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Back-Stepping Sliding Mode Controller for Uncertain Chaotic Colpitts Oscillator with no Chattering

Maryam Ghorbani, Hamid Ghadiri*

Department of Electrical, Biomedical and Mechatronics Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran

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Abstract

By introducing Colpitts oscillator as a chaotic system, this paper deals with back-stepping control method and investigates the restrictions and problems of the controller where non-existence of a suitable response in the presence of uncertainty is the most important problem to note. In this paper, the back-stepping sliding mode method is introduced as a robust method for controlling nonlinear Colpitts oscillator system with chaotic behavior. Thereafter, we simulated the proposed method and compared its advantages with that of the previous method. The experimental results show that the most important advantages of the proposed method are making system robust in case of uncertainties and disturbances, and also having a fast response.

Keywords: *Back-Stepping Sliding Mode Controller, Chattering Phenomena, Chaos, Uncertainty, MAE, MSE.*

1. Introduction

Colpitts oscillator was named after Edwin Colpitts, the researcher who invented that device. This oscillator is only one of various designs for electronic circuits and utilizes a combination of inductor and capacitor for describing gain frequency characteristics, for it is called inductive-capacitive oscillator. One key feature of these types of oscillators is their simplicity, which needs only a single inductor. Another feature which discriminates these oscillators from the others is their application in chaotic behavior and robustness against uncertainty [1].

Chaotic theory has many applications in extensive fields such as in communication and telecommunications, safe data transfer, physical systems and ecological systems [2]. Due to the importance of this subject, researchers have carried out extensive studies in different areas associated with this issue, where one of such areas is designing that type of oscillators having chaotic behavior which is due to the commercial applications of chaotic signals in some specific areas such as decoding, communication, telecommunication, and radars [4, 5]. In fact, due to the need for chaotic signals, it is vital to design the circuits that can generate these signals.

Chaotic behavior of Colpitts oscillator was first observed by Kennedy [3]. One of basic applications of Colpitts chaotic oscillators is for transmitting and receiving data in binary telecommunications. In this application, the transmitter system is equipped with a chaotic Colpitts oscillator which contains a parameter modulated with information signal. Every symbol that is to be transmitted is coded as an attractor in Colipitts oscillator.

The unique feature of Colpitts is that its feedback signal is obtained from a voltage divider consisting of two series capacitors. These oscillators are favorable in radio

* Corresponding author. Email: h.ghadiri@qiau.ac.ir

frequencies related applications, and it has been years since they are used in these fields [4].

In the recent years, due to the importance of Colpitts system, numerous studies have been performed in the field of oscillator system control. In one of the simplest methods, using the error system definition as a linear dynamics, first, sufficient conditions are provided for the existence of the controller and then it is designed [6]. Since the considered system is nonlinear, this method is not efficient enough to control the system. The next method is based on back-stepping control method. Lyapunov function is chosen recursively and by using a kinematics method, and desired control input is calculated [2-4]. This method does not ensure appropriate efficiency in case of uncertainties. In Colpitts system, the capacitors used in the circuit may have depreciation errors, or resistor values may change due to becoming hot, or even parameters of the transistor may change over time. Colpitts oscillator system is influenced by the above mentioned factors, and system dynamics may experience variations. Now, if these variations

Are not considered in designing the controller, it may result in steady-state error and since the applied controller in [7] in not robust against uncertainty, practically, using this controller will

Cause some problems. In [2] the general layout of back-stepping sliding mode controller will be explained. Deficiency of this method is occurrence of vibration phenomenon. Vibration is a common damaging event in sliding mode controller. Control input should have very changes in a short period of time in order to yield to a desirable output, where not any system could apply a control input with these characteristics and these variations would result in more loss for the system. A method has been proposed in [8] to reduce vibration phenomenon. The proposed method was very successful in solving the vibration problem. Due to deteriorative effects of buzzing phenomenon in practical systems and because of plentiful benefits of using sliding mode control, researchers instead of putting aside this controlling method, should find a solution to remove or reduce this phenomenon. In the years after introducing sliding mode control, different methods have been presented to counteract the buzzing phenomenon including fuzzy model for estimating switching function [9, 10], [11] estimating mentioned function with use of sigmoid function [12, 13], high-order sliding mode control [14], using low-pass filter [15], [16] and etc. each of these methods have their own deficiencies that make them not ideal to be used. For instance, fuzzy approximation would lead to transient and steady-state errors in the closed-loop response. Also, forming of type-2 fuzzy and optimization of its membership functions increases the calculation burden. Another method of reducing buzzing is application of fractional-order sliding mode control [17], [18], [19]. This method makes the design complex and increases the calculations. The proposed method in this paper uses sigmoid function.

