Planning Arm with 5 Degrees of Freedom for Moving Objects Based on Geometric Coordinates and Color

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Abstract

Skilled mechanical arms of consanguine relationship formed by joints the relative motion of the adjacent interfaces enable, have been connected. Ability to perform a variety of pre-programmed robotic manipulator in various industries. Skilled mechanical arms in recent years as a significant progress has been completed. House repair and easier to work with them as well and fit and optimal relationship between power, controllability and skill building. In this respect, there are a variety of skilled mechanical arms and a wide variety of different industrial applications and also to cover investigative. These applications perform various activities including construction, underwater cutting and welding production lines to perform various operations - such as the installation of underwater robots - such as the pursuit of a cable or wire, or locked objects in complex samples such as establishing connections or electrical or hydraulic lines .The mechanical arms that what matters is the simplest example may be assigned to duty at the desired time to do the selection. Robot design complexity while increasing performance capabilities can be difficult to control, guidance and ensuring the accuracy and maintenance of the cause. Selection and completion of the complex and sophisticated robotic manipulator robot designer should consider the many points. Number of arms required, location, type of controller, the access point and the maximum and minimum access point, space and arms control by the user, including the following.

Keywords: Skilled mechanical arms, End-effector, Robot joint.

1. Introduction

Mechanical arms rigid form of relationship that Mfslhayy by the relative motion of adjacent interfaces enable the connection each task. A variety of pre-programmed robotic arms capable of operating in various industries. Skilled mechanical arms in recent years has been substantially completed and have progressed. As well as repair and feeding them easier to work with and fit and optimal relationship between power, controllability and skills they have been created. At the end of a relationship chain end-forming and mechanical arm that can be expected in terms of applications of robots or other tools such as clamps or hooks for cutting, welding and like it. In this respect, there are various types of skilled mechanical arms wide variety of industrial applications and also cover investigative. These applications include assembly of diverse activities, diverse underwater cutting and welding production lines to operate - such as the installation of underwater robot - like getting and looking a cable or wire or imprisonment or of sophisticated objects such as electric or hydraulic lines are connections Contact .Analytical solutions for inverse kinematics there are different methods and practices as they are with DOF. In practice, the consent of the three-dimensional orientation of the feature vector and the host is quite logical. For example, when the machine is used for welding, welding tube along the tip it should be good values. If the machine is installed on the robot fluid, the fluid robots that can act as the fluid machine and free status and robot end-fluid along the fluid off. This fundamental to the overall number DOF, five are controlled.

2. Analysis Frameworks

The PArm is an accessory for the family of Pioneer Mobile Robots, as shown in Fig.1. It is a 5-DOF manipulator driven by six open-loop servomotors and has a gripper as an end-effector. The gripper has foam- lined fingers for firm grasping and manipulation of objects as large as a soda can and as heavy as 150 grams throughout the arm’s envelope of operation.

Fig.1 Robot arm PArm 5-DOF.

Coordinate frames for the PArm are assigned as shown in Fig.2. They are established using the principles of the Denavit - Hartenbarg (D-H) convention. Frame O5 is an auxiliary frame attached to the end- effector at the place of joint 5, whose coordinate directions are the same as O6. Using the auxiliary frame O5 to maintain consistency in the orientation for the last link of the manipulator keeps frames O0 to O5 in the same plane and this allows us to apply the geometric projection method to the derivation of the inverse kinematics.

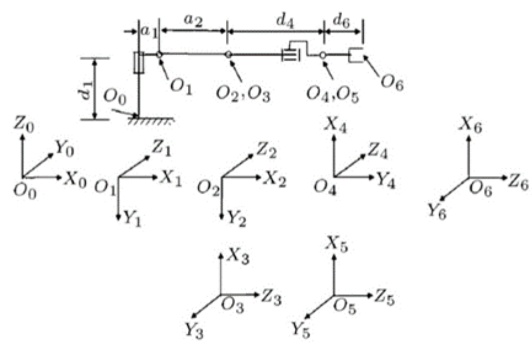


Fig.2. Coordinate Frames for the PArm

3. Forward kinematics for the Parm

3.1 Forward kinematics

The forward kinematics is a set of equations that calculates the position and orientation of the end- effector in terms of given joint angles. This set of equations is generated by using the D-H parameters obtained from the frame assignation. The parameters for the PArm are listed in Table 1, where θi represents rotation about the Z-axis, αi rotation about the X - axis, di transition along the Z-axis, and ai transition along the X -axis.

