon-constraint method is used. In this method, the number of objective functions decreases, and the number of problem constraints increases. In the epsilon-constraint method, among the available objective functions, one of them is selected for an objective function. The other objective functions are added to constraints by taking into account values such as the epsilon (ε), which is determined by the decision-maker. The problem then becomes a single-objective programming model. In the minimization problem, a lower bound (epsilon value) is considered for those objective functions that are added to the problem as a new constraint. Then, the problem is solved with a single-objective function (Rahimi et al. 2017). Therefore, for this reason, we consider an initial value of epsilon (ε) that can be changed to create the Pareto frontier to find the constraints of the objective function. Then, the desired problem is divided into two sections: the master problem (MP) and the sub-problem (SP) using the Benders decomposition algorithm.

4.2 Implementation details

In this subsection, the presented model is solved using the Benders decomposition algorithm method. According to the model, $x_{ijhds}, y_{jhds}, z_{shd}$ are part of discrete variables and mixed variables, while the variables $C_{max}, CM_{max}, C_{jh}, f_{jhd}$, $over_{hd}$ are simple continuous variables. As a result, the master problem (MP) and the sub-problem (SP) are as follows:

Master problem:

$$\min \sum_{j=1}^{n}\sum_{h=1}^{H} cost_{jh}.\, y_{jhds} \tag{26}$$

Constraints (4) until (18). (27)

Equation (26) is part of the discrete variable of Equation (3). The solutions obtained from the master problem are given in the form of $(\bar{y},\bar{x})$ with fixed values to the sub-problem to continue solving the model.

Sub-problem:

$$\min \sum_{h=1}^{H}\sum_{d=1}^{D} cos_{hd}\, .\, over_{hd} \tag{28}$$

$$s.t.: \sum_{j=1}^{n}\sum_{h=1}^{n} c_{jh} \ \geq \varepsilon_1 \tag{29}$$

$$CM_{max} \geq \varepsilon_2 \tag{30}$$

$$f_{jhd} \geq Tt_{js} \sum_{\substack{i=0\\ i\neq j}}^{n} \bar{x}_{ijhds} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{31}$$

$$f_{jhd} - f_{ihd} \geq Tt_{js} - M\left(1-\bar{x}_{ijhds}\right) \qquad \forall i,j \in \{1,\dots,n\}, i\neq j, \qquad h \in \{1,\dots,n\}, d \in \{1,..,n\}, s \in \{1,\dots,S\} \tag{32}$$

$$c_{jh} \geq w_{jh}\left(f_{jqd} + 2t(h,q).\,\bar{y}_{jqds}\right) \qquad \forall\, j \in \{1,\dots,n\}, h,q \in \{1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{33}$$

$$cmax_{\mathrm{h}} \geq w_{jh}\left(f_{jqd} + 2t(h,q).\,\bar{y}_{jqds}\right) \qquad \forall\, j \in \{1,\dots,n\}, h,q \in \{n+1,\dots,H\}, d \in \{1,\dots,D\}, s \in \{1,\dots,S\} \tag{34}$$

$$CM_{max} \geq cmax_h \qquad \forall\, h \in \{n+1,\dots,H\} \tag{35}$$

$$over_{hd} \geq f_{jhd} - B_{hd} \qquad \forall\, j \in \{1,\dots,n\}, h \in \{1,\dots,H\}, d \in \{1,\dots,D\} \tag{36}$$

$$C_{max} \geq 0, CM_{max} \ \geq 0, C_{jh} \geq 0, f_{jhd} \geq 0, over_{hd} \geq 0 \tag{37}$$

Dual Benders sub-problem is presented below:

$$\begin{aligned}\max zdsp = \varepsilon_1 lo + \varepsilon_2 ro &+ \sum_{\substack{i=0\\ i\neq j}}^{n}\sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S}\left(Tt_{js}.\,\bar{x}_{ijhds}\right).\, wo_{jhds}\\ &+ \sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S}\left(Tt_{js} - M\left(1-\bar{x}_{ijhds}\right)\right).\, uo_{ijhds}\\ &+ \sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{q=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S}\left(w_{jh}.\,2t_{hq}.\,\bar{y}_{jqds}\right).\, ao_{jhqds}\\ &+ \sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{q=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S}\left(w_{jh}.\,2t_{hq}.\,\bar{y}_{jqds}\right).\, bo_{jhqds} + \sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{d=1}^{D} B_{hd} * eo_{jhd}\end{aligned} \tag{38}$$