In this paper, after introducing Colpitts system, its circuit and equations and expressing the chaos concept for it, first, designing back-stepping controller, output waveforms and state error are investigated. By adding uncertainty to the system it is seen that back-stepping controller is not able to completely remove the steady-state error in case of uncertainties. Then, based on combination of both back-stepping and sliding mode methods a controller will be designed to achieve a stable system in the case of uncertainty, in addition to having a more suitable response. Results of the simulations show that in addition to stabilizing Colpitts chaotic system, back-stepping sliding mode controller reduces the rising time, settling time and MAE and MSE error, and also gives better response. Moreover, robustness of back-stepping sliding mode controller in the case of uncertainty and external disturbances will be studied where back-stepping controller solely is not such capable. In contrast, the deficiency of this method is buzzing phenomenon which due to deteriorative effects it should be removed. By using sigmoid function in this paper, the mentioned problem is obviated.

This paper is organized as below. The circuit and equations of chaotic Colpitts system are presented in section 2. Section 3 describes the original back-stepping method. The performance of the system in the presence of uncertainty is studied in this section. Introducing the slip mode controller and its advantages and disadvantages are also given in section 3. Combining the back-stepping method and slip mode method to design a new controller is presented in section 4, where to reduce the chattering, the switching function is changed. The simulation results are expressed in this section along with studying the effect of disturbance on Colpitts system. Section 5 provides the results of statistical comparison. Finally, conclusions are given in section 6.

2. Colpitts Oscillator

2.1. Circuit and Equations

Colpitts oscillator is a nonlinear system due to exponential behavior of the transistor. This oscillator has generally three order equations. Circuit of the Colpitts oscillator is as follows:

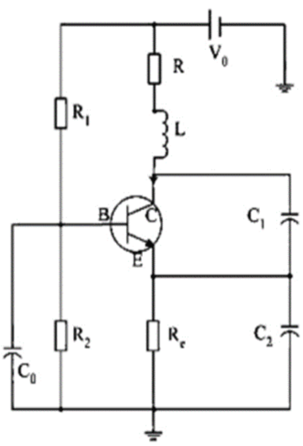

Fig. 1. Circuit diagram of the chaotic Colpitts oscillator [7]

In the considered oscillator there is a BJT (Bipolar Junction Transistor) transistor as an amplifying element, and the circuit consists of a resonance network comprised of an inductor and a pair of capacitors. The circuit's dynamics are explained using the below differential equations [7]:

$$\frac{dx}{dt} = y - aF(z)$$
$$\frac{dy}{dt} = c - x - by - z \qquad (1)$$
$$\frac{dz}{dt} = y - dz$$

In the above equation, parameters are defined as in (2), and piecewise linear characteristics of the current and voltage curves of a BJT have been utilized in these parameters.

$$x = \frac{V_{cl}}{V^*}, y = \frac{\rho I_L}{V^*}, z = \frac{V_{c2}}{V^*}, t = \sqrt{LC_1} \qquad (2)$$
$$\rho = \sqrt{\frac{L}{C_1}} a = \frac{\rho}{r}, b = \frac{R}{\rho}, c = {}^{V_0}\!/_{V^*}$$
$$d = \rho / R_e, e = \frac{R_2}{R_1 + R_2} c$$
$$F(z) = \begin{cases} e - 1 - 1 & z < e - 1 \\ 0 & z \geq e - 1 \end{cases}$$
$$V^* = 0.73$$

2.2. Chaos in Colpitts Oscillator and its Application

Chaotic behavior of Colpitts oscillator was first observed by Michael Peter Kennedy [8]. When a Colpitts oscillator is affected by a chaos phenomenon it means that:

- The system behavior is heavily dependent on its initial conditions;
- It has a pseudo random and unpredictable signal from the observer view;
- Long-term prediction of the system behavior using generalization of the past to future procedure cannot be realized;
- Divergence of close together trajectories;
- has bizarre attractors and variable phase portraits.

This system in terms of values in (3) will have chaotic behavior.

$$C_1 = C_2 = 470 \text{ nF}$$
$$C_0 = 47 \mu\text{F}, R = 36 \Omega, R_e = 510 \Omega \qquad (3)$$
$$R_1 = R_2 = 3 \text{ k}\Omega, V_0 = 15V$$

These parameters will result in coefficients in (4).

$$a = 30, b = 0.8, c = 20$$
$$d = 0.08, e = 10 \qquad (4)$$

One of basic applications of Colpitts chaotic oscillators is in transmitting and receiving data in binary telecommunications. In this application, the transmitter has a chaotic Colpitts oscillator which contains a parameter modulated with information signal. Every symbol that is going to be transmitted is coded as an attractor in the Colpitts oscillator.

Phase portrait of Colpitts is shown in the below figure. As it can be seen from the figure, in terms of some specific parameters, not only does system oscillate with the main frequency, but oscillations with infinite kinds of periods are created which shows the occurrence of chaos phenomenon.