$$A_i = Rot(z,\ \theta_1)Trans(0,0,\ d_i)Trans(\ a_i, 0,0)Rot(x,\ a_i)$$
$$= \begin{bmatrix} \cos(\theta_i) & -\sin(\theta_i)\cos(\ a_i) & \sin(\theta_i)\sin(\ a_i) & a_i\cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i)\cos(\ a_i) & -\cos(\theta_i)\sin(\ a_i & a_i\sin(\theta_i) \\ 0 & \sin(\ a_i) & \cos(\ a_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Frame	θ_i	$d_i(mm)$	$a_i(mm)$	α_i(degree)
$O_0 - O_1$	θ_1	$d_1 =$	$a_1 =$	-90
$O_1 - O_2$	θ_2	0	$a_2 = 160$	0
$O_2 - O_3$	$-90 + \theta_3$	0	0	-90
$O_3 - O_4$	θ_4	$d_4 = 137.75$	0	90
$O_4 - O_5$	θ_5	0	0	-90
$O_5 - O_6$	0	$D_6 = 113.21$	0	0

The forward kinematics describe the transformation from one frame to another, starting at the base and ending at the end-effector. The transformation matrix Ai between two neighboring frames O_{i-1} and O_i is expressed as (1), found at the bottom of the page .By substituting the D-H parameters in Table 1 into (1), we can obtain the individual transformation matrices A_1 to A_6, and a global matrix of transformation 0T6 as in (2)

$$T_6 = A_1\ A_2\ A_3\ A_4\ A_5\ A_6 = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Where (px, py, pz) represents the position, and ({nx, ny, nz }, {ox, oy, oz },{ax, ay, az }) the orientation of the end-effector. The orientation and position of the end-effector can be calculated in terms of joint angles and the D-H parameters of the manipulator, as shown in (3) to (14)

$$n_x = c_1 s_{23} c_4 c_5 + s_1 s_4 s_5 + c_1 c_{23} s_5 \tag{3}$$

$$n_y = s_1 s_{23} c_4 c_5 - c_1 s_4 s_5 + s_1 c_{23} s_5 \tag{4}$$

$$n_z = c_{23} c_4 c_5 - s_{23} s_5 \tag{5}$$

$$o_x = -c_1 s_{23} s_4 + s_1 c_4 \tag{6}$$

$$o_y = -s_1 s_{23} s_4 - c_1 c_4 \tag{7}$$

$$o_z = -c_{23} s_4 \tag{8}$$

$$a_x = -c_1 s_{23} c_4 s_5 - s_1 s_4 s_5 + c_1 c_{23} c_5 \tag{9}$$

$$a_y = -s_1 s_{23} c_4 s_5 + c_1 s_4 s_5 + s_1 c_{23} c_5 \tag{10}$$

$$a_z = -c_{23} c_4 s_5 - s_{23} s_5 \tag{11}$$

$$p_x = -d_6 c_1 s_{23} c_4 s_5 - d_6 s_1 s_4 s_5 + d_6 c_1 c_{23} c_5 + d_4 c_1 s_{23} + a_2 c_1 c_2 + a_1 c_1 \tag{12}$$

$$p_y = -d_6 s_1 s_{23} c_4 s_5 - d_6 c_1 s_4 s_5 + d_6 s_1 c_{23} c_5 + d_4 s_1 s_{23} + a_2 s_1 c_2 + a_1 s_1 \tag{13}$$

$$p_z = -d_6 c_{23} c_4 s_5 - d_6 s_{23} c_5 - d_4 s_{23} - a_2 s_2 + d_1 \quad (14)$$

where ci = cos(θi), si = sin(θi), c23 = cos(θ2 + θ3), and s23 = sin(θ2 + θ3)

3.2 Discussion of the singular position problem

It should be noted that if joint angle $\theta 4$ equals zero, frames $O1$, $O2$, and $O4$ would rotate around parallel axes, which means that all links of the arm would be on the same plane. A singular position problem might emerge in this case. This has to be checked for, before deriving the inverse kinematics. If $\theta 4 = 0$, then

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

and the transform frame O2 to O5 and O1 to O5, are as follows:

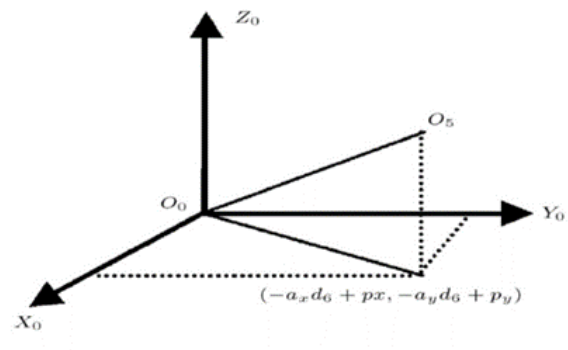
$$^2T_5 = A_3 A_4 A_5 = \begin{bmatrix} s_{35} & 0 & c_{35} & c_3 d_4 \\ -c_{35} & 0 & s_{35} & s_3 d_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad (16)$$

Obviously, the solutions for joint angles θ1 to θ5 can be determined for a given desired matrix 0T6. In other words, there exists no singular problem in the PArm in the case of θ4 = 0.