$s.t.$:

$$\sum_{s=1}^{S} wo_{jhds} - \sum_{\substack{i=1\\ i\neq j}}^{n}\sum_{s=1}^{S} uo_{ijhds} + \sum_{\substack{k=1\\ k\neq j}}^{n}\sum_{s=1}^{S} uo_{kjhds} - \sum_{\mathrm{q}'=1}^{H} w_{j\mathrm{q}'} \times \left(ao_{jhqds} - bo_{jhgds}\right) - eo_{jhd} \leq 0 \quad \forall\, j \in \{1,\dots,n\}, h,q \in \{1,\dots,H\}, d \in \{1,\dots,D\} \tag{39}$$

$$\sum_{j=1}^{n}\sum_{h=1}^{H}\sum_{q=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} ao_{jhqds} \leq 0 \tag{40}$$

$$\sum_{j=1}^{n}\sum_{q=1}^{H}\sum_{d=1}^{D}\sum_{s=1}^{S} w_{jh} * (bo_{jhqds} - po_h) \leq 0 \qquad \forall\, h \in \{1, \dots, H\} \tag{41}$$

$$po_h + lo \leq 0 \qquad \forall\, h \in \{1, \dots, H\} \tag{42}$$

$$\sum_{j=1}^{n} eo_{jhd} \leq cos_{hd} \qquad \forall\; h \in \{1, \dots, H\}, d \in \{1, \dots, D\} \tag{43}$$

$$lo, ro, wo_{jhds}, uo_{ijhds,} ao_{jhqds}, po_h, eo_{jhd} \geq 0 \tag{44}$$

In the above model, the dual variables $wo_{jhds}, uo_{ijhds}, ao_{jhqds}, bo_{jhqds}, po_h, eo_{jhd}$ lo, ro, respectively, corresponding to Constraints (29) to (36) in the sub-problem. The number of cuts can be exponential, but only a few of them are active in the optimal solution. After each iteration of the Benders algorithm, and checking the conditions of optimality and feasibility, the mentioned cuts are added to the master problem. Adding Benders cuts improves the lower bound (LB) and leads the algorithm to convergence.

5. Computational Results

In this section, in order to validate the proposed algorithm and model, in two small and medium sizes, the Benders decomposition algorithm and solver CPLEX GAMS software were tested and analyzed for sensitivity analysis. The results obtained from the above two methods with a one-day and two-day planning horizon are compared.

5.1 Numerical results

For this reason, the data used to solve the problem are randomly generated. In order to make the assumed conditions more realistic, the speed of each surgeon can be different and in the range of $V_s \in [1,2]$ is considered. Also, the number of operating rooms in each hospital is displayed as *O(h)=(O₁,O₂,…)*.