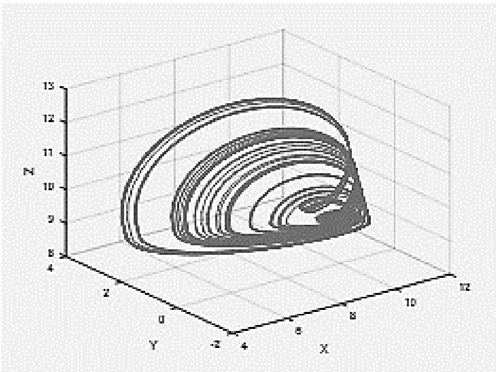

Fig. 2. Phase portrait of chaotic Colpitts system [8]

3. Controlling Chaotic Colpitts System

3.1. Back-Stepping Control of Chaotic Colpitts System

In back-stepping control method, dynamics of the system error [7] are obtained as (5):

$$\begin{aligned}\frac{de_3}{dt} &= e_2 - de_3 \\ \frac{de_2}{dt} &= -e_1 - be_2 - e_3 \\ \frac{de_1}{dt} &= e_2 - aF(z) + aF(z+e_3) - u\end{aligned} \tag{5}$$

$\boldsymbol{e_1}$, $\boldsymbol{e_2}$ and $\boldsymbol{e_3}$ are steady state error.

In the first step of designing by Lyapunov function, e2 is designed in such a way that value of e3 reaches to zero. For this purpose, Lyapunov function is defined as (6).

$$\begin{aligned}V_1 &= \frac{1}{2}e_3^2 \\ \dot{V}_1 &= e_3.\dot{e}_3\end{aligned} \tag{6}$$

The presented Lyapunov function is in such a way that its value is positive for all values of e_3. For satisfaction of stability criterion, the derivative of Lyapunov function should be negative. So, if relation (7) is satisfied, derivative of Lyapunov function will be negative.

$$e_3 = -\dot{e}_3 = -e_2 + de_3 \tag{7}$$

Thus, for e_2 we have:

$$e_2^* = (d-1)e_3 \tag{8}$$

In the seconf step, e1 should be chosen in such a way that $e_2 = e_2^*$. Therefore, the second Lyapunov function is defined as (9).

$$\begin{aligned}V_2 &= V_1 + \frac{1}{2}(e_2 - e_2^*)^2 \\ \dot{V}_2 &= \dot{V}_1 + (\dot{e}_2 - \dot{e}_2^*)(e_2 - e_2^*) < 0\end{aligned} \tag{9}$$

For the sake of negative derivative of the mentioned Lyapunov function:

$$e_1^* = (2-d-b)e_2 = (d^2 - 2d)e_3 \tag{10}$$

In the third step, *u* should be calculated and applied in such a way that $e_1 = e_1^*$ is satisfied. To obtain *u*, first, the appropriate Lyapunov function should be selected and then *u* will be calculated.

$$\begin{aligned}&V_3 = V_2 + \frac{1}{2}(e_1 - e_1^*)^2 \Rightarrow \\ &\dot{V}_3 = \dot{V}_2 + (\dot{e}_1 - \dot{e}_1^*)(e_1 - e_1^*) \\ &if\ \ (\dot{e}_1 - \dot{e}_1^*) = -(e_1 - e_1^*) \quad \Rightarrow \\ &\dot{V}_3 = \dot{V}_2 - (e_1 - e_1^*)^2 < 0\end{aligned} \tag{11}$$

Now Lyapunov function can be negative when:

$$\begin{aligned}&u = -aF(z) + aF(z+e_3) + \\ &(3-d-b)e_1 + (-1+3b-db-b^2-d^2+3d) \\ &e_2 + (-3d+2-b+d^3-3d^2)e_3\end{aligned} \tag{12}$$

This way the control input applied to the system will be achieved. Now, we deal with applying control input and investigate system's response. Below figure shows the triple states.

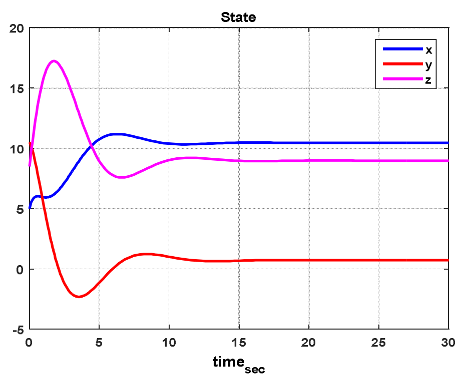

Fig. 3. Triple states diagram for back-stepping controller

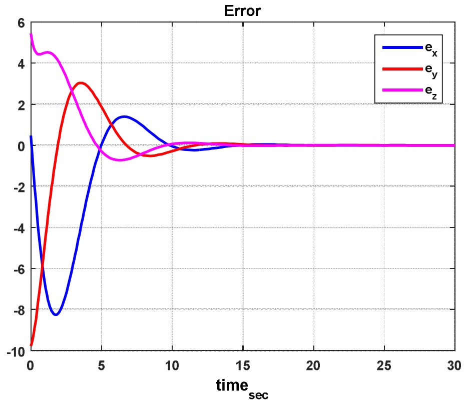

Fig. 4. Triple error states diagram for back-stepping controller

It can be seen that in case there is no uncertainty, the modes error is converged to zero after several seconds.

