A Robust Optimization Approach for the Hub Arc Location Problem

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Abstract

Hub networks play a crucial role in optimizing transportation flow and reducing overall costs by efficiently connecting origins and destinations through strategically placed hub nodes. The decision of hub location carries significant long-term implications and necessitates consideration of various factors within an uncertain environment. This paper addresses the hub arc location problem in hub networks, considering setup costs, isolated hubs, and uncertain flows between nodes. To tackle this challenge, a two-stage stochastic programming model is formulated to incorporate the uncertainty in flow volumes. Additionally, a robust optimization approach is proposed to enhance the resilience of hub location decisions against uncertain scenarios. The problem is solved using a tailored Genetic algorithm, which achieves optimal solutions with high quality and reasonable computational time. The results demonstrate the effectiveness of the proposed methodology in handling the uncertain nature of the hub location problem, contributing to the advancement of transportation planning and logistics optimization. The findings provide valuable insights for practical applications in real-world scenarios, offering a framework for decision-makers to make informed choices regarding hub network design and location. By integrating uncertainty and robust optimization techniques, this paper offers a comprehensive approach to address complex transportation network problems and improve overall efficiency in transportation systems.

Keywords

Hub arc location, Isolated hub, Uncertainty, Robust Optimization, Min-Max Regret Model, Genetic Algorithm.

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1. Introduction

The transportation, logistics, and telecommunications industries grapple with the efficient movement of passengers, goods, and information from their origins to destinations (Farahani, Hekmatfar et al. 2013). To enhance this process and achieve economies of scale, hub networks play a crucial role by facilitating the transfer of flow and improving service levels. Traditional hub location problems assume a complete graph with discounted hub arcs, connecting all hub nodes (Campbell, Ernst et al. 2005). However, these assumptions often yield unrealistic outcomes, prompting the development of a new problem class referred to as Hub Arc Location problems (Campbell, Ernst et al. 2005). These problems aim to determine the ideal positioning of hub arcs and the allocation of non-hub nodes to hubs, thereby minimizing both the total transportation cost and deployment.

Real-world problems often suffer from the limitation of assuming fixed input parameters, resulting in suboptimal and infeasible solutions (Alumur, Campbell et al. 2021). Parameters such as set-up cost, transportation cost, demand, distance, and density are prone to non-deterministic behavior, as they can change after decisions have been made. Traditional methods employ sensitivity analysis to account for minor uncertainties in data, but this approach falls short in producing robust results (Chou 2010). Hence, addressing the inherent uncertainty in hub arc location problems becomes necessary. This paper aims to introduce a novel approach to robust hub arc location problems, requiring a comprehensive exploration of the pertinent literature.

When it comes to optimization under uncertainty, three research methods are commonly discussed: stochastic optimization, robust optimization, and fuzzy optimization (Contreras, Cordeau et al. 2011). Stochastic programming is often modeled through two-stage programming, where an initial decision is made, followed by the observation of random events and subsequent decisions to mitigate their effects or improve outcomes (Bashiri and Mehrabi 2010): In contrast, robust optimization considers indefinite possibilities through discrete scenarios or continuous intervals to estimate uncertain parameters (Hekmatfar and Pishvaee 2009). Discrete scenarios refer to different numbers suggested for each parameter based on past experiences or feasibility studies, while continuous intervals represent ranges within which uncertain parameters are defined (Chou 2010). These scenarios depict the most likely states that may arise in the future.

Our research addresses significant limitations in classical hub location problems related to the consideration of a standard discount factor for all hub arcs. This approach, commonly used to achieve cost savings and economies of scale for flows on hub arcs, tends to yield unrealistic outcomes. Optimal solutions may lead to disproportionately smaller flows on hub arcs when compared to non-hub arcs, especially as the discount factor is applied solely between hub points (Li, Bing et al. 2023). Moreover, the traditional assumption of complete connectivity among hub points in classical hub location problems can restrict model flexibility and realism. While this assumption may simplify network design and flow routing, it enforces specific cost structures and topologies that may not align well with practical scenarios (Rahmati, Neghabi et al. 2024).

In contrast to air transport networks where direct flights between all hubs optimize passenger travel time, large transportation and long-haul communication networks typically deviate from this direct connectivity model. Consequently, actual hub networks often lack full interconnections between hub nodes, affecting the efficiency and dynamics of the network as a whole (Lasemi, Arabkoohsar et al. 2022).

Recognizing that location decisions play a crucial role in strategic decision-making processes and demand substantial time investment for implementation, we have incorporated uncertainty into our hub location modeling. By relaxing the assumptions inherent in classical hub location problems, we have tailored our approach to address real-world complexity and optimize decision-making processes under uncertainty.

The innovative application of hub location modeling in our research not only alleviates restrictive assumptions but also enhances the practical relevance and applicability of our findings in real-world settings. By emphasizing the necessity and significance of our work in overcoming these research challenges and promoting innovation, our study contributes to advancing the field of hub location optimization and its practical implications.

The structure of this paper is organized as follows: Section 2 involves literature review. Section 3 provides the problem definition, assumptions, notations, and modeling approach. In Section 4, we present the solution methodology and computational results, including a comparison between traditional and novel approaches. Moreover, it consists of a sensitivity analysis to elucidate the underlying concepts. Finally, in Section 5, we summarize the results obtained from the preceding sections and propose directions for future research.

2. Literature Review

In this section, the literature related to hub location problem considering uncertainty is reviewed. There are various methods to deal with this uncertainty. However, proper categorization of these methods is necessary for a better analysis. In order to tackle the uncertain hub arc location problem, two common approaches to ensure robustness in combinatorial optimization problems, particularly in location problems, are minimizing the maximum cost and minimizing the maximum regret (Klincewicz 1998). Regret refers to the difference between the quality of a given strategy and the quality that would have been chosen if the future were certain (Berman, Drezner et al. 2007).

Minmax Regret models aim to minimize the maximum regret across all scenarios. This field of research finds applications in scheduling problems, production planning, location and allocation, resource allocation, and various other domains. (Klincewicz 1998) first applied the Minmax Regret model to the 1-median location problem on a tree. O'Kelly and Bryan (1998) extended this approach to the weighted p-center facility location problem, incorporating uncertain interval weights. They also explored the formulation of Minmax Regret models for handling uncertainties in edge lengths and node weights in the 1-center location problem. Klincewicz (1998) further presented the 1-median Minmax regret location problem by considering interval uncertainties in each node's demands, providing a polynomial algorithm to solve it.

In the literature addressing hub location problems, Campbell, Ernst et al. (2005) introduced four types of Hub Arc Location Problems, defining and comparing these models against classic hub median problems. They also formulated related integer programming models and proposed an enumeration-based algorithm to solve them. (Campbell, Ernst et al. 2005). Campbell, Stiehr et al. (2003) applied the first type of problem to a cluster of workstations, considering service level constraints in formulating time-definite transportation problems (Campbell 2009). Subsequently, Sasaki, Campbell et al. (2009) explored hub arc location problems involving competitive conditions. Martins de Sá, Contreras et al. (2015) addressed a specific hub line location problem focused on minimizing total travel time from origin to destination and devised an exact algorithm to solve it. The research on isolated hubs was explored in the works of (Korani and Eydi 2021) and (Atay, Eroglu et al. 2023). Furthermore, a rigorous examination of a path-based formulation for the tree of hub location problem was conducted in the studies by (Fernández and Sgalambro 2020), (Bütün, Petrovic et al. 2021), (Espejo, Marín et al. 2023, Khaleghi and Eydi 2024) and , while introducing a valid inequality in (Contreras, Fernández et al. 2010). Additionally, alternative modeling approaches utilizing a tree structure for hub location optimization were investigated and resolved using the minimum spanning tree method by (Mohajeri and Taghipourian 2011) and an enhanced Benders decomposition algorithm by (Muffak and Arslan 2023) and (Ramamoorthy, Vidyarthi et al. 2024).