Table 2
Compare results with a one-day planning horizon

Planning horizon (*d*=1)													
						Benders decomposition				CPLEX			
No	Number of elective patients	Number of emergency patients	Number of hospitals	Number of operating rooms	Number of surgeons	CM_{max}	C_{jh}	Network objective function	Runtime (seconds)	CM_{max}	C_{jh}	Network objective function	Runtime (seconds)
1	6	4	2	(2,1)	5	340	923.21	66675	19	340	932.21	66675	19
2	6	4	2	(2,2)	5	251.25	832.14	44470	26	251.25	832.14	44770	27
3	6	4	2	(3,2)	7	172.64	492.64	29967	26	168.33	510	29467	31
4	6	4	3	(1,2,1)	7	230.41	419.46	56864	42	235	420.5	58882	86
5	6	4	3	(3,2,3)	9	162.5	596.88	34920	45	166.16	522.41	35259	103
6	9	6	2	(2,2)	6	540.71	1550.61	129570	131	421.94	1691.37	129948	300
7	9	6	2	(3,4)	10	344.09	1623.12	108817	136	412.9	1693.02	109511	425
8	9	6	3	(3,2,2)	10	352.13	1186.77	71058	198	390.14	1227.87	71627	662
9	9	6	3	(4,3,4)	14	201.48	998.32	10073	209	240.76	1032.98	11715	694
10	13	7	2	(4,3)	10	494.4	2680.08	23422	428	469.38	2678.33	23923	1740
11	13	7	3	(3,3,4)	12	399.16	1017.07	19306	801	422.4	1786.92	20305	2152
12	13	7	4	(4,3,4,3)	17	284.01	1967.42	19889	1232	306.89	2042.33	20112	2820
13	13	7	4	(4,5,4,3)	18	201.99	1036.92	19021	1359	218.27	1083.4	19734	3364
14	15	10	3	(3,3,4)	14	414	1956.45	25633	1980	450.36	2019.81	26002	3600
15	15	10	4	(3,5,4,3)	20	482.86	2037.22	19567	2107	521.77	2100.19	21617	4007
			Average			322.97	1334.08	45303.46	585.6	334.37	1371.16	45969.7	1335.33

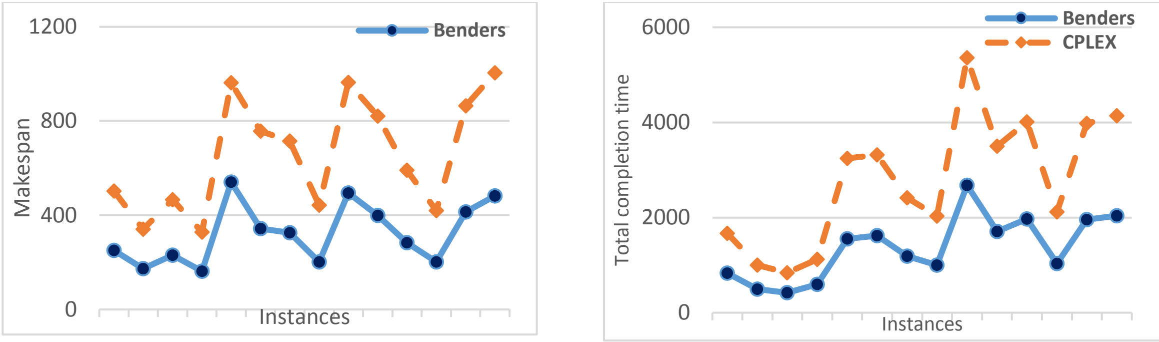
Table 3
Compare results with a two-day planning horizon

Planning horizon (d=2)													
						Benders decomposition				CPLEX			
No.	Number of elective patients	Number of emergency patients	Number of hospitals	Number of operating rooms	Number of surgeons	CM_{max}	C_{jh}	Network objective function	Runtime (seconds)	CM_{max}	C_{jh}	Network objective function	Runtime (seconds)
1	6	4	2	(2,1)	5	167.5	520.71	16850	22	167.5	520.71	16850	22
2	6	4	2	(2,2)	5	153	510	3225	45	153	510	3225	45
3	9	6	2	(2,2)	7	294.36	1338.18	7797	198	296.77	1303.49	7831	701
4	9	6	2	(4,3)	10	145.49	1206.42	7547	203	18065	1355.5	7562	768
5	9	6	3	(2,3,2)	12	257.23	1806.84	7567	316	317.66	1812	7545	1224
6	13	7	2	(3,4)	10	316.36	1744.09	7614	683	356.74	1382.4	7669	2349
7	13	7	3	(2,2,4)	12	238.85	801.42	9439	861	257.23	732.13	9467	3022
8	13	7	4	(3,1,3,2)	14	278.33	1667.52	9487	1375	282.06	1896.4	9495	3657
9	15	10	2	(6,5)	12	258.62	1997.94	11101	1844	203.73	2099.7	11148	4886
10	15	10	3	(3,3,2)	14	328.46	1846.17	9660	1260	380.38	1528.1	9715	7323
11	15	10	4	(2,3,2,2)	16	415.24	2445.89	9803	1046	430.84	2507	9817	9011
12	25	10	2	(4,3)	12	991.17	5002.25	109547	1321	-	-	-	-
13	25	20	3	(4,2,3)	15	687.81	3990.87	80068	1860	-	-	-	-
14	25	20	4	(4,2,2,3)	10	532.4	5095.7	47451	2593	-	-	-	-
15	25	20	5	(2,2,3,2,3)	25	817.13	3391.77	172038	4320	-	-	-	-
16	50	20	4	(6,5,6,7)	15	702.09	7006.95	103961	6312	-	-	-	-
17	50	20	4	(4,5,3,4,3)	20	951.68	6227.13	129010	7995	-	-	-	-
18	50	20	4	(6,5,6,7)	15	985.17	7996.11	200504	13808	-	-	-	-
19	50	20	5	(5,6,4,3,4)	30	1073.92	6245.06	308592	14496	-	-	-	-
Average						510.202	3202.17	65445	3178.26	275.14	1422.53	9120.36	3000.72