3.2. Performance Evaluation of Back-Stepping Method in Case of Uncertainty

Every system made and used in the real world is affected by environmental factors. However, the level of being affected by environment differs in different systems. Human error and structural changes of the devices used in the system over time are some factors along with environmental factors that change the proposed dynamic equations of the system and limit the band parametric uncertainty. For instance, the capacitor used in the circuit may experience depreciation error or resistor values change because of getting hot. Also, in Colpitts oscillator system due to using devices such as resistor and capacitor the possibility of existence of uncertainty in the system dynamics is very high. To investigate the performance of back-stepping controller in controlling Colpitts system the values of resistors and capacitors will be changes 10% and the mentioned controller will be applied to the system.

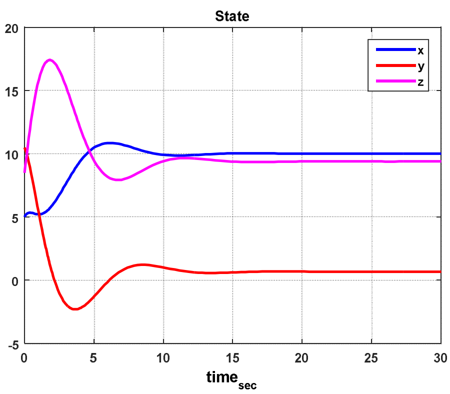

Fig. 5. Triple states diagram for back-stepping controller with uncertainty

In addition, below figure shows triple error states of the desired values:

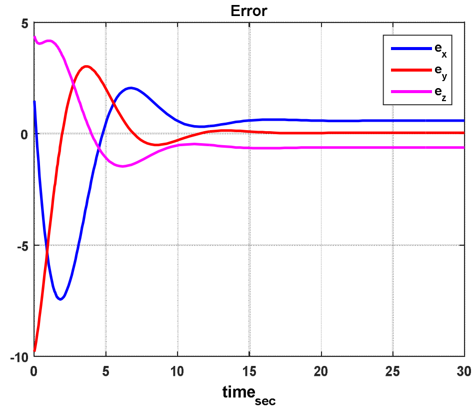

Fig. 6. Triple error states diagram for back-stepping controller with uncertainty

It is obvious from Fig.6 that in spite of uncertainty there is a steady-state error in the system which back-stepping controller is not able to reach this error to zero. Therefore, it is essential to use a controller that is robust against the parametric variations in the system dynamics. Nevertheless, since capabilities and advantages of using a back-stepping controller is that much important that it is not rational to put it aside. So, in this paper a robust controller such as sliding mode control is amalgamated with the back-stepping control to resolve the problems related to the uncertainty.

3.3. Sliding Mode Control: Pros and Cons

Sliding mode controller is among those nonlinear controllers which is able to suitably control the system in the presence of structural or nonstructural uncertainties.

This type of controller using a control law with high speed switching between two controlling structures will assign the system state variables in a specific surface knows as sliding surface. This surface is defined in such a way that always by actuating the system sates towards that, it is possible to satisfy all the objectives from control view of point. In this method, control law usually consists of two separate parts. The first part actuates the system states towards the sliding surface, and the second part is responsible for maintaining the states on the sliding surface. Sliding mode control in addition to being robust against the uncertainties has some other advantages including insensitivity with respect to external disturbances, fast transient response and simplicity in design and run.

Nonetheless, sliding mode control has some problems where the most important one is big control signal and chattering phenomenon.

In the design procedure of sliding mode controller, in practice, limitation on high frequency switching and presence of uncertainties in the system cause the system states not to be maintained on the sliding surface they oscillate around it. These oscillations are called chattering which is normally an undesirable phenomenon [20] because it may cause increased control activities and motivate not modeled high frequency system dynamics or even make the system unstable. So far, different methods have been utilized to solve these unwanted oscillations. In this paper, optimized designing of discrete part of the law control using sigmoid functions is used.