In recent years, there has been a growing interest in addressing stochastic hub arc location problems. Several studies have been conducted by Hamid, Bastan et al. (2019), Shang, Yang et al. (2020), Wang, Chen et al. (2020), Hu, Hu et al. (2021), Rostami, Kämmerling et al. (2021), Taherkhani, Alumur et al. (2021), Ghaffarinasab and Kara (2022), Ghaffarinasab (2022), Rahmati, Neghabi et al. (2023)and Sener and Feyzioglu (2023). These studies proposed uncertain models and employed various approaches to mitigate uncertainties. They utilized both exact and heuristic algorithms to solve the proposed models, with details provided in **Table 1**. In **Table 1**, we provide a comprehensive review of articles that focus on the consideration of uncertainty in hub location problems. Additionally, we present the research gap investigated in this study in **Table 2**, highlighting the contributions of this paper in relation to the related and most similar papers.

In our manuscript, we involve the complexities of hub facility location under uncertain parameters, especially focusing on arc hubs, which have been relatively understudied compared to traditional hub location models. We recognize that hub location decisions are crucial strategic choices with long-term implications, necessitating a comprehensive understanding of the impact of uncertainty on various parameters such as costs, demand, and distance.

By incorporating uncertainty into our model, we aim to provide a more realistic and robust solution that can adapt to dynamic changes in the operating environment. Unlike classical models that treat data as certain and may lead to suboptimal outcomes when faced with uncertainty, our method accounts for the inherent uncertainties in hub location decisions, leading to more reliable and resilient solutions.

Furthermore, our approach considers the uncertainties surrounding flow parameters between nodes and the implications of isolated hubs, offering a more comprehensive perspective on arc hub location problems. This novel approach not only fills an important research gap but also paves the way for future advancements in modeling and optimizing hub facilities under uncertain conditions.

A key highlight of this research is the inclusion of isolated hubs alongside connected hub facilities. Isolated hubs, which operate independently without direct connections to hub arcs, play a vital role in streamlining transportation and flow transfer processes. They facilitate the creation of more efficient origin-destination routes, thus elevating service standards significantly.

Moreover, this paper introduces a metaheuristic algorithm, specifically a Genetic Algorithm, to address the hub arc location problem in high-dimensional sizes—a pioneering venture. Notably, this is the first instance in which this problem has been tackled under conditions of uncertainty. Leveraging minimax regret theory adds a robust layer to the proposed algorithm, which is validated by meticulous comparisons of solutions concerning accuracy and computational efficiency against exact solutions. Additionally, the development of a two-stage stochastic programming model, juxtaposed with a robust approach, accentuates the novelty and impact of this study.

In essence, this work's innovations stem from its exploration of isolated hubs, the application of metaheuristic algorithms in complex dimensions, and the resolution of hub arc location considering uncertainty. These breakthroughs not only fill important research lacunae but also provide valuable insights for practical implementations, propelling the realms of transportation planning and logistics optimization forward. Now, a concise summary of the original contributions made by our research:

- Integration of isolated hubs alongside connected hub facilities to enhance transportation efficiency and service quality.
- Introduction of a Genetic Algorithm to solve the hub arc location problem in large-dimensional spaces, marking a novel advancement in addressing complex logistics challenges.
- Pioneering exploration of uncertainty in hub arc location problems, supported by minimax regret theory for algorithm robustness.
- Innovation in developing a two-stage stochastic programming model with a robust methodology to deliver accurate and efficient solutions.

Overall, our study breaks new ground by examining isolated hubs, applying metaheuristic algorithms to complex scenarios, and tackling uncertainty in hub arc location challenges. These advancements not only contribute to academic knowledge but also offer practical insights for improving transportation planning and logistics optimization strategies.

Table 1. Literature review on hub location problems under uncertainty

Papers	Hub features			Uncertain parameters					Solution method			Description
	Single-hub location	P-hub location	Isolated hub location	Demand	Flow	Delivery time	Costs	others	Exact	Heuristic	Meta-heuristic	
Makui, Rostami et al. (2002)	✓			✓				✓				Robust programming
Marianov and Serra (2003)	✓			✓							✓	Tabu search
Ramos, Ramos et al. (2004)	✓				✓				✓			Improved Lagrangian decomposition
Yang (2009)	✓			✓					✓			2-stage stochastic programming
Bashiri and Mehrabi (2010)	✓					✓					✓	GA with Chance constraint
Miranda Junior, Camargo et al. (2011)	✓			✓					✓			Generalized Benders decomposition
Contreras, Cordeau et al. (2011)	✓			✓			✓		✓			Monte Carlo simulation-based algorithm with Benders' algorithm
Yang, Liu et al. (2011)		✓				✓						MILP programming
Alumur, Nickel et al. (2012)	✓			✓			✓					Robust and stochastic optimization
Rostami, Farahani et al. (2012)	✓			✓								Robust optimization with goal programming
Zhai, Liu et al. (2012)	✓			✓								Minimum risk criteria 2-stage stochastic optimization
Mohammadi, Razmi et al. (2013)	✓					✓				✓		Invasive weeds optimization
Wang, Meng et al. (2013)	✓			✓								SAA chance constraint
Mohammadi, Jolai et al. (2013)	✓							✓			✓	NSGA-11 and PAES
Rahmaniani, Ghaderi et al. (2013)	✓			✓				✓		✓		Neighborhood search with robust optimization
Hult, Jiang et al. (2014)		✓						✓	✓			Exact solution algorithm based on separation
Mohammadi, Torabi et al. (2014)	✓			✓							✓	SA, ICA, restrictions chance and fuzzy programming

Table 1. Literature review on hub location problems under uncertainty

Papers	Hub features			Uncertain parameters					Solution method			Description
	Single-hub location	P-hub location	Isolated hub location	Demand	Flow	Delivery time	Costs	others	Exact	Heuristic	Meta-heuristic	
Qin and Gao (2017)	✓				✓							GA with stochastic programming
Habibzadeh Boukani, Farhang Moghaddam et al. (2016)	✓						✓	✓				Robust optimization
Shahabi and Unnikrishnan (2014)	✓			✓								Robust optimization
Ghaffari-Nasab, Ghazanfari et al. (2015)	✓			✓								Robust optimization
Ahmadi, Karimi et al. (2015)		✓		✓								Robust and stochastic programming
Adibi and Razmi (2015)	✓			✓			✓					Stochastic programming
Ghaffari–Nasab, Ghazanfari et al. (2015)	✓				✓							Robust optimization
Zhai, Liu et al. (2016)	✓			✓								Two-stage stochastic programming
Gao and Qin (2016)		✓						✓				Chance constrained programming
Meraklı and Yaman (2016)		✓		✓								Robust optimization
Ghaderi and Rahmaniani (2016)		✓		✓				✓			✓	Robust optimization
Zetina, Contreras et al. (2017)	✓			✓					✓			Branch and cut algorithm with Robust Optimization
Meraklı and Yaman (2016)	✓			✓					✓			Benders' decomposition and Robust Optimization
Talbi and Todosijević (2017)		✓			✓							Robust Optimization
de Sá, Morabito et al. (2018)	✓			✓					✓			Benders' decomposition and Robust Optimization
Ghaffarinasab (2018)		✓		✓							✓	Tabu Search based metaheuristic with Robust Optimization
Wang and Qin (2020)	✓				✓			✓				Chance constrained programming