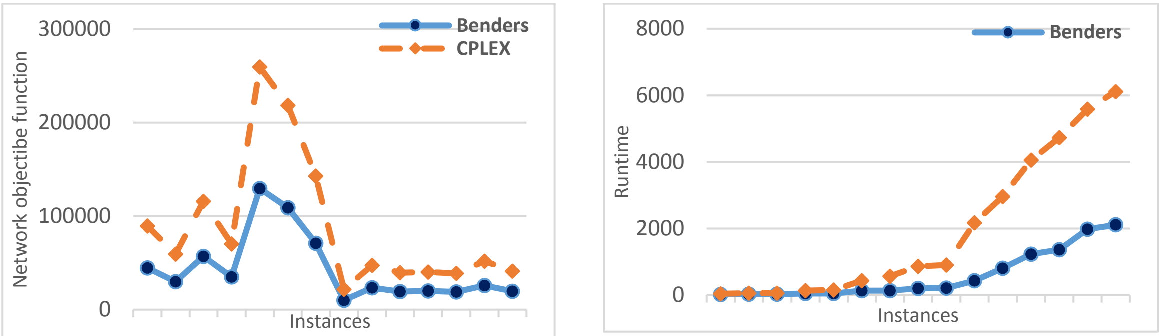
According to the results obtained from tables (2) and (3), the high number of collaborations in hospitals in collaborative hospitals increases the distributed resources, including the number of operating rooms, the number of surgeons, and emergency transportation. The mentioned results have relatively reduced the total completion time of surgeries, the completion time of the last surgery, and the reduction of overtime of operating rooms in each hospital. It also reduces the costs of operating room surgeries in hospitals.

5.2 Analysis of results

Figures (2) and (3) show the extent to which the objective functions of the problem change relative to the distributed resources of operating rooms, the number of surgeons, and emergency transportation within the network with a one and two-day planning horizon. According to these figures, it can be obtained that by decreasing and increasing the distributed resources in relation to the number of patients, the values of the objective function become relatively worse and better, respectively. As a result, with the collaboration of more hospitals, the objective function of each of them independently and separately will be more desirable and better than the situation in which they operate independently.