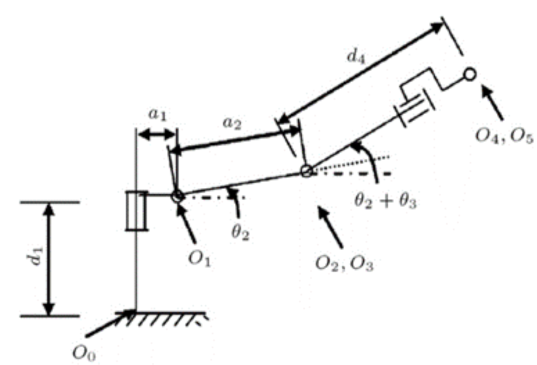
4. Inverse kinematics for the Parm

4.1 Solutions for θ1 to θ3

It can be observed that the origins of frames O0 to O5 are all in the same plane, as shown in Fig.3 (b). Based on this observation, in order to simplify the derivation of the inverse kinematics, an auxiliary frame O5 is added, which is a rotation of the frame O4. The geometric projection that relates frame O5 to frame O1 is shown in Fig.3(a).



a)The sketch of links in the same plane



b) The sketch of links in the same plane)

Fig.3. The sketch of coordinate projection for the PArm

Obviously, the transformation matrix from frame O0 to O5 is as follows:

$$T_5 = {}^0T_6 A_6^{-1} = \begin{bmatrix} n_x & o_x & a_x & -a_x d_6 + p_x \\ n_y & o_y & a_y & -a_y d_6 + p_y \\ n_z & o_z & a_z & -a_z d_6 + p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

And from Fig.3(a) it is easy to find and solution frame O0 to O5 is as flow:

$$\begin{cases} d_1 - a_2 s_2 - d_4 s_{23} = -a_z\, d_6 + p_z \\ a_1 + a_2 c_2 + d_4 c_{23} = \pm\sqrt{(-a_x\, d_6 + p_x)^2 + (-a_y\, d_6 + p_y)^2} \end{cases} \quad (19)$$

$$\begin{cases} \theta_{11} = arctan2(-a_y\, d_6 + p_y, -a_x\, d_6 + p_x) \\ \theta_{12} = arctan2(a_y\, d_6 + p_y, a_x\, d_6 + p_x) \end{cases} \quad (18)$$

Where the positive sign on the right hand side of the second equation is for the front arm orientation, and the negative sign for the rear arm orientation. In other words, the positive corresponds to $\theta 11$ and the negative to $\theta 12$. For the sake of convenience, (19) is rewritten as(20)

$$\begin{cases} d_4 s_{23} = a_z\, d_6 + p_z + d_1 - a_2 s_2 = B_1 - a_2 s_2 \\ d_4 c_{23} = \pm\sqrt{(-a_x\, d_6 + p_x)^2 + \left(-a_y\, d_6 + p_y\right)^2} - a_1 - a_2 c_2 = B_2 - a_2 c_2 \end{cases} \quad (20)$$

Where

$$\begin{cases} B_1 = a_z\, d_6 + p_z + d_1 \\ B_2 = \pm\sqrt{(-a_x\, d_6 + p_x)^2 + \left(-a_y\, d_6 + p_y\right)^2} - a_1 \end{cases} \quad (21)$$

By applying a square sum to the two equations in (20), a new equation that contains $\theta 2$ alone can be formed as follows:

$$\mathrm{B_1 s_2 + B_2 c_2 = \frac{B_1^2 + B_2^2 + a_2^2 - d_4^2}{2\, a_2}} \quad (22)$$

It is easy to establish that the values of $B1$ and $B2$ are not zero at the same time. Otherwise (22) would not hold because of the different values of $a2$ and $d4$. Therefore, it is reasonable to introduce an auxiliary angle γ which is defined by:

$$\gamma = \mathrm{arctan2(\, B_2,\, B_1)} \quad (23)$$

By combining (23) and (22), a solution for $\theta 2$ can be derived as follows:

$$\begin{cases} \theta_{21} = \arcsin\left(\dfrac{B_1^2 + B_2^2 + a_2^2 - d_4^2}{2\, a_2\sqrt{B_1^2 + B_2^2}}\right) - \gamma \\ \theta_{22} = \pi - \arcsin\left(\dfrac{B_1^2 + B_2^2 + a_2^2 - d_4^2}{2\, a_2\sqrt{B_1^2 + B_2^2}}\right) - \gamma \end{cases} \tag{24}$$

where θ21 and θ22 are two candidate values for θ2. From (20), we can obtain a solution for θ3:

$$\theta 3 = \arctan 2(B1 - a2s2, B2 - a2c2) - \theta 2 \tag{25}$$

4.2 Solutions for $\theta 4$ and $\theta 5$

After obtaining solutions for θ_1 , θ_2 , and θ_3 , it should be straightforward to obtain solutions for θ4 and θ_5 by using the equations for the forward kinematics However, various conditions with different values for solved joint angles have to be considered. This paper proposes a useful strategy to solve $\theta 4$ time obtain useful criteria for checking the correctness of solutions. From (2), we have

$$A_3^{-1}A_2^{-1}A_1^{-1}\,{}^0T_6A_6^{-1} = A_4A_5 \tag{26}$$

$$A_4A_5 = \begin{bmatrix} c_4c_5 & -s_4 & -c_4s_5 & 0 \\ s_4c_5 & c_4 & -s_4s_5 & 0 \\ s_5 & 0 & c_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

$$A_3^{-1}A_2^{-1}A_1^{-1}\,{}^0T_6A_6^{-1} = \begin{bmatrix} n_xc_1s_{23} + n_ys_1s_{23} + n_zc_{23} & o_xc_1s_{23} + o_ys_1s_{23} + o_zc_{23} & a_xc_1s_{23} + a_ys_1s_{23} + a_zc_{23} \\ n_xs_1 - n_yc_1 & o_xs_1 - o_yc_1 & a_xs_1 - a_yc_1 \\ n_xc_1c_{23} + n_ys_1c_{23} + n_zs_{23} & o_xc_1c_{23} + o_ys_1c_{23} + o_zc_{23} & a_xc_1c_{23} + a_ys_1c_{23} + a_zs_{23} \end{bmatrix} \tag{28}$$

$$A_3^{-1}A_2^{-1}A_1^{-1}\,{}^0T_6A_6^{-1} = \begin{bmatrix} n_xc_1s_{23} + n_ys_1s_{23} + n_zc_{23} & o_xc_1s_{23} + o_ys_1s_{23} + o_zc_{23} & a_xc_1s_{23} + a_ys_1s_{23} + a_zc_{23} & 0 \\ n_xs_1 - n_yc_1 & o_xs_1 - o_yc_1 & a_xs_1 - a_yc_1 & 0 \\ n_xc_1s_{23} + n_ys_1s_{23} + n_zc_{23} & o_xc_1s_{23} + o_ys_1s_{23} + o_zc_{23} & a_xc_1s_{23} + a_ys_1s_{23} + a_zc_{23} & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{29}$$

$$\begin{cases} v_{z31} = [c_1c_{231} \quad s_1c_{231} \quad -s_{231}]^T, for\ \theta_{11},\ \theta_{21},\ \theta_{231} \\ v_{z32} = [c_1c_{232} \quad s_1c_{232} \quad -s_{232}]^T, for\ \theta_{11},\ \theta_{22},\ \theta_{232} \\ v_{z33} = [-c_1c_{233} \quad -s_1c_{233} \quad -s_{233}]^T, for\ \theta_{12},\ \theta_{21},\ \theta_{233} \\ v_{z34} = [-c_1c_{234} \quad -s_1c_{234} \quad -s_{234}]^T, for\ \theta_{12},\ \theta_{22},\ \theta_{234} \end{cases} \tag{32}$$

and the left side is easily derived as (28). By substituting $s1$, $c1$, $s23$, and $c23$ obtained from (18), (24), and (25) into the last column in (28), we have(29).

By equating the first and second terms of the second column in (27) and (29) respectively, we can obtain a solution for $\theta 4$. Similarly, by equating the first and third terms of the third row in (27) and (29) respectively, we can obtain a solution for θ_5. The solutions for θ_4 and θ_5 are as follows:

$$\begin{cases} \theta_4 = arctan2(-o_x c_1 s_{23} - o_y s_1 s_{23} - \\ \quad o_z c_{23}, o_x s_1 o_y c_1) \\ \theta_5 = arctan2(n_x c_1 c_{23} + n_y s_1 c_{23} - n_z s_{23} \\ \quad -a_x c_1 c_{23} + a_y s_1 s_{23} - a_z s_{23}) \end{cases} \tag{30}$$

4.3 The existence of correct solute ions

It is evident that the inverse kinematics for a 5-DOF manipulator such as the PArm will have no solution for some given positions and orientations of the end- effector. How can we know in which cases there will be no solution to the inverse kinematics? We notice that the term in the third row and second column in (27) is zero and the corresponding term in (29) is determined by $\theta 1$ to $\theta 3$, so that we have the following criterion–the solutions for $\theta 1$, $\theta 2$, and $\theta 3$ have to satisfy:

$$o_x c_1 c_{23} + o_y s_1 c_{23} - o_z s_{23} = 0 \tag{31}$$

Proof

a) The necessity

b) The sufficiency

If (31) is satisfied by any of the four groups of solutions given in (18), (24), and (25), then the solutions for $\theta 4$ and $\theta 5$ given by (30) will satisfy (26), because of the orthogonally of the rotation transformation matrix for the orientation. That is, the desired position and orientation of the end-effector can be realized. There are four groups of candidate solutions given above.