Table 1. Literature review on hub location problems under uncertainty

Papers	Hub features			Uncertain parameters					Solution method			Description
	Single-hub location	**P-hub location**	**Isolated hub location**	**Demand**	**Flow**	**Delivery time**	**Costs**	**others**	**Exact**	**Heuristic**	**Meta-heuristic**	
Hamid, Bastan et al. (2019)		✓					✓	✓		✓		Heuristic solution with a single-scenario stochastic optimization
Shang, Yang et al. (2020)							✓	✓			✓	Chance-constrained programming and expected-value, Memetic algorithm
Wang, Chen et al. (2020)	✓			✓			✓					Distributionally robust optimization
Hu, Hu et al. (2021)	✓			✓					✓			Chance constrained and Cone programming
Taherkhani, Alumur et al. (2021)	✓			✓				✓	✓			Benders’ decomposition and Robust Optimization
Rostami, Kämmerling et al. (2021)	✓			✓					✓			L-shaped decomposition with Branch and cut and stochastic optimization
Ghaffarinasab and Kara (2022)		✓		✓					✓			Benders’ decomposition and Stochastic programming
Ghaffarinasab (2022)	✓			✓					✓			Benders’ decomposition, Lagrangian relaxation and 2-stage stochastic programming
Rahmati, Neghabi et al. (2023)	✓			✓					✓			L-shaped and Benders’ decomposition with Two-stage stochastic programming
Sener and Feyzioglu (2023)	✓			✓			✓		✓			L-shaped decomposition and stochastic programming
This paper			✓		✓		✓		✓	✓		Tailored GA with Min-max regret modelling in 2-stage programming

Table 2. Research gaps

Hub Arc Location Problem Literature	Other costs	Releasing discount factor restriction	Releasing full connectivity restriction	Problem size	Solution approach	Uncertainty
Hub Arc Location Problems:Part I—Introduction and Results (Campbell, Ernst et al. 2005)	-	✓	✓	-	-	-
Hub Arc Location Problems:Part II—Formulations and Optimal Algorithms (Campbell, Ernst et al. 2005)	-	✓	✓	Small	Enumeration based algorithm	-
Solving Hub Arc Location Problems on a Cluster of Workstations (Campbell, Stiehr et al. 2003)	-	✓	✓	Medium	Parallel Implementation of enumeration-based algorithms	-
Hub location for Time Definite Transportation (Campbell 2009)	-	✓	✓	Small	Exact solution from GAMS optimization software	-
Hub Arc Location with Competition (Sasaki, Campbell et al. 2009)	-	✓	✓	Large	Exact solution	-
Uncertain and sustainable hub location problem (Mohammadi, Torabi et al. 2014)	✓	✓	✓	Large	Heuristic solution	✓
Stochastic uncapacitated P-hub median problem (Hamid, Bastan et al. 2019)	-	-	✓	Small	Heuristic solution	✓
Stochastic hub profit maximization problem (Taherkhani, Alumur et al. 2021)	-	-	✓	Large	Heuristic solution	✓
Maximal covering, median and center P-hub location problem (Ghaffarinasab and Kara 2022)	-	✓	✓	Large	Exact solution	✓
Stochastic and sustainable hub location problem (Rahmati, Neghabi et al. 2023)	-	-	✓	Large	Exact solution	✓
This paper	Set-up cost	✓	✓	Small, Medium, Large	Exact solution Metaheuristic algorithm	✓ **(flow)**

3. Research Framework

The objective of this study is to minimize the total transportation cost and deployment by determining the optimal location of hub arcs and allocating non-hub nodes to the hubs. Hub arcs are defined as arcs connecting two hub nodes and providing cost-efficient flow transportation through economies of scale. Unlike traditional hub location problems, this research departs from the assumption of full connectivity in the hub network and instead introduces a discount factor exclusively applied to hub arcs, aiming to align with real-world scenarios. Moreover, the focus is shifted towards locating hub arcs rather than hub nodes.

To formulate a hub arc location problem under uncertainty, it is necessary to first discuss the deterministic problem. Consequently, this study proposes a mixed-integer linear formulation for hub arc location problems. In the formulation, the objective function incorporates set-up costs, and the hub network structure accounts for the presence of isolated hubs, facilitating a comprehensive analysis of the problem at hand.

Consider a graph $G = (V, E)$ with vertex set $V = 1, ..., N$. The demand or flow between node o and node d is W_{od}. The cost of traveling from node i to node j is dis_{ij} and follows triangle inequality theorem. The hub arcs that connect hub nodes i and j have αdis_{ij} discounted costs per flow unit ($0 < \alpha < 1$). The number of hub arcs and the maximum number of hub nodes on the network are exogenous and hub node capacity is assumed unlimited. Also, the origin-destination route contains at least one and at most two hub facilities.

Assumptions and Notation

Indices

o	Set of origin nodes
d	Set of destination nodes
i	Set of first hub nodes
j	Set of second hub nodes (if needed)

Parameters

W_{od}: The flow between origin o and destination d

dis_{ij}: The distance between node i and j

f_i: Set-up cost for hub i

α: Discount factor between two hubs

q: Number of hub arcs

p: Maximum number of hubs

Decision variables

X_{ijod}: If the flow from origin i to destination j passes through hub i, 1; otherwise, 0.

Y_{ij}: If there is a hub arc between hub i and hub j, 1; otherwise, 0.

Z_i: If node i is a hub node, 1; otherwise, 0.

$$Min \sum_{\substack{o,d,i,j \\ o<d}} (W_{od} + W_{do})\big(dis_{oi} + \propto dis_{ij} + dis_{jd}\big)X_{ijod} + \sum_i f_i Z_i \tag{1}$$

St.

$$\sum_i \sum_j X_{ijod} = 1, \quad \forall o, d, o < d \tag{2}$$

$$\sum_i \sum_j Y_{ij} = q \tag{3}$$

$$\sum_i Z_i \leq p \tag{4}$$

$$X_{iiod} + \sum_{\substack{j \\ j \neq i}} (X_{ijod} + X_{jiod}) \leq Z_i, \quad \forall i, o, d, o < d \tag{5}$$

$$X_{ijod} \leq Y_{ij}, \quad \forall i, j, o, d, o < d, i \neq j \tag{6}$$

$$Y_{ij} \leq Z_i, \quad \forall i, j, i \neq j \tag{7}$$

$$Y_{ij} \leq Z_j, \quad \forall i, j, i \neq j \tag{8}$$

$$X_{ijod} \geq 0 \tag{9}$$

$$Z_i, Y_{ij} \in \{0,1\} \tag{10}$$

The objective of this paper is to minimize the total transportation and set-up costs. The constraints are designed to ensure specific conditions for the routing of flows through hub arcs. Constraint (2) guarantees that flow between any origin and destination must pass through a pair of hubs, or in some cases, a single hub X_{iiod}), disallowing direct paths between origins and destinations. Constraints (3) and (4) restrict the number of hub arcs and the maximum number of hub nodes, respectively. For each route that utilizes a specific hub, constraint (5) ensures that the corresponding hub is opened. Furthermore, constraint (6) ensures that a hub arc must be opened for each flow transmission routed through it. Constraints (7) and (8) stipulate that both ends of a hub arc must be hub nodes. Lastly, constraints (9) and (10) define the characteristics of the decision variables.

It is important to clarify that although the routing decision variable X_{ijod} is formulated as a continuous variable, it effectively takes binary values. When the active hub nodes and hub arcs are known, the optimal route between each origin-destination pair is determined based on the path with the least transportation cost. As the hub facilities do not have capacity restrictions, only one path is selected for the current routing. Therefore, despite the variable being continuous in its formulation, the problem's formulation inherently forces it to take binary values.

3.1.1 Flow Uncertainty

To address the challenge of flow uncertainty in the hub arc location problem, two modeling approaches, namely min-max regret and two-stage stochastic programming, are employed. In addition to the previously mentioned assumptions, the following assumptions are introduced:

1. Each pair of nodes is subject to uncertain flow.
2. The flow uncertainty is represented by discrete scenarios. In the robust approach, the scenarios are considered based on an unknown probability distribution, while the stochastic approach assumes a uniform probability distribution with equal probability assigned to each scenario.
3. In the robust approach, the objective function aims to minimize the maximum deviation between the proposed solution and the optimal solution under each scenario.
4. In the stochastic approach, the objective function aims to minimize both the total cost of hub establishment and the expected value of the flow allocation cost.