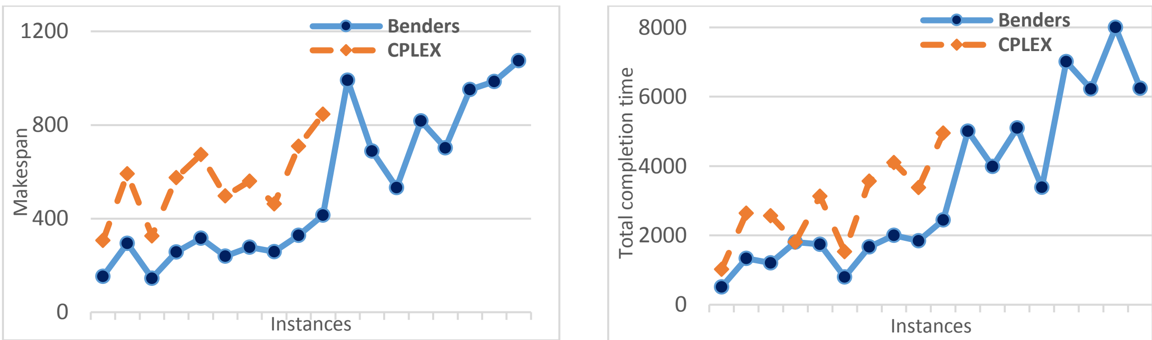
(a) Makespan (b) Total completion time of surgeries



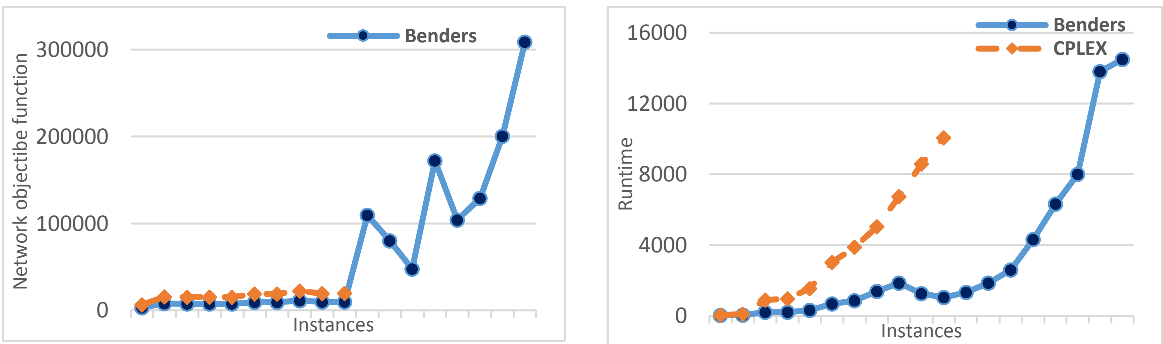
(c) Objective function of the network (d) Runtime

Fig. 2. Comparison of the results with a one-day planning horizon

According to the values of the objective function and the problem-solving runtime in Tables (2) and (3) and Figures (2) to (3), it can be concluded that the Benders decomposition algorithm has better performance than CPLEX, and in some cases, CPLEX solver cannot solve problems with more than 40 patients ($n \geq 40$). Figure (4) shows the convergence of the Benders decomposition algorithm for an example of an operating room scheduling problem with 25 patients, four hospitals, and a two-day planning horizon. This figure shows the upper and lower bounds of the algorithm per number of iterations.



(a) Makespan *(b)* Total completion times of surgeries



(c) Objective function of the network *(d)* Runtime

Fig. 3. Comparison of the results with a two-day planning horizon

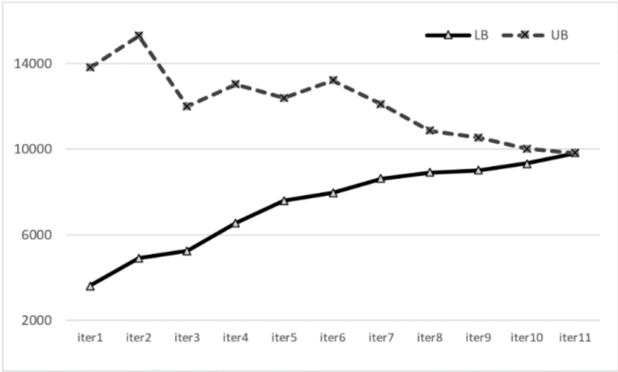


Fig. 4. The convergence of the proposed algorithm
(for example with 25 patients and four hospitals with *d*=2)