4. Back-Stepping Sliding Mode Controller

4.1. Design of Back-Stepping Sliding Mode Controller for Colpitts Oscillator

Dynamics of the system error are considered as (13).

$$\begin{aligned}\frac{de_3}{dt} &= e_2 - de_3\\ \frac{de_2}{dt} &= -e_1 - be_2 - e_3\\ \frac{de_1}{dt} &= e_2 - aF(z) + aF(z+e_3) - u\end{aligned} \tag{13}$$

In the first step, the first sliding mode S_1 is defined as $S_1 = e_3$**.** Since the values of S_1 will approach to zero, consequently e_3 will be zero. To obtain the proper control input, a Lyapunov function defined in (14) will be applied.

$$\begin{aligned}V_1 &= \frac{1}{2}(S_1)^2\\ \dot{V}_1 &= -t_1(e_3)^2\end{aligned} \tag{14}$$

Derivative of this Lyapunov function should be a negative value. In the above equation, t_1 is positive. The procedure of obtaining e_2 to make e_3 value equal to zero is as (15).

$$\begin{aligned}&\dot{V}_1 = \dot{s}_1 s_1 = \dot{e}_3 e_3 = (e_2 - de_3)e_3 = -t_1(e_3)^2\\ &\Rightarrow\ e_{2_{eq}} = (d - t_1)e_3\end{aligned} \tag{15}$$

In this step, the switching control input is considered as a sign function.

$$e_{2_d} = k_1 sign(s_1) \tag{16}$$

Sliding mode control inputs include u_{eq} and u_d. So, the proper value for e_2 is obtained as in (17).

$$e_2^* = e_{2_{eq}} + e_{2_d} = (d - t_1)e_3 + k_1 sign(s_1) \tag{17}$$

In the second step, the second sliding surface is considered as $S_2 = e_2 - e_2^*$. By specifying appropriate Lyapunov function, control input will be obtained, where S_2 will reach zero. Proper Lyapunov function is defined as (18).

$$\begin{aligned}V_2 &= V_1 + \frac{1}{2}(S_2)^2\\ \dot{V}_2 &= \dot{V}_1 + \dot{s}_2 s_2 = \dot{V}_1 - t_2(e_2 - e_2^*)^2\end{aligned} \tag{18}$$

In the above equation t_2 is positive. In this recursive way, e_1^* is obtained in such a way that brings e_2 to e_2^* and as a result e_3 to e_3^*. In the third and final step, control input u is calculated so that e_1 reaches to e_1^*. The proper Lyapunov function for obtaining the mentioned control input is considered as (19).

$$\begin{aligned}V_3 &= V_2 + \frac{1}{2}(s_3)^2\\ \dot{V}_3 &= \dot{V}_2 + \dot{s}_3 s_3\end{aligned} \tag{19}$$

In the above equation $\boldsymbol{s_3}$ is considered as $\boldsymbol{s_3 = e_1 - e_1^*}$. Now Lyapunov function can be negative when:

$$\dot{s}_3 s_3 = -t_3(e_1 - e_1^*)^2 \tag{20}$$

A control input which can satisfy equation (20) is obtained as (21).

$$\begin{aligned}&\dot{s}_3 s_3 = (\dot{e}_1 - \dot{e}_1^*)(e_1 - e_1^*) = -t_3(e_1 - e_1^*)^2\\ &(\dot{e}_1 - \dot{e}_1^*) = -t_3(e_1 - e_1^*)\\ &(\dot{e}_1 - \dot{e}_1^*) = (e_2 - aF(z) + aF(z + e_3) - u\\ &\quad -((t_2 - d + t_1 - b)(-e_1 - be_2 - e_3)\\ &\quad -(-1 + d^2 - dt_1 + t_1 t_2 - dt_2)(e_2 - de_3)\\ &\quad = -t_3(e_1 - e_1^*)\\ &\quad \Rightarrow u_{eq} = (t_2 - d + t_1 - b + t_3)e_1\\ &\quad +(1 + bt_2 - bd + bt_1 - b^2 + 1 - d^2\\ &\quad + dt_1 - t_1 t_2 + dt_2)e_2\\ &\quad +(t_2 - d + t_1 - b - d + d^3 - d^2 t_1\\ &\quad + d\, t_1 t_2 - d^2 t_2)e_3\\ &\quad - aF(z) + aF(z + e_3) - t_3 e_1^*\end{aligned} \tag{21}$$

Control inputs for back-stepping sliding mode control include u_{eq} and u_d ($u = u_{eq} + u_d$). In this relation, u_d is switching control input and is calculated as (22).

$$u_d = k_3\, sign(s_3) \tag{22}$$

So, to calculate control input from the previous steps, u_d and u_{eq} are replaced.

$$\begin{aligned} u = {} & (t_2 - d + t_1 - b + t_3)e_1 \\ & +(1 + bt_2 - bd + bt_1 - b^2 + 1 \\ & -d^2 + dt_1 - t_1 t_2 + dt_2)e_2 \\ & +(t_2 - d + t_1 - b - d + d^3 \\ & -d^2 t_1 + d\, t_1 t_2 - d^2 t_2)e_3 \\ & -aF(z) + aF(z + e_3) - t_3 e_1^* + k_3 sign(s_3) \end{aligned} \tag{23}$$

The block-diagram of back-stepping sliding mode controller from computation of S1 to control input signal is as Fig. 7.

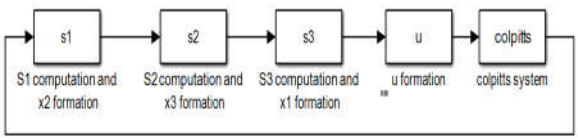

Fig. 7. Back-stepping sliding method

4.2. Simulation Results

The obtained results from applying back-stepping sliding mode controller are as Fig. 8.