As stated earlier, location decisions regarding hub arcs are long-term strategic decisions, and one way to address the uncertainty of future events is by defining alternative future scenarios (Ravelo 2021). Since incorporating uncertainty in hub arc location problems is a novel area of study, there is currently no existing research in the literature that addresses this aspect. The introduction of modeling uncertain flows as limited discrete scenarios, using robust optimization and two-stage stochastic programming, represents a pioneering contribution in this regard. To this end, the previous deterministic model is expanded by introducing the following variables and parameters.

Indices

s: Set of scenarios S

Parameters

W_{sod}: Demand from origin o to destination d under scenario s

$C^{*}(s)$: The optimal value related to scenario s

P_s: The probability of scenario s

Decision variables

R_s: The regret related to scenario s

X_{ijod} (s): If the flow under scenario s is transferred from origin o to destination d through hubs i and j, 1; otherwise, 0.

Y_{ij} (s): If under scenario s there is hub arc between i and j, 1; otherwise, 0.

3.1.2 Min-max Regret Modeling

As previously mentioned, the primary objective of a robust approach is to identify decisions that perform well across all possible scenarios. Regret refers to the disparity between the quality of a given strategy and the quality that would have been chosen if future events were certain (Berman, Drezner et al. 2007). A robust solution is one that minimizes the maximum regret among all scenarios. In other words, regret represents the difference between the output obtained under a given scenario and the optimal output under the same scenario (Hekmatfar and Pishvaee 2009). Minimizing the maximum regret is a common strategy for achieving robustness in combinatorial optimization problems, particularly in location problems.

Models that minimize the maximum regret across all scenarios are known as *Minmax Regret* models. These models incorporate uncertainty into the objective function and are referred to as robust deviation because they minimize the deviation between the objective function of the best possible solution for a scenario and the objective function of the given solution. Instead of minimizing the worst performance of a solution, they focus on minimizing the difference in objective function values that can arise in different scenarios.

$$Min \max_{s \in S} R_s \tag{11}$$

St.

Constraints (2 - 10)

$$R_s = \left[\sum_{\substack{o,d,i,j \\ o<d}} (W_{od}^s + W_{do}^s)\left(dis_{oi} + \propto dis_{ij} + dis_{jd}\right) X_{ijod} + \sum_i f_i Z_i \right] - C^*(s), \quad \forall s \in S \tag{12}$$

Equation (11) represents minimizing maximum regret. Constraints (12) are related to each scenario’s regret. Constraints (2) to (10) are the same as the previous section.

The above model linearization is as follows:

$$Min\ R \tag{13}$$

St.

Constraints $(2-10)$

$$R \geq R_s\,, \quad \forall s \in S \tag{14}$$

3.1.3 Two-stage Stochastic Programming

In this section, we make the assumption that imprecise flows can be modeled by a discrete uniform probability distribution. This distribution is characterized by a restricted set of scenarios, each assigned a probability of Ps. We address the hub arc location problems in the context of uncertainty by employing a two-stage stochastic programming approach with recourse. The first stage involves determining the locations of hub facilities, while the second stage focuses on optimizing the allocation of non-hub nodes through hub nodes and hub arcs.

A random variable ξ represents a scenario. This means that $\xi \in S$ and $P_s = P\ [\xi = s],\ s \in S$. Besides, $W_{od}\ (\xi)$ represents a random variable of flow from origin to destination.

First Stage:

$$Min_Z \sum_i f_i Z_i + E_\xi[Q(Z,\xi)] \tag{15}$$

St.

$$\sum_i Z_i \leq \text{p} \tag{4}$$

$$Z_i \in \{0,1\} \tag{16}$$

Second Stage:

$$Q(Z,\xi) = Min_{X,Y} \sum_{\substack{o,d,i,j \\ o<d}} (W_{od}(\xi) + W_{do}(\xi))\big(dis_{oi} + \propto dis_{ij} + dis_{jd}\big) X_{ijod} \tag{17}$$

$$\sum_i \sum_j X_{ijod} = 1, \quad \forall o,d,o<d, \tag{2}$$

$$\sum_i \sum_j Y_{ij} = q \tag{3}$$

$$X_{iiod} + \sum_{\substack{j \\ j \neq i}} (X_{ijod} + X_{jiod}) \leq Z_i, \quad \forall i,o,d,o<d \tag{5}$$

$$X_{ijod} \leq Y_{ij}, \quad \forall i,j,o,d,o<d,i \neq j \tag{6}$$

$$Y_{ij} \leq Z_i, \quad \forall i,j,i \neq j \tag{7}$$

$$Y_{ij} \le Z_j, \quad \forall i, j, i \neq j \tag{8}$$

$$X_{ijod} \ge 0, \quad \forall i, j, o, d, o < d \tag{9}$$

$$Y_{ij} \in \{0,1\}, \quad \forall i, j, i \neq j \tag{18}$$

Equation (15) represents the total cost of setting up a hub facility plus the expected cost of allocation and routing between the origin-destination pairs. In the second stage, Equation (17) variables Z_i are fixed and W_{od} *(ξ)* are random variables for all *i* and *j*. X_{ijod} variables are recourse variables because the accurate routing through the network is determined only after specifying the flows. Moreover, Equation (16) assures Z_i to be binary.

Because there is no capacity constraint in the model, for each feasible solution of the first stage (network configuration), there is at least a feasible solution for the second stage. Since the optimal routing between the origin-destination is determined after the possible occurrence of scenario *s*, their values must be determined under each scenario. The decision variables Y_{ij} *(s)* and X_{ijod} *(s)* are used as variables related to the arc between hub *i* and hub *j* and the route between *o* and *d* via hub *i* and hub *j* under scenario *s*, respectively. Now we can provide a certain equivalent of hub arc location problem with uncertain flows.

$$Min \sum_{s \in S} P_s \sum_{\substack{o,d,i,j \\ o<d}} \left(W_{od}(s) + W_{do}(s)\right)\left(dis_{oi} + \propto dis_{ij} + dis_{jd}\right) X_{ijod}(s) + \sum_i f_i Z_i \tag{19}$$

$$\sum_i Z_i \le \mathrm{p} \tag{4}$$

$$Z_i \in \{0,1\} \tag{16}$$

$$\sum_i \sum_j X_{ijod}(s) = 1, \quad \forall o, d, o < d, s \in S \tag{20}$$

$$\sum_i \sum_j Y_{ij}(s) = q \tag{21}$$

$$X_{iiod}(s) + \sum_{\substack{j \\ j \neq i}} \left(X_{ijod}(s) + X_{jiod}(s)\right) \le Z_i, \quad \forall i, o, d, o < d, s \in S \tag{22}$$

$$X_{ijod}(s) \le Y_{ij}(s), \quad \forall i, j, o, d, o < d, i \neq j, s \in S \tag{23}$$

$$Y_{ij}(s) \le Z_i, \quad \forall i, j, i \neq j, s \in S \tag{24}$$

$$Y_{ij}(s) \leq Z_j, \quad \forall i, j, i \neq j, s \in S \tag{25}$$

$$X_{ijod}(s) \geq 0, \quad \forall i, j, o, d, o < d, s \in S \tag{26}$$

$$Y_{ij}(s) \in \{0,1\}, \forall i, j, i \neq j, s \in S \tag{27}$$

The objective function (19) represents minimizing the total set-up cost of hub facilities and the expected value of transportation and allocation costs. Constraints (20-27) are similar to constraints of the main model that are explained in the previous section, except that these are considered under scenario *s*. Moreover, equation (16) assures Z_i to be binary.