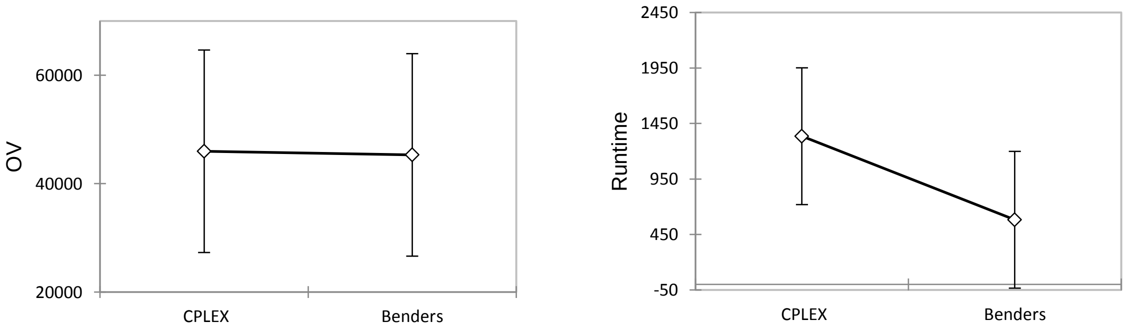
To further examine the results, statistical analysis has been performed and the results are shown in Tables 4 to 7. Figures 4 and 5 also show the changes in the behavior of the studied algorithm according to the value of the objective function and the runtime for one-day and two-day planning horizons.

Table 4
Analysis of variance to compare network objective functions with a one-day planning horizon

Source	DF	Sum of squares	Mean squares	F	Pr > F
Model	1	3330000.833	3330000.833	0.003	0.959
Error	28	34912515936.133	1246875569.148		
Corrected Total	29	34915845936.967			

Table 5
Analysis of variance to compare runtime with a one-day planning horizon

Source	DF	Sum of squares	Mean squares	F	Pr > F
Model	1	4249556.033	4249556.033	3.132	0.088
Error	28	37989524.933	1356768.748		
Corrected Total	29	42239080.967			



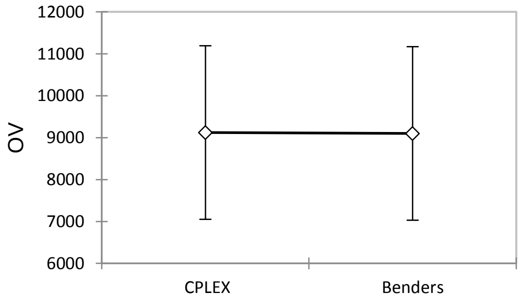
(a) Network objective functions *(b)* Runtime
Fig. 5. Comparison of means with a one-day planning horizon

Table 6
Analysis of variance to compare network objective functions with a two-day planning horizon

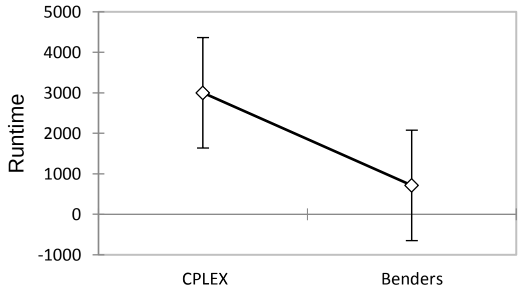
Source	DF	Sum of squares	Mean squares	F	Pr > F
Model	1	2488.909	2488.909	0.000	0.988
Error	20	216715005.455	10835750.273		
Corrected Total	21	216717494.364			

Table 7
Analysis of variance to compare runtime with a two-day planning horizon

Source	DF	Sum of squares	Mean squares	F	Pr > F
Model	1	28762455.682	28762455.682	6.119	0.022
Error	20	94012221.091	4700611.055		
Corrected Total	21	122774676.773			



(a) Network objective functions



(b) Runtime

Fig. 6. Comparison of means with a two-day planning horizon

As shown in Table 4 and Table 6, the difference between the results is not significant in terms of the overall network objective function of the network. The next important point in these analyses is the significant difference between the runtime results, which are shown in Tables 5 and 7. Here, there is complete superiority with the Benders decomposition algorithm, so that this algorithm was able to achieve a better solution in a completely shorter time.