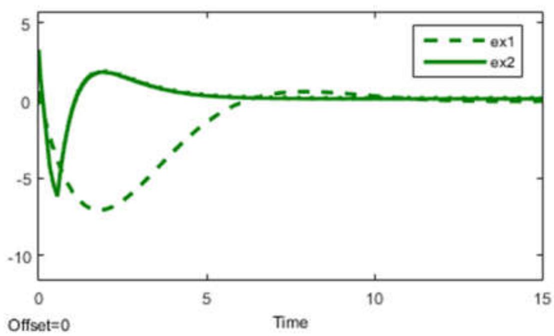

Fig. 8. a. comparison of x state error diagrams

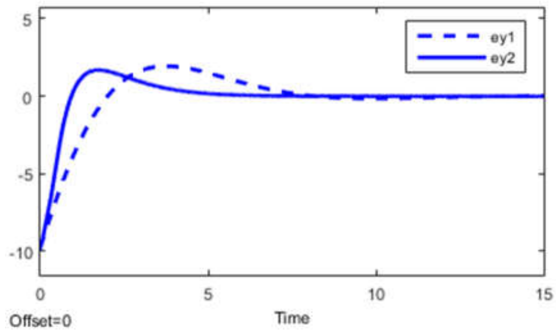

Fig. 8. b. comparison of y state error diagrams

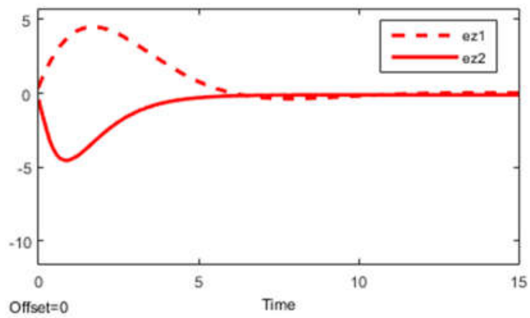

Fig. 8. c. comparison of z state error diagrams

Figure 7. Comparison of triple states error diagrams of back-stepping mode (ey1) and back-stepping sliding mode (ey2) controllers.

This figure shows the errors of triple states, where the smooth and dotted lines respectively belong to the cases when back-stepping sliding mode and back-stepping controllers are used. It is seen that back-stepping sliding mode controller method reduces the settling time compared to that of back-stepping controller and this is its advantages. However, the very important reason for application of sliding mode controller in the back-stepping control structure is need to a robust controller against the parametric variations (uncertainty). As a result, in this step, capability of the designed controller in case the system's dynamics have uncertainties should be assessed. Changing 10% of the values of resistor and capacitors are considered as the applied uncertainties. Now, by applying the mentioned variations, system's response is studied.

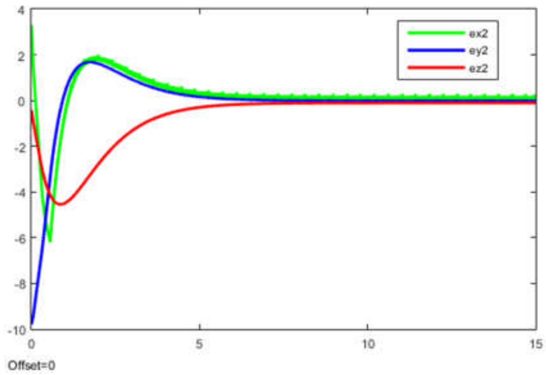

Fig. 9. Error diagrams of triple states at back-stepping sliding mode controller with uncertainty

It is seen that when using back-stepping sliding mode controller, the steady-state error reached to zero, and the designed controller is capable of suitably controlling the system when system has uncertainties. There is not such capability in back-stepping controller and there will be steady-state error. Nevertheless, by investigating the control input applied to the system we notice that the instantaneous changes in the control input is apparent.

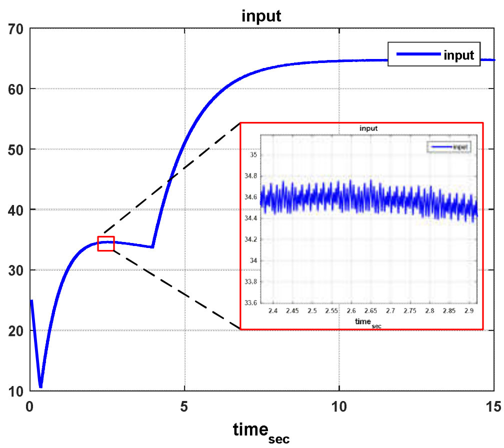

Fig. 10. Control input signal applied to back-stepping sliding mode controller

As it mentioned before, this phenomenon is called buzzing. Since applying the control input is difficult in this form, we will try to reduce the buzzing phenomenon. Among the buzzing reduction methods is using tanh functions instead of sign function, which will be carried out at the next sections.