4. Research Methodology

The hub arc location problem addressed in this article is known to be NP-Hard, as highlighted by (Campbell, Ernst et al. 2005). Moreover, considering the vast search space involved, the problem becomes even more complex when uncertainties are taken into account. To overcome these challenges within a reasonable timeframe and on a large scale, employing a metaheuristic method is a suitable choice. Among the various metaheuristics utilized in hub location literature, the genetic algorithm (GA) has gained significant popularity and achieved favorable results in terms of solution quality. The GA algorithm initiates with an initial population comprising randomly generated answers, referred to as chromosomes. Each chromosome represents a potential solution to the problem, whether feasible or infeasible. These chromosomes are composed of a fixed number of genes, which encapsulate the information contained within each answer. Illustrated in **Fig 1**, each chromosome undergoes flourishing processes to form subsequent generations. Within each generation, chromosomes are assessed based on a fitness measure, enabling the preservation and progression of better solutions to the next generation. Regarding the representation of solutions, since there can be a maximum of two hub nodes and at least one hub node for each origin-destination route, the solution representation takes the form of a 2*q matrix. The first row corresponds to the first hub, the second row represents the second hub (if non-repetitive), and the number of columns aligns with the required hub arcs in the problem.

	1	2	...	*Q*
First Hub				
Second Hub				

Fig 1. The structure of a chromosome

Initial population: the first generation of genetic algorithm is created randomly. First, p number of hub nodes are chosen and non-repetitive different pairwise states of them are formed to obtain possible hub arcs. Then q numbers of these non-repetitive binary combinations will be chosen randomly as hub arcs.

Fitness function: fitness function is used as an evaluation function for each chromosome. All of the obtained results for each origin-destination pair are stored in an auxiliary variable called X that is a matrix of 4 rows and $\binom{N}{2}$ columns. Each chromosome that has a better objective function value will have more utility and is more likely to be transmitted to the next generation.

Selection operator: This operator is the selection process of two qualified parents of the population to lead to the procreation of children with high fitness. Each member with a probability proportional to its utility, or its fitness function value, has the chance of being selected. We use the Roulette Wheel method in the way that the wheel is partitioned to the number of the population members and the surface of each section is proportional to the fitness value of each chromosome. Then the wheel is rotated to stop somewhere randomly. This point identifies the selected chromosome.

Crossover operator: Crossover is a process of producing new children from parents. In this paper, the Single-Point Crossover method is used in the way that two chromosomes randomly break from a point and both parts are displaced with each other from the fracture point. (See **Fig 2**)

Mutation operator: Using this operator leads to releasing from local optimum, improving the probability of finding the global optimum responses and maintaining diversity. In this method, a single parent is selected, and then one of the hubs is randomly selected and replaced with a non-hub node. Then each arc that is connected to the replaced hub will be updated to the new hub. (See **Fig 3**)

Stopping criterion: Genetic algorithm stops when the specified number of generations occurred.

Parameter calibration: Inexact algorithms result in a variety of answers, so an acceptable algorithm is an algorithm that its solutions converge to the global optimal solution. For this purpose, we use the Taguchi method to calibrate the algorithm parameters. The Taguchi method seeks to minimize the effect of noise factors and find the optimal level of control factors using the *S/N* ratio. The goal is to maximize this ratio.

$$\frac{S}{N}\text{ratio} = -10\log\left(\frac{1}{n}\sum_{i=1}^{n}\tau_i\right)^2 \quad (27)$$

				Crossover point	
Parent 1	1	2	2	1	3
	3	3	5	4	5
Parent 2	1	1	1	3	2
	6	2	3	4	4
Offspring 1	1	2	2	3	2
	3	3	5	4	4
Offspring 2	1	1	1	1	3
	6	2	3	4	5

Fig 2. The structure of crossover operation

Parent	1	2	2	1	3
	3	3	**5**	4	**5**
offspring	1	2	2	1	3
	3	3	**6**	4	**6**

Fig 3. The structure of mutation operation

The complete form of the Genetic algorithm is showed in the **Fig 4**. The controllable factors of this model include population size, the maximum number of iterations, crossover probability and mutation probability. The first group with index S is related to a small scale with a maximum of 20 nodes, and the second group with index L is related to a large scale with more than 20 nodes. Different factors are defined in four levels and are provided in **Table 3**.

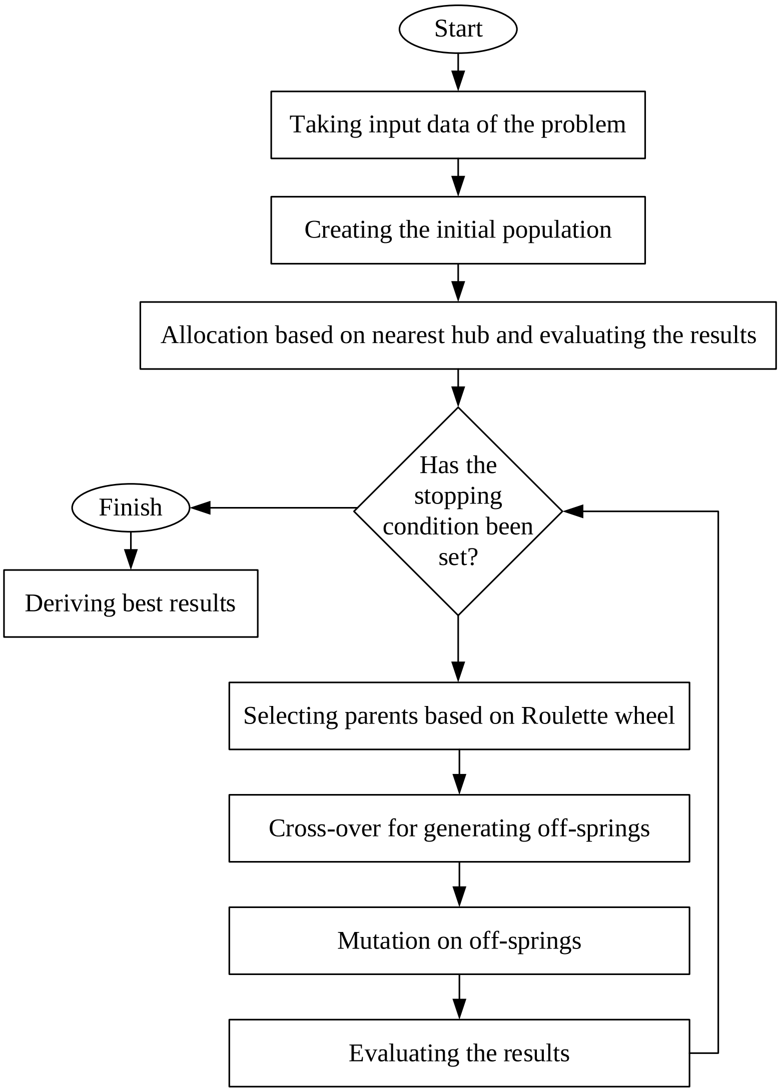

Fig 4. The proposed Genetic algorithm flowchart

Table 3. Proposed Factors and levels

Factor 1		Factor 2		Factor 3	Factor 4
Population Size (A)		**Maximum Number Of Repetitions (B)**		**Crossover Probability (C)**	**Mutation Probability (D)**
A_S	A_L	B_S	B_L		
$A_S(1)$: 50	$A_L(1)$: 100	$B_s(1)$: 100	$B_L(1)$: 150	C(1): 0.8	D(1): 0.9
$A_S(2)$: 70	$A_L(2)$: 130	$B_s(2)$: 150	$B_L(2)$: 200	C(2): 0.6	D(2): 0.8
$A_S(3)$: 100	$A_L(3)$: 170	$B_s(3)$: 200	$B_L(3)$: 300	C(3): 0.4	D(3): 0.7
$A_S(4)$: 150	$A_L(4)$: 230	$B_s(4)$: 250	$B_L(4)$: 400	C(4): 0.2	D(4): 0.6

The *S/N* ratio is used to determine the appropriate levels of controllable factors as the main priority. The average value of the objective function and average runtime is used as the second and third priority respectively to strengthen the assessment. The more the *S/N* ratio and the less the average runtime and the objective function, the more appropriate is the parameter. Table 4 and **Fig 5 - Fig 10** are obtained from *Minitab* software for small and medium/large scale problems.