5.3 Managerial insights

According to the research conducted in healthcare systems, it can be claimed that operating room scheduling is one of the most practical and popular topics among operation research researchers. In the meantime, proper scheduling of surgeries in operating rooms as expensive resources of the hospital has a significant impact on increasing the service delivery rate to patients and increasing their satisfaction. In this regard, in most of the operating room scheduling research, simple assumptions have been used, while the development process in health institutions, the huge increase in cost and the need to use shared resources (considering the importance of health and people's lives) caused health systems to move from single agent planning to network planning. In fact, this research showed that the distributed planning of operating rooms and sharing the operating room block in the hospital network has become a cost-effective method to improve the efficiency and productivity of operating rooms and surgeons. Based on tactical management (allocation of surgeons to the operating room) and based on the planning of the operating room (allocation of surgeries to the operating room), the results of this research indicate that collaborative planning and scheduling of operating rooms between hospitals improves the better use of shared resources and allows the allocation of surgeries to surgeons in the entire network in an optimal manner. At the same time, the sensitivity analysis shows that the higher the number of participants in the hospital network, it will relatively reduce the total completion time of the surgeries, the makespan and the overtime of the operating rooms in each of the participating hospitals in the network. Therefore, in order to facilitate network management decision-making and optimal use of distribution resources, the model presented in this research can be used as a suitable management tool for health center managers in all public and private hospitals.

6. Conclusions and Future Suggestions

Many researchers have considered operating room scheduling due to the high costs involved in this area. In most of these studies, operating room scheduling is considered independently in a hospital. While today, due to efforts to reduce system costs by reducing the cost of medical systems, hospital managers try to seek to increase patient satisfaction and increase resource efficiency. For this reason, they try to share the resources and the creation of parallel connections between hospitals and the formation of collaborative hospitals. In this regard, here, operating room planning and scheduling in collaborative hospitals with virtual alliances have been considered to increase flexibility and reduce hospital costs. In this network, a set of hospitals is scheduled by sharing operating rooms, surgeons, and emergency transport in a network to reduce patients' surgery completion time. In a virtual alliance, each hospital may have a different objective function, which has a priority over the overall objective function of the network. Patients are of both elective and emergency types. Considering the patient's transfer from one hospital to another, according to the patient's transportation time, the conditions of the system under-study have been tried to be as close as possible to the real-world systems. Due to the Np-hard nature of the problem, the Benders decomposition algorithm is proposed and the results are compared with the CPLEX solver of the GAMS software. The runtime and the results of the problem objective function indicate the relative efficiency of the proposed algorithm. Comparing the results showed that the runtime of the Benders algorithm is quite competitive and based on the analysis of variance, there is a significant difference between the output of GAMS and the proposed model. Due to the large optimization gap that existed in solving the model, the Benders algorithm in larger dimensions has been able to achieve the optimal solution. In comparison, GAMS has not been able to solve large-size instances. Due to the novelty of the research investigated in this paper, there are many fields of future study for those interested in the field of health systems, which can be mentioned on the lateness and cancellation of selected patients' surgeries. Furthermore, examining the types of effective objective functions in the hospital network can also be a suitable suggestion for further research. Considering the limited number of emergency transport vehicles for moving patients among networked hospitals can also be one of the suggestions in this field. Considering nurses' shifts and nurses' sharing as resources in the distributed scheduling of operating rooms and the scheduling of surgical theaters along with related resources can be attractive topics for future research. Studying the conditions of uncertainty in the time of patients' transportation, surgery and the arrival of emergency patients are other suitable future suggestions to bring the conditions of the problem closer to the real world. In addition, considering the complexity of the investigated problem and the inability of exact methods to solve these problems in a reasonable time, the use of methods based on heuristics and metaheuristics can be a suitable research field to continue this work. As the last suggestion, we can mention the combination of exact methods with heuristic methods as matheuristic algorithms to solve the problem to simultaneously achieve the speed of the heuristic algorithm and the accuracy of the exact methods.

Availability of data and materials: The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.

Competing Interests: The authors declare no competing interests.

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