4.3. Design of Back-Stepping Sliding Mode Controller without Chattering

4.3.1. Changing Switching Function

In this method to reduce the chattering phenomenon, *tanh* function is used instead of sign function in order to obtain the control input. After evaluating the system's response pros and cons of the controller will be dealt with.

In the first step, the first sliding surface S_1 is defined as $S_1 = e_3$, and Lyapunov function similar to the precious case is defined as $V_1 = ½(S_2)^2$, and the previous steps are exactly repeated, which are not mentioned here, and it continues from the step of changing switching control input to *tanh* function.

$$\begin{aligned} e_{2_d} &= k_1\, tanh(s_1) \\ e^*{}_2 &= e_{2_{eq}} + e_{2_d} = (d - t_1)e_3 + k_1 tanh(s_1) \end{aligned} \quad (24)$$

In the second step, the second sliding surface is considered as $S_2 = e_2 - e_2^*$. Lyapunov function of this step is defined as below:

$$\begin{aligned} V_2 &= V_1 + \frac{1}{2}(S_2)^2 \\ \dot{V}_2 &= \dot{V}_1 + \dot{s}_2 s_2 = \dot{V}_1 - t_2\,(e_2 - e_2^*)^2 \end{aligned} \quad (25)$$

To the similar way, using a recursive method the value of S_3 is defined as $S_3 = e_1 - e_1^*$.

$$\begin{aligned} V_3 &= V_2 + \frac{1}{2}(S_3)^2 \\ \dot{V}_3 &= \dot{V}_2 - t_3\,(e_1 - e_1^*)^2 < 0 \end{aligned} \quad (26)$$

The control inputs of back-stepping sliding mode controller include u_{eq} and u_d ($u = u_{eq} + u_d$). In this equation, u_d which is switching control input is calculated as (27).

$$u_d = k_3\, tanh(s_3) \quad (27)$$

Therefore, equations (28) are satisfied for the control input.

$$\begin{aligned} u =& (t_2 - d + t_1 - b + t_3)e_1 \\ &+(1+bt_2 - bd + bt_1 - b^2 + 1 - d^2 + dt_1 \\ &- t_1 t_2 + dt_2)e_2 + (t_2 - d + t_1 - b - d + d^3 \\ &- d^2 t_1 + d\,t_1 t_2 - d^2 t_2)e_3 - aF(z) \\ &+ aF(z + e_3) - t_3 e_1^* - t_2 k_1 \dot{s}_1 (1 - \tanh^2(s_1)) \\ &- k_1 \ddot{s}_1 (1 - \tanh^2(s_1)) + 2k_1 \dot{s}_1 \tanh(s_1)(1 - \tanh^2(s_1)) \\ &+ k_2 \dot{s}_2 (1 - \tanh^2(s_2)) + k_3 \tanh(s_3) \end{aligned} \tag{28}$$

4.3.2.Simulation Results

The obtained results from applying the controller to Colpitts system is as follows:

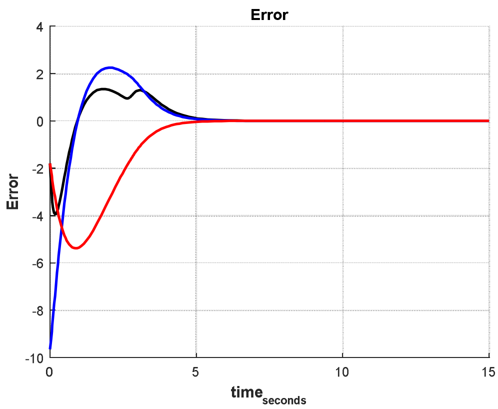

Fig. 11. Triple states error signal at back-stepping sliding mode controller

Fig. 11 shows the error between triple modes and the desired values. It is apparent that error curve has approached zero in a proper time. Now, the controller is applied to a system with uncertainty to evaluate the obtained responses.

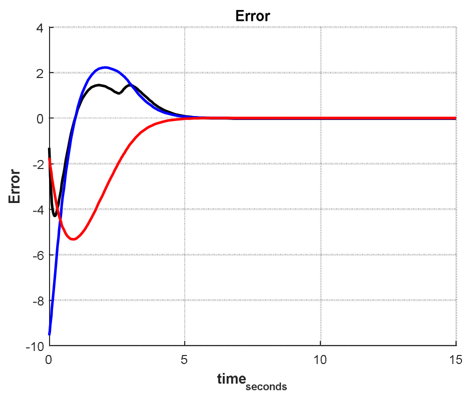

Fig. 12. Triple states error signal in back-stepping sliding mode controller with uncertainty

It can be seen that with the applied changes in system parameters, the designed controller does not lose its ability for making the error values equal to zero.