Table 4. Taguchi experimental design

Experiment	Factors			
	Mutation Probability	Crossover Probability	Maximum Number Of Repetitions	Population Size
1	D(1)	C(1)	B(1)	A(1)
2	D(2)	C(2)	B(2)	A(1)
3	D(3)	C(3)	B(3)	A(1)
4	D(4)	C(4)	B(4)	A(1)
5	D(3)	C(2)	B(1)	A(2)
6	D(4)	C(1)	B(2)	A(2)
7	D(1)	C(4)	B(3)	A(2)
8	D(2)	C(3)	B(4)	A(2)
9	D(4)	C(3)	B(1)	A(3)
10	D(3)	C(4)	B(2)	A(3)
11	D(2)	C(1)	B(3)	A(3)
12	D(1)	C(2)	B(4)	A(3)
13	D(2)	C(4)	B(1)	A(4)
14	D(1)	C(3)	B(2)	A(4)
15	D(4)	C(2)	B(3)	A(4)
16	D(3)	C(1)	B(4)	A(4)

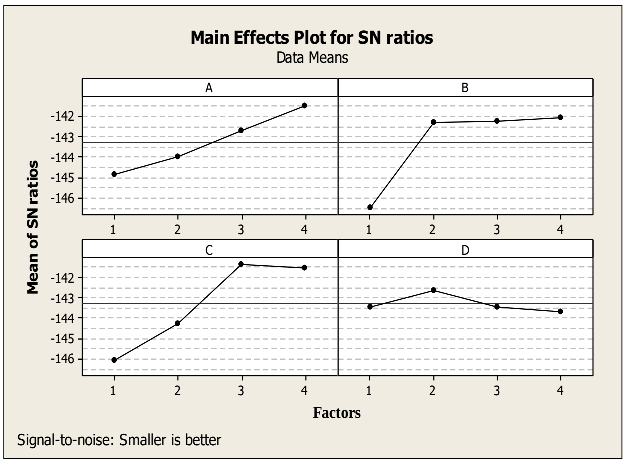

Fig 5. Mean of S/N ratio for small size

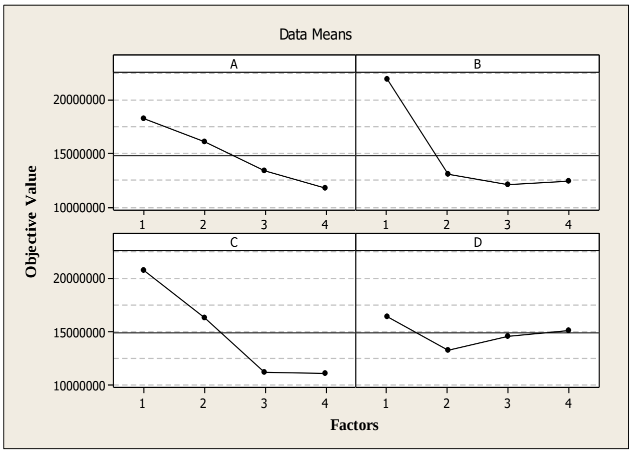

Fig 6. Average response variable for small size

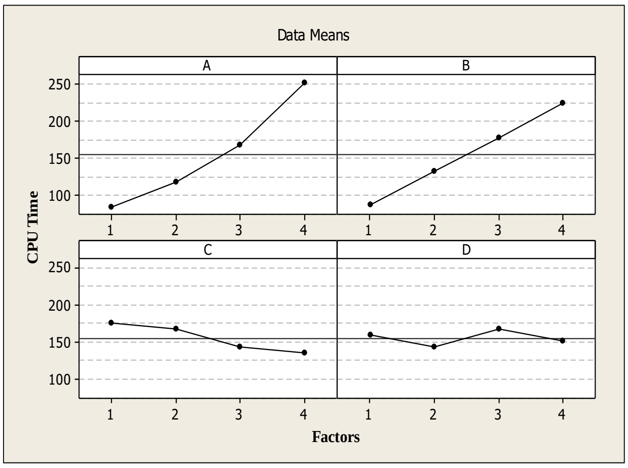

Fig 7. Mean of runtime for small size

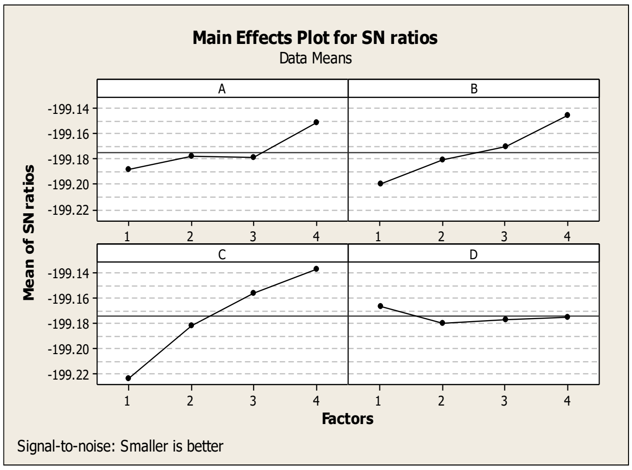

Fig 8. Mean of S/N ratio for large size

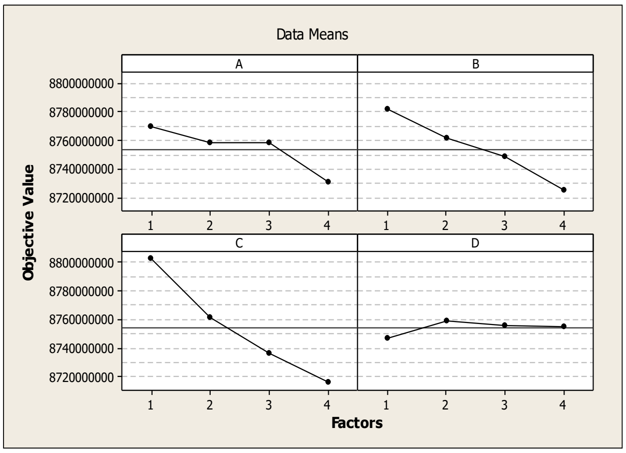

Fig 9. Average response variable for large size

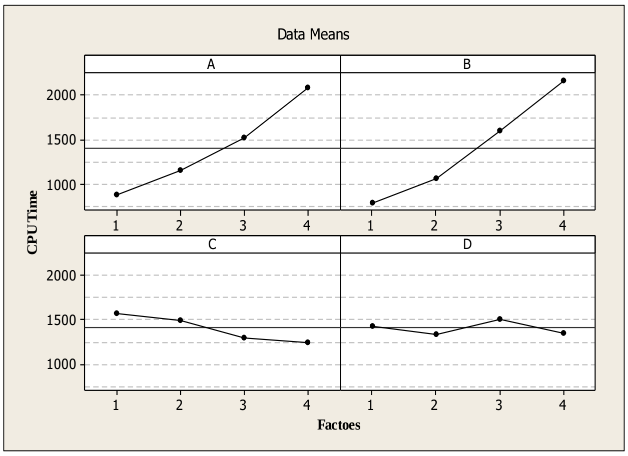

Fig 10. Mean of runtime for large size

The appropriate parameters for the small and medium/large scale problems are shown in **Table 5.**

Table 5. Optimum level for proposed GA controllable factors

Factors	Symbol	Optimum Level for Large Size	Optimum Level for Small Size
Population Size	A	A(4) = 230	A(4) = 150
Maximum Number of Repetitions	B	B(4) = 400	B(3) = 200
Crossover Probability	C	C(4) = 0.2	C(4) = 0.2
Mutation Probability	D	D(1) = 0.9	D(2) = 0.8

5. Result

The hub arc location problem under uncertainty will be solved by applying the proposed genetic algorithm to the robust model. To validate the meta-heuristic methods, the obtained results are compared to the exact optimal solution from *GAMS* optimization software in terms of average and worst error and mean run time.