Although above criterion can be employed to check how many of those groups belong to correct solutions, an easier way is to predict the number of correct solutions in terms of the desired position and orientation of the end-effector. The direction vector of the Z-axis of frame O3 can be indicated using different candidate solutions for θ1 and θ2 as follows:

Where $vz31$ to $vz34$ represent the direction of the Z-axis of frame O3 in frame O0.

$$v_{3ij} = v_{z3i} \times v_{z3j} = \left[-s_1 k_{ij} \quad c_1 k_{ij} \quad 0\right]^T \tag{33}$$

$$v_{3ij} \times v_{y5} = \left[k_{ij} o_z c_1 \quad k_{ij} o_z s_1 \quad - k_{ij}(o_x c_1 + o_y s_1)\right]^T \tag{34}$$

$$\begin{cases} o_x(p_x - a_x d_6) + o_y\left(p_y - a_y d_6\right) = 0 \\ o_z = 0 \end{cases} \tag{35}$$

$$(a_2 - d_4)^2 \leq B_1^2 + B_2^2 \leq (a_2 + d_4)^2 \tag{36}$$

Condition (35) is derived from (34) and (31) under the condition that s1 and c1 are not zero at the same time. To ensure that the solutions for θ1 to θ3 given in (18), (24), and (25) have real values, condition (36) must be satisfied. If conditions (35) and (36) are satisfied, there will be two or four solution groups that can achieve the desired position and orientation of the end-effector. More specifically, the following conclusions can be drawn:

- In the case that (35) and (36) are atisfied for two selections of $B2$, four groups of correct solutions can be obtained.
- If (35)is satisfied, but(36) is true for only one selection of $B2$, then two groups of correct solutions can be obtained.
- If (35) is satisfied, but (36) is false for two se- lections of $B2$, there will be no correct solutions, because of the mechanical constraints.
- If (35) is not satisfied, but (31) is satisfied, there will be a unique group of solutions to realize the desired position and orientation of the end-effector.
- If (35) and (31) are not satisfied, there will be no solution to realize the desired position and orientation of the end-effector.

5. Inverse Kinematics for the Parm with one Dof of the End-Effector Unsatisfied

5.1 Solutions that satisfy the position and Z-axis orientation

As discussed in Section 1, the position of the end-effector has to be controlled accurately, but only the desired Z-axis orientation should be matched in some applications, i.e. the X or Y -axis orientation is ignored. In this case, the solutions for $\theta 1$ to $\theta 3$ are the same as given in Section 4.1, because they are able to realize the desired position of the end-effector.

To take only the Z-axis orientation into account, the terms in the third row and third column in (27) and (29) are used to obtain a solution for $\theta 5$ as follows:

$$\boldsymbol{\theta_5} = \pm\mathbf{arccos}(\boldsymbol{a_x c_1 c_{23}} + \boldsymbol{a_y s_1 c_{23}} - \boldsymbol{a_z s_{23}}) \tag{37}$$

Then, the first and second terms in the third column in (27) and (29) can be used to solve $\theta 4$. Here three cases are presented:

If $s5 > 0$ then the solution for $\theta 4$ is:

$$\theta_{41} = \mathrm{arctan2}(-a_x s_1 + a_y c_1, -a_x c_1 s_{23} - a_y s_1 s_{23} - a_z c_{23}) \tag{38}$$

If s5 < 0 then the solution for θ4 is:

$$\theta_{42} = \mathrm{arctan2}(a_x s_1 - a_y c_1, a_x c_1 s_{23} + a_y s_1 s_{23} + a_z c_{23}) \tag{39}$$

If $s5 = 0$, then ax and ay are independent of $\theta 4$, which can be checked in (9) and (10). Therefore, the solution of $\theta 4$ can be given arbitrarily in this case.

5.2 Solutions to satisfy all orientations

Now consider the situation where all orientations can be satisfied and the position coordinates in the Y and Z axes

are also taken into account. For example, if the manipulator is installed on a mobile robot, then the position coordinate on the X -axis could be realized by the mobile robot.
It can be noted that the first term is the product of the second and third terms in (27). Therefore, an equation including only $\theta 1$ to $\theta 3$ can be formed as follows:

$$n_x c_1 s_{23} + n_y s_1 s_{23} + n_z c_{23} = (o_x s_1 - o_y c_1)(a_x c_1 c_{23} + a_y s_1 c_{23} - a_z s_{23}) \quad (40)$$