Here, the control input applied to the system is studied. As it was expected, using *tanh* function instead of *sign* function removes the vibration phenomenon.

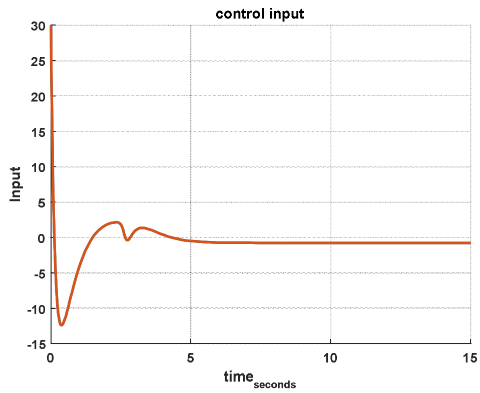

Fig. 13. Applied control input signal

4.4. Investigation of Disturbance

In the nature, most systems are faced with disturbances and chaotic Colpitts system is also involving external disturbances. According to the fact that the aim of using back-stepping sliding mode controller is achieving a robust method, effect of disturbance should be investigated in this step. To do so, by feeding the disturbance signal, which is considered as a step function, into the designed system with back-stepping sliding mode controller, this effect is applied. Waveforms of the system without disturbance are shown in Fig. 13.

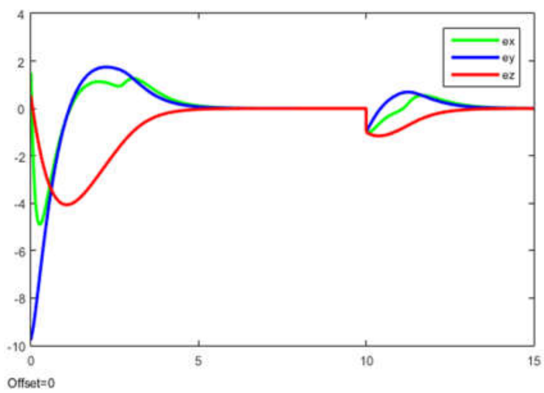

Fig. 14. Steady-states error diagrams with disturbance in back-stepping sliding mode controller

It is observed that in the case of feeding disturbance at time 10 sec, the system loses its normal condition for a short time, but it returns back to its previous state. So, this system is capable of removing the disturbances.

5. Statistical Comparison

For better and tangible comparison of the controllers used in this paper, Table 1 and 2 was prepared. The criteria used in this comparison are mean average error (MAE) and mean squared error (MSE). The first rows of tables are based on Li method or back-stepping method. Formulation of achieving these parameters is as follows.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|e_i| \tag{29}$$

$$MSE = \frac{1}{n}\sum_{i=1}^{n}{e_i}^2 \tag{30}$$

The statistical comparison given in Table 1 and 2 show that error values of the error signal for sliding mode controller are less than those of back-stepping controller. This is one of advantages of combining back-stepping and sliding mode methods.

Table. 1. MAE standard comparison

controller	**MAE**	**MAE (with uncertainty)**
Back-stepping	4.8075	5.4914
Back-stepping sliding mode	1.8068	1.8605
Back-stepping sliding mod without chattering	4.3646	4.5231

Table. 2. MSE standard comparison

controller	**MSE**	**MSE (with uncertainty)**
Back-stepping	21.957	22.067
Back-stepping sliding mode	3.795	4.192
Back-stepping sliding mod without chattering	18.319	19.239

As it was mentioned before, using *tanh* function instead of *sign* function, as the switching function in the sliding mode controller, reduces the efficiency, however due to its ability at reducing the chattering, *tanh* function has been used.

The statistical comparison given in the above table shows that MAE and MSE values of the error signal with either *sign* or *tanh* switching functions in the both cases of with or without uncertainty are less than those of back-stepping controller. Also, it is observed that sliding mode controller with *sign* function perform better than that with *tanh* function.

6. Conclusions

In this paper, first, we described back-stepping control method, introduced by Li and et al., for Colpitts oscillator. According to the simulation results, an appropriate response was not obtained in the case of occurrence of uncertainty, and a steady-state error is formed. . Next, by combining back-stepping method with sliding mode method a novel method for controlling Colpitts oscillator is found. The obtained results show that in this case the errors are reduced significantly. Therefore, in the next step, back-stepping sliding mode controller was designed and applied. This controller showed that it is capable of making the system error equal to zero even in the presence of parametric uncertainty, while it has faster response with less MAE and MSE values. However, chattering phenomenon appears in the waveforms. To eliminate chattering in the proposed method, *tanh* switching function was used instead of Sign function, and chattering humming is eliminated. This way, a new control signal without chattering is obtained, where the most superiority of the proposed method compared to the preceding methods is its robustness against uncertainty and stability against disturbances, in addition to eliminating the steady state error.

With this approach, we have a controller which has three main features: 1) making the system error equal to zero, 2) removing chattering phenomenon, and 3) stability against disturbance.

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