To evaluate the proposed solution method, it is necessary to test their performance in different sample problems. We use four sample problems that are taken from data sets *AP* (Ernst and Krishnamoorthy 1996). The proposed heuristic and meta-heuristic algorithms are implemented in *Matlab* software *R2014b* and exact solutions are obtained from *GAMS* software 24.1.2 *Cplex* solver. All calculations related to the sample problems have been done on a computer with specs *Intel Core i7-2670 QM 2.2 GHz* Memory 8 *GB.*

As the genetic algorithm starts with a random initial solution, we repeat the algorithm three times for each sample problem and each size and report the average results. We show the uncertainty with five different discrete scenarios. According to (Gelareh and Nickel 2007), to create series of different flow scenarios, for each pair of $i, j \in N$ nodes, five random values between interval $[0.01w_{ij}, 10w_{ij}]$ is produced, such that w_{ij} is the available flow between nodes in the standard data set *AP*. To reduce the symmetry around the generated scenario's mean, the flow range is split into two parts $[0.01w_{ij}, 5w_{ij}]$ and $[5w_{ij}, 10w_{ij}]$. In each scenario, for each pair of $i, j \in N$ nodes, flow is a random value with the probability of 2/3 from the first interval and with the probability of 1/3 from the second interval (Gelareh and Nickel 2007).

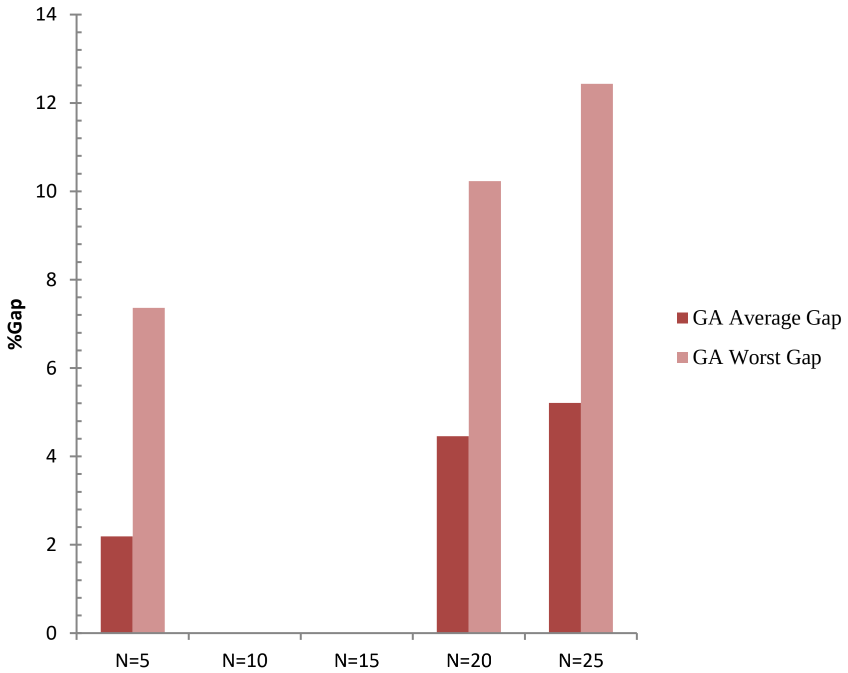

Fig 11. GA error

To solve the problem under uncertainty, each scenario is solved as a deterministic problem in *GAMS* optimization software, and then their optimal values are entered into the Minmax Regret model. According to **Table 6** and **Fig 11** by increasing the size of the problem, the optimal solution's runtime increases exponentially and **Fig 12** presents that average and worst error of the proposed Genetic algorithm increases as the size the problem rises. Moreover, the proposed genetic algorithm average error of 2.44% and the maximum error of 6% represent an acceptable solution quality in reasonable run time.

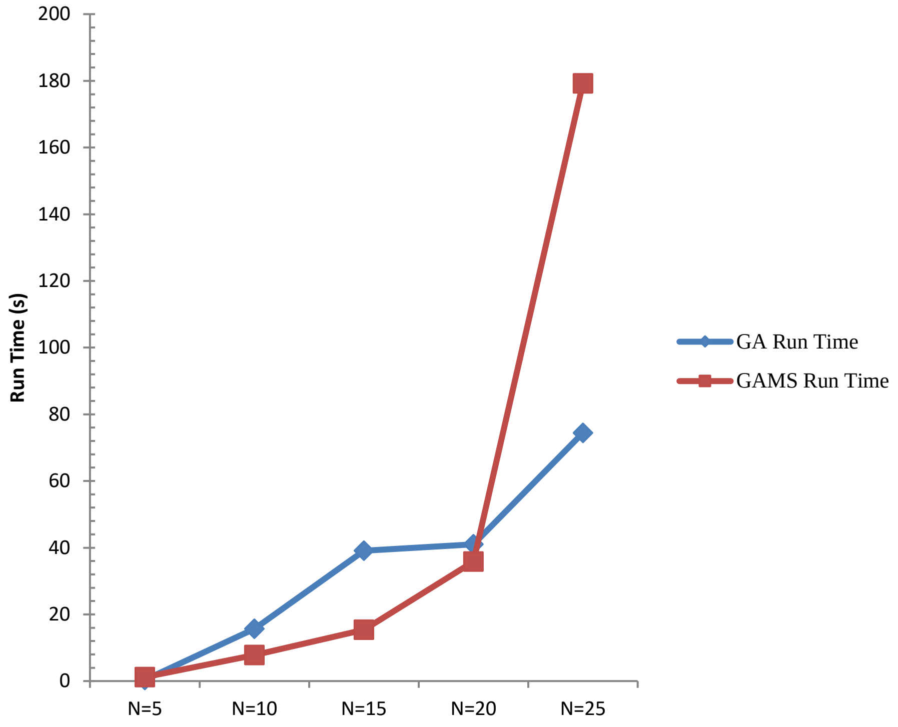

Fig 12. Comparing run times

Table 6. Average error and runtime

α =0.2		Gams Solution Average CPU Time (s)	Genetic Algorithm Solution % Gap Average	Worst	Average CPU Time (s)
***N*=5**	***q*=1, *p*=2**	1.19	2.55	7.36	0.35
***N*=10**	***q*=2, *p*=4**	7.80	0	0	15.68
***N*=15**	***q*=3, *p*=6**	15.42	0	0	39.05
***N*=20**	***q*=4, *p*=8**	35.85	4.46	10.23	41.04
***N*=25**	***q*=5, *p*=10**	179.20	5.21	12.43	74.44
Average		47.89	2.44	6.00	34.11

In this part, an equivalent expected value of the scenarios, named Base case problem, is defined by replacing the average value of the parameters with the inexact parameter. The resulting solution of the Base case problem is compared with the Minmax Regret model results.