Another equation including only θ1 to θ3 is (31). If oz f= 0 then s23 can be expressed using c23, s1, and c1. Combining (31) and (40) leads to:

$$n_y o_y (o_z - 1) + \left(n_x o_x - n_y o_y\right)(o_z - 1) c_1^2 \quad (41)$$

$$+\left(n_x o_y + n_y o_x\right)(o_z - 1) c_1 s_1 = 0$$

If oz f= 1, then the common term oz − 1 can be eliminated from (41). By applying a double angle formula to (41), a typical triangle equation can be obtained as follows:

$$\left(n_x o_x - n_y o_y\right)\cos(2\,\theta_1) + \left(n_x o_y + n_y o_x\right) sin(2\,\theta_1) = n_z o_z \quad (42)$$

and possible solutions for θ1 derived as:

$$\begin{gathered} \gamma_1 = arctan2\left(n_x o_x - n_y o_y, n_x o_y + n_y o_x\right) \\ \gamma_2 = arcsin\frac{n_z o_z}{\sqrt{(1 - n_z^2)(1 - o_z^2)}} \\ \theta_{11} = (\,\gamma_2 - \gamma_1)/2 \\ \theta_{12} = (\pi - \gamma_2 - \gamma_1)/2 \\ \theta_{13} = \pi + (\,\gamma_2 - \gamma_1)/2 \\ \theta_{14} = \pi + (\pi - \gamma_2 - \gamma_1)/2 \end{gathered} \quad (43)$$

where γ1 and γ2 are auxiliary angles, and θ11 to θ14 are four candidate solutions for θ1.
By substituting θ1 in (31) using(43) the value of θ23 can then be determined as follows:

$$\begin{cases} \theta_{231} = arctan2(a_x c_1 + a_y s_1 - a_z) \\ \theta_{232} = \pi + \theta_{231} \end{cases} \quad (44)$$

$$\begin{cases} d_1 - a_2 s_2 - d_4 s_{23} = -a_z d_6 + p_z \\ (a_1 + a_2 c_2 + d_4 c_{23}) s_1 = -a_y d_6 + p_y \\ (a_1 + a_2 c_2 + d_4 c_{23}) c_1 = -a_x d_6 + p_x \end{cases} \quad (45)$$

By submitting $\theta 23$ to the first equation in (45), then solutions for $\theta 2$ and $\theta 3$ can be obtained as (46) and (47):

$$\begin{cases} \theta_{21} = arcsin[(d_1 - d_4 s_{23} + a_z d_6 + p_z), \\ \theta_{22} = \pi - \theta_{21} \end{cases} \quad (46)$$

$$\theta_3 = \theta_{23} - \theta_2 \quad (47)$$

The solutions for θ4 and θ5 can be given as in (30). In this case, the reachable positions px and py can be calculated from the last two equations in (45), which may be far away from the desired positions.
If oz = 1, then ox = 0 and oy = 0. The result θ23 = 0 can be deduced from (31). By submitting θ23 in (45), the solutions for θ1 to θ3 can be obtained as in(48) and (49):

$$\begin{cases} \theta_{21} = arcsin\left[(d_{1+}a_z d_6 - p_y)/a_2\right] \\ \theta_{22} = \pi - \theta_{21} \\ \theta_3 = -\theta_2 \end{cases} \quad (48)$$

$$\begin{cases} \gamma_4 = arcsin\left[(-a_y d_6 + p_y)/(a_1 + a_2 c_2 + d_4 c_{23})\right] \\ \theta_{11} = \gamma_4 \\ \theta_{12} = \pi - \gamma_4 \end{cases} \quad (49)$$

where $\theta 11$ and $\theta 12$ are two candidate solutions for $\theta 1$.

If $oz = 0$, then $c23 = 0$ or $s4 = 0$. If $c23 = 0$, then the solution of $\theta 1$ to $\theta 3$ can be obtained from (45). If $s4 = 0$, $\theta 1$ can be resolved from (31), and $\theta 2$ and $\theta 3$ deduced from (45). The solution of $\theta 4$ and $\theta 5$ can be given as (30). The processes of resolution in detail for these cases are omitted here.

6. Experimental Results

Several experiments were conducted to validate the derived inverse kinematics. An experiment was first conducted to check the correctness of the method de- rived in Section 4, with reachable positions and ori- entations. The desired positions and orientations of the end-effector were created using forward kinematics with random joint angles. Therefore, these positions and orientations were reachable by the manipulator. Experiment results show that the derived inverse kinematics provide completely accurate solutions in this experiment. In addition, criteria 1 and 2 described by (31), (35), and (36) are also proved correct.

6.1 Experiment of trajectory following

In this experiment, desired positions and orientations of the end-effector were generated using:

$${}^0T_6 = \begin{bmatrix} 1 & 0 & 0 & p_x \\ 0 & -1 & 0 & p_y \\ 0 & 0 & -1 & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (50)$$

$$\begin{cases} p_x = 298 + t/5 \\ p_y = 50\cos(\pi t/36) \\ p_z = 100 - 40sin(\pi t/36) \end{cases} \quad (51)$$

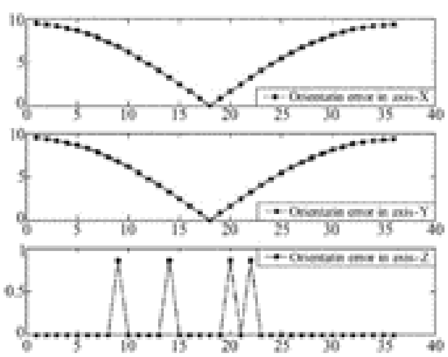
Where t(=1,2,•••, 36) is an independent variable. The orientations were fixed, and the positions formed a smooth trajectory. Obviously, there were unreachable positions. This experiment was conducted to test solutions derived in both Sections 4 and 5.1.

Experiment results for the simulated PArm are given in Fig. 4, and those for the real PArm in Fig.5. Figs.4(a) and 5(a) show desired positions with stars, and reached positions with circles. Coordinate units are in millimeters. Figs.4(b) and 5(b) show orientation errors in terms of Euler angles. Figs.4(c) and 5(c) il- lustrate orientation errors in terms of angles between desired and reached axis directions. Figs.4(d) and 5(d) display joint angles from the inverse kinematics. From Figs.4 and 5 it is clear that all desired positions match with mm accuracy and the Z-axis orientation is well satisfied. However, this is not the case with the X and Y -axis orientations, because the meth- ods applied here satisfy the positions and the Z-axis orientation without considering the remained DOF in orientation.

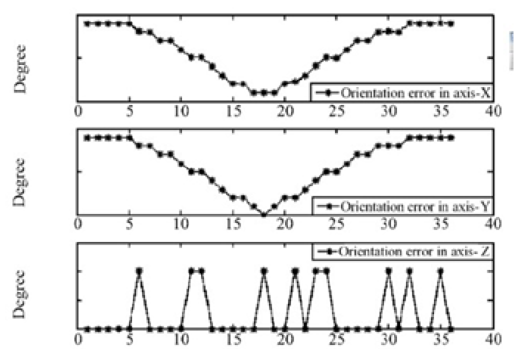
6.2 Experiment for unreachable positions and orientations near reachable ones

This simulation experiment was used to check if a group of reasonable solutions could be obtained for Positions and orientations near reachable ones. Reachable positions and orientations were generated using the forward kinematics with random joint angles, as conducted in experiment 1. Their Euler angles indicating orientations were calculated and then added using small random angles in the range [−1.5 1.5] degrees. Positions were also added using random values, ranging between [−2 2] mm.

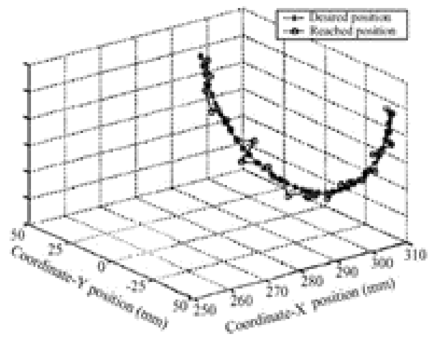
In this way, positions and orientations in a small neighborhood of reachable ones were generated. The values of joint angles were strictly limited in their actual working range, and their accuracy set the same as for the real PArm, i.e. one degree. When values of solutions exceeded this range, a simple optimal search method using a grade-descent strategy was employed to find a group of approximate solutions in the range. It used the calculated solutions as initial starting points. It is clear that a group of satisfied approximate solutions can be found for any unreachable positions and orientations near reachable ones by using the inverse kinematics calculation combined with the grade-descent searching strategy. Inverse kinematics results are shown in Fig.6 for positions, and Fig.7 for errors of positions and the Z-axis orientation, respectively. It can be concluded that the three- dimensional positions and the Z-axis orientation can be effectively satisfied, whether the orientation of the end-effector is reachable or not.



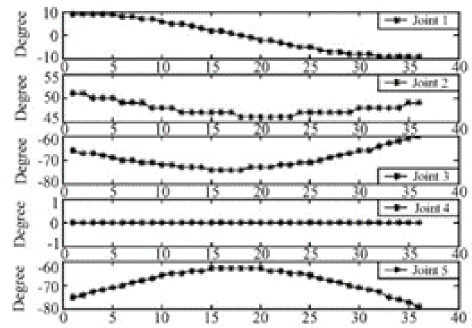
(a)Desired and reached position



(b)Orientation errors terms of Euler angles

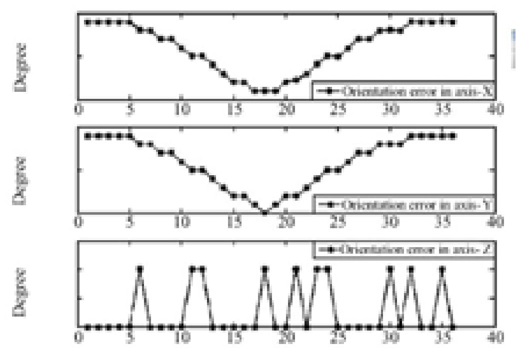


(c)Orientation errors in terms angles between desired and reached axis directions