Table 7. Comparing results

Problem Size	α=0.2	Best Cost	Best Hubs	Best Hub Arcs
	Scenario 1	8300731	2, 3	2-3
	Scenario 2	23910042	2, 3	2-3
N = 5	**Scenario 3**	42866688	2, 3	2-3
Q = 1	**Scenario 4**	42380636	1, 3	1-3
P = 2	**Scenario 5**	40615964	2, 3	2-3
	Base Case	32356591	2, 3	2-3
	Minmax Regret	**GAMS Solution**	2, 3	2-3
	Scenario 1	43297891	1, 4, 7, 8	7-1, 7-4
	Scenario 2	206248180	2, 3, 7, 8	2-8, 3-7
N = 10	**Scenario 3**	245296135	2, 3, 7, 8	2-7, 3-7
Q = 2	**Scenario 4**	172996832	1, 4, 7, 8	1-8, 4-7
P = 4	**Scenario 5**	202295798	2, 3, 7, 8	2-8, 3-7
	Base Case	174745740	2, 3, 7, 8	2-8, 3-7
	Minmax Regret	**GAMS Solution**	2, 3, 7, 8	2-8, 3-7
	Scenario 1	28491846	2, 6, 8, 9, 10, 14	2-14, 6-14, 8-14
	Scenario 2	167581080	2, 4, 6, 9, 11, 14	2-14, 4-14, 11-14
N = 15	**Scenario 3**	145085099	2, 3, 6, 8, 9, 14	2-14, 6-14, 8-14
Q = 3	**Scenario 4**	146273345	2, 4, 6, 11, 13, 14	2-14, 4-11, 13-14
P = 6	**Scenario 5**	136570832	2, 6, 8, 9, 10, 14	2-10, 6-14, 8-14
	Base Case	126474449	2, 6, 8, 9, 10, 14	2-14, 6-14, 8-14
	Minmax Regret	**GAMS Solution**	2, 5, 6, 8, 10, 14	2-14, 6-14, 8-14
	Scenario 1	214460233	2, 4, 6, 11, 13, 14, 15, 16	2-14, 4-11, 6-14, 14-16
	Scenario 2	225702878	2, 4, 7, 9, 12, 13, 14, 15	2-14, 4-14, 9-14, 12-14
N = 20	**Scenario 3**	208691489	2, 4, 6, 7, 9, 14, 15, 16	2-14, 4-15, 6-14, 15-16
Q = 4	**Scenario 4**	208699272	2, 4, 6, 11, 12, 13, 14, 15	2-14, 4-11, 6-14, 12-15
P = 8	**Scenario 5**	220534032	2, 4, 6, 11, 12, 13, 14, 20	4-14, 6-14, 12-14, 13-14
	Base Case	217147998	2, 4, 6, 11, 12, 13, 14, 20	2-14, 4-11, 6-14, 12-14
	Minmax Regret	**GAMS Solution**	2, 4, 6, 11, 12, 13, 14, 20	4-14, 6-14, 12-14, 13-14
	Scenario 1	42575653	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 17-18, 18-20
	Scenario 2	172223571	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 17-18, 18-20
N = 25 **Q = 5** **P = 10**	**Scenario 3**	178572808	2, 5, 7, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 16-18, 18-20
	Scenario 4	172554879	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 17-18, 18-20
	Scenario 5	173923241	2, 5, 6, 8, 13, 16, 17, 18, 20, 25	2-18, 5-13, 8-18, 17-18, 18-20

Table 7. Comparing results

Problem Size	α=0.2	Best Cost	Best Hubs	Best Hub Arcs
N = 30 Q = 6 P = 12	**Base Case**	148100336	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 16-18, 18-20
	Minmax Regret	**GAMS Solution**	2, 5, 6, 8, 9, 13, 16, 17, 18, 20	2-18, 5-13, 8-18, 16-18, 18-20
	Scenario 1	27596542	3, 4, 7, 8, 10, 12, 13, 18, 20, 23, 26, 28	3-28, 4-12, 8-20, 13-28, 18-28, 23-28
	Scenario 2	115002418	3, 4, 7, 8, 10, 12, 13, 18, 20, 23, 26, 28	3-28, 4-12, 8-20, 13-28, 18-28, 23-28
	Scenario 3	113368312	3, 4, 7, 8, 10, 12, 14, 18, 20, 23, 26, 28	3-28, 4-12, 8-20, 14-28, 18-28, 23-28
	Scenario 4	114104418	3, 4, 7, 8, 10, 12, 13, 17, 20, 23, 26, 28	3-28, 4-12, 8-13, 13-28, 23-28, 26-28
	Scenario 5	113843763	3, 4, 7, 8, 10, 12, 13, 17, 20, 23, 26, 28	3-28, 4-12, 8-13, 13-28, 23-28, 26-28
	Base Case	96883012	3, 4, 7, 8, 10, 12, 13, 18, 20, 23, 26, 28	3-28, 4-12, 8-20, 13-28, 18-28, 23-28
	Minmax Regret	**GAMS Solution**	3, 4, 7, 8, 10, 12, 13, 18, 20, 23, 26, 28	3-28, 4-12, 8-20, 13-28, 18-28, 23-28

In the min-max regret models, it's important to note that the transportation costs and consequently the total cost vary across different scenarios. However, **Table 7**, fails to reflect these variations. In the AP standard data set, a significant portion of the demand originates from a small number of nodes. As a result, the range associated with these nodes, which is used to generate random scenarios, is considerably wide. Accordingly, these nodes frequently emerge as potential hub facilities in the majority of scenarios. For instance, Nodes #7 and #8 consistently handle substantial flow volumes and are often selected as hub nodes.

The results indicate that the optimal network structure, including the locations of hubs and hub arcs, differs among scenarios, as well as in comparison to the Base Case problem and the optimal solutions obtained from the robust models. Consequently, it becomes imperative to carefully consider the most effective approach for locating hubs and hub arcs in order to minimize future costs in the face of uncertainties. The solutions obtained through the robust model adopt a pessimistic perspective by minimizing the regret associated with the worst-case scenario. This implies that the results achieved are deemed satisfactory for each individual scenario.

To provide a comprehensive summary of the conducted analyses, it is essential to emphasize, as previously mentioned in this article, that in order to solve the problem under conditions of uncertainty, the flow parameter has been considered in the form of limited discrete scenarios, and we have utilized three solution approaches. The primary approach under investigation is a robust approach using the minimax regret model, which we have solved using a Genetic algorithm. By comparing the solution time and the errors of the obtained solutions with respect to the exact solution, we have successfully validated the proposed solution method. The results indicate a satisfactory quality of the obtained solutions and a reasonable solution time.

Furthermore, for the analysis of the primary solution approach under conditions of uncertainty, we have employed a two-stage stochastic programming and assumed that the uncertain parameter follows a discrete uniform probability distribution. We have solved the problem and compared both the robust and probabilistic approaches with the solution obtained from the expected value problem. The results demonstrate that the optimal solutions of the problem vary when confronted with inaccurate parameters. Therefore, it can be stated that the optimal solutions are sensitive to flow values. Consequently, in situations with high uncertainty regarding these parameters and lack of knowledge about the distribution information of the inaccurate parameter, the utilization of the robust approach is advisable. However, when the probability distribution information of the inaccurate parameter is known, the probabilistic programming approach is highly preferable to inaccurate parameter estimation or utilization of mean values. This is because using deterministic values in uncertain conditions leads to suboptimal solutions that impose additional costs on the system in the long run.

6. Conclusion and Discussion

In this paper, we have developed a hub arc location problem considering flow uncertainty, isolated hubs, and setup costs. Our model allows for a flexible hub network structure, where the presence of isolated or individual hubs is permitted. Additionally, we have incorporated a discount factor exclusively on hub arcs. To tackle the problem under uncertainty, we have considered discrete limited scenarios for the flow parameters and employed three approaches to solve it.

The primary method employed is a robust model utilizing the min-max regret approach, which we have solved using a genetic algorithm. To validate the solution, we have compared the runtime and errors against the exact solution. The obtained results demonstrate the satisfactory quality of the answers and reasonable runtimes achieved.

Based on our findings, several future research directions have been proposed. Firstly, incorporating additional parameters such as cost and distance between origin-destination pairs, or their combinations, under uncertain conditions should be explored. Representing inexact parameters using continuous or interval scenarios can provide more realistic models. Additionally, investigating exact solution methods like branch and bound, cutting branches, and branch and price techniques can offer more precise solutions. Furthermore, optimizing alternative methods such as a fuzzy optimization approach can be explored to solve the problem under uncertainty.

Expanding our understanding of modeling under uncertainty and enhancing the effectiveness of solution approaches will contribute to the advancement of transportation planning and logistics optimization. By addressing these avenues of investigation, practical applications in real-world scenarios can benefit from improved decision-making and cost minimization in hub arc location problems.

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