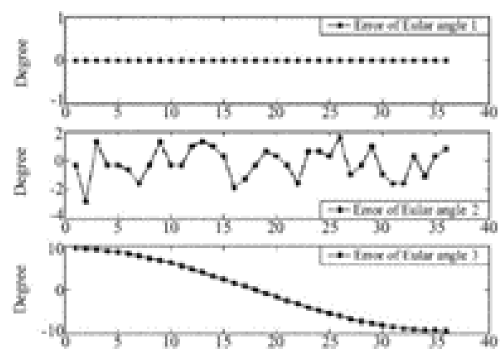


(d)Joint angles

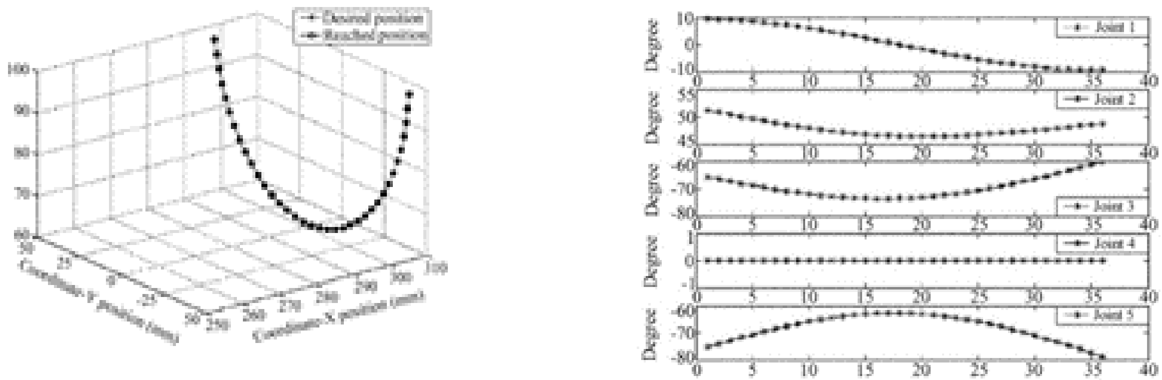
Fig.4 Experimental results using the simulated PArm



(a)Desired and reached position



(b)Orientation errors terms of Euler angles



(c)Orientation errors in terms angles between desired and reached axis directions

(d)Joint angles

Fig.5.Experimental results using the real PArm

6.3 comparison of methods described in Section 5

The last experiment compared the methods derived in Sections 5.1 and 5.2 with unreachable positions and orientations of the end-effector. Experiment results are provided below as Td, T1, and T2, where Td is the desired unreachable position and orientation, T1 is the reached position and orientation obtained by using the method in Section 5.1, and T2 is the reached position and orientation obtained by using the method in section 5.2. It can be seen that the position and Z-axis orientation in T1 match Td very well and the remaining orientation is close to that in Td. The orientations in T2 are almost the same as those in Td, but the positions are far from those in Td. Therefore, the method in section 5.1 provides advantages in comparison to the method in section 5.2.

7. Conclusions and Future Work

In this paper, a strategy based on geometric projection was proposed to resolve the inverse kinematics of a 5-DOF manipulator. Sufficient and necessary criteria were provided to determine whether correct solutions existed without using forward kinematics. Once the four groups of candidate solutions were calculated for the first three joint angles, one criterion was employed to check which group was correct. In addition, it was possible using another criterion to check if there were multiple groups of correct solutions for a given position and orientation before resolving the inverse kinematics. Both criteria were derived for the first time in this paper. They provide significant convenience for the resolving process of inverse kinematics. This is an im- portant advancement, since the singular problem often exists in 6-DOF manipulators.

The effectiveness of the derived methods has been demonstrated through experiment results. For any reachable positions and orientations of the end-effector, correct solutions for the inverse kinematics of the PArm could be obtained from (18), (21), (23), (24), (25), and (30). Their correctness can be checked using criterion (31). For dealing with unreachable positions an orientations of an end-effector, solutions that satisfy the positions and the Z-axis orientation are preferred and can be used to satisfy applications such as those in [11] and [12]. We have currently focused on the problem associated with a mobile manipulator, i.e. a combination of a manipulator and a mobile robot. Although the manipulator has insufficient DOF, the addition of a 3-DOF mobile platform makes its DOF redundant; this in turn creates new problems for us to address in the future.

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